LHC Higgs Cross Section Working Group 2 (Higgs Properties).

Next to Leading Order in the Standard Model Effective Field Theory.

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Abstract

We review the status of calculations in the Standard Model Effective Field Theory (SMEFT) beyond leading order (LO). Improving the SMEFT beyond LO allows theoretical errors to be characterized and reduced when considering SMEFT interpretations of the data, which is essential considering the improving experimental precision at LHC. Next to leading order (NLO) results also allow a more consistent analysis of measurements with different effective scales in the SMEFT. Going beyond LO is clearly important in the event that deviations from the SM are large enough that experimental indications of physics beyond the SM emerge. We discuss a consistent and well defined approach to LO in the SMEFT, so that the improvement to NLO is straightforward. We discuss the basic issues involved in improving calculations to NLO in the SMEFT, and review the advances in this direction that have been achieved to date.

1 Introduction

Almost all Quantum Field Theories can be regarded as examples of Effective Field Theory (EFT), the treatment of which was pioneered in [1–3]. The predictions of the leading order (LO) semi-classical Lagrangian of any EFT is an approximation, of limited applicability and precision. As exact non-perturbative solutions to QFTs are rarely known, approximate solutions that expand observables perturbatively in a small coupling constant, or non-perturbatively in a ratio of scales are generally developed.[2] Developing such expansions beyond leading order is straightforward, if the LO EFT is well defined, and is in general extremely useful. Going beyond LO can also be required for consistent interpretations of the data, if the theoretical error of a LO description exceeds the experimental error. It is the ability of EFT's, as consistent field theories, to be systematically improved that (largely) explains why they have become the standard approach to interpreting data sets of constraints on the Standard Model (SM). A great strength of a serious EFT approach is that vague statements on theoretical errors and inaccuracies

[1]Note that this document is currently a work in progress. Appearing in acknowledgements does not imply full or partial endorsement. Further contributions, comments, and (scientific) criticisms are welcome.

[2]Formally, a purely LO treatment of a process in an EFT is also intrinsically ambiguous, as the scale dependence (μ) of the parameters contributing to a process is not defined.

can be replaced by a more precise quantification of the limitations of a LO analysis, using next to leading order (NLO) calculations. As a corollary, LO descriptions of deviations from the SM that cannot be systematically improved, are not EFT analyses, and are of limited use. In this section, we will discuss extending the EFT approach to physics beyond the SM to NLO. We will also briefly outline how the standard straightforward LO formulation is usually defined.

We will restrict our attention to the case of the Standard Model Effective Field Theory (SMEFT), with a linear realization of ElectroWeak Symmetry Breaking (EWSB). This means we assume that $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ is spontaneously broken to $\mathrm{U(1)_{em}}$ by the vacuum expectation value of the Higgs field (v) and that the observed 0^+ scalar is embedded in the Higgs doublet. It is assumed in this approach that to capture the low energy (IR) physics of an underlying physics sector, it is sufficient to add $\mathrm{SU(3)} \times \mathrm{SU(2)_L} \times \mathrm{U(1)}_Y$ invariant higher dimensional operators, built out of the SM fields, to the renormalizable SM interactions.[3] The Lagrangian is schematically

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}_5 + \mathcal{L}_6 + \mathcal{L}_7 + \mathcal{L}_8 + \cdots \tag{1.1}$$

$\mathcal{L}_5$ has one operator suppressed by one power of the cut off scale (Λ) [4]. $\mathcal{L}_6$ has 76 parameters that preserve Baryon number [5,6] in the $N_f = 1$ limit[4], and 2499 parameters in the case $N_f = 3$ [7]. Four operators suppressed by $1/\Lambda^2$ violate Baryon number [4,8]. $\mathcal{L}_7$ contains thirty operators that all violate Lepton or Baryon number [9,10] (and also $B - L$ [10]) for $N_f = 1$ and $\mathcal{L}_8$ [10,11] counts 993 $N_f = 1$ operators.

We label the Wilson coefficients of the operators in $\mathcal{L}_5$ as C_i^5, operators in $\mathcal{L}_6$ as C_i^6 etc., and have implicitly absorbed the appropriate power of $1/\Lambda$ into the definition of the C_i. When $1/\Lambda$ is made explicit, and pulled out of the Wilson coefficient we will use the tilde superscript as a notation to indicate this, for example $\tilde{C}_i/\Lambda^2$.

The construction of the SMEFT, to all orders, is not based on assumptions on the size of the Wilson coefficients of higher dimensional operators. Constructing a NLO SMEFT result means performing a complete NLO calculation *in the SMEFT*. This means including all operators at a fixed order in the power counting of the theory[5], or performing a complete one loop calculation for a process, including all of the operators in $\mathcal{L}_6$ that can contribute. We emphasise that the existence of NLO corrections is a necessary consequence of the SMEFT being a well defined field theory. The *numerical size of the higher order terms* depends upon the high energy (UV) scenario dictating the $\tilde{C}_i$ and Λ, which is unknown. Restricting to a particular UV case is not an integral part of a general SMEFT treatment, and it is a strength of a general SMEFT approach that various cases can be chosen once the general calculation is performed.

Whether or not a NLO treatment of the data is required in the SMEFT is defined by three considerations:

[3] It is not guaranteed that this choice is the correct one to capture the low energy phenomenology of the underlying physics beyond the SM, see the Nonlinear EFT section for an alternative formulation.

[4] Here N_f counts the number of fermion generations.The distinct use of parameters and operators in this paragraph is not accidental

[5] Generally in this document LO in the SMEFT refers to the effects of $\mathcal{L}_6$ while NLO in the Lagrangian expansion refers to the effects of $\mathcal{L}_8$

- What is the cut off scale, and what is the matching pattern of Wilson coefficients into the SMEFT?
- What is the experimental precision that will be reached in a measurement?
- How will a bound projected into the SMEFT formalism at LO be used?

Considering the first question, it is interesting to consider the cases where $1\,\mathrm{TeV} \lesssim \Lambda/\sqrt{\tilde{C}_i} \lesssim 3\,\mathrm{TeV}$.[6] In these cases, deviations in processes measured at LHC could possibly be observable. It is also interesting to consider cut off scales of this form, as they are (roughly) a loop factor above the known 0^+ scalar mass. This is an interesting range of mass scales considering the Hierarchy problem. Finally, model building exercises for decades have indicated that such mass scales are not robustly ruled out, when considering ElectroWeak Precision data (EWPD). If the ratio $\Lambda/\sqrt{\tilde{C}_i}$ lies in this interesting range, the effect of NLO corrections are not negligible [12–25].

There is no guarantee that $\Lambda/\sqrt{\tilde{C}_i}$ will be this low. There is no clear and absolute answer to the second question above, and the answer to the third differs among analyses and authors. As such, it must be addressed that very significant differences of opinion exist regarding the need and use of NLO SMEFT analyses.[7] An advantage of the SMEFT is that, putting opinions aside, NLO corrections can just be systematically determined and used, and theoretical errors can be reduced as required. This effort is well underway, utilizing the NLO improvement of the straightforward LO approach briefly outlined in this note.

Directly related to NLO improvements and analysis of the SMEFT we will address the following points:

- To what extent can experimental limits on dimension 6 operators be affected by the presence of dimension 8 operators?
- When is it justified to calculate the SMEFT predictions at tree level? In what circumstances may including 1-loop corrections modify the predictions in an important way?

As well as addressing the above points, it is necessary to emphasize our main point very directly. The experimental data must be reported in a manner that allows NLO theoretical improvements and studies of the data in the future. *It is absolutely critical that the data is reported outside of the attempt at a purely LO formalism – known as the "Higgs basis" , for this to occur.* It is also important to emphasize that there is no barrier to reporting the data in a LO SMEFT formalism that does not have the intrinsic problems of "the Higgs basis". We also discuss how this can be done.

[6] It is very important to stress that the point of the SMEFT is to also capture the possibility of deviations from the SM actually being observed, not just to express experimental limits in a compact notation in terms of Wilson coefficients.

[7] Our claim is that almost all of these disagreements are due to different implicit UV assumptions and the data should be reported in a manner that maximizes its potential use in the future, including its use in NLO analyses.

2 Basics of the SMEFT

The SMEFT is a different theory than the SM. The SMEFT has local contact operators suppressed by powers of $1/\Lambda$ that are not present in the SM. In the SMEFT a (lepton number preserving) amplitude can be written as

$$\mathcal{A} = \sum_{n=\mathrm{N}}^{\infty} g_{SM}^{n}\, \mathcal{A}_n^{(4)} + \sum_{n=\mathrm{N}_6}^{\infty} \sum_{l=1}^{n} \sum_{k=1}^{\infty} g_{SM}^{n} \left[\frac{1}{(\sqrt{2}\, G_F\, \Lambda^2)^k}\right]^l \mathcal{A}_{n\,l\,k}^{(4+2\,k)}\,, \tag{2.2}$$

where g_{SM} is a SM coupling. G_F is the Fermi coupling constant and Λ is again the cut off scale. l is an index that indicates the number of SMEFT operator insertions leading to the amplitude, and k indicates the inverse mass dimension of the Lagrangian terms inserted. N is a label for each individual process, that indicates the order of the coupling dependence for the leading non vanishing term in the SM (e.g. $N = 1$ for $\mathrm{H} \to \mathrm{VV}$ etc. but $N = 3$ for $\mathrm{H} \to \gamma\gamma$). $N_6 = N$ for tree initiated processes in the SM. For processes that first occur at loop level in the SM, $N_6 = N - 2$ when operators in the SMEFT can mediate such decays directly thought a contact operator, for example, through a $\mathcal{L}_6$ operator for $\mathrm{H} \to \gamma\gamma$. For instance, the $\mathrm{H}\gamma\gamma$ (tree) vertex is generated by $O_{HB} = H^\dagger\, H\, B^{\mu\nu}\, B_{\mu\nu}$, by $O^8_{HW} = \; H^\dagger\, B^{\mu\nu}\, B_{\mu\rho}\, \mathrm{D}^\rho\, \mathrm{D}_\nu\, H$ etc. An example of the Feynman diagrams leading to $\mathcal{A}$ is given in Fig. 1.

An example of how the SMEFT orders a double expansion in the non-perturbative power counting parameter and the perturbative expansion is as follows. Consider a tree level 2 body decay of a single field. The double expansion of such a process is given as the following Table [8]:

$$\begin{array}{llll} g\,/\,\mathrm{dim} \longrightarrow & & & \\ \downarrow & g\,\mathcal{A}_1^{(4)} & +\, g\, g_6\, \mathcal{A}_{1,1,1}^{(6)} & +\, g\, g_8\, \mathcal{A}_{1,1,2}^{(8)} \\ & g^3\,\mathcal{A}_3^{(4)} & +\, g^3\, g_6\, \mathcal{A}_{3,1,1}^{(6)} & +\, g^3\, g_6^2\, \mathcal{A}_{3,2,1}^{(6)} \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots \end{array} \tag{2.3}$$

The combination of parameters $g\, g_6\, \mathcal{A}_{1,1,1}^{(6)}$ defines the LO SMEFT expression for the process, including the leading insertion of a higher dimensional operator, and is generally well known. $g^3\, g_6\, \mathcal{A}_{3,1,1}^{(6)}$ defines the NLO SMEFT amplitude in the perturbative expansion, and $g\, g_8\, \mathcal{A}_{1,1,2}^{(8)}$ defines the NLO SMEFT amplitude in the non-perturbative expansion. NLO terms in the double expansion present in the SMEFT are generally unknown, in almost every process that is of interest phenomenologically. The discussion here generalizes to cases other than two body decays of a single field directly.

Generalizing analyses in the SM at LO in the SMEFT is essentially a solved problem in the literature. Although LO analyses are challenging, it is necessary for theorists to further develop the SMEFT to NLO. NLO analyses are required to consistently map lower energy measurements in the SMEFT to the cut off $\mu = \Lambda$, or to consistently combine data sets measured at different effective scales ($\mu_1 \neq \mu_2$). Learning precisely

[8] Here we have introduced short hand notation where $g_{4+2\,k} = 1/(\sqrt{2}\, G_F\, \Lambda^2)^k$, so that g_6 denotes a single $\mathcal{O}^{(6)}$ insertion, g_8 denotes a single $\mathcal{O}^{(8)}$ insertion, g_6^2 denotes two, distinct, $\mathcal{O}^{(6)}$ insertions, etc..

about the underlying theory using EFT methods, and combining data sets consistently are (usually) core goals of an analysis of deviations in the SMEFT. For these goals to be reached consistently requires NLO analyses in the SMEFT.

