The SMEFT at NLO

Higgs XS Working Group



"To misname things is to add to the misery of the world." -A. Camus (re basis debate) "Absence of evidence is not evidence of absence." -The BSM physics mantra. (Unfortunately, you also can't prove a negative.)

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Basic Outline

- NLO EFT note under development for WG2 will be reviewed. Basic points covered will be as follows:
- 1. What is the linear SMEFT? What is LO and NLO in the SMEFT?
- 2. Why are calculations in the NLO SMEFT being done? When are NLO effects on specific processes or physics results of interest? When does this matter?
- 3. Some details/reminders on theoretical errors and why this matters in the LO SMEFT formalism brief mention of the standard approach to LO in the SMEFT.
- 4. Some details on the systematic renormalization programs that have been done, allowing NLO advances.
- 5. Discussion of the complete $h \rightarrow \gamma \gamma$ result, how it is laying a path in the NLO jungle for future work.



What is the linear SMEFT

Linear EFT - built of H doublet + higher D ops

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda_{\delta B = 0}^2} \mathcal{L}_6 + \frac{1}{\Lambda_{\delta B \neq 0}^2} \mathcal{L}_6' + \frac{1}{\Lambda_{\delta L \neq 0}^3} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots$$

Linear SMEFT: Global sym structure

Linear EFT - built of H doublet + higher D ops

$$\mathcal{L} = \mathcal{L}_{SM} + \left(\frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5\right) + \frac{1}{\Lambda_{\delta B=0}^2} \mathcal{L}_6 + \frac{1}{\Lambda_{\delta B\neq 0}^2} \mathcal{L}_6' + \left(\frac{1}{\Lambda_{\delta L \neq 0}^3} \mathcal{L}_7\right) + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots$$

Lepton number violating, associated with neutrino mass and higher suppression scale. Operator dimension and global sym structure related - see 1404.4057 Gouvea, Herrero-Garcia, Kobach

Linear SMEFT: Global sym structure

Linear EFT - built of H doublet + higher D ops

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda_{\delta B = 0}^2} \mathcal{L}_6 + \underbrace{\frac{1}{\Lambda_{\delta B \neq 0}^2} \mathcal{L}_6'}_{\Lambda_{\delta B \neq 0}^2} + \frac{1}{\Lambda_{\delta L \neq 0}^3} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots$$

Baryon number violating, experimentally known to be small

SMEFT:development cycle

Linear EFT - built of H doublet + higher D ops



Glashow 1961, Weinberg 1967 (Salam 1967)

- Weinberg 1977
- - Leung, Love, Rao 1984, Buchmuller Wyler 1986, Grzadkowski, Iskrzynski, Misiak, Rosiek 2010
 - Weinberg 1979, Abbott Wise 1980
- \bigcirc
- Lehman 1410.4193, Henning et al. 1512.03433
- \bigcirc
- Lehman, Martin 1510.00372, Henning et al. 1512.03433

The Lagrangian expansion theory technology is essentially a solved problem

Linear EFT - built of H doublet + higher D ops

 $\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_{5} + \frac{1}{\Lambda_{\delta B = 0}^{2}} \mathcal{L}_{6} + \frac{1}{\Lambda_{\delta B \neq 0}^{2}} \mathcal{L}_{6} + \frac{1}{\Lambda_{\delta L \neq 0}^{3}} \mathcal{L}_{7} + \frac{1}{\Lambda^{4}} \mathcal{L}_{8} + \cdots$

14 operators, or 18 parameters (+ 1 op and then 19 with strong CP)

1 operator, and 7 extra parameters

Dim 6 counting is a bit non trivial.

С	lass	i	N_{op}	CP-even			$CP ext{-odd}$		
_				n_g	1	3	n_g	1	3
	1	$g^{3}X^{3}$	4	2	2	2	2	2	2
	2	H^{6}	1	1	1	1	0	0	0
	3	H^4D^2	2	2	2	2	0	0	0
	4	$g^{2}X^{2}H^{2}$	8	4	4	4	4	4	4
	5	$y\psi^2H^3$	3	$3n_g^2$	3	27	$3n_g^2$	3	27
	6	$gy\psi^2 XH$	8	$8n_g^2$	8	72	$8n_g^2$	8	72
	7	$\psi^2 H^2 D$	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_{g}(9n_{g}-7)$	1	30
	8	$:(\overline{L}L)(\overline{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$rac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
	8	$: (\overline{R}R)(\overline{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
ψ^4	8	$: (\overline{L}L)(\overline{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
т	8	$:(\overline{L}R)(\overline{R}L)$	1	n_g^4	1	81	n_g^4	1	81
	8	$:(\overline{L}R)(\overline{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
	8	: All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$rac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Т	otal		59	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 213n_g^2 + 30n_g + 72)$	53	1350	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 57n_g^2 - 30n_g + 48)$	23	1149

