

## Morphing

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## Introduction

### Plans for Run 2 and beyond

- Perform combined studies of **many (all) parameters** in the matrix element
- Take **all correlations** between different operators into account
- Use constraining power from **rate & shape information**
- Combine results from different channels

→ Challenge: **large parameter space**

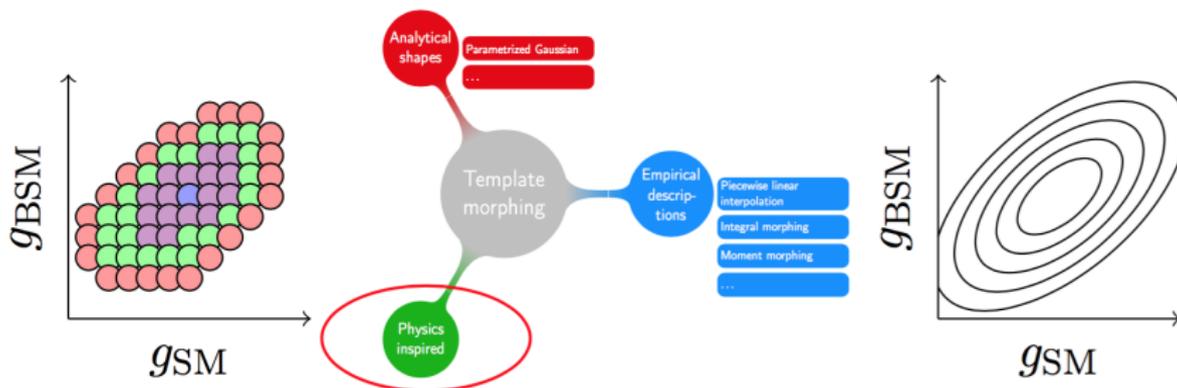
→ For properties necessary to build a signal model taking **all parameters** into account **simultaneously** & modelling all interference effects → **Morphing**

→ Test VBF sensitivity of the input parameters in a VBF  $H \rightarrow \mu\mu$  study

→ Look at optimisation of Basis sample set

# Introduction

- Measurements based on formulation of Likelihood  $L(x|\theta)$
- Predict observable distribution from a composite model:  
HEP model  $\times$  soft physics  $\times$  detector response  $\times$  reconstruction
- Provide smooth interpolation of the Likelihood



## Effective field theory framework implemented in Higgs Characterisation model

- Effective Lagrangian for the interaction of scalar and pseudo-scalar states with vector bosons

$$\mathcal{L}_0^V = \left\{ \begin{array}{l}
 c_\alpha \kappa_{SM} \left[ \frac{1}{2} \tilde{g}_{HZZ} Z_\mu Z^\mu + \tilde{g}_{HWW} W_\mu^+ W^{-\mu} \right] \\
 - \frac{1}{4} \left[ c_\alpha \kappa_{H\gamma\gamma} \tilde{g}_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} \tilde{g}_{A\gamma\gamma} A_{\mu\nu} \tilde{A}^{\mu\nu} \right] \\
 - \frac{1}{2} \left[ c_\alpha \kappa_{HZ\gamma} \tilde{g}_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} \tilde{g}_{AZ\gamma} Z_{\mu\nu} \tilde{A}^{\mu\nu} \right] \\
 - \frac{1}{4} \left[ c_\alpha \kappa_{Hgg} \tilde{g}_{Hgg} G_{\mu\nu}^a G^{a,\mu\nu} + s_\alpha \kappa_{Agg} \tilde{g}_{Agg} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \right] \\
 - \frac{1}{4} \frac{1}{\Lambda} \left[ c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] \\
 - \frac{1}{2} \frac{1}{\Lambda} \left[ c_\alpha \kappa_{HWW} W_{\mu\nu}^+ W^{-\mu\nu} + s_\alpha \kappa_{AWW} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu} \right] \\
 - \frac{1}{\Lambda} c_\alpha \left[ \kappa_{H\partial\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\partial Z} Z_\nu \partial_\mu Z^{\mu\nu} + \kappa_{H\partial W} (W_\nu^+ \partial_\mu W^{-\mu\nu} + h.c.) \right] \end{array} \right\} \mathcal{X}_0$$

Used in Run 1  
 Plan Run 2

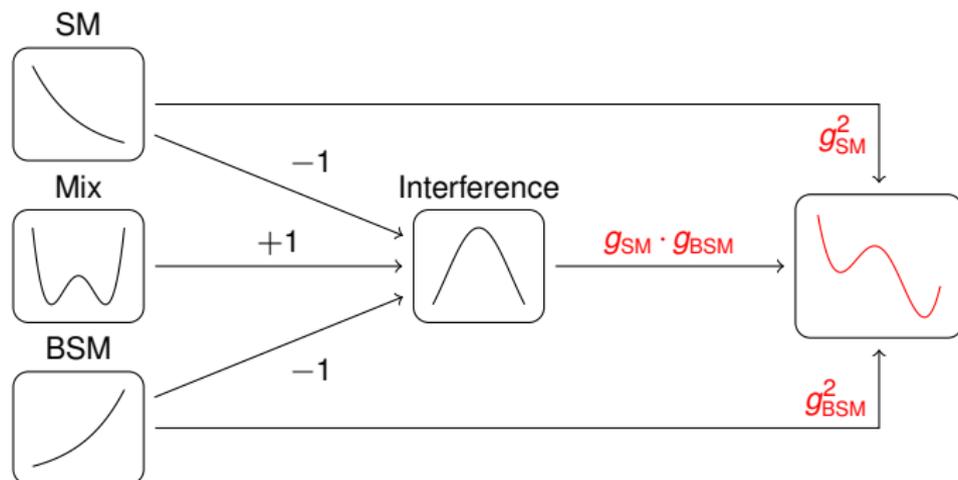
- Implemented in MADGRAPH5\_AMC@NLO
- $\Lambda = 1 \text{ TeV}$ ,  $\cos \alpha = \frac{1}{\sqrt{2}}$  fixed
- Define full coupling parameter as  $g_x$  (e.g.  $g_{AWW} = s_\alpha \kappa_{AWW} / \Lambda$ )