2.1 Power counting

Our use of EFT terminology is standard, but different than some WG documents. For this reason we define here some of our key concepts such as power counting. Power counting in EFT's is subtle. The term "power counting" is also frequently misused. Our use of this term is consistent with almost all of the historical literature [1–3,26–36]. Power counting in an EFT is a means by which the size of the local operators present in $\mathcal{L}_{SMEFT}$ are estimated. An operator in the SMEFT expansion is schematically of the form

$$\frac{\bar{\psi}^a \, \psi^b \, \partial^c \, H^d \, (H^\dagger)^e \, (A)^f}{\Lambda^n}. \tag{2.4}$$

Here detailed Lorentz structure, flavour and group indicies are being suppressed. ψ stands for a generic fermion field, and A for a generic Gauge field. The mass dimensions are such that $3(a+b)/2+c+d+e+f-n=4$. This defines a power counting scheme based on the mass dimensions of the operators. As the SM fields obtain their mass by the vev (v) the numerator can have an explicit power of v, and an expansion in $(v/\Lambda)^m$ is present relative to SM interactions. Further, due to the presence of derivatives an expansion in $(E/\Lambda)^m$ is present, relative to the SM interactions.[9] At high orders in the expansion of $\mathcal{L}_{SMEFT}$ both of these ratios are generically present. The most general power counting for the SMEFT is to suppress all operators simply by a generic power of Λ, dictated by the mass dimension of the operator. This can always be done. By definition any remaining coupling dependence, or alternate scales present in the EFT, can always be absorbed into the Wilson coefficients in the matching procedure. Alternative power counting schemes can be defined, and can be self consistent, however they are limited in their applicability.

2.1.1 Pole observables vs tails of distributions

The two expansions discussed above are generically present in the SMEFT. $(v/\Lambda)^m$ contributions are of interest for data dictated by the presence of a pole, for example, the Higgs pole. $(E/\Lambda)^m$ contributions are particularly relevant for tails of distributions.

When analyzing data near poles, scaling arguments that apply to the suppression of local contact (non resonant) four fermion operators in $\mathcal{L}_6$ also apply to NLO $\mathcal{L}_8$ corrections. This is fortunate. The very large number of parameters present in $\mathcal{L}_8$ and $\mathcal{L}_6$ are primarily present in four fermion operators. In the case of $\mathcal{L}_6$ 2205 of the 2499 parameters

[9] Higher derivative terms are systematically traded in favour of other operators without derivatives for a number of technical reasons in well defined bases (such as the Warsaw basis [6]). However, this point should be kept in mind if such higher derivative terms are actually systematically retained in an operator basis.

present in $\mathcal{L}_6$ [7] are due to four fermion operators. NLO power corrections in $\mathcal{L}_8$, higher order in $(v/\Lambda)^m$, are suppressed compared to $\mathcal{L}_6$ by the power counting parameter v^2/Λ^2,which varies from $\sim 6\%$ to $\sim 0.6\%$ for $\Lambda/\sqrt{\tilde{C}_i} = 1, 3\,\mathrm{TeV}$ respectively.

The suppression of NLO terms that scale as p^2/Λ^2 can be far less in the tails of distributions. The SMEFT expansion breaks down when $p^2/\Lambda^2 \sim 1$, and Pseudo Observable/form factor [19,37–40] methods are required to characterize the data in this case. Tails of distributions have a very large number of SMEFT parameters contributing due to non-resonant fermion pair (and higher multi-body) production processes.

It is also worth noting that unlike the case of pole data, NLO corrections to tails of distributions are complicated in their analysis, as the p^2/Λ^2 terms are in general not gauge invariant alone, and need to always be combined with the interference with non-resonant part of the SM, and SMEFT background processes. The requirement for joint analysis including SMEFT corrections on the background that results, further complicates the analysis of non-pole data. These are some of the reasons it is generally more promising to study pole data, and related distributions, when pursuing deviations from the SM in the SMEFT formalism.

2.2 Matching

If the underlying theory is known, then the Wilson coefficients would be precisely dictated by matching onto the SMEFT. Functionally the matching procedure is performed by calculating on-shell amplitudes in the UV theory, and in the SMEFT. The low energy limit, $E/\Lambda << 1$ is taken. The two theories are separately renormalized. The IR poles cancel between the theories, and the mismatch of the finite terms that remain defines the Wilson coefficient in the matching condition. Performing matching calculations requires that the UV theory be well defined and specified. The utility of this approach is that the Wilson coefficients are universal, and can be calculated only once, ideally in a particularly simple process.

If the value of the Wilson coefficients in broad UV scenarios could be inferred – if a meta-matching could be done – this would be of significant scientific value. One example of an influential classification scheme of some value is the Artz-Einhorn-Wudka "potentially-tree-generated" (PTG) scenario [41,42]. In this scheme, it is argued that classes of Wilson coefficients for operators in $\mathcal{L}_6$ can be argued to be tree level, or loop level (suppressed by $g^2/16\,\pi^2$), essentially using topological matching arguments. This classification scheme corresponds only to a subset of weakly coupled and renormalizable UV physics cases, as the topologies considered are (effectively) limited by Lorentz invariance and renormalizability. This scheme does not apply to scenarios where the UV physics is strongly interacting. Generally such theories are in their confining phase, and are examples of UV EFT's where the PTG classification scheme does not apply. For discussion clarifying this issue see Ref. [33].

One can study the Wilson coefficients using dimensional analysis, by restoring $\hbar \neq 1$ in the Lagrangian. See the recent discussions on this approach in Refs. [43,44]. Doing so one cannot unambiguously identify that powers of hypothetical UV couplings present in the $\tilde{C}_i$. This is due to the fact that the SM couplings also carry $\hbar$ dimensions, and the UV theory is not known. For this reason, and the fact that the matching procedure introduces order one constant terms that can be as large as, or dominant over the coupling dependence, it is generally not useful to treat the $\tilde{C}_i$ as anything other than parameters to be fit to and constrained by experiment. This is the recommended procedure when using the SMEFT at LO or NLO.

2.3 The SMEFT at LO

In this section we briefly review the standard approach to LO analyses in the literature. The dimension six terms in the SMEFT, in the Warsaw basis [6], are given in Table 1. It is important to emphasize a great strength of this basis is that it is actually completely and precisely defined. Any well defined basis can be used, however, we are not aware of competing bases in the literature that have been completely specified (including flavour indicies) as the Warsaw basis has been defined.[10]

Expanding around the vev, the LO modification of the SM interactions comes about in the SMEFT in a straightforward manner. The following section is largely taken in whole from Ref. [7] and is not intended to be a complete treatment, but simply an illustrative discussion of some Lagrangian terms in a standard LO implementation. As the theory should be canonically normalized, we denote coupling parameters in the canonically normalized SMEFT with bar superscripts. This use of the bar notation is distinct from bar superscripts on fermion fields, that have the standard interpretation $\bar{\psi} = \psi^\dagger \gamma^0$.

2.3.1 SM Lagrangian

We define the SM Lagrangian as

$$\begin{aligned}\mathcal{L}_{\rm SM} = &-\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \overline{\psi}\, i \not{D}\, \psi \\ &- \lambda \left(H^\dagger H - \frac{1}{2} v^2\right)^2 - \left[H^{\dagger j} \overline{d}\, Y_d\, q_j + \widetilde{H}^{\dagger j} \overline{u}\, Y_u\, q_j + H^{\dagger j} \overline{e}\, Y_e\, l_j + \text{h.c.}\right], \end{aligned} \tag{2.5}$$

which implicitly defines our notational conventions.

[10] A different operator set is referred to as the Warsaw basis in the “Higgs Basis” note. This is at variance with the initial paper, and essentially all published literature that references this basis. On the other hand, a very positive aspect of the “Higgs Basis” note is that it actually, for the first time, defined a version of the “SILH basis” including flavour indicies. This addresses the need for this to occur, as was pointed out in Ref. [7]. The “Higgs Basis” construction itself is still undefined at LO, as indicated by the presence of a $\mathcal{L}_{other}$ Lagrangian term.

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde W B}$	$H^\dagger \tau^I H\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu}$
Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde H\, G^A_{\mu\nu}$
Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde H\, W^I_{\mu\nu}$
Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar q_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

$8: (\bar L L)(\bar L L)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$

$8: (\bar R R)(\bar R R)$	
Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$
Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$
Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$
Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$
Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar L L)(\bar R R)$	
Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar L R)(\bar R L) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$

$8: (\bar L R)(\bar L R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$

Table 1: The $\mathcal{L}_6$ operators built from Standard Model fields which conserve baryon number, as given in Ref. [6]. The flavour labels of the form p, r, s, t on the Q operators are suppressed on the left hand side of the tables.

2.3.2 Higgs mass and self-couplings

The potential in the SMEFT is

$$V(H) = \lambda \left(H^\dagger H - \frac{1}{2}v^2\right)^2 - C_H \left(H^\dagger H\right)^3, \tag{2.6}$$

yielding the new minimum

$$\langle H^\dagger H\rangle = \frac{v^2}{2}\left(1 + \frac{3C_H v^2}{4\lambda}\right) \equiv \frac{1}{2}v_T^2. \tag{2.7}$$

The scalar field can be written in unitary gauge as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \left[1 + c_{H,\,kin}\right] h + v_T \end{pmatrix}, \tag{2.8}$$

where

$$c_{H,\,kin} \equiv \left(C_{H\Box} - \frac{1}{4}C_{HD}\right)v^2, \qquad\qquad v_T \equiv \left(1 + \frac{3C_H v^2}{8\lambda}\right)v. \tag{2.9}$$

The coefficient of h in Eq. (2.8) is no longer unity, in order for the Higgs boson kinetic term to be properly normalized when the dimension-six operators are included. The kinetic terms

$$\mathcal{L} = (D_\mu H^\dagger)(D^\mu H) + C_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + C_{HD}\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right), \tag{2.10}$$

and the potential in Eq. (2.6) yield

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{c_{H,\,kin}}{v_T^2}\left[h^2(\partial_\mu h)^2 + 2vh(\partial_\mu h)^2\right] - \lambda v_T^2\left(1 - \frac{3C_H v^2}{2\lambda} + 2c_{H,\,kin}\right)h^2 \\ &- \lambda v_T\left(1 - \frac{5C_H v^2}{2\lambda} + 3c_{H,\,kin}\right)h^3 - \frac{1}{4}\lambda\left(1 - \frac{15C_H v^2}{2\lambda} + 4c_{H,\,kin}\right)h^4 \\ &+ \frac{3}{4}C_H v h^5 + \frac{1}{8}C_H h^6,\end{aligned} \tag{2.11}$$

for the h self-interactions. The Higgs boson mass is

$$m_H^2 = 2\lambda v_T^2\left(1 - \frac{3C_H v^2}{2\lambda} + 2c_{H,\,kin}\right). \tag{2.12}$$

2.3.3 Yukawa couplings

The Lagrangian terms in the unbroken theory

$$\mathcal{L} = -\left[H^{\dagger j}\overline{d}_r\ [Y_d]_{rs}\ q_{js} + \widetilde{H}^{\dagger j}\overline{u}_r\ [Y_u]_{rs}\ q_{js} + H^{\dagger j}\overline{e}_r\ [Y_e]_{rs}\ l_{js} + \text{h.c.}\right]$$

$$+\left[\underset{sr}{C^*_{dH}}\left(H^\dagger H\right)H^{\dagger j}\overline{d}_r q_{js}+\underset{sr}{C^*_{uH}}\left(H^\dagger H\right)\tilde{H}^{\dagger j}\overline{u}_r q_{js}+\underset{sr}{C^*_{eH}}\left(H^\dagger H\right)H^{\dagger j}\overline{e}_r l_{js}+\text{h.c.}\right], \tag{2.13}$$

yield the fermion mass matrices

$$\left[M_\psi\right]_{rs}=\frac{v_T}{\sqrt{2}}\left(\left[Y_\psi\right]_{rs}-\frac{1}{2}v^2\underset{sr}{C^*_{\psi H}}\right), \qquad \psi=u,d,e \tag{2.14}$$

in the broken theory. The coupling matrices of the h boson to the fermions $\mathcal{L}=-h\,\overline{u}\,\mathcal{Y}\,q+\ldots$ are

$$\begin{aligned}\left[\mathcal{Y}_\psi\right]_{rs}&=\frac{1}{\sqrt{2}}\left[Y_\psi\right]_{rs}\left[1+c_{H,\,kin}\right]-\frac{3}{2\sqrt{2}}v^2\underset{sr}{C^*_{\psi H}}\\&=\frac{1}{v_T}\left[M_\psi\right]_{rs}\left[1+c_{H,\,kin}\right]-\frac{v^2}{\sqrt{2}}\underset{sr}{C^*_{\psi H}}, \qquad \psi=u,d,e\end{aligned} \tag{2.15}$$

and are not simply proportional to the fermion mass matrices, as is the case in the SM.

