Heavy Quarks in Global Fits

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Charm $\sim 1.5\mathrm{GeV}$ and bottom $\sim 4.3\mathrm{GeV}$ are considered as as heavy flavours since $m_H^2 \gg \Lambda_{QCD}^2$.

Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Note that n_f is effective number of light quarks. Can be $3, 4$ or 5.

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen et al), but are not defined at NNLO – $\alpha_S^3 C_{2,Hg}^{FF,3}$ unknown.

So far top quark usually considered in FFNS, even at LHC.

Variable Flavour

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down (strange always in this regime. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza et al) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2)$ $\rightarrow$ correct evolution for both.

Want a **General-Mass Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Note though recent improved ZM-VFNS (Nadolsky, Tung).

The GM-VFNS can be defined by demanding equivalence of the n_f light flavour and $n_f + 1$ light flavour descriptions at all orders – above transition point $n_f \to n_f + 1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the massless limits as $Q^2/m_H^2 \to \infty$.

However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Original ACOT prescription determined heavy quark coefficient functions by calculating scattering diagram for single particle, i.e. $c + \gamma^{\star} \rightarrow c$.

Problem, violated threshold $W^2 > 4m_H^2$ since only needed one quark in final state rather than quark-antiquark pair.

Effects cancel between terms of different order

(TR-VFNS) recognised ambiguity in definition of $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and removed it by making $(d\,F_2/d\,\ln Q^2)$ continuous at transition.

Smoothness guaranteed at $Q^2 = m_H^2$ – but complicated.

At higher orders can only be applied for gluons, not singlet quarks.

Various other alternatives.

Quite recently Tung, Kretzer, Schmidt have come up with the ACOT(χ) prescription which I interpret as

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) = \delta(z - Q^2/(Q^2 + 4m_H^2)).$$

$$\rightarrow F_2^{H,0}(x, Q^2) = (h + \bar{h})(x/x_{max}, Q^2), \qquad x_{max} = Q^2/(Q^2 + 4m_H^2)$$

$\rightarrow C^{ZM,0}_{2,HH}(z) = \delta(1 - z)$ for $Q^2/m_H^2 \rightarrow \infty$.

Also $W^2 = Q^2(1 - x)/x \geq 4m_H^2$, i.e. correct kinematics built into each term.

For VFNS to remain simple (and physical) at all orders is necessary to choose

$$C^{VF,n}_{2,HH}(Q^2/m_H^2, z) = C^{ZM,n}_{2,HH}(z/x_{max}).$$

MSTW have also adopted this.

One more problem in defining VFNS. Ordering for $F_2^H(x,Q^2)$ different above and below transition point.

	Below	**Above**
LO	$\frac{\alpha_S}{4\pi} C_{2,Hg}^{FF,n_f,(1)} \otimes g^{n_f}$	$C_{2,HH}^{VF,n_f+1,(0)} \otimes (h+\bar{h})$
NLO	$\left(\frac{\alpha_S}{4\pi}\right)^2 (C_{2,Hg}^{FF,n_f,(2)} \otimes g^{n_f} + C_{2,Hq}^{FF,n_f,(2)} \otimes \Sigma^{n_f})$	$\frac{\alpha_S}{4\pi} (C_{2,HH}^{VF,n_f+1,(1)} \otimes h^+ + C_{2,Hg}^{VF,n_f+1,(1)} \otimes g^{n_f+1})$
NNLO	$\left(\frac{\alpha_S}{4\pi}\right)^3 \sum_i C_{2,Hi}^{FF,n_f,(3)} \otimes f_i^{n_f}$	$\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_j C_{2,Hj}^{VF,n_f+1,(2)} \otimes f_j^{n_f+1}$.

Switching direct from fixed order to same order when going from n_f to n_f+1 flavours $\longrightarrow$ discontinuity.

Must make some decision how to deal with this.

ACOT type schemes have used e.g.

NLO $\quad \frac{\alpha_S}{4\pi} C^{FF,n_f,(1)}_{2,Hg} \otimes g^{n_f} \rightarrow \frac{\alpha_S}{4\pi} (C^{VF,n_f+1,(1)}_{2,HH} \otimes (h+\bar{h}) + C^{VF,n_f+1,(1)}_{2,Hg} \otimes g^{n_f+1})$,

i.e., same order of α_S above and below, and same order as light quark contributions.

Simple – may be thought of as **Minimal prescription**.

TR have used e.g.