## Signal model construction: Morphing

- Morphing function** for an observable  $T_{out}$  at any coupling point  $\vec{g}_{target}$  constructed from weighted sum of input samples  $T_{in}$  at fixed coupling points  $\vec{g}_i$

$$T_{out}(\vec{g}_{target}) = \sum_{i=1}^{N_{input}} w_i(\vec{g}_{target}; \vec{g}_i) \cdot T_{in}(\vec{g}_i)$$

e.g.  $T = \Delta\phi_{jj}$



## Example for 2 free parameters in one vertex

- Process with **two parameters** applied in **one vertex**:  $g_{\text{SM}}$  and  $g_{\text{BSM}}$
- Matrix element can be **factorized**:

$$\mathcal{M}(g_{\text{SM}}, g_{\text{BSM}}) = g_{\text{SM}} \mathcal{O}_{\text{SM}} + g_{\text{BSM}} \mathcal{O}_{\text{BSM}}$$

$$|\mathcal{M}(g_{\text{SM}}, g_{\text{BSM}})|^2 = g_{\text{SM}}^2 |\mathcal{O}_{\text{SM}}|^2 + g_{\text{BSM}}^2 |\mathcal{O}_{\text{BSM}}|^2 + 2g_{\text{SM}} g_{\text{BSM}} \mathcal{R}(\mathcal{O}_{\text{SM}}^* \mathcal{O}_{\text{BSM}})$$

- Distribution** of a kinematic observable **proportional to the matrix element squared**

$$T(g_{\text{SM}}, g_{\text{BSM}}) \propto |\mathcal{M}(g_{\text{SM}}, g_{\text{BSM}})|^2$$

- 3 generated distributions** needed to obtain distribution with arbitrary parameters
- E.g. generate MC events for  $T(1, 0)$ ,  $T(0, 1)$ ,  $T(1, 1)$

$$T_{in}(1, 0) \propto |\mathcal{O}_{\text{SM}}|^2$$

$$T_{in}(0, 1) \propto |\mathcal{O}_{\text{BSM}}|^2$$

$$T_{in}(1, 1) \propto |\mathcal{O}_{\text{SM}}|^2 + |\mathcal{O}_{\text{BSM}}|^2 + 2\mathcal{R}(\mathcal{O}_{\text{SM}}^* \mathcal{O}_{\text{BSM}})$$

- Distribution with **arbitrary parameters** ( $g_{\text{SM}}, g_{\text{BSM}}$ )

$$T_{out}(g_{\text{SM}}, g_{\text{BSM}}) = \underbrace{(g_{\text{SM}}^2 - g_{\text{SM}} g_{\text{BSM}})}_{=w_1} T_{in}(1, 0) + \underbrace{(g_{\text{BSM}}^2 - g_{\text{SM}} g_{\text{BSM}})}_{=w_2} T_{in}(0, 1) + \underbrace{g_{\text{SM}} g_{\text{BSM}}}_{=w_3} T_{in}(1, 1)$$

## Example for 2 free parameters in one vertex: generalisation of input parameter

- Generalize to **arbitrary input parameters**  $\vec{g}_i$  used to generate input distributions  $T_{in}(\vec{g}_i)$

$$T_{in}(\mathbf{g}_{SM,i}, \mathbf{g}_{BSM,i}) \propto \mathbf{g}_{SM,i}^2 |O_{SM}|^2 + \mathbf{g}_{BSM,i}^2 |O_{BSM}|^2 + 2\mathbf{g}_{SM,i}\mathbf{g}_{BSM,i} \mathcal{R}(O_{SM}^* O_{BSM}),$$

$$i = 1, \dots, 3$$

- Ansatz for **output distribution**

$$\begin{aligned} T_{out}(\mathbf{g}_{SM}, \mathbf{g}_{BSM}) &= \underbrace{(a_{11}\mathbf{g}_{SM}^2 + a_{12}\mathbf{g}_{BSM}^2 + a_{13}\mathbf{g}_{SM}\mathbf{g}_{BSM})}_{w_1} T_{in}(\mathbf{g}_{SM,1}, \mathbf{g}_{BSM,1}) \\ &+ \underbrace{(a_{21}\mathbf{g}_{SM}^2 + a_{22}\mathbf{g}_{BSM}^2 + a_{23}\mathbf{g}_{SM}\mathbf{g}_{BSM})}_{w_2} T_{in}(\mathbf{g}_{SM,2}, \mathbf{g}_{BSM,2}) \\ &+ \underbrace{(a_{31}\mathbf{g}_{SM}^2 + a_{32}\mathbf{g}_{BSM}^2 + a_{33}\mathbf{g}_{SM}\mathbf{g}_{BSM})}_{w_3} T_{in}(\mathbf{g}_{SM,3}, \mathbf{g}_{BSM,3}) \end{aligned}$$

## Example for 2 operators in one vertex

- $T_{out}$  should be equal to  $T_{in}$  for  $\vec{g}_{target} = \vec{g}_i$

$$1 = a_{11}g_{SM,1}^2 + a_{12}g_{BSM,1}^2 + a_{13}g_{SM,1}g_{BSM,1}$$

$$0 = a_{21}g_{SM,1}^2 + a_{22}g_{BSM,1}^2 + a_{23}g_{SM,1}g_{BSM,1}$$

...

- Constraints in **matrix form**

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} g_{SM,1}^2 & g_{SM,2}^2 & g_{SM,3}^2 \\ g_{BSM,1}^2 & g_{BSM,2}^2 & g_{BSM,3}^2 \\ g_{SM,1}g_{BSM,1} & g_{SM,2}g_{BSM,2} & g_{SM,3}g_{BSM,3} \end{pmatrix} = \mathbb{1}$$

$$\Leftrightarrow A \cdot G = \mathbb{1}$$

- **Definite solution**  $A = G^{-1}$  requires the samples to have parameters such that  $\det(G) \neq 0$
  - Very flexible in choosing the parameters for the input distributions
- Can be chosen to **reduce statistical uncertainty** in considered parameter space

## General morphing and number of input distributions

- More complicated when processes share amplitudes between **production and decay**, for example VBF  $H \rightarrow VV$
- General matrix element squared at **LO** & assuming **narrow-width-approximation** (ignoring the effect on the total width)  
 $\Rightarrow$  **polynomials** of 2nd order in production and 2nd order in decay

$$T(\vec{g}) \propto |\mathcal{M}(\vec{g})|^2 = \underbrace{\left( \sum_{i=1}^{n_p+n_s} g_i O_i \right)^2}_{\text{production vertex}} \cdot \underbrace{\left( \sum_{j=1}^{n_d+n_s} g_j O_j \right)^2}_{\text{decay vertex}}$$

with number of parameters in **production vertex** ( $n_p$ ), **decay vertex** ( $n_d$ ) and **shared in vertices** ( $n_s$ )

- **Number of required input distributions** equal to number of different terms in expanded matrix element squared  
 $\rightarrow$  dependent on process and considered parameters  
 $\rightarrow N_{input}$  function of  $n_p$ ,  $n_d$  and  $n_s$
- Example: 13 free parameters for VBF  $H \rightarrow ZZ$  process:
  - $n_p = 4$  operators in production:  $\mathcal{G}_{HWW}, \mathcal{G}_{AWW}, \mathcal{G}_{H\partial W}, \mathcal{G}_{H\partial W}^*$
  - $n_s = 9$  operators in both vertices:  $\mathcal{G}_{SM}, \mathcal{G}_{HZZ}, \mathcal{G}_{AZZ}, \mathcal{G}_{H\partial Z}, \mathcal{G}_{H\gamma\gamma}, \mathcal{G}_{A\gamma\gamma}, \mathcal{G}_{HZ\gamma}, \mathcal{G}_{AZ\gamma}, \mathcal{G}_{H\partial\gamma}$
  - $n_d = 0$ , no operators only in decay

$\rightarrow$  **1605 samples** needed!