2.3.4 Gauge boson masses and couplings

The relevant dimension-six Lagrangian terms are

$$\begin{aligned}\mathcal{L}^{(6)}&=C_{HG}H^\dagger H G^A_{\mu\nu}G^{A\mu\nu}+C_{HW}H^\dagger H W^I_{\mu\nu}W^{I\mu\nu}+C_{HB}H^\dagger H B_{\mu\nu}B^{\mu\nu}\\&\quad+C_{HWB}H^\dagger\tau^I H W^I_{\mu\nu}B^{\mu\nu}+C_G f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho+C_W\epsilon^{IJK}W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho\,.\end{aligned} \tag{2.16}$$

The gauge fields need to be redefined, so that the kinetic terms are properly normalized and diagonal. The first step is to redefine the gauge fields

$$G^A_\mu=\mathcal{G}^A_\mu\left(1+C_{HG}v_T^2\right), \quad W^I_\mu=\mathcal{W}^I_\mu\left(1+C_{HW}v_T^2\right), \quad B_\mu=\mathcal{B}_\mu\left(1+C_{HB}v_T^2\right). \tag{2.17}$$

The modified coupling constants are

$$\overline{g}_3=g_3\left(1+C_{HG}\,v_T^2\right), \qquad \overline{g}_2=g_2\left(1+C_{HW}\,v_T^2\right), \qquad \overline{g}_1=g_1\left(1+C_{HB}\,v_T^2\right), \tag{2.18}$$

so that the products $g_3 G^A_\mu=\overline{g}_3\mathcal{G}^A_\mu$, etc. are unchanged. The mass eigenstate basis is given by [45]

$$\begin{bmatrix}\mathcal{W}^3_\mu\\ \mathcal{B}_\mu\end{bmatrix}=\begin{bmatrix}1 & -\frac{1}{2}\,v_T^2\,C_{HWB}\\ -\frac{1}{2}\,v_T^2\,C_{HWB} & 1\end{bmatrix}\begin{bmatrix}\cos\overline{\theta} & \sin\overline{\theta}\\ -\sin\overline{\theta} & \cos\overline{\theta}\end{bmatrix}\begin{bmatrix}\mathcal{Z}_\mu\\ \mathcal{A}_\mu\end{bmatrix}, \tag{2.19}$$

where the rotation angle is

$$\tan\overline{\theta}=\frac{\overline{g}_1}{\overline{g}_2}+\frac{v_T^2}{2}\,C_{HWB}\left[1-\frac{\overline{g}_1^{\,2}}{\overline{g}_2^{\,2}}\right]. \tag{2.20}$$

The W and Z masses are

$$M_W^2=\frac{\overline{g}_2^{\,2}v_T^2}{4},$$

$$M_Z^2 = \frac{v_T^2}{4}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{8}v_T^4 C_{HD}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{2}v_T^4 \overline{g}_1 \overline{g}_2 C_{HWB}. \tag{2.21}$$

The covariant derivative is

$$D_\mu = \partial_\mu + i\,\frac{\overline{g}_2}{\sqrt{2}}\left[\mathcal{W}_\mu^+ T^+ + \mathcal{W}_\mu^- T^-\right] + i g_Z \left[T_3 - \overline{s}^2 Q\right] \mathcal{Z}_\mu + i\,\overline{e}\,Q\,\mathcal{A}_\mu, \tag{2.22}$$

where $Q = T_3 + Y$, and the effective couplings are given by

$$\begin{aligned}
\overline{e} &= \frac{\overline{g}_1\overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}}\left[1 - \frac{\overline{g}_1\overline{g}_2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} v_T^2 C_{HWB}\right] = \overline{g}_2\,\sin\overline{\theta} - \frac{1}{2}\,\cos\overline{\theta}\,\overline{g}_2\,v_T^2\,C_{HWB}, \\
\overline{g}_Z &= \sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1\overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} v_T^2 C_{HWB} = \frac{\overline{e}}{\sin\overline{\theta}\cos\overline{\theta}}\left[1 + \frac{\overline{g}_1{}^2 + \overline{g}_2{}^2}{2\overline{g}_1\overline{g}_2} v_T^2 C_{HWB}\right], \\
\overline{s}^2 &= \sin^2\overline{\theta} = \frac{\overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1\overline{g}_2(\overline{g}_2{}^2 - \overline{g}_1{}^2)}{(\overline{g}_1{}^2 + \overline{g}_2{}^2)^2} v_T^2 C_{HWB}.
\end{aligned} \tag{2.23}$$

2.3.5 $h \to WW$ and $h \to ZZ$

The relevant CP-even Lagrangian terms are

$$\begin{aligned}
\mathcal{L} &= (D_\mu H)^\dagger (D^\mu H) - \frac{1}{4}\left(W^I_{\mu\nu}W^{I\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right), \\
&+ C_{HW}\,Q_{HW} + C_{HB}\,Q_{HB} + C_{HWB}Q_{HWB} + C_{HD}\,Q_{HD},
\end{aligned} \tag{2.24}$$

which lead to the interactions

$$\mathcal{L} = \frac{1}{4}\overline{g}_2{}^2 v_T h\left[(\mathcal{W}^1_\mu)^2 + (\mathcal{W}^2_\mu)^2\right]\left[1 + c_{H,\,kin}\right] + C_{HW} v_T h\left[(\mathcal{W}^1_{\mu\nu})^2 + (\mathcal{W}^2_{\mu\nu})^2\right] \tag{2.25}$$

for the W, and

$$\begin{aligned}
\mathcal{L} &= \frac{1}{4}(\overline{g}_2{}^2 + \overline{g}_1{}^2) v_T h(\mathcal{Z}_\mu)^2\left[1 + c_{H,\,kin} + v_T^2 C_{HD}\right] + \frac{1}{2}\overline{g}_1\overline{g}_2 v_T^3 h(\mathcal{Z}_\mu)^2 C_{HWB} \\
&+ v_T h(\mathcal{Z}_{\mu\nu})^2\left(\frac{\overline{g}_2{}^2 C_{HW} + \overline{g}_1{}^2 C_{HB} + \overline{g}_1\overline{g}_2 C_{HWB}}{\overline{g}_2{}^2 + \overline{g}_1{}^2}\right)
\end{aligned} \tag{2.26}$$

for the Z.

2.3.6 TGC parameters

The effect on the off-shell Triple gauge coupling parameter is given by

$$\left(-\mathcal{L}_{TGC}\right)/\bar{g}_{VWW} = i\bar{g}_1^V\left(\mathcal{W}^+_{\mu\nu}\mathcal{W}^{-\mu} - \mathcal{W}^-_{\mu\nu}\mathcal{W}^{+\mu}\right)\mathcal{V}^\nu + i\bar{\kappa}_V + \mathcal{W}^+_\mu\mathcal{W}^-_\nu\mathcal{V}^{\mu\nu}, \tag{2.27}$$

$$+ i\frac{\bar{\lambda}_V}{\bar{M}_W^2}\mathcal{V}^{\mu\nu}\mathcal{W}_\nu^{+\rho}\mathcal{W}_{\rho\mu}^-$$

where $\mathcal{V} = \{\mathcal{Z}, \mathcal{A}\}$ and the shifts compared to the SM are [46]

$$\delta g_1^A = -\delta\kappa_A = -\frac{v_T^2}{2}\frac{c_{\bar{\theta}}}{s_{\bar{\theta}}}C_{HWB}, \qquad \delta g_1^Z = -\delta\kappa_Z = \frac{v_T^2}{2}\frac{s_{\bar{\theta}}}{c_{\bar{\theta}}}C_{HWB}, \tag{2.28}$$

and

$$\delta\lambda_A = 6s_{\bar{\theta}}C_W\frac{M_W^2}{g_{AWW}}, \qquad \delta\lambda_Z = 6c_{\bar{\theta}}C_W\frac{M_W^2}{g_{ZWW}}. \tag{2.29}$$

Here $g_{AWW} = \bar{e}$ and $g_{ZWW} = \bar{g}_2\, c_{\bar{\theta}}$.

2.4 Input parameters

The parameters v_T, $\bar{g}_1$, $\bar{g}_2$, $\bar{e}$ in the Lagrangian terms in the previous section (and related quantities with bar superscripts) have to be assigned numerical values consistent with some input parameter set (IPS). This is sometimes known as a "finite renormalization", as we discuss below. This is distinct from simply rotating to the mass eigenstate fields in the canonically normalized SMEFT, which is discussed in the previous section.

The relationship to the IPS is in general different in the SMEFT and the SM, as the theories are different. An operator basis can be related to any IPS, and is not limited in its relationship to a particular IPS, or a LO implementation. Once an IPS is chosen, relationships between observables are derivable in a well defined formalism at LO and at NLO. Of course, if one were to make some relationships to a particular IPS the same in the SM and the SMEFT, with algebraic manipulations[11] that were only defined at LO, this would make such a construction an example of a "phenomenological effective Lagrangian" that was then limited to LO. If the algebraic manipulations were not the same for all possible IPS sets that could be chosen, this would tie such a construction to a specific IPS. Such a construction would not properly be referred to as equivalent to a well defined operator basis, such as the Warsaw basis, and such a design feature is exactly what would make such a proposal more difficult to improve to NLO.[12] As our purpose is to discuss the NLO improvement of the SMEFT, we first illustrate how a straightforward LO implementation is related to the IPS α_{ew}, G_F, M_Z.

G_F is defined as the following parameter measured in μ decay, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. Define the local effective interaction for muon decay as

$$\mathcal{L}_{G_F} = -\frac{4G_F}{\sqrt{2}}\left(\bar{\nu}_\mu\,\gamma^\mu P_L\mu\right)\left(\bar{e}\,\gamma_\mu P_L\nu_e\right). \tag{2.30}$$

[11] In particular a series of field redefinitions and EOM manipulations whose sole purpose is to obscure the difference between the SM and the SMEFT at LO with respect to certain measurements.

[12] Such a construction in the SMEFT is in fact a gauge dependent parameterization of some terms in the phenomenological effective Lagrangian, and tied to a specific IPS at a fixed order in perturbation theory. The "Higgs Basis" is an example of such a construction.

In the SMEFT,[13]

$$-\frac{4G_F}{\sqrt{2}}=-\frac{2}{v_T^2}+\left(C_{\substack{ll\\ \mu ee\mu}}+C_{\substack{ll\\ e\mu\mu e}}\right)-2\left(C^{(3)}_{\substack{Hl\\ ee}}+C^{(3)}_{\substack{Hl\\ \mu\mu}}\right). \tag{2.31}$$

The parameter α_{ew} is measured in the Thompson ($p^2 \to 0$) limit and discused in Section 2.5.1, and M_Z is defined in the resonance pole scale of LEP measurements.

Relating to the IPS α_{ew}, G_F, M_Z at LO in the $U(3)^5$ case of the SMEFT, one finds straightforwardly the following results [17,18]. For the development of this approach see Refs. [45,47,48]. Here our notational conventions are that shifts due to the SMEFT are denoted as $\delta X = (X)_{SMEFT} - X_{SM}$ for a parameter X. Measured input observables, or parameters directly defined by combinations of input observables are denoted with hat superscripts. The shifts in the commonly appearing Lagrangian parameters M_Z, M_W, G_F, s^2_θ in the Warsaw basis at LO are given by

$$\delta M_Z^2 \equiv \frac{1}{2\sqrt{2}}\frac{\hat{m}_Z^2}{\hat{G}_F}C_{HD}+\frac{2^{1/4}\sqrt{\pi}\,\sqrt{\hat{\alpha}}\,\hat{m}_Z}{\hat{G}_F^{3/2}}C_{HWB}, \tag{2.32}$$

$$\delta M_W^2 = -\hat{m}_W^2\left(\frac{\delta s^2_{\hat{\theta}}}{s^2_{\hat{\theta}}}+\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}\sqrt{2}\hat{G}_F}C_{HWB}+\sqrt{2}\delta G_F\right), \tag{2.33}$$

$$\delta G_F = \frac{1}{\sqrt{2}\,\hat{G}_F}\left(\sqrt{2}\,C^{(3)}_{Hl}-\frac{C_{ll}}{\sqrt{2}}\right), \tag{2.34}$$

$$\delta s^2_\theta = -\frac{s_{\hat{\theta}}\,c_{\hat{\theta}}}{2\sqrt{2}\,\hat{G}_F(1-2s^2_{\hat{\theta}})}\left[s_{\hat{\theta}}\,c_{\hat{\theta}}\,(C_{HD}+4\,C^{(3)}_{Hl}-2\,C_{ll})+2\,C_{HWB}\right]. \tag{2.35}$$

These shifts lead to modifications of the W,Z couplings with the normalization

$$\mathcal{L}_{Z,eff} = g_{Z,eff}\,\left(J_\mu^{Z\ell}Z^\mu + J_\mu^{Z\nu}Z^\mu + J_\mu^{Zu}Z^\mu + J_\mu^{Zd}Z^\mu\right), \tag{2.36}$$

where $g_{Z,eff} = -2\,2^{1/4}\sqrt{\hat{G}_F}\,\hat{m}_Z$, $(J_\mu^{Zx})^{pr} = \bar{x}_p\,\gamma_\mu\left[(\bar{g}^x_V)^{pr}_{eff}-(\bar{g}^x_A)^{pr}_{eff}\,\gamma_5\right]x_r$ for $x = \{u,d,\ell,\nu\}$. In general, these currents are matricies in flavour space. When we restrict our attention to the case of a minimal linear minimal flavor violation (MFV) [49,50] scenario $(J_\mu^{Zx})_{pr} \simeq (J_\mu^{Zx})\delta_{pr}$. In the Warsaw basis, the effective axial and vector couplings are modified from the SM values by a shift defined as

$$\delta(g^x_{V,A})_{pr} = (\bar{g}^x_{V,A})^{eff}_{pr} - (g^x_{V,A})^{SM}_{pr}, \tag{2.37}$$

where

$$\delta(g^\ell_V)_{pr} = \delta\bar{g}_Z\,(g^\ell_V)^{SM}_{pr} - \frac{1}{4\sqrt{2}\hat{G}_F}\left(C_{\substack{He\\ pr}}+C^{(1)}_{\substack{Hl\\ pr}}+C^{(3)}_{\substack{Hl\\ pr}}\right)-\delta s^2_\theta, \tag{2.38}$$

$$\delta(g^\ell_A)_{pr} = \delta\bar{g}_Z\,(g^\ell_A)^{SM}_{pr} + \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(C_{\substack{He\\ pr}}-C^{(1)}_{\substack{Hl\\ pr}}-C^{(3)}_{\substack{Hl\\ pr}}\right), \tag{2.39}$$

$$\delta(g^\nu_V)_{pr} = \delta\bar{g}_Z\,(g^\nu_V)^{SM}_{pr} - \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(C^{(1)}_{\substack{Hl\\ pr}}-C^{(3)}_{\substack{Hl\\ pr}}\right), \tag{2.40}$$

