Top (pair) cross section calculations for the LHC

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Work with:
Michael Czakon
arXiv:0812.0353
arXiv:0811.4119
in progress …
George Sterman and Ilmo Sung
arXiv:0903.3241
in progress …

Current status

The state of the art is NLO QCD corrections.

Original results derived long ago (20 years):

Nason, Dawson, Ellis (1988-90)
Beenakker, Kuijf, van Neerven, Smith (1989)
Beenakker, van Neerven, Meng, Schuler, Smith (91)
Mangano, Nason, Ridolfi (1992)
Bernreuther et al. (2004)

New calculations/results (3-4 months ago):

M. Czakon, A.M. (2008)

- Various observables:

  a) Differential:
  single particle inclusive,
  pair-invariant mass distribution,
  etc.
  b) Fully inclusive (until few months ago – numerical; now analytic)

- Relevance of the differential vs the total cross section:
  For not too strong cuts, the NLO effect is on normalization, not shapes !

Current status

Second source: NLL soft gluon (threshold) resummation.

The only source of new information in top production in the last > 10 years

- Various observables:

  a) Differential:
  single particle inclusive,
  pair-invariant mass distribution,
  etc.

  Developed: Sterman et al mid-90's
  Applied: Kidonakis, Laenen, Moch, Vogt

  b) Fully inclusive

  Developed (NLL): Bonciani, Catani, Mangano, Nason '98
  Applied: Cacciari et al, Moch Uwer, Czakon AM

First step in promoting these to NNLL:

A.M., Sterman, Sung '09

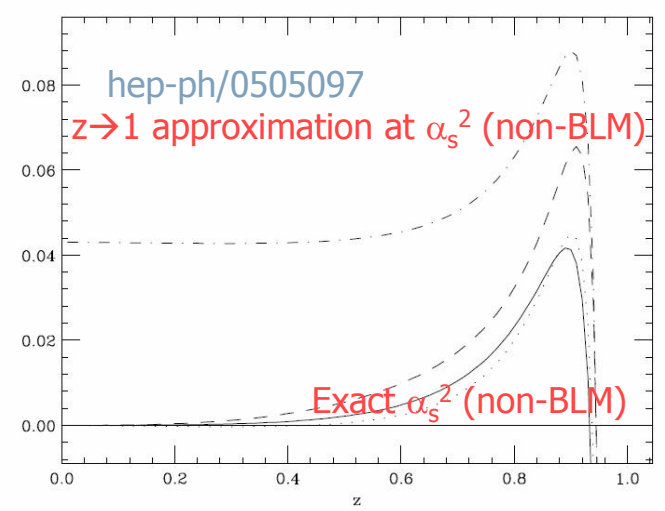
The relation between the two pictures is still unclear !

Top quark: what do we need/want to know?

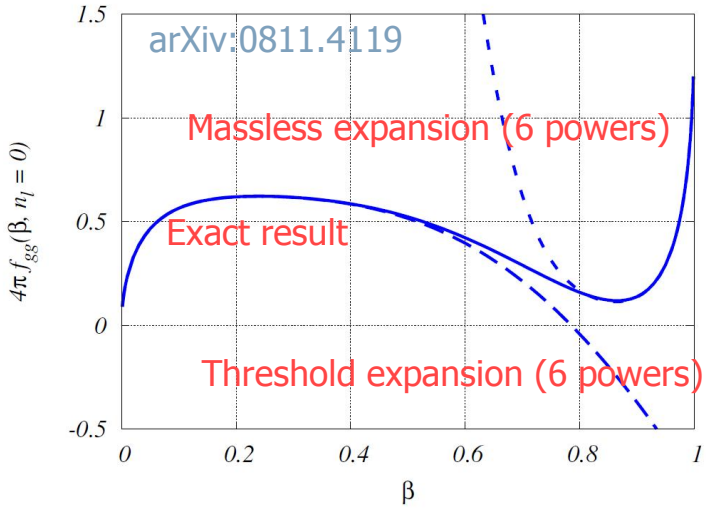
- Understanding true (scale) uncertainty requires full NNLO calculation !
- The appropriate observable is the total inclusive cross-section.
- Some NNLO terms can be obtained by truncating all-order resummation.
  - is this a systematic approximation?