$\rightarrow$  Reduction of considered operators favourable  $\rightarrow$  see VBF study

## Generality of the method

- Morphing only requires that any differential cross section can be expressed as **polynomial in BSM couplings**
- Method can be used on **any generator** that allows one to vary input couplings
- Works on **truth** and **reco-level** distributions
- **Independent of physics process**
- Works on distributions and cross sections

## Comparison of methods

- **Needed:** MC samples covering wide range of values for coupling parameters
- Run 1 HWW and HZZ analyses: **Matrix Element Reweighting**  
(Event by event matrix element reweighting of one source MC sample with large statistics)

$$w(\vec{g}_{target}) = w(\vec{g}_i) \frac{|\mathcal{M}(\vec{g}_{target})|^2}{|\mathcal{M}(\vec{g}_{source})|^2}$$

### ME Reweighting

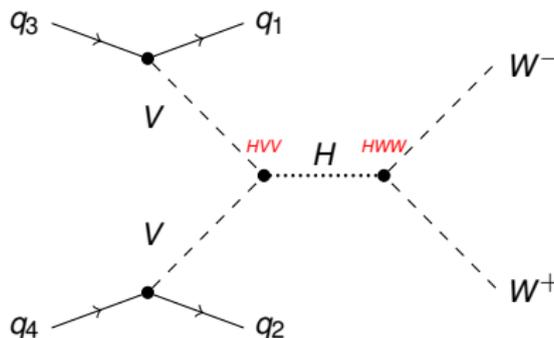
For every configuration point

- rerun analysis
  - write event weights to disk
  - additional interpolation
- **Morphing function:** Instead of “matrix element reweighting” use morphing to obtain a distribution with arbitrary coupling parameters
  - Can be applied directly and without change to
    - Cross sections
    - Distributions (before or after detector simulation)
    - MC events
  - **Exact continuous analytical description of rates and shapes**
  - Even possible to **fit** coupling parameters to data & derive limits

### Morphing

- only calculates linear sums of coefficients
- all other inputs are pre-computed once
- computationally fast & convenient tool

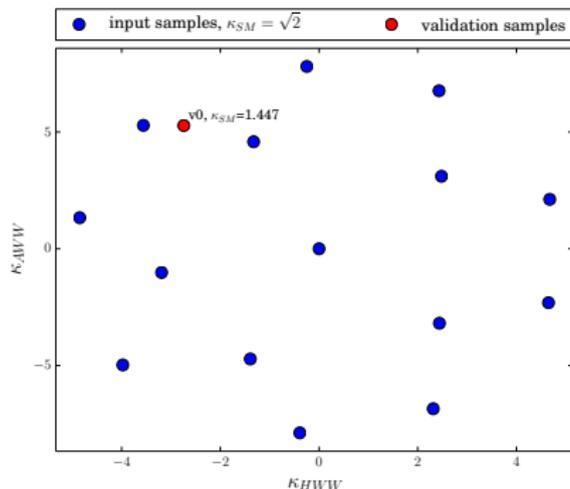
## VBF H→WW example



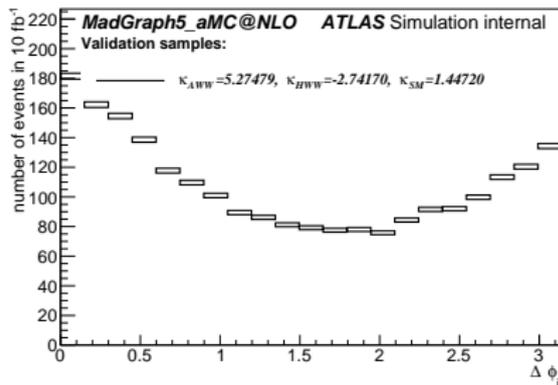
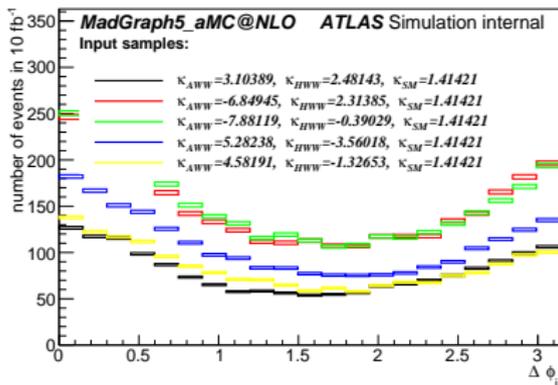
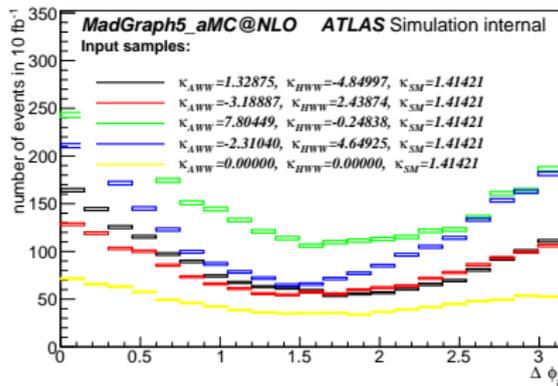
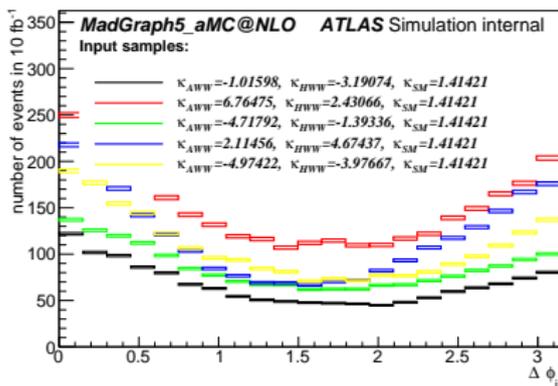
- VBF H→WW process with **SM** ( $g_{SM}$ ) and **2 BSM** operators ( $g_{HWW}$ ,  $g_{Aww}$ )
- **15 samples** with different parameters needed
- 50k events generated for each sample
- Kinematic observable used:  $\Delta\phi_{jj}$
- Only signal considered