[13] e and μ are generation indices 1 and 2, and are not summed over.

$$\delta(g_A^{\nu})_{pr} = \delta\bar{g}_Z\,(g_A^{\nu})_{pr}^{SM} - \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(C^{(1)}_{\substack{Hl\\pr}} - C^{(3)}_{\substack{Hl\\pr}}\right), \tag{2.41}$$

$$\delta(g_V^{u})_{pr} = \delta\bar{g}_Z\,(g_V^{u})_{pr}^{SM} + \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(-C^{(1)}_{\substack{Hq\\pr}} + C^{(3)}_{\substack{Hq\\pr}} - C_{\substack{Hu\\pr}}\right) + \frac{2}{3}\delta s_\theta^2, \tag{2.42}$$

$$\delta(g_A^{u})_{pr} = \delta\bar{g}_Z\,(g_A^{u})_{pr}^{SM} - \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(C^{(1)}_{\substack{Hq\\pr}} - C^{(3)}_{\substack{Hq\\pr}} - C_{\substack{Hu\\pr}}\right), \tag{2.43}$$

$$\delta(g_V^{d})_{pr} = \delta\bar{g}_Z\,(g_V^{d})_{pr}^{SM} - \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(C^{(1)}_{\substack{Hq\\pr}} + C^{(3)}_{\substack{Hq\\pr}} + C_{\substack{Hd\\pr}}\right) - \frac{1}{3}\delta s_\theta^2, \tag{2.44}$$

$$\delta(g_A^{d})_{pr} = \delta\bar{g}_Z\,(g_A^{d})_{pr}^{SM} + \frac{1}{4\sqrt{2}\,\hat{G}_F}\left(-C^{(1)}_{\substack{Hq\\pr}} - C^{(3)}_{\substack{Hq\\pr}} + C_{\substack{Hd\\pr}}\right), \tag{2.45}$$

where

$$\delta\bar{g}_Z = -\frac{\delta G_F}{\sqrt{2}} - \frac{\delta M_Z^2}{2\hat{m}_Z^2} + \frac{s_{\hat{\theta}}\,c_{\hat{\theta}}}{\sqrt{2}\hat{G}_F}\,C_{HWB}, \tag{2.46}$$

and similarly the W couplings are defined as

$$\delta(g_V^{W_\pm,\ell})_{rr} = \delta(g_A^{W_\pm,\ell})_{rr} = \frac{1}{2\sqrt{2}\hat{G}_F}\left(C^{(3)}_{\substack{Hl\\rr}} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\,C_{HWB}\right) + \frac{1}{4}\frac{\delta s_\theta^2}{s_{\hat{\theta}}^2}, \tag{2.47}$$

$$\delta(g_V^{W_\pm,q})_{rr} = \delta(g_A^{W_\pm,q})_{rr} = \frac{1}{2\sqrt{2}\hat{G}_F}\left(C^{(3)}_{\substack{Hq\\rr}} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\,C_{HWB}\right) + \frac{1}{4}\frac{\delta s_\theta^2}{s_{\hat{\theta}}^2}. \tag{2.48}$$

Here our chosen normalization is $(g_V^x)^{SM} = T_3/2 - Q^x\,s_\theta^2$, $(g_A^x)^{SM} = T_3/2$ where $T_3 = 1/2$ for u_i, ν_i and $T_3 = -1/2$ for d_i, ℓ_i and $Q^x = \{-1, 2/3, -1/3\}$ for $x = \{\ell, u, d\}$. The set of δX parameters are *not an operator basis*, they simply parameterize some terms in the phenomenological effective Lagrangian, as a particular IPS is chosen. The operator basis is given in Table I.

2.4.1 TGC parameters

As another straightforward example, relating the TGC parameters to the same set of inputs one finds in the Warsaw basis [46]

$$\delta g_Z^1 = \frac{\delta s_\theta^2}{2}\left(\frac{1}{c_{\hat{\theta}}^2} + \frac{1}{s_{\hat{\theta}}^2}\right) + \frac{1}{2\sqrt{2}\,G_F}(\cot_{\hat{\theta}} + \tan_{\hat{\theta}})\,C_{HWB}, \tag{2.49}$$

$$\delta g_\gamma^1 = 0, \tag{2.50}$$

$$\delta\kappa_Z = \frac{\delta s_\theta^2}{2}\left(\frac{1}{c_{\hat{\theta}}^2} + \frac{1}{s_{\hat{\theta}}^2}\right) + \frac{1}{2\sqrt{2}\,G_F}(\cot_{\hat{\theta}} - \tan_{\hat{\theta}})\,C_{HWB}, \tag{2.51}$$

$$\delta\kappa_\gamma = \frac{\cot_{\hat{\theta}}}{\sqrt{2}G_F}\,C_{HWB}, \tag{2.52}$$

$$\delta\lambda_Z = \delta\lambda_\gamma = 6\, s_{\hat\theta}\, c^2_{\hat\theta}\, \frac{\hat{M}_Z^2}{\sqrt{4\,\pi}\,\sqrt{\hat\alpha_{ew}}}\, C_W, \tag{2.53}$$

Note that some TGC parameters are clearly not physical parameters. They correspond to off-shell verticies that are not trivially related to asymptotic S matrix elements. In general, Lagrangian parameters are not physical[14], but as they are usually closely related to S matrix elements, or the properties of asymptotic states in the field theory, this distinction is not critical to emphasise. When considering SMEFT modifications of the SM, this distinction can have a more important role. For more discussion on resolving related issues, see Ref. [51].

The differences between a standard LO treatment, as sketched above, and more overly complicated proposals are easy to determine by inspection. Introducing layers of complications beyond the straightforward LO implementation is not necessary and is best avoided. The reason this is true is such complications can make it much more challenging, or even formally impossible, to improve an approach beyond LO. If the SMEFT is never improved beyond LO this introduces theoretical errors in a SMEFT analysis of the data which are not necessarily small. We discuss this in more detail in the next section.

2.5 The SMEFT beyond leading order, theoretical errors

In the SM, when a particular process is calculated, a common practice is that a theoretical error is assigned. For example, for parametric and theoretical uncertainties within the SM, see Tab. 1 of Ref. [52]. It can be subtle to assign such an error [53] due to the neglect of missing higher order perturbative terms in the SM. If an expansion in a ratio of the mass scales is used to define a prediction in the SM[15] higher order "non-perturbative" corrections are also present, and a further theoretical error to characterise neglected local contact operators is generally introduced.

Unknown NLO corrections in the SMEFT can (and should) be treated in analogy to the treatment of theoretical errors in the SM. Introducing some theoretical error for the SMEFT is essential in LO analyses. This is an additional source of theoretical error when the data is interpreted in the SMEFT, see Refs. [17–19] for recent detailed discussion on theory errors in the SMEFT.

To properly characterize the perturbative error, it is essential to calculate at least to one loop order in the SMEFT, including the leading insertion of operators in $\mathcal{L}_6$. Until such calculations are performed, conservative theoretical errors should be applied to theoretical relations in the SMEFT. Further, the introduction of a "non-perturbative" error, due to $\mathcal{L}_8$ when bounding $\mathcal{L}_6$ should be done. In Eqn.2.3, the $g^3\, g_6^2\, \mathcal{A}^{(6)}_{3,2,1}$ terms can be used as estimators of missing higher order non-perturbative terms in the SMEFT. This approach is not particularly novel, but is simply the obvious extension of the widely accepted approach to assigning theoretical error in the SM to the SMEFT. Early works calculating and articulating the need of such corrections to consistently interpret the data in the SMEFT include Refs. [12–16]. Generally, at low Λ the neglect of $\mathcal{L}_8$ dominates, while as Λ gets larger, the neglect of perturbative corrections begins to dominate.

[14] This is still the case when mass eigenstate fields are used.

[15] For example, in the case of the heavy m_t limit when calcucating $gg \to h$.

An excellent example of the importance of theory errors is provided by another effective field theory, NRQED, as discussed in Refs. [54–60]. The Hydrogen hyperfine splitting is measured to fourteen digits, but only computed to seven digits. This introduces a theoretical error when using this measurement. Comparatively, the Positronium hyperfine splitting is measured and computed to eight digits. It would simply be a mistake to give the H hyperfine splitting a weight 10^6 larger than the P_s hyperfine splitting in a global fit to the fundamental constants, and to totally ignore theory errors. A careful consideration of NLO effects can help in avoiding similar errors when using the SMEFT formalism.

2.5.1 Theory errors in a LO formalism on the IPS

As a specific example, any LO approach does not take into account that the scales characterizing the measurements of the input parameters α_{ew}, G_F, M_Z differ. Consider the error introduced due to the neglect of this NLO effect in the SMEFT, compared to the errors quoted on α_{ew} in the SM. This parameter is measured at low energies in the $p^2 \to 0$ limit.[16] The value of this input parameter is given in Table 2. In the SMEFT, the running of α_{ew} is modified compared to the SM as given in Ref. [63]. As a simple approximation of the error introduced in the SMEFT, one finds that the neglected NLO SMEFT correction to α_{ew} is then

$$\frac{(\Delta\alpha_{ew})_{SMEFT}}{(\Delta\alpha_{ew})_{SM}} \simeq -250\left(\frac{1\text{TeV}}{\Lambda}\right)^2 \tilde{C}_{HB} - 80\left(\frac{1\text{TeV}}{\Lambda}\right)^2 \tilde{C}_{HW}, \tag{2.54}$$

running from $p^2 \sim 1\,\text{GeV}^2$ to m_h.[17] Here $(\Delta\alpha_{ew})_{SM}$ is the SM error quoted in the Table. Depending on $\tilde{C}_{HB}$ and $\tilde{C}_{HW}$ and Λ, which are unknown, the neglected NLO SMEFT effects can lead to an error on this input parameter far larger than in the SM. This should be completely unsurprising. Neglected NLO effects in the SMEFT in this case include corrections of order $g_{1,2}^2 v_T^2/(16\,\pi^2)\,\Lambda^2$. The theoretical errors due to such neglected effects can obviously compete with the SM theoretical errors, introduced in a QED calculation out to *tenth order* in the SM. Similarly, neglected NLO corrections on the other input parameters modify their theoretical error.

If one adopted a formalism that claims to make the relationship to some input parameters the same in the SM and in the SMEFT (at LO), one might be tempted to not seriously consider the effect of NLO corrections, such as given in this example. This could lead to incorrect interpretations of the data. Being able to define and reduce such errors is a reason that a straightforward approach to the SMEFT at LO, that can be directly improved to NLO, is of significant value.

[16] α_{ew} is frequently extracted in the Thompson limit $p^2 \to 0$ when probing some Coulomb potential of a charged particle, for example in a measurement of $g-2$ for the electron or muon. Recently, extractions with a competitive error budget have emerged where α_{ew} is extracted from the measured ratio of $\hbar/M_{atom}$ via the recoil velocity for a stable atom, such as Rb^{87} [61] or C_s [62]. The important point is to realize that this input parameter differs in the SM and in the SMEFT at NLO.

[17] This is only an approximation, as formally all of the SM states with masses $m^2 \gg p^2$ should be integrated out in sequence when running down from the high scale. This significantly complicates the analysis, but the effect of the SMEFT modification of the running, illustrated with this simple expression, is still present.

Parameter	Input Value	Ref.
$\hat{m}_Z$	91.1875 ± 0.0021	[64–66]
$\hat{G}_F$	$1.1663787(6) \times 10^{-5}$	[65,66]
$\hat{\alpha}_{ew}$	$1/137.035999074(94)$	[61,65–67]

Table 2: Current best estimates of α_{ew}, G_F, M_Z.