Table 2. Number of *CP*-even and *CP*-odd coefficients in $\mathcal{L}^{(6)}$ for n_g flavors. The total number of coefficients is $(107n_g^4 + 2n_g^3 + 135n_g^2 + 60)/4$, which is 76 for $n_g = 1$ and 2499 for $n_g = 3$.

arXiv:1312.2014 Alonso, Jenkins, Manohar, Trott

Linear EFT - built of H doublet + higher D ops



LO SMEFT = dim 6 shifts

Warsaw basis: 1008.4884 Grzadkowski, Iskrzynski, Misiak, Rosiek

X^3			$arphi^6$ and $arphi^4 D^2$	$\psi^2 arphi^3$		
Q_G	$f^{ABC}G^{A u}_\mu G^{B ho}_ u G^{C\mu}_ ho$	Q_{arphi}	$(\varphi^{\dagger}\varphi)^{3}$	$Q_{e\varphi}$	$(arphi^\dagger arphi) (ar l_p e_r arphi)$	
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A u}_{\mu}G^{B ho}_{ u}G^{C\mu}_{ ho}$	$Q_{arphi\square}$	$(\varphi^{\dagger}\varphi)\Box(\varphi^{\dagger}\varphi)$	$Q_{u\varphi}$	$(arphi^\dagger arphi) (ar q_p u_r \widetilde arphi)$	
Q_W	$\varepsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$	$Q_{arphi D}$	$\left(arphi^\dagger D^\mu arphi ight)^\star \left(arphi^\dagger D_\mu arphi ight)$	$Q_{d \varphi}$	$(arphi^\dagger arphi) (ar q_p d_r arphi)$	
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$					
$X^2 \varphi^2$		$\psi^2 X arphi$		$\psi^2 arphi^2 D$		
$Q_{arphi G}$	$arphi^\dagger arphi G^A_{\mu u} G^{A\mu u}$	Q_{eW}	$(ar{l}_p \sigma^{\mu u} e_r) au^I arphi W^I_{\mu u}$	$Q^{(1)}_{arphi l}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r})$	
$Q_{arphi \widetilde{G}}$	$arphi^\dagger arphi \widetilde{G}^A_{\mu u} G^{A\mu u}$	Q_{eB}	$(ar{l}_p \sigma^{\mu u} e_r) arphi B_{\mu u}$	$Q^{(3)}_{arphi l}$	$(arphi^\dagger i \overleftrightarrow{D}^I_\mu arphi) (ar{l}_p au^I \gamma^\mu l_r)$	
$Q_{\varphi W}$	$\varphi^{\dagger} \varphi W^{I}_{\mu u} W^{I\mu u}$	Q_{uG}	$(ar q_p \sigma^{\mu u} T^A u_r) \widetilde arphi G^A_{\mu u}$	$Q_{arphi e}$	$(arphi^\dagger i \overleftrightarrow{D}_\mu arphi) (ar{e}_p \gamma^\mu e_r)$	
$Q_{arphi \widetilde{W}}$	$arphi^\dagger arphi \widetilde{W}^I_{\mu u} W^{I\mu u}$	Q_{uW}	$(ar q_p \sigma^{\mu u} u_r) au^I \widetilde arphi W^I_{\mu u}$	$Q^{(1)}_{arphi q}$	$(arphi^\dagger i \overleftrightarrow{D}_\mu arphi) (ar{q}_p \gamma^\mu q_r)$	
$Q_{arphi B}$	$arphi^\dagger arphi B_{\mu u} B^{\mu u}$	Q_{uB}	$(ar q_p \sigma^{\mu u} u_r) \widetilde arphi B_{\mu u}$	$Q^{(3)}_{arphi q}$	$(\varphi^{\dagger}i \overleftrightarrow{D}_{\mu}^{I} \varphi)(ar{q}_{p} au^{I} \gamma^{\mu} q_{r})$	
$Q_{arphi \widetilde{B}}$	$arphi^\dagger arphi \widetilde{B}_{\mu u} B^{\mu u}$	Q_{dG}	$(ar q_p \sigma^{\mu u} T^A d_r) arphi G^A_{\mu u}$	$Q_{arphi u}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(ar{u}_{p}\gamma^{\mu}u_{r})$	
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(ar{q}_p \sigma^{\mu u} d_r) au^I arphi W^I_{\mu u}$	$Q_{arphi d}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{d}_{p}\gamma^{\mu}d_{r})$	
$Q_{\omega \widetilde{W}B}$	$\varphi^{\dagger} \tau^{I} \varphi \widetilde{W}^{I}_{\mu u} B^{\mu u}$	Q_{dB}	$(ar{q}_p \sigma^{\mu u} d_r) arphi B_{\mu u}$	$Q_{arphi u d}$	$i(\widetilde{arphi}^{\dagger}D_{\mu}arphi)(ar{u}_{p}\gamma^{\mu}d_{r})$	

Table 2: Dimension-six operators other than the four-fermion ones.