## VBF H→WW example: Samples

- Expect only **small deviations from SM**
- $g_{SM} = 1$  for all input samples ( $\Lambda = 1 \text{ TeV}$ ,  $\cos \alpha = \frac{1}{\sqrt{2}}$ )
- BSM parameter limits chosen such that  $\sigma_{\text{pure BSM}} \sim \sigma_{SM}$
- all other BSM parameters set to 0
- Scatter plot shows **blue** points in  $(g_{AWW}, g_{HWW})$  space used to generate **input samples**
- A **validation sample** is produced at the **red** point for cross-check
  - **morphing** can reproduce the distribution there
  - **fit** can reproduce the parameters from the validation sample



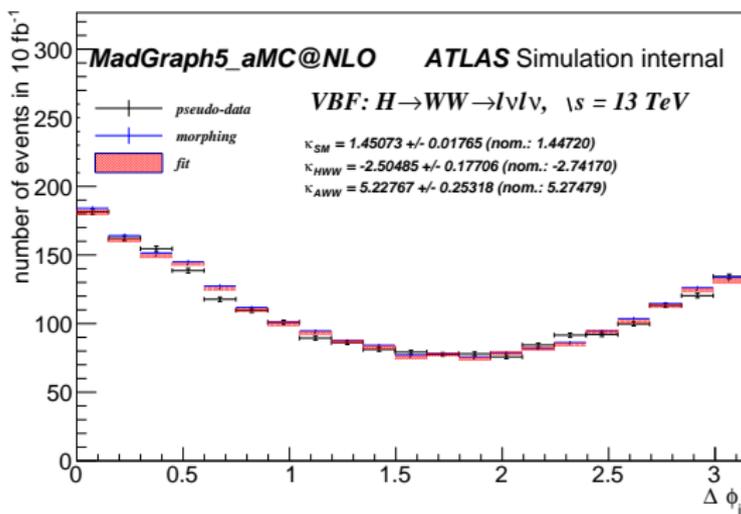
# VBF H $\rightarrow$ WW example: Input and validation distributions



## VBF H $\rightarrow$ WW example: Morphing and fit to validation sample

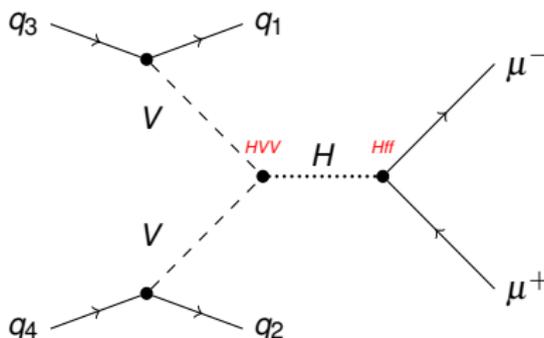
- **Morphing** and **fit** to validation distr. (pseudo-data)
- Validation and morphed distribution stat. independent
  - Agreement in morphing within MC stat. uncertainty
  - Fit results match nominal values within fit uncertainties
- **Sensitivity** on parameters shown in fit uncert.
- **Correlations** vary at different parameter point

	$\kappa_{SM}$	$\kappa_{HWW}$	$\kappa_{Aww}$
$\kappa_{SM}$	1.00	0.20	-0.95
$\kappa_{HWW}$	0.20	1.00	0.09
$\kappa_{Aww}$	-0.95	0.09	1.00



## Study of the VBF vertex

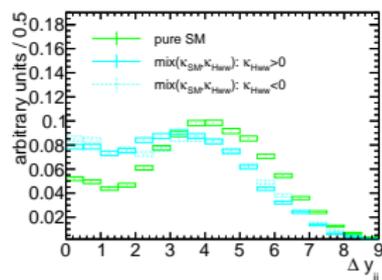
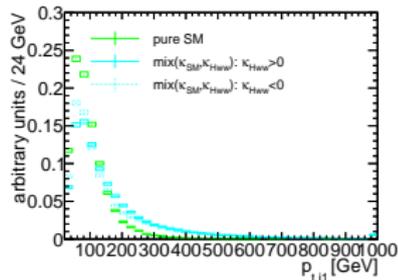
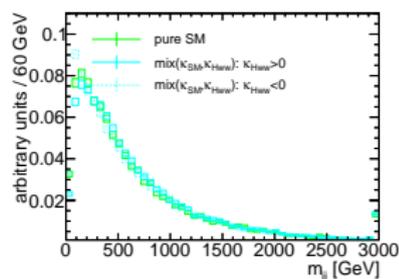
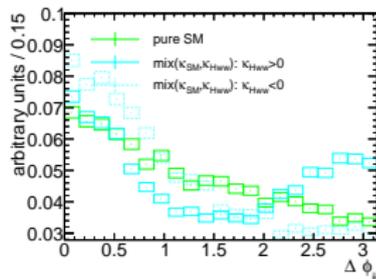
- Many operators enter VBF
- Goal: **neglect operators** without losing generality to **minimize number of needed samples**



- Technical choice of  $H \rightarrow \mu\mu$  decay: no crossover between production and decay
- Full set of 13 VBF prod. op. leads to **91 samples** to produce: still manageable
- Generator level, signal only samples used with 30k events each
- Setup **fit to SM input sample**
  - Learn **correlations** between operators
  - Explore **sensitivity**
  - uses observables:  $\Delta\phi_{jj}$ ,  $p_T^{j1}$ ,  $m_{jj}$ ,  $\Delta\eta_{jj}$
  - Understand which operators have negligible influence on VBF

