QCD matrix elements and truncated showers

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SM discoveries with early LHC data

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[1] In collaboration with F. Krauss, S. Schumann, F. Siegert; see arXiv:0903.1219 [hep-ph]

Combining ME & PS

Why do we combine ME and PS ... ?

Because accelerated QCD charges radiate !

Well-defined schemes to account for the bulk of radiation effects in certain regions of phase space exist (DGLAP, BFKL, ...)
Shower generators implement these schemes to simulate QCD events

But this is not the end of the story !

All resummation calculations are, in the end, approximate
If we are interested in a particular QCD final state, however,

We should correct this approximation with a matrix element without spoiling the inclusive picture of the event

Designing the method

The starting point: QCD evolution

$$\frac{\partial}{\partial \log(t/\mu^2)} \frac{g_a(z,t)}{\Delta_a(\mu^2,t)} = \frac{1}{\Delta_a(\mu^2,t)} \int_z^{\zeta_{\max}} \frac{\mathrm{d}\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}(\zeta,t)\, g_b(z/\zeta,t)$$

Defines backward no-branching probability for showers

$$\mathcal{P}^{(B)}_{\mathrm{no},\,a}(z,t,t') = \frac{\Delta_a(\mu^2,t')\,g_a(z,t)}{\Delta_a(\mu^2,t)\,g_a(z,t')} = \exp\left\{-\int_t^{t'} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int_z^{\zeta_{\max}} \frac{\mathrm{d}\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}(\zeta,\bar{t})\, \frac{g_b(z/\zeta,\bar{t})}{g_a(z,\bar{t})}\right\}$$

Requirements for ME-PS merging

- Above equation for shower evolution is preserved
- Hardest emissions are described by matrix elements through

$$\mathcal{K}_{ab}(z,t) \to \frac{1}{\sigma_a^{(N)}(\Phi_N)} \frac{\mathrm{d}^2\sigma_b^{(N+1)}(z,t;\Phi_N)}{\mathrm{d}\log(t/\mu^2)\,\mathrm{d}z}$$

How does it work ?

Slicing the phase space

$$\mathcal{K}_{ab}^{\rm ME}(\xi,\bar{t}) = \mathcal{K}_{ab}(\xi,\bar{t})\;\Theta[Q_{ab}(\xi,\bar{t}) - Q_{\rm cut}\,] \qquad \mathcal{K}_{ab}^{\rm PS}(\xi,\bar{t}) = \mathcal{K}_{ab}(\xi,\bar{t})\;\Theta[Q_{\rm cut} - Q_{ab}(\xi,\bar{t})\,]$$

Patching it up

Let us veto the shower

$$\tilde{\mathcal{P}}^{(B)\,\rm PS}_{{\rm no},\,a}(z,t,t') = \frac{\Delta_a^{\rm PS}(\mu^2,t')\,\tilde{g}_a(z,t)}{\Delta_a^{\rm PS}(\mu^2,t)\,\tilde{g}_a(z,t')} = \exp\left\{ -\int_t^{t'} \frac{{\rm d}\bar{t}}{\bar{t}} \int_z^{\zeta_{\rm max}} \frac{{\rm d}\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}^{\rm PS}(\zeta,\bar{t})\, \frac{\tilde{g}_b(z/\zeta,\bar{t})}{\tilde{g}_a(z,\bar{t})} \right\}$$

At first glance we obtain a different evolution ...

... but this is easily corrected by adding the missing part

$$\mathcal{P}^{(B)}_{{\rm no},\,a}(z,t,t') = \frac{\Delta^{\rm ME}(\mu^2,t')}{\Delta^{\rm ME}(\mu^2,t)}\; \mathcal{P}^{(B)\,\rm PS}_{{\rm no},\,a}(z,t,t') \quad \text{where} \quad \mathcal{P}^{(B)\,\rm PS}_{{\rm no},\,a}(z,t,t') = \frac{\Delta_a^{\rm PS}(\mu^2,t')\,g_a(z,t)}{\Delta_a^{\rm PS}(\mu^2,t)\,g_a(z,t')}$$

Note

- Method is independent of the definition of Q
- Phase space is by definition completely filled

Truncated showers

Why is a standard shower not enough ?

Assume we have a ME, predefining a branching at t with hard scale t'.
Filling the remaining phase space means computing

$$\mathcal{P}^{(B)\,\mathrm{PS}}_{\mathrm{no},\,a}(z,t,t') = \frac{\Delta^{\mathrm{PS}}_a(\mu^2,t')\,g_a(z,t)}{\Delta^{\mathrm{PS}}_a(\mu^2,t)\,g_a(z,t')}$$

$\Rightarrow$ We need a shower evolving between t' and t, i.e. a “truncated” one.

What is the catch of it ?

The ME branching at t sets the evolution-, splitting and angular variable of a predefined node to be inserted later.
After any emission above t, this node must be reconstructed.