LO $\quad \frac{\alpha_S(Q^2)}{4\pi} C^{FF,n_f,(1)}_{2,Hg}(Q^2/m_H^2) \otimes g^{n_f}(Q^2) \rightarrow \frac{\alpha_S(M^2)}{4\pi} C^{FF,n_f,(1)}_{2,Hg}(1) \otimes g^{n_f}(M^2)$

$+ C^{VF,n_f+1,(0)}_{2,HH}(Q^2/m_H^2) \otimes (h+\bar{h})(Q^2)$,

i.e. freeze higher order α_S term when going upwards through $Q^2 = m_H^2$.

Identical to FFNS at same order for $Q^2 \leq m_H^2$

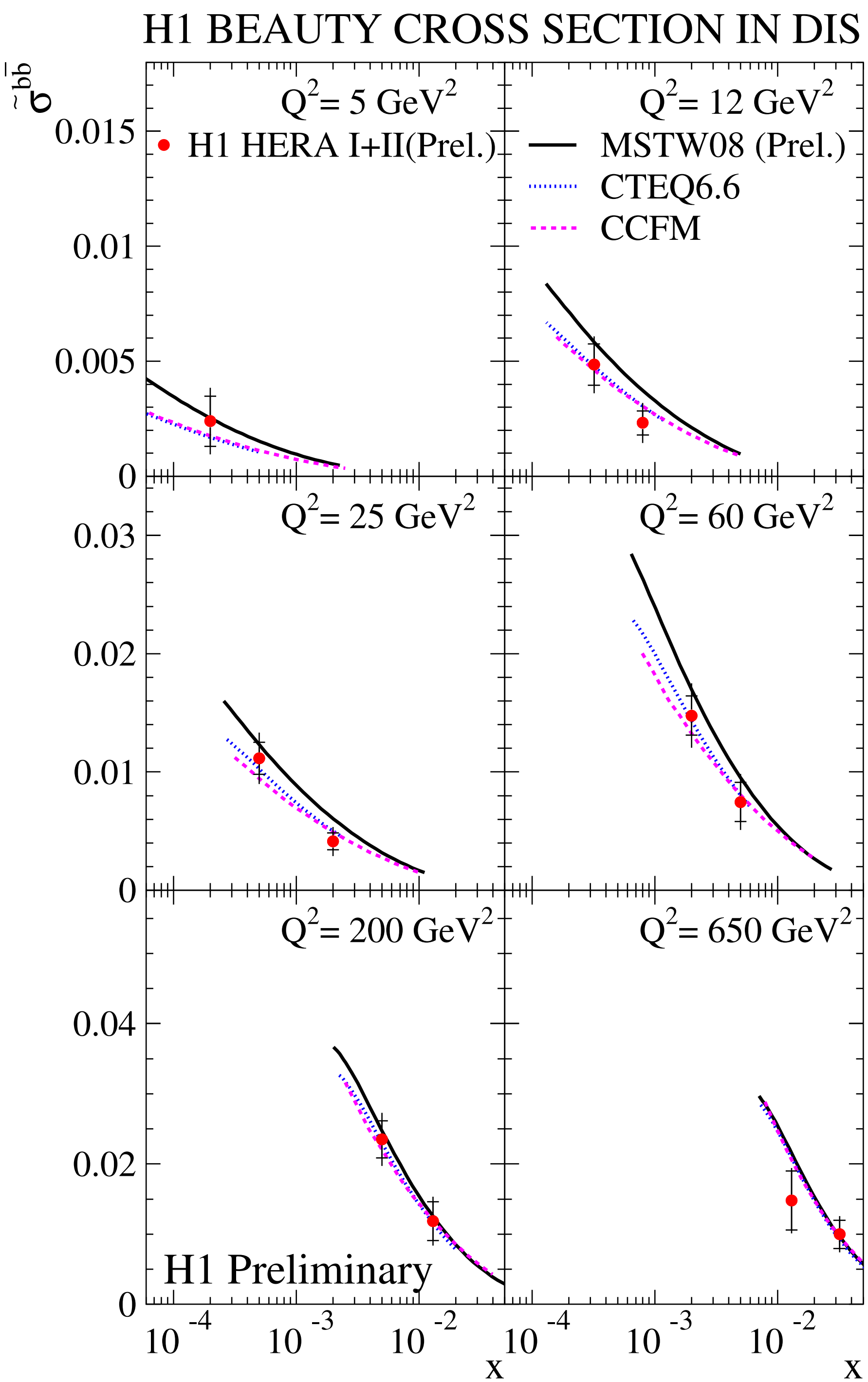
Includes as much information as possible at given order, but more complicated – **Maximal prescription**.