6 gauge dual ops
28 non dual operators
25 four fermi ops
59 + h.c.
operators
NOTATION:
$\widetilde{X}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} X^{\rho\sigma} (\varepsilon_{0123} = +1)$
$\widetilde{arphi}^{j}=arepsilon_{jk}(arphi^{k})^{\star}$ $arepsilon_{12}=+1$
$arphi^{\dagger}i \overleftrightarrow{D}_{\mu} arphi \equiv i arphi^{\dagger} \left(D_{\mu} - \widecheck{D}_{\mu} ight) arphi$
$arphi^{\dagger}i \overleftrightarrow{D}_{\mu}^{I} arphi \equiv i arphi^{\dagger} \left(au^{I} D_{\mu} - \widecheck{D}_{\mu} au^{I} ight) arphi_{\mu}$

LO SMEFT = dim 6 shifts

Four fermion operators: 1008.4884 Grzadkowski, Iskrzynski, Misiak, Rosiek

	$8:(ar{L}L)(ar{L}L)$		$8:(ar{R}R)(ar{R}$	(R)		$8:(\bar{L}L)(\bar{R}R)$
Q_{ll}	$(ar{l}_p \gamma_\mu l_r) (ar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(ar{e}_p \gamma_\mu e_r)$	$(ar{e}_s \gamma^\mu e_t)$	Q_{le}	$(ar{l}_p\gamma_\mu l_r)(ar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{\left(1 ight) }$	$(ar{q}_p \gamma_\mu q_r) (ar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(ar{u}_p \gamma_\mu u_r)$	$(ar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(ar{l}_p\gamma_\mu l_r)(ar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{\left(3 ight) }$	$(ar{q}_p \gamma_\mu au^I q_r) (ar{q}_s \gamma^\mu au^I q_t)$	Q_{dd}	$(ar{d}_p \gamma_\mu d_r)$	$(ar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(ar{l}_p\gamma_\mu l_r)(ar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{\left(1 ight) }$	$(ar{l}_p\gamma_\mu l_r)(ar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(ar{e}_p \gamma_\mu e_r)$	$(ar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(ar q_p \gamma_\mu q_r) (ar e_s \gamma^\mu e_t)$
$Q_{lq}^{\left(3 ight) }$	$(ar{l}_p \gamma_\mu au^I l_r) (ar{q}_s \gamma^\mu au^I q_t)$	Q_{ed}	$(ar{e}_p \gamma_\mu e_r)$	$(ar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{\left(1 ight)}$	$(ar q_p \gamma_\mu q_r) (ar u_s \gamma^\mu u_t)$
		$Q_{ud}^{\left(1 ight) }$	$(ar{u}_p \gamma_\mu u_r)$	$(ar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{\left(8 ight)}$	$(ar q_p \gamma_\mu T^A q_r) (ar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{\left(8 ight)}$	$(ar{u}_p \gamma_\mu T^A u_r)$	$(ar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{\left(1 ight)}$	$(ar q_p \gamma_\mu q_r) (ar d_s \gamma^\mu d_t)$
					$Q_{qd}^{\left(8 ight)}$	$(ar{q}_p\gamma_\mu T^A q_r)(ar{d}_s\gamma^\mu T^A d_t)$
	$8:(ar{L}R)(ar{R}$	L) + h.c	. 8	$(\bar{L}R)(\bar{L}R) +$	h.c.	
	Q_{ledq} $(ar{l}_p^j e$	$(\bar{d}_s q_{tj})$) $Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk}$	$(\bar{q}_s^k d_t)$	
			$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk}$	$(\bar{q}_s^k T^A d$)
			$Q_{lequ}^{\left(1 ight)}$	$(ar{l}_p^j e_r) \epsilon_{jk}$	$(ar{q}_s^k u_t)$	Over 20 years?!
			$Q_{lequ}^{\left(3 ight)}$	$(ar{l}_p^j\sigma_{\mu u}e_r)\epsilon_{jk}$	$(\bar{q}_s^k \sigma^{\mu u} u$	f_{t} FOM reduction?
						Our priorities were
						elsewhere

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$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda_{\delta B = 0}^2} \mathcal{L}_6 + \frac{1}{\Lambda_{\delta B \neq 0}^2} \mathcal{L}_6' + \frac{1}{\Lambda_{\delta L \neq 0}^3} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots$$



Can reduce the number of relevant parameters to about 50 or so using flavour symmetry and neglecting CP violation, using scaling when near resonances..

- WE CAN DO THE RELEVANT GENERAL CASE!
- Consistent power counting can also be done.
- There is no need for extra model dependence to be introduced or vague assumptions..

Can always restrict to less general case AFTER general analysis.

Straightforward LO

- Expand around the vev the dim 6 operators, go to mass eigenstates
- Canonically normalize the field theory.
- Choose some input parameters to relate to:

 (α, G_F, M_Z) a choice than can be made is an alpha scheme

 (m_W, G_F, M_Z) equally you can choose to use a Gf scheme (associated with an onshell renormalization scheme usually)

The choice is yours. This is not part of the Basis definition. Relation to input parameters differs as the SMEFT is a different theory than the SM. For example

$$\delta M_Z^2 \equiv \frac{1}{2\sqrt{2}} \frac{\hat{m}_Z^2}{\hat{G}_F} C_{HD} + \frac{2^{1/4} \sqrt{\pi} \sqrt{\hat{\alpha}} \, \hat{m}_Z}{\hat{G}_F^{3/2}} C_{HWB},$$

These differences taken into account with straightforward expansion. Trivial to do LO SMEFT directly, in a manner that can be improved to NLO.