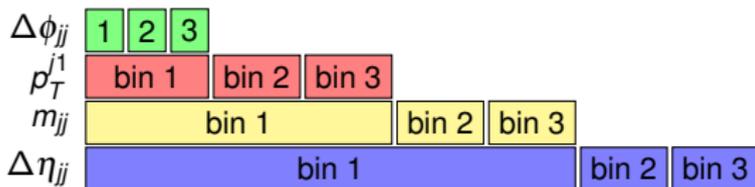
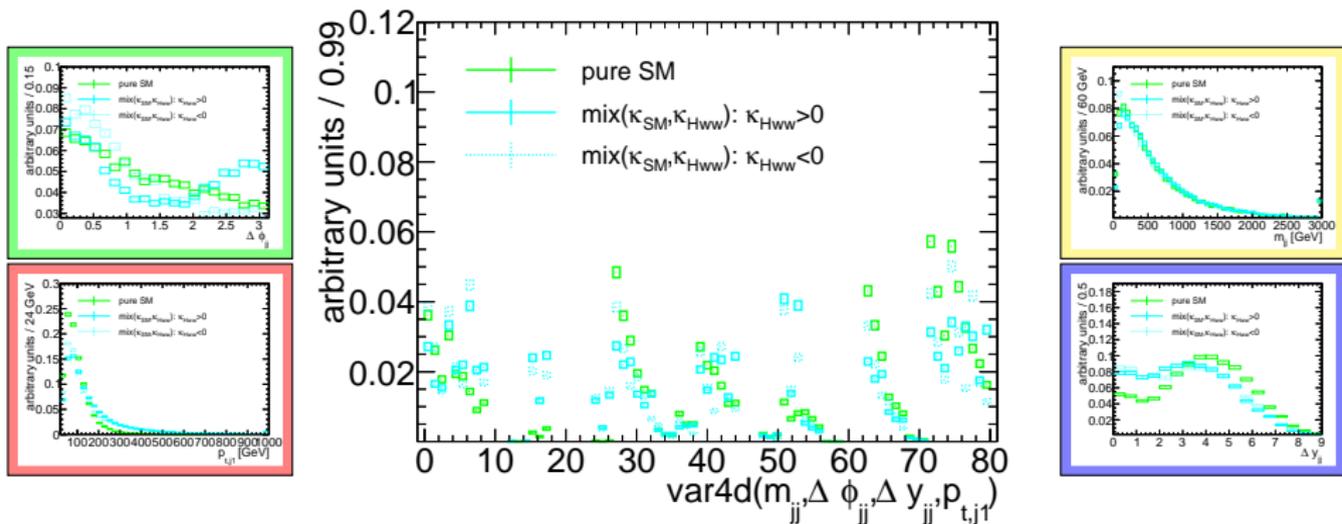
# Cross sections and shapes depending on VBF couplings

- BSM coupling parameters chosen such that one-operator pure BSM samples correspond to SM VBF cross section

 $\kappa_{HZZ}$  $\kappa_{AAZ}$  $\kappa_{HWW}$  $\kappa_{AWW}$  $\kappa_{HZ\gamma}$  $\kappa_{AZ\gamma}$  $\kappa_{H\gamma\gamma}$  $\kappa_{A\gamma\gamma}$  $\kappa_{H\partial Z}$  $\kappa_{H\partial\gamma}$  $\kappa_{H\partial W}$  $\kappa_{H\partial W^*}$ 

- Production of 91 samples ( $\Lambda = 1000 \text{ GeV}$ ,  $\cos \alpha = 1/\sqrt{2}$ ,  $\kappa_{SM} = \sqrt{2}$ )
  - 1 pure SM sample
  - 24 samples with SM + 1 BSM ( $\pm \kappa_{BSM}$ )
  - 66 samples with SM + 2 BSM ( $+ \kappa_{BSM}$ )

# Combined distribution of four observables



## Fit result sensitivity on VBF couplings

parameter	post-fit value	+	-
$\Lambda$	1000.		
$\cos \alpha$	0.71		
$\kappa_{H\ell\ell}$	1.41		
$\kappa_{A\gamma\gamma}$	0	+219	-408
$\kappa_{Aww}$	0	+1.2	-2.6
$\kappa_{AZ\gamma}$	0	+442	-399
$\kappa_{AZz}$	0	+7.7	-0
$\kappa_{H\gamma\gamma}$	0	+115	-338
$\kappa_{H\partial\gamma}$	0	+0.7	-0.6
$\kappa_{H\partial w\ell}$	0	+0.4	-1.4
$\kappa_{H\partial wR}$	0	+0.5	-0.6
$\kappa_{H\partial z}$	0	+1.2	-0.5
$\kappa_{Hww}$	0	+2.8	-0
$\kappa_{Hz\gamma}$	0	+21	-49
$\kappa_{Hzz}$	0	+9.8	0
$\kappa_{SM}$	1.41	+0.22	-0.11

- Simultaneously to all parameters to SM distribution
- Assuming 8% VBF cross section uncertainty
- **Fit uncertainties** give information on **sensitivity**
- Sensitivity on  $\gamma\gamma$  and  $Z\gamma$  couplings small
- Close to the SM, 4 couplings ( $H\gamma\gamma$ ,  $A\gamma\gamma$ ,  $HZ\gamma$ ,  $AZ\gamma$ ) can be ignored for VBF without loss of generality
- Other measurements will limit this operator to far smaller values
- To be tested that the  $\gamma\gamma$  and  $Z\gamma$  operators don't influence any other VBF observable

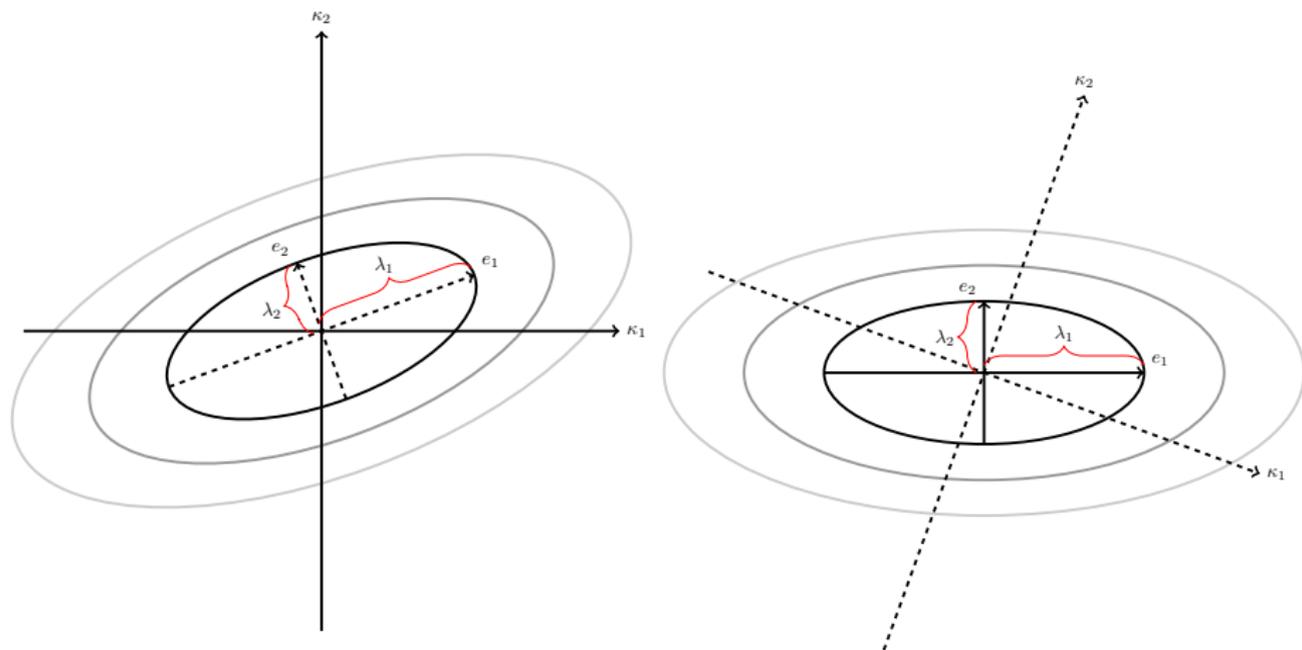
## Fit result correlations of VBF couplings