2.5.2 Approximating unknown SMEFT theory errors

A reasonable approximation of a theoretical error to introduce for an observable i when fitting to the leading parameters in $\mathcal{L}_6$, is given by [18,17]

$$\Delta^i_{SMEFT}(\Lambda) \simeq \sum_j x_{ij}\, \tilde{C}^8_{ij}\, \frac{v_T^4}{\Lambda^4} + \sum_j \frac{(g^{ij}_{SM})^2}{16\,\pi^2}\, \tilde{C}^6_{ij}\, y_{ij}\, \ln\left[\frac{\Lambda^2}{v_T^2}\right] \frac{v_T^2}{\Lambda^2}\,. \qquad (2.55)$$

Non log dependence in the second term is also present, but is suppressed for a simplifying approximation. Here x_{ij}, y_{ij} label the observable dependence and are $\mathcal{O}(1)$. One can further define

$$x'_i \sqrt{N^i_8} = \sum_j \sqrt{x^2_{ij}\, (\tilde{C}^8_{ij})^2}, \qquad y'_i \sqrt{N^i_6} = \sum_j \sqrt{y^2_{ij}\, (\tilde{C}^6_{ij})^2}\,, \qquad (2.56)$$

as the product of $\mathcal{O}(1)$ numbers that characterize the multiplicity of the operators that contribute to a process ($N_{6,8}$) and the typical numerical dependence x'_i, y'_i. The square root is because errors are assumed to add in quadrature. As an alternative, a Bayesian uniform prior for the C_i could be used.

Although the number of operators is large, the relevant number of operators that contribute in a process is far less then the full operator set; in known examples $N_{6,8} \sim \mathcal{O}(10)$. This error is multiplicative and the absolute error is obtained as $\Delta^i_{SMEFT}(\Lambda)$ times the SM prediction for an observable.

For cut off scales and Wilson coefficients in the range $1\,\mathrm{TeV} \lesssim \Lambda/\sqrt{\tilde{C}_i} \lesssim 3\,\mathrm{TeV}$ and order one numbers for $x_i, y_i, N_{6,8}$ the value of $\Delta^i_{SMEFT}(\Lambda)$ is in the range of few $\mathcal{O}(\%)$ to $\mathcal{O}(0.1\%)$ [17–19]. It is widely considered to be the case that the precision expected in LHC analyses can be expected to approach a few percent in well measured channels, see Ref. [68,69]. NLO corrections and the corresponding theoretical errors should be considered when the precision of an experimental analysis descends below $\mathcal{O}(10\%)$ to be conservative. This level of *experimental* precision was already reached by the LEP experiments so projecting consistently the constraints derived from the LEP program into LHC analyses, and encoded in the LO SMEFT parameters, requires a proper treatment of theoretical errors [17–19].

2.5.3 More on theoretical uncertainty

The need to include theoretical errors when perturbatively expanding the SMEFT is tied to the fact that different truncations of such expansions can be constructed. Suppose that a given quantity $\mathrm{Q}(a)$ is given in perturbation theory by the following expansion:

$$\mathrm{Q} = a + g\left[a^2 + f_1(a)\right] + g^2\left[a^3 + f_2(a)\right] + \mathcal{O}(g^3) = \bar{a} + g\,f_1(a) + \mathcal{O}(g^2), \tag{2.57}$$

where $\bar{a} = a/(1 - ga)$. Suppose that only the f_1 term is actually known. It could be decided that $\bar{a}$ is the effective expansion parameter (or that in the full expression we change variable $a \to \bar{a}$). This is equivalent, in the truncated expansion, to introducing

$$\mathrm{Q} = \bar{a} + g\,f_1(a) = \bar{a} + g\,f_1(\bar{a}), \tag{2.58}$$

which gives $\Delta\mathrm{Q} = g^2\,f_1'(a)$, the difference in the two results due to neglected higher order terms is an estimate of the associated theoretical uncertainty. A fit to observables defined in a perturbative expansion must always include an estimate of the missing higher order terms [70], which specifies a theoretical uncertainty. Various ways exist to estimate this uncertainty. Once can compute the same observable with different "options", e.g. linearization or quadratization of the squared matrix element, resummation or expansion of the (gauge invariant) fermion part in the wave function factor for the external legs, variation of the renormalization scale, G_{F} renormalization scheme or α-scheme, etc.

A conservative estimate of the associated theoretical uncertainty is obtained by taking the envelope over all "options"; the interpretation of the envelope is a log-normal distribution (this is the solution preferred in the experimental community) or a flat Bayesian prior [71,53] (a solution preferred in a large part of the theoretical community).

These general considerations apply to fits in the SMEFT, where missing higher order terms include, in most cases, all NLO perturbative corrections, and all NLO higher dimensional operators. A fit performed at LO in the SMEFT that does not include any estimate of the missing higher order terms, and never specifies a theoretical error, is not a serious effort. In particular, in EWPD the modifications of the W mass, the ρ parameter and the effective weak-mixing angle are loop-induced quantities and a study of their SM deviations require an analysis at NLO in the SMEFT.

This is another reason why it is important to preserve the original data, not just the interpretation results, as the estimate of the missing higher order terms can change over time, modifying the lessons drawn from the data and projected into the SMEFT.

2.6 Effects of NLO SMEFT power corrections

No complete operator basis of $\mathcal{L}_8$ has ever been encoded in a Monte-Carlo program and used to fit the data. It is a remarkable achievement that in recent years a full reduction of the operator basis to describe $\mathcal{L}_6$ was determined (for the first time) in 2010 [6]. The rapid complete characterization of a fully reduced $\mathcal{L}_7$ and $\mathcal{L}_8$ bases in recent years now allows more precise error estimates to be made due to neglected $\mathcal{L}_8$ corrections. In principle, it is now possible to characterize a theoretical error by varying the complete set of C_8 parameters in a "reasonable" range when a C_6 parameter bound is extracted. In practice, this has never been accomplished in the literature. Rough error estimates as detailed previously should be used until more complete error analyses are available.

2.7 NLO SMEFT loop corrections

Note that including loop corrections in the SMEFT context is more crucial than for a pure SM calculation. One loop corrections can introduce a dependence on Wilson coefficients that do not contribute at tree level to a particular process, and some of these Wilson coefficients are very poorly bounded. This is different from the SM where all of the Lagrangian terms are extremely well known. We will refer to the introduction of such dependence as "non-factorizable" corrections.

Loop corrections also introduce a perturbative rescaling of the dependence on an operator's Wilson coefficient, if the operator contributed at LO to a process. These later corrections can be naively interpreted to only modify an unknown parameter by a perturbative correction, and of limited interest. However such corrections are crucial to perform data analyses based on measurements performed at different scales, which is required due to the large number of operators present in the SMEFT.

Improving the SMEFT to one loop requires a renormalization scheme be defined, a systematic renormalization of the SMEFT be carried out on the new parameters in $\mathcal{L}_6$, and loop corrections be performed in a particular chosen gauge. We now discuss each of these steps in the NLO program in more detail.

2.8 SMEFT: renormalization in practice

In this Section we describe a general renormalization procedure in the SMEFT. The results presented have been developed in Refs. [15,25], based on the conventional formalism widely used in the SM [72–75]. To perform renormalization in an EFT it is appropriate to use a dimensionless regulator, see Refs. [28] for a review discussion. We work with dimensional regularization and define

$$\Delta_{\mathrm{UV}} = \frac{2}{4-d} - \gamma - \ln\pi - \ln\frac{\mu_{\mathrm{R}}^2}{\mu^2}\,, \tag{2.59}$$

where d is space-time dimension, the loop measure is $\mu^{4-d}\,d^n q$ and μ_{R} is the renormalization scale; γ is the Euler-Mascheroni constant. Counterterms for SM parameters and fields are defined by

$$\mathrm{Z}_i = 1 + \frac{g^2}{16\,\pi^2}\,\left(d\mathrm{Z}_i^{(4)} + g_6\,d\mathrm{Z}_i^{(6)}\right)\,\Delta_{\mathrm{UV}}\,. \tag{2.60}$$

With field/parameter counterterms we can make UV finite the self-energies and the corresponding Dyson resummed propagators. However, these counterterm subtractions are not enough to make UV finite the Green's functions with more than two legs (at

$\mathcal{O}(g^{\mathrm{N}_6} g_6))$. A mixing matrix among Wilson coefficients is needed:

$$C_i = \sum_j \mathrm{Z}^{\mathrm{W}}_{ij}\, C^{\mathrm{ren}}_j\,, \qquad \mathrm{Z}^{\mathrm{W}}_{ij} = \delta_{ij} + \frac{g^2}{16\,\pi^2}\, d\mathrm{Z}^{\mathrm{W}}_{ij}\, \Delta_{\mathrm{UV}}\,. \tag{2.61}$$

For example, in this way we can renormalize the (on-shell) S-matrix for $\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{A}_\nu(p_2)$ and $\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{Z}_\nu(p_2)$ which have only one (transverse) Lorentz structure. By on-shell S-matrix for an arbitrary process (involving unstable particles) we mean the corresponding (amputated) Green's function supplied with LSZ factors and sources, computed at the (complex) poles of the external lines [76–78]. For processes that involve stable particles this can be straightforwardly transformed into a physical observable.

The connection of the HVV, V = Z, W (on-shell) S-matrix with the off shell vertex $\mathrm{H} \to \mathrm{VV}$ and the full process $\mathrm{pp} \to 4\,\psi$ is more complicated and is discussed in some detail in Sect. 3 of Ref. [19]. The "on-shell" S-matrix for HVV, being built with the the residue of the H$-$V$-$V poles in $\mathrm{pp} \to 4\,\psi$ is gauge invariant by construction (it can be proved by using Nielsen identities [79]) and represents one of the building blocks for the full process: in other words, it is a pseudo-observable [37,38,19]. Technically speaking the "on-shell" limit for external legs should be understood "to the complex poles" (for a modification of the LSZ reduction formulas for unstable particles, see Ref. [80]) but, as well known, at one loop we can use on-shell masses (for unstable particles) without breaking the gauge parameter independence of the result. Residues of complex poles are what matters, as far as renormalization is concerned.

The $\mathrm{H}(P) \to \mathrm{Z}_\mu(p_1)\mathrm{Z}_\nu(p_2)$ (on-shell) matrix contains a part of the amplitude proportional to $g^{\mu\nu}$ (referred to as $\mathcal{D}_{\mathrm{HZZ}}$ below) and a part of the amplitude proportional to $p_2^\mu\, p_1^\nu$ (referred to as $\mathcal{P}_{\mathrm{HZZ}}$ below). Both of these terms get renormalized through a mixing.

Consider now the $\mathrm{H}(P) \to \mathrm{W^-}_\mu(p_1)\mathrm{W^+}_\nu(p_2)$ (on-shll) matrix: it has the same Lorentz decomposition of $\mathrm{H} \to \mathrm{ZZ}$ and it is UV finite in the dim = 4 part. The $\mathcal{D}_{\mathrm{HWW}}$ part at dim = 6 is renormalized through a mixing; however, there are no Wilson coefficients in $\mathcal{P}_{\mathrm{HWW}}$ that are not also present in $\mathcal{P}_{\mathrm{HZZ}}$, so that the UV finiteness of this term is related by gauge symmetry to the renormalization of $\mathcal{P}_{\mathrm{HZZ}}$. This is the first part of the arguments used in Refs. [15,25] in proving closure of NLO SMEFT under renormalization.

The (on-shell) decays $\mathrm{H}(P) \to \mathrm{b}(p_1)\overline{\mathrm{b}}(p_2)$ and $\mathrm{Z}(P) \to \bar{\psi}(p_1)\psi(p_2)$ are more involved to improve to NLO in the SMEFT. The SM contribution to these amplitudes are rendered finite by the SM counterterms, however renormalizing the contributions due to $\mathcal{L}_6$ requires an extensive treatment of this operator mixing.

Some structure present in the SM is not preserved when extending an analysis into the SMEFT. Manifestly, processes that first appear at one loop in the SM can occur at tree level in the SMEFT, due to the presence of local contact operators. However, some symmetries of the SM are preserved. For example, consider the universality of the electric charge. In pure QED there is a Ward identity [81] telling us that e can be renormalized in terms of vacuum polarization (which is a way to understand the universality of the coupling), and Ward-Slavnov-Taylor (WST) identities [81–83] allow us to generalize the argument to the full spontaneously broken SM symmetry group. The previous statement means that the contribution from vertices (at zero momentum transfer) in the full SM exactly cancel those from (fermion) wave function renormalization factors. Therefore, by directly computing the vertex $A\,\bar{\psi}\,\psi$ (at $q^2 = 0$) and the Z_{ψ} wave function factor in the SMEFT, one can directly prove (or check) that the WST identity is extended to the SMEFT at $\mathcal{L}_6$. This is expected as the corresponding identities are the consequence of symmetries. However, this is technically non-trivial even after the previous steps in the renormalization program discussed above. Once (non-trivial) finiteness of this vertex is established, the finiteness of $\mathrm{e}^+\mathrm{e}^- \to \bar{\psi}\,\psi$ (including the four-point functions in the non resonant part) follows. This is the second part in proving closure of the NLO SMEFT under renormalization, using the arguments of Refs. [15,25].

At NLO one first has to render all SM and SMEFT parameters finite. Considering the arguments above, and the complete renormalization results of all the operators in $\mathcal{L}_6$ reported in Refs. [7,16,63,84] in the Warsaw basis, this step in the NLO program has been accomplished. This result has not been established in any other basis to date.