Straightforward NLO

- Expand around the vev the dim 6 operators, go to mass eigenstates at one loop.
- Canonically normalize the field theory to one loop.
- Choose some input parameters to relate to:

 (α, G_F, M_Z) a choice than can be made is an alpha scheme

 (m_W, G_F, M_Z) equally you can choose to use a Gf scheme (associated with an onshell renormalization scheme usually)

Only change is the next order term in the perturbative expansion calculated for observables and the input parameters.

This is not easy, the note discusses the details, but this is straightforward. Over time is seems likely that the Gf scheme will be widely used. As is the case in the high precison SM calculating community, not the alpha scheme.

No matter what the inputs different

- Any LO SMEFT approach does not have the same theoretical errors as in the SM.
- The SM parameters run differently in the SM and the SMEFT.

$$\mu \frac{\mathrm{d}g_2}{\mathrm{d}\mu} = -4 \frac{m_H^2}{16\pi^2} g_2 C_{HW} \,, \qquad \mu \frac{\mathrm{d}g_1}{\mathrm{d}\mu} = -4 \frac{m_H^2}{16\pi^2} g_1 C_{HB} \,,$$

We show an example of how theory errors are modified on the input parameters in the note due to this running. One finds

$$\frac{(\Delta \alpha_{ew})_{SMEFT}}{(\Delta \alpha_{ew})_{SM}} \simeq -250 \, \left(\frac{1 \text{TeV}}{\Lambda}\right)^2 \tilde{C}_{HB} - 80 \, \left(\frac{1 \text{TeV}}{\Lambda}\right)^2 \tilde{C}_{HW},$$

This is how much bigger the SMEFT theory error is for this input compared to the SM. In any LO scheme, extra error is at least this big.

The SMEFT is a DIFFERENT theory than the SM. No matter what. NLO informs us of this difference.

Warsaw basis standard for LO and NLO

Warsaw basis: 1008.4884 Grzadkowski, Iskrzynski, Misiak, Rosiek



- Very easy and straightforward set of rules for EOM to reduce if you can get rid of a derivative - get rid of it!
- This is why this basis is the only one that has been renormalized at one loop allowing perturbative NLO - need to know what to do with divergences systematically. Essential for systematic improvement.

Warsaw basis standard for LO and NLO

One of the Yukawa results, full 3 generation result, nontrivial flavour structure in the RGEs :

$$\begin{split} \dot{C}_{prst}^{(1)} &= \frac{1}{2} [Y_{u}^{\dagger}Y_{u} - Y_{d}^{\dagger}Y_{d}]_{pr} C_{Hq}^{(1)} + \frac{1}{2} [Y_{u}^{\dagger}Y_{u} - Y_{d}^{\dagger}Y_{d}]_{st} C_{Hq}^{(1)} \\ &= r \\ &+ \frac{1}{4N_{c}} \left([Y_{u}^{\dagger}]_{pv} [Y_{u}]_{wr} C_{qu}^{(8)} + [Y_{u}^{\dagger}]_{sv} [Y_{u}]_{wr} C_{qu}^{(8)} \\ &= r \\ &+ \frac{1}{4N_{c}} \left([Y_{u}^{\dagger}]_{pv} [Y_{u}]_{wr} C_{stw}^{(8)} + [Y_{u}^{\dagger}]_{sv} [Y_{u}]_{wr} C_{qu}^{(8)} \\ &= r \\ &- \frac{1}{8} \left([Y_{u}^{\dagger}]_{pv} [Y_{u}]_{wt} C_{stw}^{(8)} + [Y_{u}^{\dagger}]_{sv} [Y_{u}]_{wr} C_{qu}^{(8)} \\ &= r \\ &- \frac{1}{8} \left([Y_{d}^{\dagger}]_{pv} [Y_{u}]_{wr} C_{stw}^{(8)} + [Y_{d}^{\dagger}]_{sv} [Y_{u}]_{wr} C_{qu}^{(8)} \\ &= r \\ &+ \frac{1}{16N_{c}} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(8)} + [Y_{d}]_{wr} [Y_{u}]_{vt} C_{quad}^{(8)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(8)} + [Y_{d}]_{wr} [Y_{u}]_{vt} C_{quad}^{(8)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(8)} + [Y_{d}]_{wr} [Y_{u}]_{vt} C_{quad}^{(8)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(8)} + [Y_{d}]_{wr} [Y_{u}]_{vt} C_{quad}^{(8)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(8)} + [Y_{d}]_{wr} [Y_{u}]_{vt} C_{quad}^{(8)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}^{\dagger}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{vr} [Y_{u}]_{vt} C_{quad}^{(1)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{vr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{wr} C_{quad}^{(1)} - \frac{1}{2} [Y_{d}]_{pv} [Y_{d}]_{wr} C_{quad}^{(1)} \\ \\ &= r \\ &= r \\ &+ \frac{1}{16} \left([Y_{d}]_{wt} [Y_{u}]_{wr} C_{quad}^{(1)} -$$

All of that due to EOM - we need to be able to deal with the theory at one loop, requires a well defined LO formalism with a clean set of rules that can be imposed in loop calculations.