	$\kappa_{A\gamma\gamma}$	$\kappa_{Aww}$	$\kappa_{AZ\gamma}$	$\kappa_{AZZ}$	$\kappa_{H\gamma\gamma}$	$\kappa_{H\partial\gamma}$	$\kappa_{H\partial Wl}$	$\kappa_{H\partial WR}$	$\kappa_{H\partial Z}$	$\kappa_{Hww}$	$\kappa_{HZ\gamma}$	$\kappa_{HZZ}$	$\kappa_{SM}$
$k_{Aaa}$	1.000	0.125	-0.092	0.027	0.117	-0.348	-0.059	0.131	-0.116	-0.178	0.064	0.072	-0.173
$k_{Aww}$	0.125	1.000	0.327	-0.303	-0.352	0.179	-0.104	-0.468	0.689	0.313	0.087	-0.263	-0.143
$k_{Aza}$	-0.092	0.327	1.000	-0.088	0.173	-0.048	-0.055	0.133	-0.001	-0.026	0.208	-0.090	-0.056
$k_{Azz}$	0.027	-0.303	-0.088	1.000	-0.262	0.230	0.405	0.124	-0.223	0.341	0.203	-0.480	0.111
$k_{Haa}$	0.117	-0.352	0.173	-0.262	1.000	-0.127	-0.221	0.159	-0.264	-0.464	-0.010	0.256	-0.008
$k_{Hda}$	-0.348	0.179	-0.048	0.230	-0.127	1.000	-0.116	-0.352	0.343	0.264	0.279	-0.338	0.202
$k_{Hdwl}$	-0.059	-0.104	-0.055	0.405	-0.221	-0.116	1.000	0.020	-0.029	0.259	-0.132	-0.194	0.012
$k_{Hdwr}$	0.131	-0.468	0.133	0.124	0.159	-0.352	0.020	1.000	-0.922	-0.151	-0.290	0.242	0.029
$k_{Hdz}$	-0.116	0.689	-0.001	-0.223	-0.264	0.343	-0.029	-0.922	1.000	0.246	0.246	-0.227	0.091
$k_{Hww}$	-0.178	0.313	-0.026	0.341	-0.464	0.264	0.259	-0.151	0.246	1.000	-0.121	-0.754	-0.041
$k_{Hza}$	0.064	0.087	0.208	0.203	-0.010	0.279	-0.132	-0.290	0.246	-0.121	1.000	-0.409	0.231
$k_{Hzz}$	0.072	-0.263	-0.090	-0.480	0.256	-0.338	-0.194	0.242	-0.227	-0.754	-0.409	1.000	0.010
$k_{SM}$	-0.173	-0.143	-0.056	0.111	-0.008	0.202	0.012	0.029	0.091	-0.041	0.231	0.010	1.000

- Some **large correlations** present **close to the standard model**
- Can likely neglect 1-2 more parameters after rotating into Eigen basis of the covariance matrix
- Correlations may change away from SM

## diagonalisation - move to Eigen basis

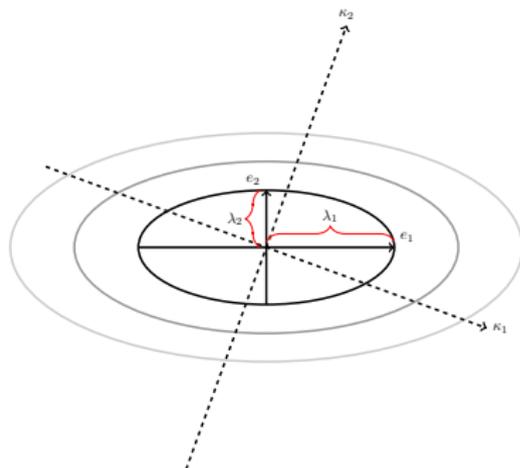
- Larger eigenvalues correspond to larger variation in the data, so less sensitivity to this parameter
- A re-parametrisation in eigenvectors is performed



## diagonalisation - move to Eigen basis

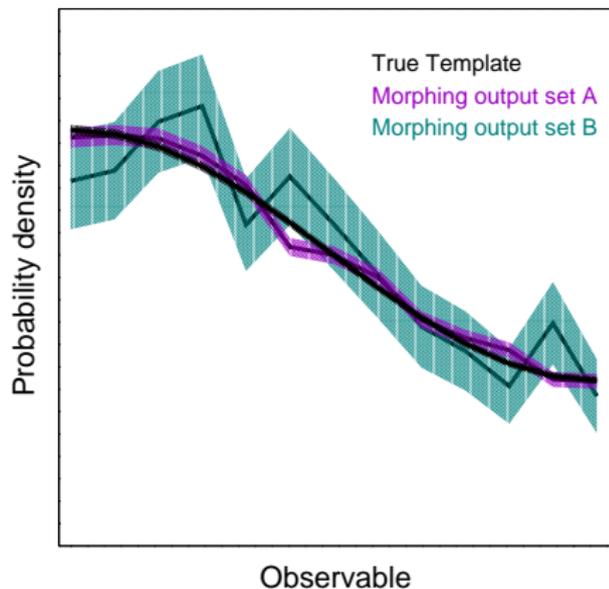
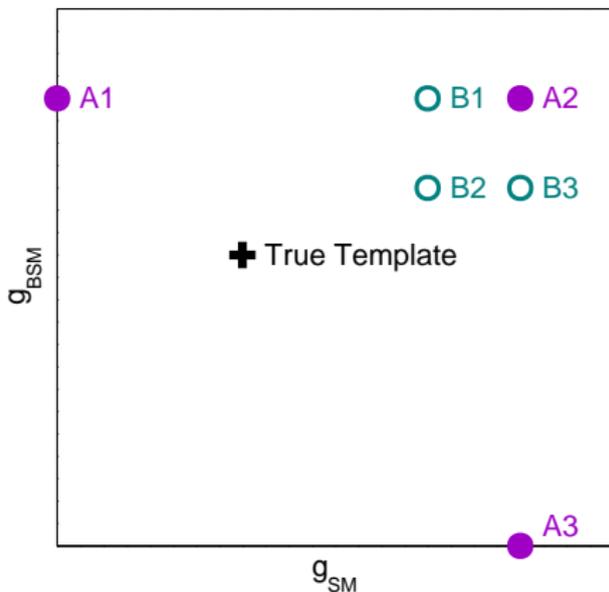
- We see that the results match with fit sensitivity result
- A re-parametrisation in eigenvectors is performed and then fit on this direction
- Fit with all (new) parameters floating where we expect a near 1 correlation matrix post fit. If the correlations are not small then this first order is not good enough and the re-parametrisation is not useful
- A second fit will be performed with a number of parameters floating (4-6) and the others fixed to the SM to see if we can take out some directions.