This difference in convention will have an effect mainly in the $Q^2 \sim m_H^2$ region. Difference dies away as order of calculation increases.

At finite order can be phenomenologically important.

Ordering is the main difference in the NLO predictions from MRST and CTEQ in the comparison to H1 data on $F_2^b(x, Q^2)$.

$\mathcal{O}(\alpha_S^2)$ part is significant for $Q^2 \sim m_H^2$.



Check effect of change in flavour prescription for NLO.

Compare MRST2004 (with 2001 uncertainties) to unofficial “MRST2006 NLO”.

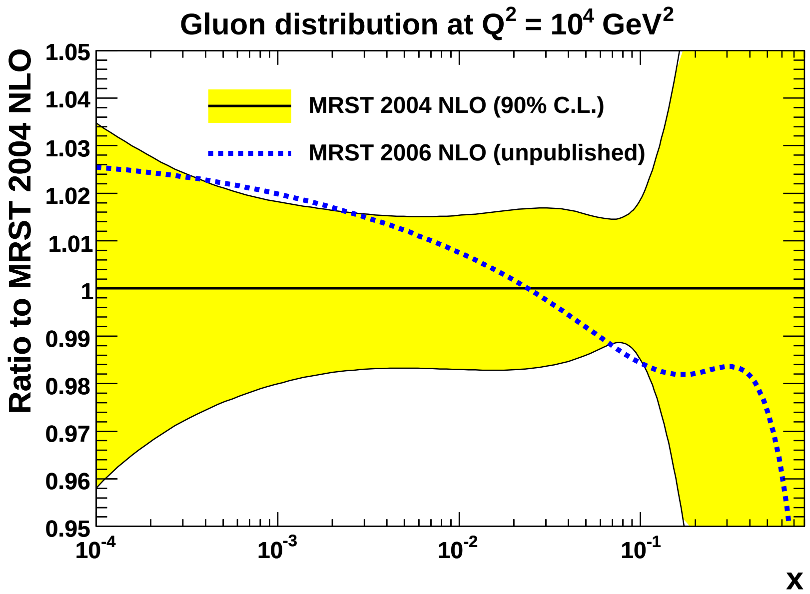
Only difference in flavour schemes (both well-defined).

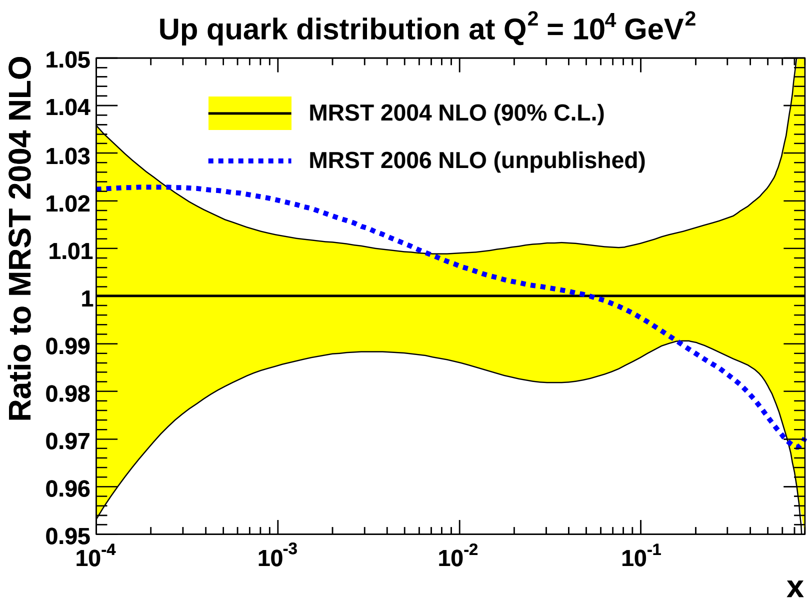
Changes of up to 2% in PDFs.

Up to 3% increase in σ_W and σ_Z at the LHC.

This is a genuine theory uncertainty due to competing but equally valid choices. Ambiguity decreases at higher orders.

Some – but probably quite little – anti-correlation with PDF uncertainties.





Similar effects noticed in variations of improved ZM-VFNS (Nadolsky, Tung).

Will be much smaller, but same type of effect in a GM-VFNS.