Why are calculations at NLO being done?



- To combine the various constraints consistently take into account they rotate as you change scale.. or introduce theory error.
- Any future discovery has to be projected back on these constraints to check consistency.

Why are calculations at NLO being done?

 It is required to study constraints at many different scales to constrain all the parameters in the LO SMEFT model independently.

Hierarchies of constraints exist. At higher scales different combinations of parameters present due to NLO effects.

$$\sqrt{2} \sqrt{\frac{d}{d\mu}C_{Hl}^{(1)}} = \frac{1}{48\pi^2}g_1^2 y_H \left(y_H C_{Hl}^{(1)} + N_c y_d C_{ld} + y_e C_{le} + 2y_l C_{ll} + y_l C_{ll} + y$$

Constraints of effective Z coupling at one scale a combination of effective Z coupling and 4 lepton operators at different scales.

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 It is required to study constraints at many different scales to constrain all the parameters in the LO SMEFT model independently.

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Constraints of effective Z coupling at one scale a combination of effective Z coupling and 4 lepton operators at different scales.

Naive LO analysis just imposes the strongest constraint!

But <u>completely unconstrained directions in 4 lepton operators</u> (Falkowski,Mimouni 1511.07434)

A consistent NLO treatment gets that right, and informs the theory error for the LO result.

Consider LEP I observables:

Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z[\text{GeV}]$	91.1875 ± 0.0021	[38]	-	-
$\hat{m}_W[\text{GeV}]$	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]
σ_h^0 [nb]	41.540 ± 0.037	[38]	41.488 ± 0.006	[41]
$\Gamma_Z[\text{GeV}]$	2.4952 ± 0.0023	[38]	2.4942 ± 0.0005	[41]
R^0_ℓ	20.767 ± 0.025	[38]	20.751 ± 0.005	[41]
R_b^0	0.21629 ± 0.00066	[38]	0.21580 ± 0.00015	[41]
R_c^0	0.1721 ± 0.0030	[38]	0.17223 ± 0.00005	[41]
A_{FB}^{ℓ}	0.0171 ± 0.0010	[38]	0.01616 ± 0.00008	[42]
A^c_{FB}	0.0707 ± 0.0035	[38]	0.0735 ± 0.0002	[42]
A^b_{FB}	0.0992 ± 0.0016	[38]	0.1029 ± 0.0003	[42]

arXiv:1311.3107. Chen et al. 1211.1320 Masso, Sanz 1209.6382 Batell et al. arXiv:1404.3667 Ellis et al. arXiv:1501.0280. Petrov et al. arXiv:1406.6070 Wells,Zhang

And Many others...

1308.2803 Pomarol, Riva.1409.7605 Trotthep-ph/0412166] Han, Skiba1411.0669 Falkowski, Riva.1503.07872 Efrati et al. arXiv:1306.4644 Ciuchini et al.

Basic point is that STU is no longer sufficient in general.

Pioneering SMEFT works: Phys.Lett. B265 (1991) 326-334 Grinstein, Wise hep-ph/0412166 Han, Skiba

Consider LEP I observables:

	Observable	Experimental Value	Ref.	SM Theoretical Value	e Ref.	
	$\hat{m}_Z[\text{GeV}]$	91.1875 ± 0.0021	[38]	- /	-	Note that
	$\hat{m}_W[\text{GeV}]$	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]	
	σ_h^0 [nb]	41.540 ± 0.037	[38]	41.488 ± 0.006	[41]	theorists
	$\Gamma_Z[\text{GeV}]$	2.4952 ± 0.0023	[38]	2.4942 ± 0.0005	42	worked
	R^0_ℓ	20.767 ± 0.025	[38]	20.751 ± 0.005		
	R_b^0	0.21629 ± 0.00066	[38]	0.21580 ± 0.00015		hard in SM
ĺ	R_c^0	0.1721 ± 0.0030	[38]	0.17223 ± 0.00005	[41]	for this to be
	A_{FB}^{ℓ}	0.0171 ± 0.0010	[38]	0.01616 ± 0.00008	[42]	
	A^c_{FB}	0.0707 ± 0.0035	[38]	0.0735 ± 0.0002	[42]	the case.
	A^b_{FB}	0.0992 ± 0.0016	[38]	0.1029 ± 0.0003	[42]	
1		•				r

Many 2 loop SM calculations

Consider LEP I observables:

	-			
Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z[\text{GeV}]$	91.1875 ± 0.0021	[38]	-	-
\hat{m}_W [GeV]	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]
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	-			

arXiv:1502.02570 Berthier,Trott

If you go beyond % constraints, LO SMEFT alone inconsistent.