2.9 Beyond one loop

The absorption of UV divergences into local counterterms is, to some extent, the easy step; finite renormalization, and the imposition of the appropriate renormalization conditions to fix the finite terms in an amplitude in general requires more attention. This is particularly the case beyond one loop order in the SMEFT. For example, beyond one loop one should not use on-shell masses as renormalization conditions, but only complex poles for all unstable particles, see Refs. [37,85]. Some examples where the concept of an on-shell mass can be employed are as follows. Suppose that we renormalize a physical (pseudo-)observable F,

$$F = F_{\mathrm{B}} + \frac{g^2}{16\,\pi^2}\left[F^{(4)}_{1\mathrm{L}}(m^2) + g_6\,F^{(6)}_{1\mathrm{L}}(m^2)\right] + \mathcal{O}(g^4)\,, \tag{2.62}$$

where m is some renormalized mass, F_{B} is the Born term and $F_{1\mathrm{L}}$ is one loop. Consider two cases: a) two-loop corrections are not included and b) m appears at one and two loops in $F_{1\mathrm{L}}$ and $F_{2\mathrm{L}}$ but does not show up in the Born term F_{B}. In these cases we can use the concept of an on-shell mass performing a finite mass renormalization at one loop. If m_0 is the bare mass for the field V we write

$$m_0^2 = M^2_{\mathrm{OS}}\left\{1 + \frac{g^2}{16\,\pi^2}\,\mathrm{Re}\,\Sigma_{\mathrm{VV}\,;\,\mathrm{fin}}\,\Big|_{s=M^2_{\mathrm{OS}}}\right\} = M^2_{\mathrm{OS}} + g^2\,\Delta M^2\,, \tag{2.63}$$

where M_{OS} is the on-shell mass and Σ is extracted from the required one-particle irreducible Green's function; Eq.(2.63) is still meaningful (no dependence on gauge parameters) and will be used inside the result.

2.10 Input parameter choices

Several choices can be made for implementing a renormalization scheme to calculate to NLO in the SMEFT. Any well defined scheme can be used, but various schemes offer different advantages and disadvantages on a technical level. The detailed fixing of poles and residues that make up precise renormalization conditions require a lengthy discussion. For detailed reviews in the case of the SM, see Refs. [86,87]. Below we summarize the results of the finite renormalization in the relationship to the input observables.

It is necessary to stress again that if one were to construct a complicated proposal that is tied to one particular IPS, for example the α scheme only at LO, this would be in stark contrast to the well defined Warsaw basis, which can be related to any IPS, at LO or NLO.

2.10.1 Using a 'G_F-scheme' with G_F, M_W, M_Z

In the 'G_F-scheme', one uses $\{G_F\,, M_{\mathrm{W}}\,, M_{\mathrm{Z}}\}$ to fix terms in the Lagrangian. In this case, we write the following equation for the g finite renormalization

$$g_{\mathrm{ren}} = g_{\mathrm{exp}} + \frac{g^2_{\mathrm{exp}}}{16\,\pi^2}\,\left(d\mathcal{Z}^{(4)}_g + g_6\,d\mathcal{Z}^{(6)}_g\right)\,, \tag{2.64}$$

where g_{exp} will be expressed in terms of the Fermi coupling constant G_F. Furthermore, $\mathrm{c}_\theta = M_{\mathrm{W}}/M_{\mathrm{Z}}$. The μ-lifetime can be written in the form

$$\frac{1}{\tau_\mu} = \frac{M^5_{\mu}}{192\,\pi^3}\,\frac{g^4}{32\,M^4}\,\left(1 + \delta_\mu\right)\,. \tag{2.65}$$

The radiative corrections are $\delta_\mu = \delta^{\mathrm{W}}_\mu + \delta_{\mathrm{G}}$ where δ_{G} is the sum of vertices, boxes etc and δ^{W}_μ is due to the W self-energy. The renormalization equation becomes

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\,M^2}\,\left\{1 + \frac{g^2}{16\,\pi^2}\,\left[\delta_{\mathrm{G}} + \frac{1}{M^2}\,\Sigma_{\mathrm{WW}}(0)\right]\right\}\,, \tag{2.66}$$

where we expand the solution for g

$$g^2_{\mathrm{ren}} = 4\,\sqrt{2}\,G_F\,M^2_{\mathrm{W}\,;\mathrm{OS}}\,\left\{1 + \frac{G_F M^2_{\mathrm{W}\,;\mathrm{OS}}}{2\,\sqrt{2}\,\pi^2}\,\left[\delta_{\mathrm{G}} + \frac{1}{M^2}\,\Sigma_{\mathrm{WW}\,;\,\mathrm{fin}}(0)\right]\right\}\,. \tag{2.67}$$

Note that the non universal part of the corrections is given by

$$\delta_{\mathrm{G}} = \delta^{(4)}_{\mathrm{G}} + g_6\,\delta^{(6)}_{\mathrm{G}} \qquad \delta^{(4)}_{\mathrm{G}} = 6 + \frac{7 - 4\,\mathrm{s}^2_\theta}{2\,\mathrm{s}^2_\theta}\,\ln \mathrm{c}^2_\theta\,, \tag{2.68}$$

but the contribution of $\mathcal{L}_6$ to muon decay at NLO is not available yet and has not be included in the calculation. It is worth noting that Eq.(2.66) defines the finite renormalization in the $\{G_F\,, M_{\mathrm{W}}\,, M_{\mathrm{Z}}\}$ IPS.

2.10.2 The 'α scheme', using $\alpha, G_{\rm F}, M_Z$

This scheme uses the fine structure constant α and is based on using $\{\alpha\,,\,G_F\,,\,M_{\rm Z}\}$ as the IPS. The new finite-renormalization equation is

$$g^2\,{\rm s}_\theta^2 = 4\,\pi\alpha\,\Bigl[1 - \frac{\alpha}{4\,\pi}\,\frac{\Pi_{\rm AA}(0)}{{\rm s}_\theta^2}\Bigr]\,, \tag{2.69}$$

where $\alpha = \alpha_{\rm QED}(0)$ and $\Pi_{\rm AA}$ defines the vacuum polarization. Therefore, in this scheme, the finite counterterms are

$$g^2_{\rm ren} = g^2_{\rm A}\,\Bigl[1 + \frac{\alpha}{4\,\pi}\,d\mathcal{Z}_g\Bigr]\,,\quad {\rm c}^{\rm ren}_\theta = \hat{\rm c}_\theta\,\Bigl[1 + \frac{\alpha}{4\,\pi}\,d\mathcal{Z}_{{\rm c}_\theta}\Bigr],\quad M_{\rm ren} = M_{\rm Z\,;\,OS}\,\hat{\rm c}^2_\theta\,\Bigl[1 + \frac{\alpha}{8\,\pi}\,d\mathcal{Z}_{M_{\rm W}}\Bigr]\,, \tag{2.70}$$

where the parameters $\hat{\rm c}_\theta$ and $g_{\rm A}$ are defined by

$$g^2_{\rm A} = \frac{4\,\pi\,\alpha}{\hat{\rm s}^2_\theta} \qquad \hat{\rm s}^2_\theta = \frac{1}{2}\,\Bigl[1 - \sqrt{1 - 4\,\frac{\pi\,\alpha}{\sqrt{2}\,G_F\,M^2_{\rm Z\,;\,OS}}}\Bigr]\,. \tag{2.71}$$

The reason for introducing this scheme is that the S, T and U parameters (see Ref. [88]) have been originally given in the $\{\alpha\,,\,G_F\,,\,M_{\rm Z}\}$ scheme, and these input parameters are very well measured. When calculating processes involving photons final states, this scheme can be transparent to adopt. For other processes, the $\{G_F\,,\,M_{\rm W}\,,\,M_{\rm Z}\}$ scheme can be more appropriate, and is in wider use in the SM, in higher order calculations. In the α-scheme, after requiring that $M^2_{\rm Z\,;\,OS}$ is a zero of the real part of the inverse Z propagator, we are left with one finite counterterm, $d\mathcal{Z}_g$. The latter is fixed by using G_F and requiring that

$$\frac{1}{\sqrt{2}}\,G_F = \frac{g^2}{8\,M^2}\,\Bigl\{1 + \frac{g^2}{16\,\pi^2}\,\Bigl[\delta_{\rm G} + \frac{1}{M^2}\,\Delta_{\rm WW}(0) - \Bigl(d{\rm Z}_{\rm W} + d{\rm Z}_{M_{\rm W}}\Bigr)\,\Delta_{\rm UV}\Bigr]\Bigr\}\,, \tag{2.72}$$

where we use the following relations for UV and finite renormalization,

$$g = g_{\rm ren}\,\Bigl(1 + \frac{g^2_{\rm ren}}{16\,\pi^2}\,d{\rm Z}_g\,\Delta_{\rm UV}\Bigr) \qquad g_{\rm ren} = g_{\rm A}\,\Bigl(1 + \frac{\alpha}{8\,\pi}\,d\mathcal{Z}_g\Bigr)\,. \tag{2.73}$$

Note that SM EW calculations available in literature generally use G_F for the pure weak part or evolve $\alpha(0) \to \alpha(M)$ and use $\alpha(M)$ as the expansion parameter at the scale M. For a comprehensive discussion see Sect. 5.3 of Ref. [89].

When α is used in the analysis of "high" energy data it is afflicted with hadronic uncertainties entering already at the one loop level and arising because it must be "run up" from low energy, crossing the hadronic resonance region. The Fermi coupling constant, obtained from the muon lifetime, does not suffer from this disadvantage (even in the full SM one loop hadronic effects are mass suppressed) [90].

2.11 Background field gauge

Any well defined gauge can be used in a calculation, see Ref. [91] for an excellent review on gauge fixing. There can be some advantage to organising a calculation in a manner that enforces relationships between counter terms due to gauge invariance. A technique that accomplishes this is known as the Background Field (BF) method [92,93]. The idea is that fields are split into classical and quantum components and a gauge fixing term is added that maintains the gauge invariance of the classical background fields, while breaking the gauge invariance of the quantum fields. Due to the resulting Ward identities, one finds the relations among the SM counterterms [86]

$$Z_{\mathrm{A}} Z_e = 1, \qquad Z_{\mathrm{H}} = Z_{\phi_\pm} = Z_{\phi_0}, \qquad Z_{\mathrm{W}} Z_{g_2} = 1\,, \tag{2.74}$$

where A, e are the photon and corresponding electric coupling e, H is the SM Higgs field, and $\phi_0, \phi_\pm$ are the Goldstone bosons. W is the SM W boson with gauge coupling g_2. The gauge fixing in the BF method can be imposed as in Ref. [86,94]. Use of the background field method can make extending the WST relations between counterterms manifest and transparent, even when including the effects of $\mathcal{L}_6$.

Extending any gauge fixing procedure to the case of the SMEFT is subtle, due to the order by order redefinition of the fields that are gauged due to terms in $\mathcal{L}_{SMEFT}$. This remains the case when using the background field method. For example, although naively unexpected, terms in $\mathcal{L}_6$ can, and must, source ghost interactions due to the redefinition of the SM fields order by order in the power counting of the SMEFT. For some discussion on these subtleties see Refs. [23,24]. Optimally resolving the technical complications that result, when using the background field method in the SMEFT, is an unsolved problem.

These subtleties are some of the reasons it is difficult to directly modifying computer programs that have been developed for automatic NLO calculations in the SM, to the case of the SMEFT. The essential challenge is again that the SMEFT *is a different theory than the SM*. The development of NLO SMEFT Monte-Carlo tools is still very much a work in progress.

3 Known results in the SMEFT to NLO

Despite all of the challenges to advancing SMEFT results to NLO, progress in this area is rapid and steady. In this section we briefly sumarize some of these theoretical developments.

3.0.1 Renormalization results

The complete renormalization of the Warsaw basis was reported in Refs. [7,16,63,84]. In the approach outlined in Section 2.8, results for the Warsaw basis operator renormalization were reported in Refs. [15,25]. Use of SMEFT renormalization results (including a subset of NLO finite terms) to leverage EWPD to bound operators not contributing at tree level was reported in Ref. [95]. Partial results for renormalizing some alternate operator sets in the so called "SILH basis" were given in Refs. [96,97]. A recent study of RGE effects on the oblique parameters, in a subset of UV models, was reported in Ref. [21].

3.0.2 *Advances in one loop matching techniques*

Recently, the covariant derivative expansion discussed in Refs. [98–100] has re-emerged in Refs. [101–103] as a powerful technique to perform matching calculations to underlying UV theories at one loop. The basic idea at work is that, the contribution to the effective action that results when integrating out a heavy field $\mathcal{X}$ at one loop is schematically given by

$$\Delta S \propto i \, \mathrm{Tr} \log \left[D^2 + m_{\mathcal{X}}^2 + U(x)\right] \tag{3.75}$$

where $m_{\mathcal{X}}$ is the mass of the $\mathcal{X}$ field integrated out, $D^2 = D_\mu D^\mu$, D_μ is the covariant derivative, and $U(x)$ depends on the SM field content. The covariant derivative expansion allows this functional trace to be directly evaluated, while keeping gauge covariance manifest. This simplifies and systematizes one loop matching calculations in the SMEFT, in many simple UV physics cases.[18]

3.0.3 *Full nonperturbative NLO results*

As discussed previously Refs. [9–11,105,106] have developed the theoretical technology (essentially advanced use of Hilbert series techniques) to characterize the number of independent operators present at each order in the SMEFT expansion. This has lead to the complete characterization of the operator sets in $\mathcal{L}_7$ and $\mathcal{L}_8$ in these works.