EV (0.0234272) = -0.000870072\*kAaa-  
 0.00137208\*kAww+0.00965106\*kAza-0.0363483\*kAzz-  
 0.615691\*kHaa+0.695891\*kHda+0.172168\*kHdwl-  
 0.012292\*kHdwR-0.291425\*kHdz-0.128922\*kHww-  
 0.0486755\*kHza-0.0381144\*kHzz+0.00886287\*kSM



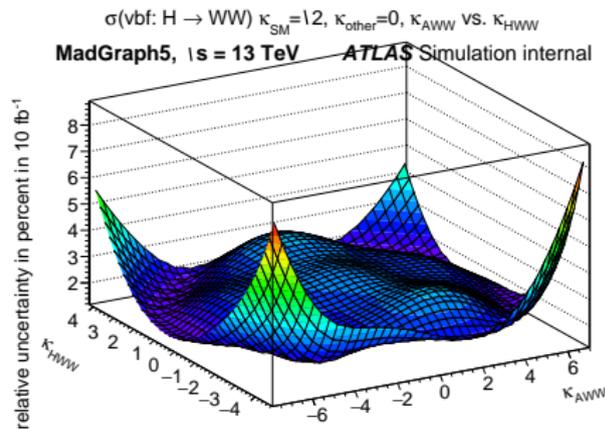
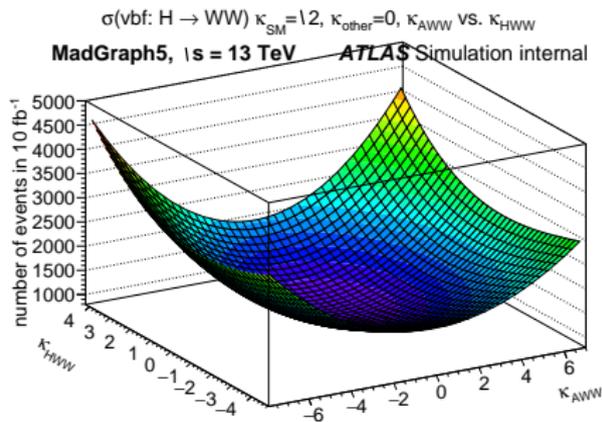
## Choice of input parameters

- Aim to **generalise morphing** to have arbitrary  $g_i$
- Can be chosen to **reduce statistical uncertainty**

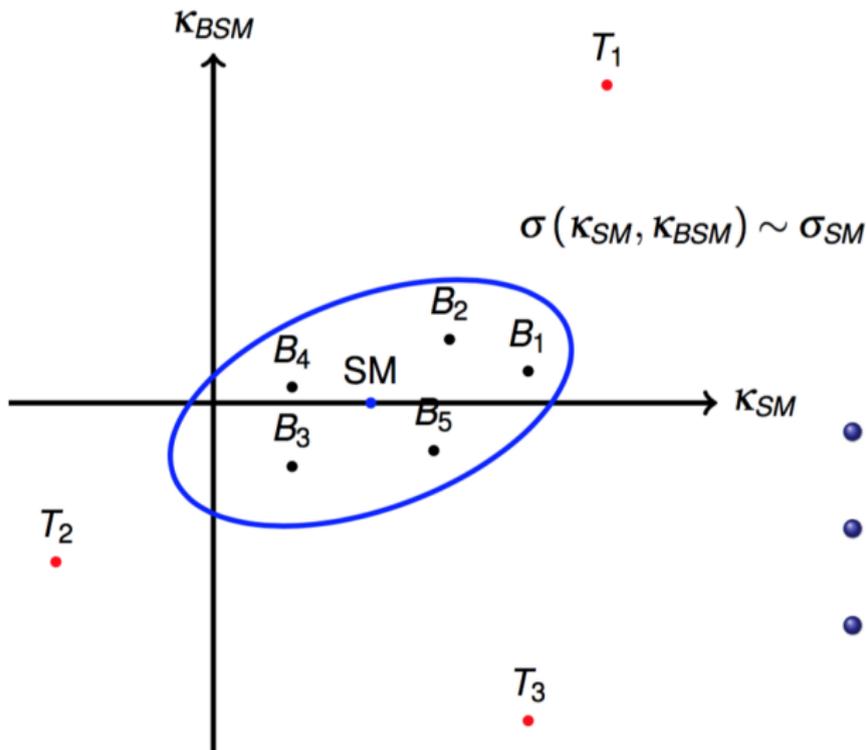


## VBF H $\rightarrow$ WW example: Rel. uncertainty on number of expected events

- Dependence of **stat. uncertainty** propagated in morphing function on generated input parameter grid
- Distribution of samples in parameter space **reduces** stat. uncertainty