Need of loops in SMEFT once measurements are 10% precise appears again and again in the literature

> 1209.5538 Passarino 1301.2588 Grojean, Jenkins, Manohar, Trott 1408.5147 Englert, Spannowsky many others..

Theory error informed by NLO

Theory error defined by what you neglect in the calculation:

All perturbative one loop corrections, LO —> NLO

$$\Delta^{i}_{SMEFT}(\Lambda) = \sqrt{\Delta^{2}_{IFI,O_{i}} + \Delta^{2}_{P} + \Delta^{2}_{P,II} + \Delta^{2}_{\mathcal{L}_{8}} + \Delta^{2}_{\text{offshell,O_{i}}}}.$$

Radiative corrections, i.e. emission, one loop, redefining input observables, correlations... in SMEFT.

Higher order dim 8 terms in the SMEFT

$$\Delta_{SMEFT}^{i}(\Lambda) \simeq \sqrt{N_8} x_i \frac{\bar{v}_T^4}{\Lambda^4} + \frac{\sqrt{N_6} g_2^2}{16 \pi^2} y_i \log \left[\frac{\Lambda^2}{\bar{v}_T^2}\right] \frac{\bar{v}_T^2}{\Lambda^2}. \quad \text{(roughly)}$$
$$\text{arXiv:I 508.05060 Berthier,Trott}$$

Error is roughly per-mille to percent level for cut off scales of interest. $\Lambda \lesssim 3 {\rm TeV}$

Theory error in SMEFT/SM

Because LEP I observables are so precise we can't ignore error in EFT:

arXiv:1508.05060 Berthier, Trott

Chances of discovery also directly tied to cut off scale. If we find a deviation NLO matters.

Recent global SMEFT analysis on 103 observables (pre LHC data).

arXiv:1502.02570, 1508.05060 Berthier, Trott

Theory errors effect subspace correlations and constraints.

Percent/per-mille precision need loops

We need loops for the SMEFT for future precision program to reduce theory error. So renormalize SMEFT as first step.

 We know the Warsaw basis is self consistent at one loop as it has been completely renormalized - DONE! Complete result, every index all couplings. Possible because this is a well defined LO formalism.

arXiv:1301.2588 Grojean, Jenkins, Manohar, Trott arXiv:1308.2627,1309.0819,1310.4838 Jenkins, Manohar, Trott arXiv:1312.2014 Alonso, Jenkins, Manohar, Trott

Some partial results were also obtained in a "SILH basis"

arXiv:1302.5661,1308.1879 Elias-Miro, Espinosa, Masso, Pomarol 1312.2928 Elias-Miro, Grojean, Gupta, Marzocca

Recent results obtained in alternate scheme approach:

arXiv:1505.03706 Ghezzi, Gomez-Ambrosio, Passarino, Uccirati

NLO EFT - Full one loop

- In SMEFT the cut off scale is not TOO high. So RGE log terms not expected to be much bigger than remaining one loop "finite terms"
- Further, no reason to expect that structure of the divergences in mixing will have to be preserved in finite terms. So lets calculate finite terms for $\Gamma(h \to \gamma \gamma)$
- Initial calc mirror initial RGE work, just use operators

$$\begin{aligned} \mathcal{O}_{HB}^{(0)} &= g_1^2 \, H^{\dagger} \, H \, B_{\mu\nu} \, B^{\mu\nu}, \\ \mathcal{O}_{HWB}^{(0)} &= g_1 \, g_2 \, H^{\dagger} \, \sigma^a H \, B_{\mu\nu} \, W_a^{\mu\nu}. \end{aligned} \qquad \qquad \mathcal{O}_{HWB}^{(0)} &= g_1^2 \, H^{\dagger} \, H \, W_{\mu\nu}^a \, W_a^{\mu\nu}, \end{aligned}$$

Hartmann, Trott 1505.02646

Full calculation with all relevant operators was then performed:

 $\begin{array}{ll} \mathcal{O}_{H}^{(0)} = \lambda (H^{\dagger}H)^{3}, & \mathcal{O}_{HW}^{(0)} = g_{2}^{2} H^{\dagger} H W_{\mu\nu}^{a} W_{a}^{\mu\nu}, & \mathcal{O}_{HB}^{(0)} = g_{1}^{2} H^{\dagger} H B_{\mu\nu} B^{\mu\nu}, \\ \mathcal{O}_{HD}^{(0)} = (H^{\dagger}D_{\mu}H)^{*}(H^{\dagger}D^{\mu}H), & \mathcal{O}_{W}^{(0)} = \epsilon^{IJK} W_{\mu}^{I\nu} W_{\nu}^{J\rho} W_{\rho}^{K\mu}, & \mathcal{O}_{HWB}^{(0)} = g_{1} g_{2} H^{\dagger} \sigma^{a} H B_{\mu\nu} W_{a}^{\mu\nu}, \\ \mathcal{O}_{uH}^{(0)} = y_{u} H^{\dagger} H(\bar{q}_{p} u_{r} \tilde{H}), & \mathcal{O}_{eB}^{(0)} = \bar{l}_{r,a} \sigma^{\mu\nu} e_{s} H_{a} B_{\mu\nu}, & \mathcal{O}_{eW}^{(0)} = \bar{l}_{r,a} \sigma^{\mu\nu} e_{s} \tau_{ab}^{I} H_{b} W_{\mu\nu}^{I}, \\ \mathcal{O}_{eH}^{(0)} = y_{e} H^{\dagger} H(\bar{l}_{p} e_{r} H), & \mathcal{O}_{eH}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu} u_{s} \tilde{H}_{a} B_{\mu\nu}, & \mathcal{O}_{uW}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu} u_{s} \tau_{ab}^{I} \tilde{H}_{b} W_{\mu\nu}^{I}, \\ \mathcal{O}_{dH}^{(0)} = y_{d} H^{\dagger} H(\bar{q}_{p} d_{r} H). & \mathcal{O}_{dB}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu} d_{s} H_{a} B_{\mu\nu}, & \mathcal{O}_{dW}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu} d_{s} \tau_{ab}^{I} H_{b} W_{\mu\nu}^{I}, \end{array}$

Hartmann, Trott 1507.03568

NLO EFT - Subtract div.

The Algorithm: Use RGE results to renormalize.

Also use SM counter term subtractions.

Recent results: Hartmann, Trott 1505.02646.pdf Ghezzi et al. 1505.03706 Pruna, Signer 1408.3565 others..

Define a scheme that fixes that asymptotic properties of states in the S matrix, this fixes the finite terms in renormalization conditions.

Gauge fix, calculate, and then check gauge independence! • Here is how this works in $\Gamma(h \to \gamma \gamma)$

NLO EFT - Subtract div.

To define the SM counter terms use background field , use R_{ξ} gauge

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}i\phi^+ \\ h + v + \delta v + i\phi_0 \end{pmatrix}$$

Background field method (with particular operator normalization) gives:

$$Z_A Z_e = 1,$$
 $Z_h = Z_{\phi_{\pm}} = Z_{\phi_0},$ $Z_W Z_{g_2} = 1.$

Also need the Higgs wavefunction and vev renorm

$$\begin{split} Z_h &= 1 + \frac{(3+\xi)\left(g_1^2+3\,g_2^2\right)}{64\,\pi^2\,\epsilon} - \frac{Y}{16\,\pi^2\,\epsilon}.\\ (\sqrt{Z_v} + \frac{\delta v}{v})_{div} &= 1 + \frac{\left(3+\xi\right)\left(g_1^2+3\,g_2^2\right)}{128\,\pi^2\,\epsilon} - \frac{Y}{32\,\pi^2\,\epsilon}. \end{split}$$

We used a clever trick involving $h \rightarrow g g$ for the latter.

NLO EFT - Loops such as this

• Calculate in BF method, in R_{ξ} gauge

NLO EFT - Fix finite terms

• Define vev of the theory as the one point function vanishing - fixes δv

$$T = m_h^2 h v \frac{1}{16\pi^2} \left[-16\pi^2 \frac{\delta v}{v} + 3\lambda \left(1 + \log \left[\frac{\mu^2}{m_h^2} \right] \right) + \frac{m_W^2}{v^2} \xi \left(1 + \log \left[\frac{\mu^2}{\xi m_W^2} \right] \right), \quad (3.3)$$
$$+ \frac{1}{2} \frac{m_Z^2}{v^2} \xi \left(1 + \log \left[\frac{\mu^2}{\xi m_Z^2} \right] \right) - \frac{1}{2} \sum_i y_i^4 N_c \frac{1}{\lambda} \left(1 + \log \left[\frac{\mu^2}{m_i^2} \right] \right),$$
$$+ \frac{g_2^2}{2} \frac{m_W^2}{m_h^2} \left(1 + 3\log \left[\frac{\mu^2}{m_W^2} \right] \right) + \frac{1}{4} (g_1^2 + g_2^2) \frac{m_Z^2}{m_h^2} \left(1 + 3\log \left[\frac{\mu^2}{m_Z^2} \right] \right) \right].$$

The finite terms that are fixed by renormalization conditions (at one loop) in the theory enter as

$$\langle h(p_h)|S|\gamma(p_a,\alpha),\gamma(p_b,\beta)
angle_{BSM} = (1+rac{\delta R_h}{2})(1+\delta R_A)(1+\delta R_e)^2 i \sum_{x=a..o} \mathcal{A}_x.$$

Many interesting technicalities

- Closed form result now known.
- Running of vev important modification of RGE results.
- Gauge fixing modified by higher D ops, higher D ops source ghosts!

Recent results: Hartmann, Trott 1505.02646.pdf Hartmann, Trott 1507.03568.pdf Ghezzi et al. 1505.03706 Pruna, Signer 1408.3565 others..