3.0.4 *Perturbative NLO results in the SMEFT*

Full results to NLO in the SMEFT have started to appear in the literature. The first pioneering calculations of this form were for the process $\mu \to e\,\gamma$ in Ref. [107] and for the process $\Gamma(\mathrm{H} \to \gamma\,\gamma)$ in Refs. [23–25]. In [24] the full NLO perturbative SMEFT result for this decay with no assumption in the underlying UV scenario was reported. Ref. [25] also reported NLO results for $\Gamma(\mathrm{H} \to \mathrm{Z}\,\gamma)$, $\mathrm{H} \to \mathrm{Z}\,\mathrm{Z}^\star$, $\mathrm{H} \to \mathrm{W}\,\mathrm{W}^\star$ under the assumption of a PTG scenario and presented results to NLO for the W mass and other EWPD parameters. Recently Ref. [22] also reported NLO perturbative results for $\mathrm{H} \to \overline{\mathrm{b}}\mathrm{b}$ and $\mathrm{H} \to \tau^-\tau^+$ in the general SMEFT, including finite terms, in the large m_{t} limit. NLO QCD results for a set of higher dimensional operators contributing to the Higgs pair production process were given in Ref. [108].

3.1 NLO theory errors and bounds on parameters in $\mathcal{L}_6$

Until deviations from the SM are observed, studies of the SMEFT are studies of constraints. Nevertheless, in a situation where no new physics effects are observed, the

[18] It is worth noting, that some questions remain about the effect of mixing between the heavy and light field content in this approach [104].

experimental results cannot be expressed trivially into the limits

$$\frac{\tilde{C}_i^6}{\Lambda^2} < \delta_i^{exp}(M_{cut}). \tag{3.76}$$

First of all, several SMEFT operators generally contribute to any one observable. The value of one operator has basically no meaning in the general SMEFT case. A complete (sub-)set of operators in the SMEFT is present unless symmetries, or knowledge of the UV theory, allows a reduction. Global constraint analyses in the SMEFT are a very active area of research. NLO SMEFT analyses have one very important point to add to such efforts, which is the characterization of the SMEFT theoretical error, when fitting to a coefficient in $\mathcal{L}_6$ at LO.

Generally, as SMEFT theoretical errors are only increasing the theoretical uncertainty, including an appropriate SMEFT theoretical error reduces the degree of constraint on parameters in $\mathcal{L}_6$. In particular, it has been shown that claims of general model independent bounds at the per-mille level, on individual operators due to the LEP experiments, do not hold, when considering SMEFT theoretical errors [17,18]. These bounds are relaxed to the percent level by such considerations. This is clearly the case in general, and the relaxing of these bounds does not correspond to the SMEFT expansion breaking down. If the SMEFT expansion were to break down, the bounds would not be relaxed only by an order of magnitude, they would not be present at all.

The reason bounds are relaxed in a consistent SMEFT approach is very easy to illustrate [17,18]. Naively incorporating a per-mille constraint in EWPD on a combination of dimension six Wilson coefficients, denoted c_6, corresponds to $c_6\,\bar{v}_T^2/\Lambda^2 \lesssim 10^{-3}$, which gives $c_6 \lesssim 0.1$ for $\Lambda \sim 2.5\,\mathrm{TeV}$. Such a naive bound neglects the effects of the large number of dimension eight operators in the SMEFT; so that schematically $c_6 + 0.01\,c_8 \lesssim 0.1$ for TeV cut off scales. Bounds of this form are difficult to consider as precise numerical limits on the inferred Wilson coefficients. It is now known that there are 993 $N_f = 1$ operators in the $\mathcal{L}_8$ SMEFT, with Wilson coefficients that are not all related to the specific combination of Wilson coefficients denoted c_6 in the above expression. Generally the SMEFT does remain predictive, and only a small subset of the $\mathcal{L}_8$ operators contribute to a particular observable. It is actually extremely conservative to only argue that LEP based constraints are relaxed to the percent level from the per-mille level, due to theoretical errors introduced due to the neglect of these effects in the SMEFT.

Per-mille constraint claims based on LEP analyses that completely neglect theoretical errors in the SMEFT, are not strong statements. The neglect of higher order terms in the SMEFT must be quantified with some theory error metric. We have advanced a number of examples in this document as to how such a theory error metric can be defined. Alternative schemes to define theory errors could also be defined, but some theory error is essential in a LO analysis. If a LO formalism makes it particularly difficult (or easy) to define such a theory error, then this is a serious consideration to take into account when choosing an approach to the LO SMEFT.

3.2 A study of constraints

As a particular example, we discuss the impact of NLO corrections on inferred LO bounds, in the case of $\Gamma(\mathrm{H} \to \gamma\,\gamma)$, using the results of Refs. [23,24]. We consider the general SMEFT case, consider unknown $\tilde{C}_i \sim 1$ and vary the unknown parameters over $0.8 \leq \Lambda \leq 3$ in TeV units. Note that $\bar{v}_T^2/(0.8\,\mathrm{TeV})^2 \sim 0.1$. Taking κ_γ from Ref. [109] to be $0.93^{+0.36}_{-0.17}$, and neglecting light fermion ($m_{\mathrm{f}} < m_{\mathrm{H}}$) effects for simplicity, one finds the $1\,\sigma$ range

$$-0.02 \leq \left(\tilde{C}^{1,NP}_{\gamma\gamma} + \frac{\tilde{C}^{NP}_i\, f_i}{16\,\pi^2}\right) \frac{\bar{v}_T^2}{\Lambda^2} \leq 0.02\,. \tag{3.77}$$

Here, the tilde superscript denotes that the scale $1/\Lambda^2$ has been factored out of a Wilson coefficient. The f_i terms correspond to the "nonfactorizable" terms, and $\tilde{C}^{1,NP}_{\gamma\gamma}$ corresponds to the one loop improvement of the Wilson coefficient that gives this decay at tree level – $\tilde{C}^{0,NP}_{\gamma\gamma}$. The difference in the mapping of this constraint to the coefficient of $\tilde{C}^{0,NP}_{\gamma\gamma}$ at tree level, and at one loop, can now be characterized.

To determine this correction we determine the percentage change on the inferred value of the bounds of $\tilde{C}^{0,NP}_{\gamma\gamma}$, while shifting the quoted upper and lower experimental bounds by the NLO SMEFT perturbative correction. The envelope of the two percentage variations on the bounds is quoted in the form [,], for values of Λ varying from [0.8, 3] TeV. For one specific choice of signs for C_i, we find the following characteristic results. The net impact of one-loop corrections (added in quadrature) due to higher dimensional operators on the bound of the tree level Wilson coefficient is

$$\Delta_{quad}\,\tilde{C}^{0,NP}_{\gamma\gamma} \sim [29, 4]\,\%\,. \tag{3.78}$$

Similarly, CMS reports $\kappa_\gamma = 0.98^{+0.17}_{-0.16}$ [110], which gives

$$\Delta_{quad}\,\tilde{C}^{0,NP}_{\gamma\gamma} \sim [52, 7]\,\%\,. \tag{3.79}$$

It is possible that these corrections could add up in a manner that is not in quadrature, as this depends on the unknown $\tilde{C}_i$ values. The impact of the one-loop corrections listed above is on *current* experimental bounds of $\Gamma(\mathrm{H} \to \gamma\gamma)$, following from our conservative treatment of unknown UV effects. As the experimental precision of the measurement of $\Gamma(\mathrm{H} \to \gamma\gamma)$ increases, the impact of the neglected corrections directly scales up. Repeating the exercise above, with a chosen projected RunII value $\kappa_\gamma = 1 \pm 0.045$ which is consistent with projected future bounds (CMS - scenario II [69,68])

$$(\Delta_{quad}\,\tilde{C}^{0,NP}_{\gamma\gamma})^{proj\,:\,RunII} \sim [167, 21]\,\%\,. \tag{3.80}$$

High luminosity LHC runs are further quoted to have a sensitivity between 2% and 5% in κ_γ [111]. Choosing a value $\kappa_\gamma = 1 \pm 0.03$ for this case, one finds

$$(\Delta_{quad}\,\tilde{C}^{0,NP}_{\gamma\gamma})^{proj\,:\,HILHC} \sim [250, 31]\,\%\,. \tag{3.81}$$

It is clear that neglected one loop corrections can have an important effect on the projection of an experimental bound into the LO SMEFT formalism, when measurements become sufficiently precise and the cut off scale is not too high.

3.3 A study of SM-deviations

Here the reference process is the off-shell gg $\to$ H production. It is important to go off-shell because the correct use of the SMEFT proves that scaling couplings on a resonance pole is not the same thing as scaling them off of a resonance pole, which has important consequences in bounding the Higgs intrinsic width, see Refs. [112–114].

In the κ approach, which was developed out of Refs. [115–117], and formalized in Ref. [118], one write the amplitude as

$$\mathrm{A}^{\mathrm{gg}} = \sum_{\mathrm{q=t,b}} \kappa^{\mathrm{gg}}_{\mathrm{q}}\, \mathcal{A}^{\mathrm{gg}}_{\mathrm{q}} + \kappa^{\mathrm{gg}}_{c}\,, \tag{3.82}$$

$\mathcal{A}^{\mathrm{gg}}_{\mathrm{t}}$ being the SM t -loop etc. The contact term (which is the LO SMEFT) is given by κ^{gg}_{c}. Furthermore $\kappa^{\mathrm{gg}}_{\mathrm{q}} = 1 + \Delta\,\kappa^{\mathrm{gg}}_{\mathrm{q}}$ Next we compute the following ratio

$$\mathrm{R} = \sigma\left(\kappa^{\mathrm{gg}}_{\mathrm{q}}\,,\,\kappa^{\mathrm{gg}}_{c}\right)/\sigma_{\mathrm{SM}} - 1 \quad [\%]\,. \tag{3.83}$$

In LO SMEFT κ_c is non-zero and $\kappa_{\mathrm{q}} = 1$. One measures a deviation and gets a value for κ_c. However, at NLO $\Delta\kappa_{\mathrm{q}}$ is non zero and one gets a degeneracy: the interpretation in terms of κ^{LO}_{c} or in terms of $\{\kappa^{\mathrm{NLO}}_{c}, \Delta\kappa^{\mathrm{NLO}}_{\mathrm{q}}\}$ could be rather different (we show an example in Fig. 4). Going interpretational we consider

$$\mathrm{A}^{\mathrm{gg}}_{\mathrm{SMEFT}} = \frac{g\,g_3}{\pi^2} \sum_{\mathrm{q=t,b}} \kappa^{\mathrm{gg}}_{\mathrm{q}}\, \mathcal{A}^{\mathrm{gg}}_{\mathrm{q}} + 2\,g_{\mathrm{S}}\,g_{_6}\,\frac{s}{M^2_{\mathrm{W}}}\,\tilde{C}_{\mathrm{H\,g}} + \frac{g\,g_3\,g_{_6}}{\pi^2} \sum_{\mathrm{q=t,b}} \mathcal{A}^{\mathrm{nfc\,;\,gg}}_{\mathrm{q}}\,\tilde{C}_{\mathrm{qg}}\,, \tag{3.84}$$

where g_3 is the $SU(3)$ coupling constant. Using Eq.(3.84) we adopt the Warsaw basis and eventually work in the (PTG) scenario [41,42]. The following options are available: LO SMEFT: $\kappa_{\mathrm{q}} = 1$ and $\tilde{C}_{\mathrm{H\,g}}$ is scaled by $1/16\,\pi^2$ being “loop-generated” (LG); NLO PTG-SMEFT: $\kappa_{\mathrm{q}} \neq 1$ but only PTG operators inserted in loops (non-factorizable terms absent), $\tilde{C}_{\mathrm{H\,g}}$ scaled as above; NLO full-SMEFT: $\kappa_{\mathrm{q}} \neq 1$ LG/PTG operators inserted in loops (non-factorizable terms present), LG coefficients scaled as above. Again we note the PTG classification scheme is not valid for all possible UV.

It is worth noting the difference between Eq.(3.82) and Eq.(3.84), showing that the original κ-framework can be made consistent at the price of adding “non-factorizable” sub-amplitudes. At NLO, $\Delta\kappa = g_{_6}\,\rho$ and

$$g^{-1}_{_6} = \sqrt{2}\,G_F\,\Lambda^2 \qquad 4\,\pi\,\alpha_{\mathrm{s}} = g_3\,, \tag{3.85}$$

$$\rho^{\mathrm{gg}}_{\mathrm{t}} = \tilde{C}_{\mathrm{H\,W}} + \tilde{C}_{\mathrm{tH}} + 2\,\tilde{C}_{\mathrm{H\,\Box}} - \frac{1}{2}\,\tilde{C}_{\mathrm{H\,D}} \qquad \rho^{\mathrm{gg}}_{\mathrm{b}} = \tilde{C}_{\mathrm{H\,W}} - \tilde{C}_{\mathrm{bH}} + 2\,\tilde{C}_{\mathrm{H\,\Box}} - \frac{1}{2}\,\tilde{C}_{\mathrm{H\,D}}\,. \tag{3.86}$$

Relaxing the PTG assumption introduces non-factorizable sub-amplitudes proportional to $\tilde{C}_{\mathrm{tH}}, \tilde{C}_{\mathrm{bH}}$ with a mixing among $\tilde{C}_{\mathrm{H\,g}}, \tilde{C}_{\mathrm{tg}}, \tilde{C}_{\mathrm{bg}}$. Meanwhile, renormalization has made one-loop SMEFT finite, e.g. in the G_F-scheme, with a residual μ_{R}-dependence.