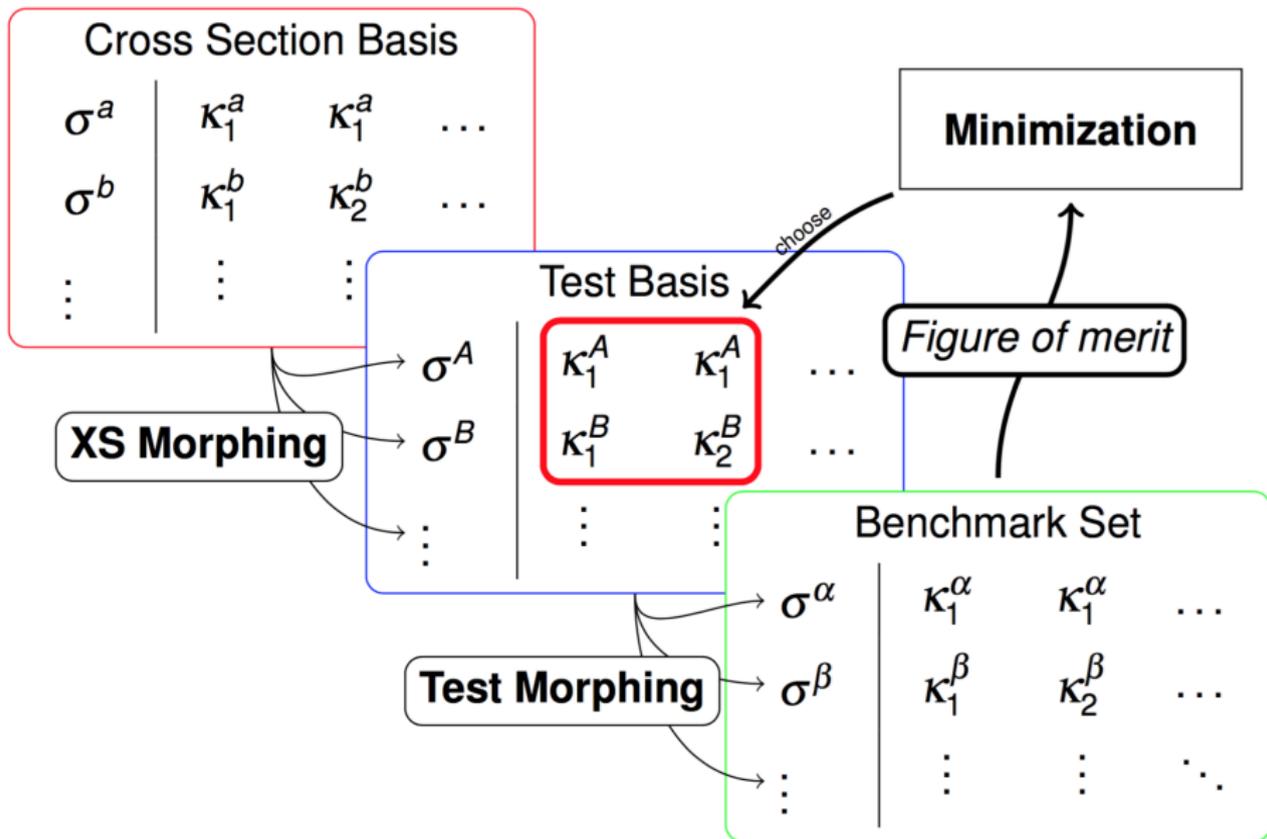


# Procedure

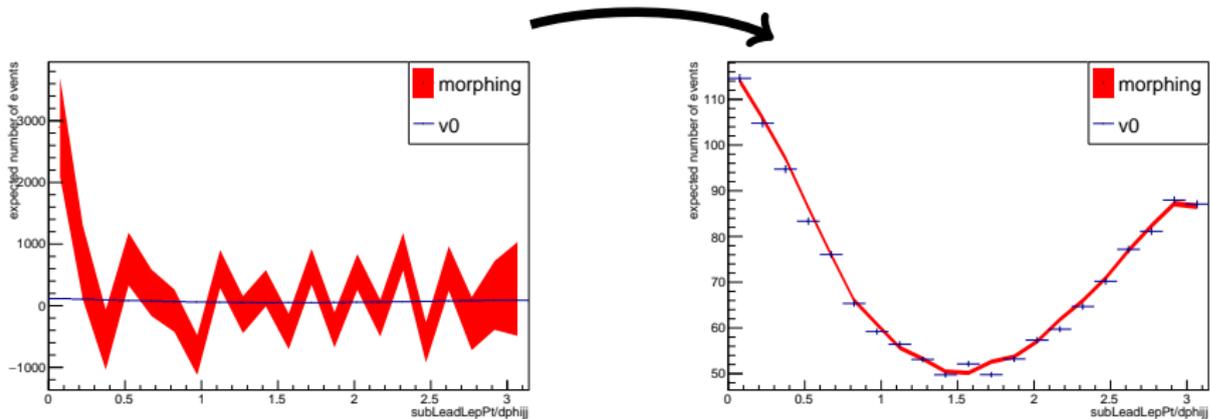


- First **define parameter space** which is interesting to morph to
- Specify a number of **benchmark points** in this parameter space
- Find **optimal input parameter basis**

# Procedure



# Improvement



## Summary

- Plan for Run 2: **Higgs coupling and properties measurements**
- Combine **rate and shape information** within effective Lagrangian framework
- First application: detailed and complete study of **VBF production**
- First attempt to find the **optimal fixed sample basis** using iterative numerical minimisation

# Backup

## Number of input distributions

$$\begin{aligned} N_{input} = & \frac{n_p(n_p+1)}{2} \cdot \frac{n_d(n_d+1)}{2} + \binom{4+n_s-1}{4} \\ & + \left( n_p \cdot n_s + \frac{n_s(n_s+1)}{2} \right) \cdot \frac{n_d(n_d+1)}{2} \\ & + \left( n_d \cdot n_s + \frac{n_s(n_s+1)}{2} \right) \cdot \frac{n_p(n_p+1)}{2} \\ & + \frac{n_s(n_s+1)}{2} \cdot n_p \cdot n_d + (n_p + n_d) \binom{3+n_s-1}{3} \end{aligned}$$

with number of parameters in **production vertex** ( $n_p$ ), **decay vertex** ( $n_d$ ) and **shared in vertices** ( $n_s$ )

## Propagation of statistical uncertainties

[noframenumbering]

- Morphing function for a bin in distribution

$$T_{out}^{bin}(\vec{g}_{target}) = \sum_i w_i(\vec{g}_{target}; \vec{g}_i) T_{in}^{bin}(\vec{g}_i)$$

- For one input distribution, the bin content is calculated as follows

$$T_{in}^{bin}(\vec{g}_i) = N_{MC,in}^{bin}(\vec{g}_i) \cdot \sigma_{in}(\vec{g}_i) \mathcal{L} / N_{MC,in}$$

- The uncertainty on that bin is  $\sqrt{N_{MC,in}^{bin}(\vec{g}_i)}$
- The propagated statistical uncertainty is

$$\Delta T_{out}^{bin} = \sqrt{\sum_i w_i^2(\vec{g}_{target}; \vec{g}_i) N_{MC,in}^{bin}(\vec{g}_i) \cdot (\sigma_{in}(\vec{g}_i) \mathcal{L} / N_{MC,in})^2}$$

- Highly **dependent** on
  - **input parameters**  $\vec{g}_i$
  - desired **target parameters**  $\vec{g}_{target}$