- Pure finite terms can be present for higher D operators at one loop.
- Finite terms not small compared to logs as cut off scale can't be too high.
- Two processes know to full one loop in SMEFT now:

 $\mu \rightarrow e \gamma$ Pruna, Signer 1408.3565 $h \rightarrow \gamma \gamma$ Hartmann, Trott 1505.02646,1507.03568 Ghezzi et al. 1505.03706

But still need to redefine input observables to one loop in SMEFT to be more consistent. Lots more work to do.

Do we need this SMEFT NLO?

- Developing the SMEFT lets you reduce theory errors in the future.
- For the current precision it is not a disaster to not have it:

Hartmann, Trott 1507.03568 Correcting tree level conclusion for 1 loop neglected effects errors introduced added in quadrature, $C_i \sim 1$:

$$\begin{array}{lll} \text{Current data for:} & -0.02 \leq \left(\hat{C}_{\gamma\gamma}^{1,NP} + \frac{\hat{C}_i^{NP} f_i}{16 \pi^2} \right) \frac{\bar{v}_T^2}{\Lambda^2} \leq 0.02. \\ & \kappa_{\gamma} = 0.93^{+0.36}_{-0.17} & \text{ATLAS data - naive map to C corrected} \\ & \kappa_{\gamma} = 0.98^{+0.17}_{-0.16} & \text{CMS data - naive map to C corrected} \end{array} \qquad \begin{bmatrix} 52,7 \end{bmatrix} \% \\ & \begin{bmatrix} 52,7 \end{bmatrix} \% \end{array}$$

The future precision Higgs phenomenology program clearly needs it: $\kappa_{\gamma}^{proj:RunII} = 1 \pm 0.045$ - naive map to C (tree level) corrected[167, 21]% $\kappa_{\gamma}^{proj:HILHC} = 1 \pm 0.03$ [250, 31]% $\kappa_{\gamma}^{proj:TLEP} = 1 \pm 0.0145$ [513, 64]%

More slides.

NLO EFT - Step 2 Renormalize

How was this renormalization done?

Calculated in the unbroken phase of the theory, using the background field method.

G. 't Hooft, Acta Universitatis Wratislaviensis No.368, Vol. 1*, Wroclaw 1976, 345-369

B. S. DeWitt, Phys.Rev. 162 (1967) 1195-1239

L. Abbott, Acta Phys. Polon. B13 (1982) 33

A. Denner, G. Weiglein, and S. Dittmaier, Nucl. Phys. B440 (1995) 95–128, hep-ph/9410338.

M. B. Einhorn and J. Wudka, Phys.Rev. D39 (1989) 2758.

A. Denner, Fortsch. Phys. 41 (1993) 307-420, [arXiv:0709.1075].

 Background field method not necessary, but a nice trick, and allowed US to succeed in avoiding gauge dependent results.
 (Some competition did not use the background field method.)

"Cool stuff" Addendum

• Gauge fixing in the SMEFT subtle compared to the SM. Consider:

$$\begin{split} \mathcal{L}_{GF} &= -\frac{1}{2\,\xi_W} \sum_a \left[\partial_\mu W^{a,\mu} - g_2 \,\epsilon^{abc} \hat{W}_{b,\mu} W^\mu_c + i\,g_2 \,\frac{\xi}{2} \left(\hat{H}^\dagger_i \sigma^a_{ij} H_j - H^\dagger_i \sigma^a_{ij} \hat{H}_j \right) \right]^2, \\ &- \frac{1}{2\,\xi_B} \left[\partial_\mu B^\mu + i\,g_1 \,\frac{\xi}{2} \left(\hat{H}^\dagger_i H_i - H^\dagger_i \hat{H}_i \right) \right]^2. \end{split}$$

$$\mathcal{L}_{FP} = -\bar{u}^{lpha} \, rac{\delta G^{lpha}}{\delta heta^{eta}} \, u^{eta}.$$

Some operators in \mathcal{L}_6 then source ghosts!

The mismatch of the mass eigenstates in the SMEFT with the SM means gauge fixing in the former results in some interesting local contact operators

$$-\frac{c_w \, s_w}{\xi_B \, \xi_W} (\xi_B - \xi_W) \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) - \frac{C_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) \cdot \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\nu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\nu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\mu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_W + c_w^2 \xi_W)}{\xi_B \, \xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\mu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_W + c_w^2 \xi_W)}{\xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\mu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_W + c_w^2 \xi_W)}{\xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\mu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 \xi_W + c_w^2 \xi_W)}{\xi_W} \left(\partial^\mu A_\mu \, \partial^\mu \, Z_\mu\right) + \frac{c_{HWB} v^2 (s_w^2 - c_w^2) (s_w^2 + c_w^2 \xi_W} + \frac{c_W^2 (s_w^2 + c_w^2 \xi_W)}{\xi_W} \left(\partial^\mu A_\mu \, Z_\mu\right) + \frac{c_W^2 (s_w^2 - c_w^2) (s_w^2 + c_w^2 \xi_W} + \frac{c_W^2$$