Defining the phase space separation

Now we need a definition of Q

$$Q^2_{ij} \;=\; 2\,p_i p_j\;\min_{k\neq i,j}\;\frac{2}{C^k_{i,j}+C^k_{j,i}}\;;\quad C^k_{i,j}\;=\;\begin{cases}\dfrac{p_i p_k}{(p_i+p_k)p_j}-\dfrac{m_i^2}{2\,p_i p_j} & \text{if } j=g\\ 1 & \text{else}\end{cases}$$

Initial state splittings: $C^k_{a,j}\to C^k_{(aj),j}$

Make sure this is sensible

- Soft limit

$$\frac{1}{Q^2_{ij}}\to\frac{1}{2\,\lambda^2}\;\frac{1}{2\,p_i\,q}\;\max_{k\neq i,j}\left[\frac{p_i p_k}{(p_i+p_k)\,q}-\frac{m_i^2}{2\,p_i q}\right]$$

- (Quasi-)Collinear limit

$$\frac{1}{Q^2_{ij}}\to\frac{1}{2\,\lambda^2}\;\frac{1}{\left|p^2_{ij}-m_i^2-m_j^2\right|}\;\left(\tilde{C}_{i,j}+\tilde{C}_{j,i}\right)\;;\quad\tilde{C}_{i,j}=\begin{cases}\dfrac{z}{1-z}-\dfrac{m_i^2}{2\,p_i p_j} & \text{if } j=g\\ 1 & \text{else}\end{cases}$$

Theoretical uncertainties of the method

Merging-related

- Choice of the jet criterion (fixed by now)
- Value of the phase-space separation cut, Q_{cut}
- Maximum number of jets from hard ME's, N_{max}

pQCD-related

- Scale uncertainties from ME's
- Scale uncertainties from PS's
- PDF uncertainties

Others

Choice of the LO process see arXiv:0903.1219 [hep-ph]

Stuffing it all into Sherpa

The ingredients

- Catani-Seymour subtraction based shower (CSS) JHEP03(2008)038
- The matrix element generator Comix JHEP12(2008)039

Why those ?

CSS provides

- very good approximation of NLO real emission ME
- invariant definitions of variables
- excellent recoil strategy

Comix provides

- explicit colour assignment
- trivial projection onto large N_C

All in all: Way better analytic control

Results: total cross sections

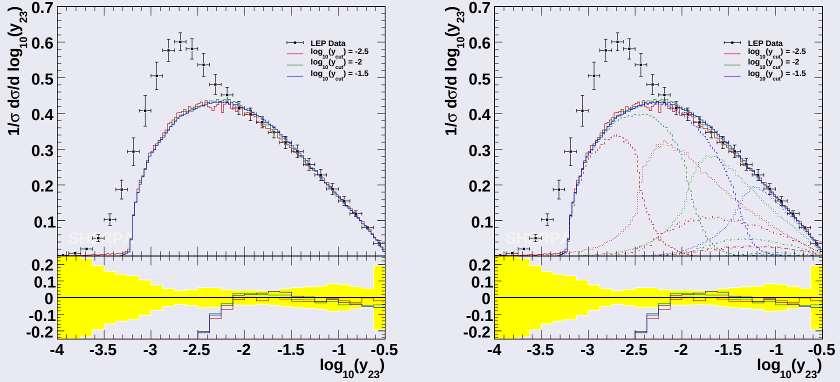
$e^+e^- \longrightarrow$ hadrons @ LEP I

		N_{max}				
		0	1	2	3	4
$\log_{10}(Q^2_{\mathrm{cut}}/s)$	-1.25	40.17(1)	39.65(3)	39.66(3)	39.66(3)	39.67(3)
	-1.75		39.38(5)	39.29(6)	39.13(5)	39.13(5)
	-2.25		39.27(8)	38.35(9)	37.89(11)	37.60(10)

Drell-Yan @ Tevatron Run II

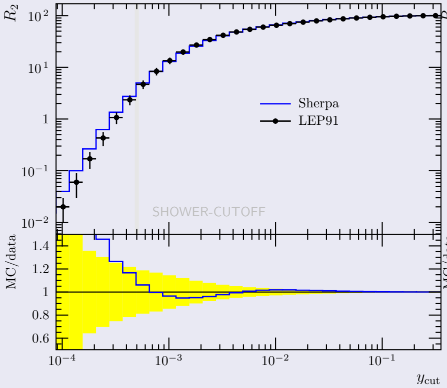
		N_{max}						
		0	1	2	3	4	5	6
Q_{cut}	20 GeV	192.6(1)	191.0(3)	190.5(4)	189.0(5)	189.4(7)	188.2(8)	189.9(10)
	30 GeV		192.3(2)	192.7(2)	192.6(3)	192.9(3)	192.7(3)	193.2(3)
	45 GeV		193.6(1)	194.4(1)	194.3(1)	194.4(1)	194.6(2)	194.4(1)