We allow each Wilson coefficient to vary in some interval $\mathrm{I}_n = [-n\,,\,+n]$ and fix a value for Λ. Next we generate points from I_n for the Wilson coefficients with uniform probability and calculate R. Finally, we calculate the R probability distribution function (pdf), as shown in Figs. 2,3.

As another example, a comparison between the LO pdf and NLO pdf for $H \to \gamma\gamma$ using the approach of this section, and the results in [25], is shown in Fig. 4.

4 Summary Comments

The takeaway lessons are as follows.

- Overall, the neglect of NLO corrections, considering the precision of RunI measurements, is (retrospectively) justified. However, considering projections for the precision to be reached in RunII analyses, LO results for interpretations of the data in the SMEFT are challenged by consistency concerns, if the cut off scale is in the few TeV range.
- NLO results are starting to become available in the SMEFT. These results allow the consistent interpretation of the data combining measurements at different scales, and can robustly accommodate the precision projected to be achieved in RunII analyses, even for lower cut off scales.
- NLO results allow the kappa-framework [118] to be extended. NLO results more consistently include kinematic deviations from the SM, and define higher order calculations in relation to a measured observable, in a well defined field theory. A properly formulated SMEFT goes beyond LO and includes EW corrections.
- The assignment of a theoretical error for LO SMEFT analyses is essential if the cut off scale is assumed to be in the "interesting range" $1\,\mathrm{TeV} \lesssim \Lambda/\sqrt{\tilde{C}_i} \lesssim 3\,\mathrm{TeV}$.
- Absorbing the effects of $\mathcal{L}_8$ corrections and/or or absorbing logarithmic NLO perturbative corrections into an "effective" parameter to attempt to incorporate NLO corrections is not recommended. Such a redefinition cannot simultaneously be made in different measurements sensitive to a LO SMEFT parameter, generally measured at different scales, unless theoretical errors are introduced. Correlating different measurements is necessary if the SMEFT is to be used in a predictive fashion for constraints on LHC measurements.
- It is not recommended that LEP constraints are interpreted to mean that effective SMEFT parameters in $\mathcal{L}_6$, or combinations of such parameters, should be set to zero in LHC analyses. Arguments leading to claims of strong bounds to justify such a step rely on LO SMEFT analyses, without any theoretical error assigned.
- In general, we recommend that the experimental collaborations restrict the bulk of their efforts to defining and reporting clean measurements that can be interpreted in any (actual) basis, and at LO or NLO in the SMEFT. By this we mean that the focus for data reporting should be on real observables, fiducial cross sections and/or pseudo-observables. If a LO interpretation of that data in the SMEFT

is reported, there is no barrier to using the straightforward LO formalism of the Warsaw basis, that is in common use in the theoretical community.

Most importantly, it is not recommended to use LO results (subject to large undetermined NLO corrections and uncertainties) when they are formulated in a framework where it is not known how to define the NLO extension, or if this is even possible. At the very least, the data must also be reported in a manner that ensures a NLO treatment of the data is always possible in the future, bypassing any ill defined LO formalism.

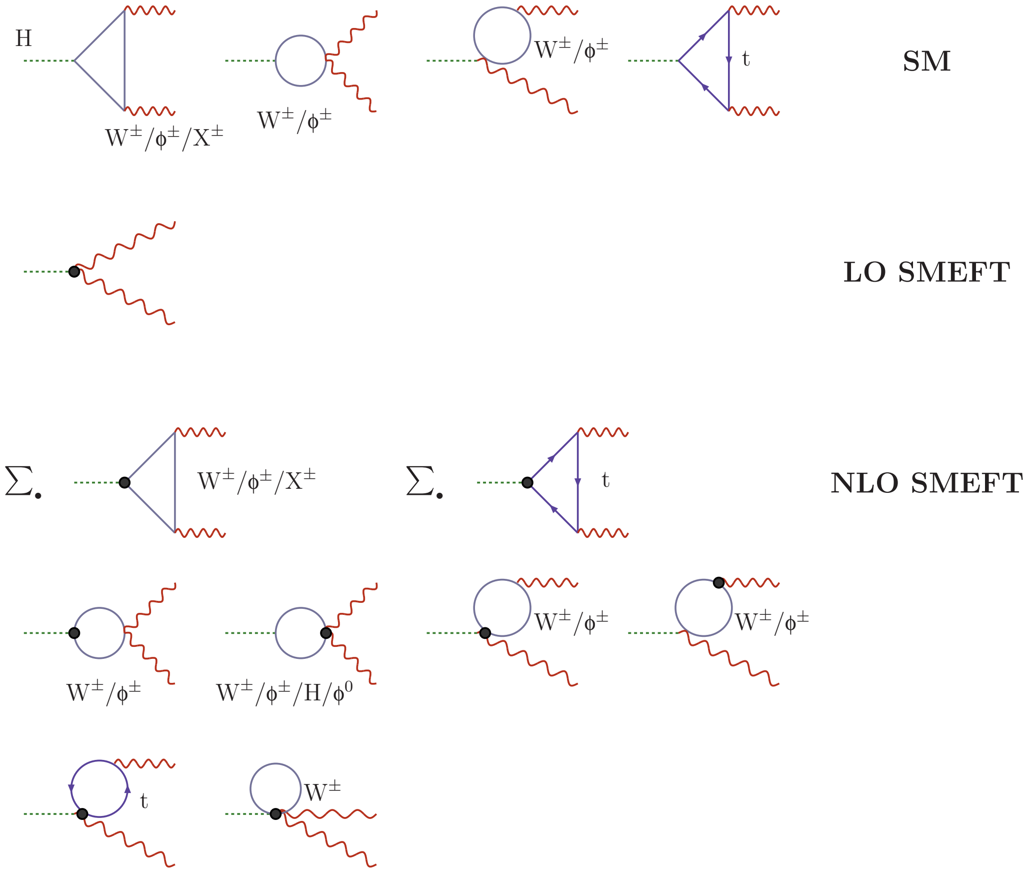

Figure 1: Diagrams contributing to the amplitude for $H \to \gamma\gamma$ in the R_ξ-gauge: SM (first row), LO SMEFT (second row), and NLO SMEFT. Black circles denote the insertion of one $\mathcal{L}_6$ operator. $\sum_\bullet$ implies summing over all insertions in the diagram (vertex by vertex). For triangles with internal charge flow $(t, W^\pm, \phi^\pm, X^\pm)$ only the clockwise orientation is shown. Non-equivalent diagrams obtained by the exchange of the two photon lines are not shown. Higgs and photon wave-function factors are not included. The Fadeev-Popov ghost fields are denoted by X.

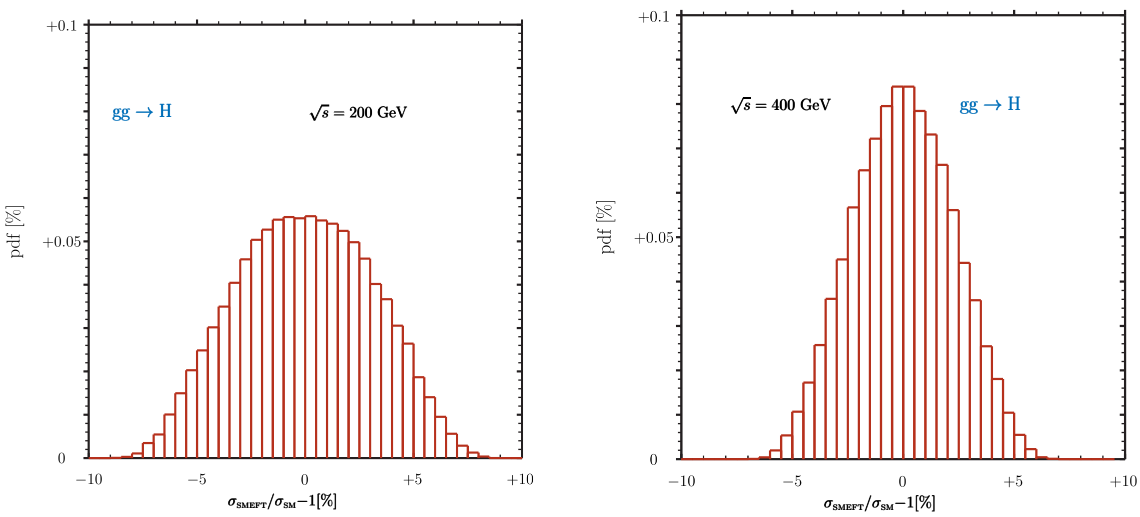

Figure 2: Probability distribution function for the off-shell process gg $\to$ H. Support is $C_i \in [-1\,,+1]$ with a uniform prior, and we have set $\Lambda = 3\,$TeV.

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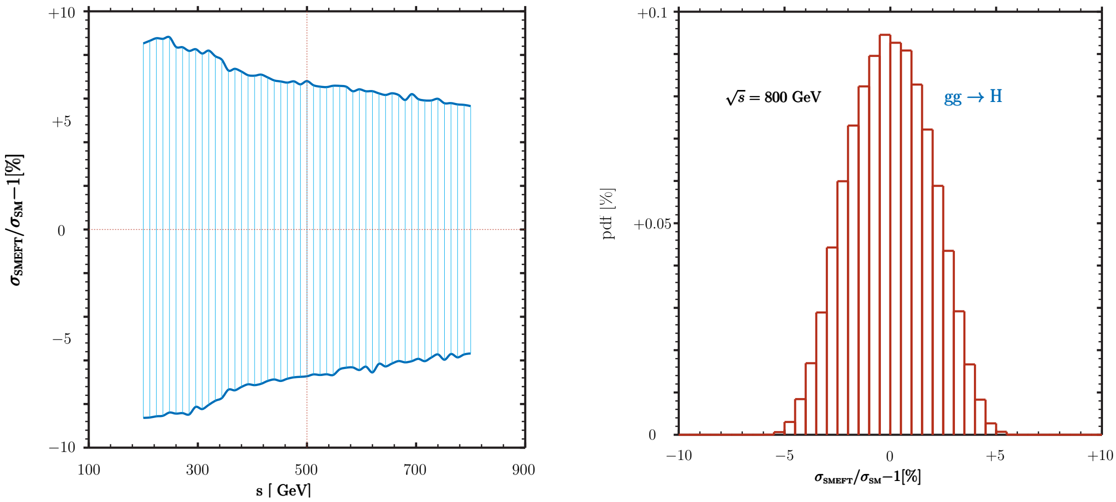

Figure 3: Probability distribution function for the off-shell process gg $\to$ H. Support is $C_i \in [-1\,,\,+1]$ with a uniform prior, and we have set $\Lambda = 3\,$TeV.

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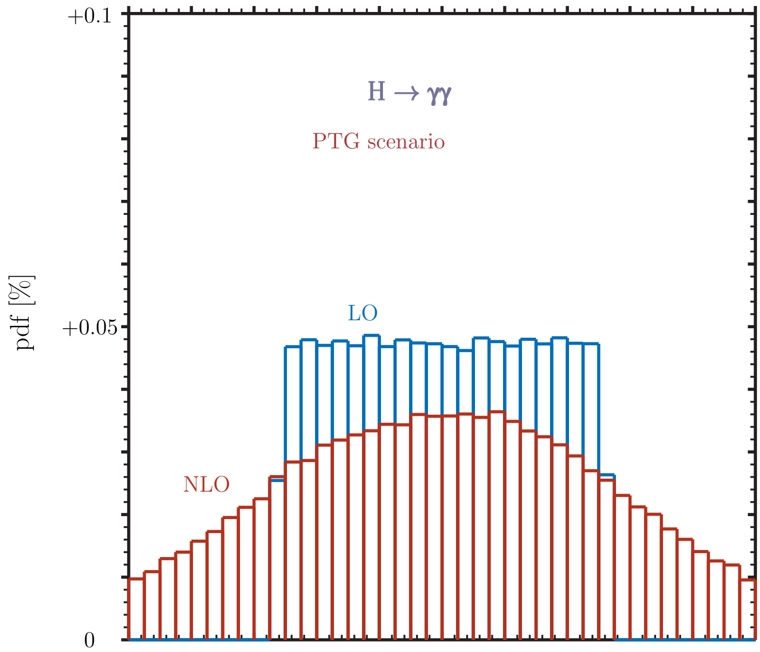

Figure 4: Probability distribution function for the decay H $\rightarrow \gamma\gamma$ with a comparison between the LO and the NLO predictions. Here $\Lambda = 3\ TeV$ and $n = 1$.

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