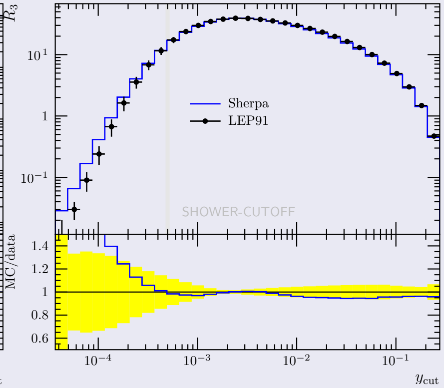
Durham 2 $\longrightarrow$ 3 jet rate in $Q_{\rm cut}$ variation (parton level) Data: EPJC17(2000)19



Results: $e^+e^- \to$ hadrons @ LEP I

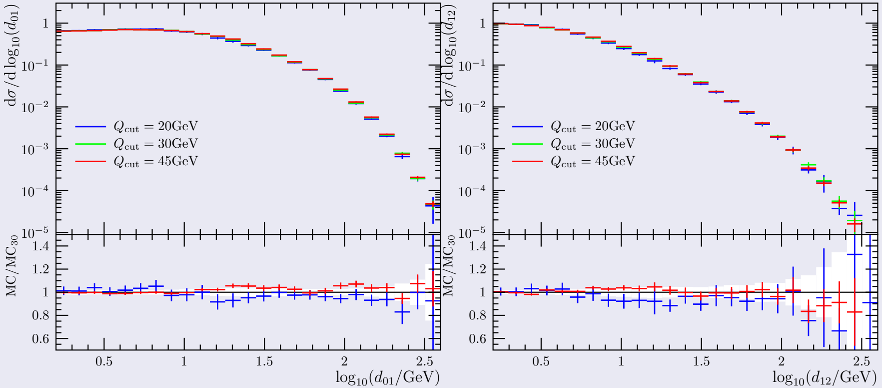
Durham jet fractions (hadron level, untuned) Data: EPJC17(2000)19





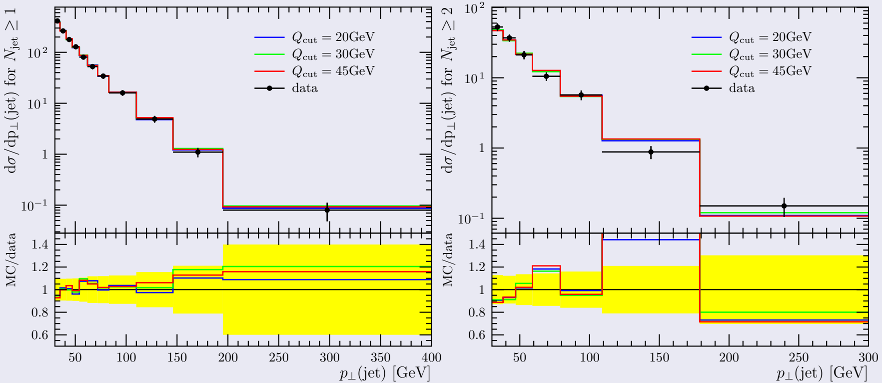
Results: Drell-Yan @ Tevatron Run II

Differential jet rates in $Q_{\rm cut}$ variation (hadron level)



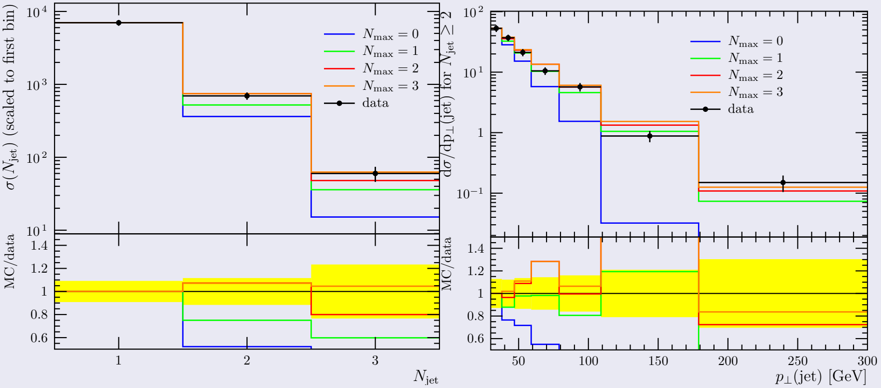
Results: Drell-Yan @ Tevatron Run II

Jet observables in Q_{cut} variation (hadron level) Data: PRL100(2008)102001



Results: Drell-Yan @ Tevatron Run II

Jet observables in $N_{\max}$ variation (hadron level) Data: PRL100(2008)102001



Summary

What has actually been improved ?

- Proof of correctness in initial-state evolution
- Freedom to define μ_F and μ_R at leading order
- Largely reduced merging systematics
- Improved phase-space separation

Uncertainties can separately be assessed

What comes next ?

- Cross-checks and applications
- Multi - leading-order prescription
- Release of the code with SHERPA v1.2