In general, this is a poor approximation to fixed order calculations:

Photon spectrum in B→ s+γ:

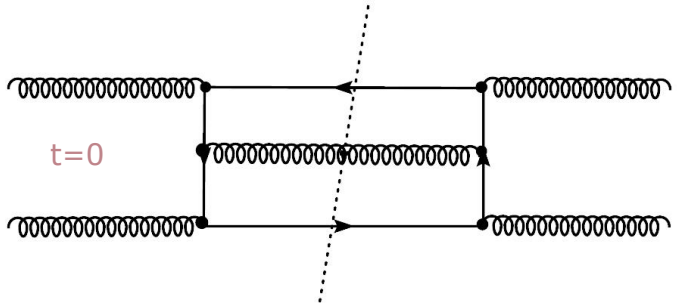


Top X-section: NLO correction

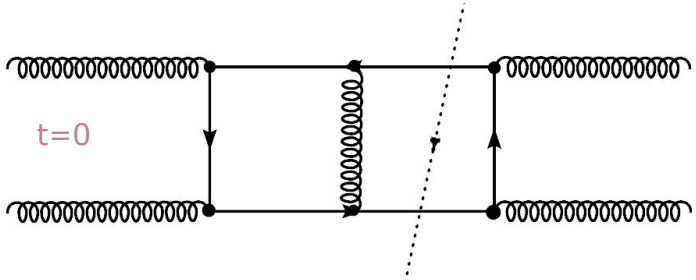


Top quark: How complicated is the NLO?

Here are few sample diagrams at NLO:



+ crossed



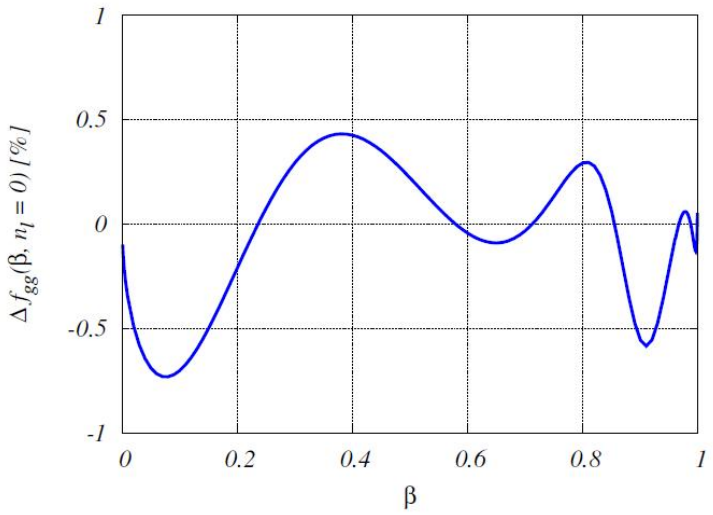
+ crossed

- Note: these are 2 loop (cut) boxes with masses.
  Not studied before.

Main details of the exact NLO calculation (1/3)

- For 20 years σ_{TOT} was known as a numerically derived fit
- Newly calculated analytical results (new techniques):
  - The whole problem is mapped into 37 masters (real+virtual)
  - We find that the cross-section develops new unphysical singularities!
  - Appearance of elliptic functions,
  - We confirm the high numerical accuracy of the earlier FO results (< 1%)

Czakon, AM '08



Details of the exact NLO calculation (2/3)

Threshold expansion $\beta \to 0$: (i.e. $4m^2 \to s$)

$$f_{gg}^{(1)}(\beta) = \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta) \left(\left(C_F - \frac{(N^2-4)\,C_A}{2(N^2-2)} \right) \frac{\pi^2}{2\beta} + 2C_A \log^2\left(8\beta^2\right) - \frac{(9N^2-20)C_A}{N^2-2} \log\left(8\beta^2\right) \right.$$
$$+ C_A \left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right)$$
$$\left. + C_F \left(-5 + \frac{\pi^2}{4} \right) + o(\beta) \right) . \qquad (27)$$

Extraction of the constant in the threshold limit:

$$C_A \left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right) + C_F \left(-5 + \frac{\pi^2}{4} \right)$$
$$= \frac{1111}{21} - \frac{283\pi^2}{168} + \frac{15\log 2}{7} - 6\log^2 2 \simeq 34.88 ,$$

Nason, Dawson, Ellis '89

$$\frac{768\pi}{7} a_0^{gg} \simeq 37.25 .$$

Czakon, AM '08

X-section better than 1%. But the constant in gg is 7% different.

Turns out, it is all consistent …

Hagiwara, Sumino, Yokoya '08

Significant (and unexpected) effect for threshold resummation!

Details of the exact NLO calculation (3/3)

From resummation, the following 2 loop logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\text{Born}}(\beta) + \frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + O(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\text{Born}}(\beta)\left(4608\log^4\beta + 1894.9\log^3\beta - 3.4811\log^2\beta + O(\log\beta)\right)$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384C_3$

where: $C_3 = 37.23$ As extracted from NDE '89 and used in ALL resummation literature

$C_3 = 34.88$ The exact value just recently derived Czakon, AM '08

Therefore the coefficient of $\ln^2(\beta)$ becomes -912.35

Note: the reason is pure numerics!

i.e. a change by a factor of 260 !

More on resummation in top-pair

The changes discussed so far are purely due to numerics.

However: there is another modification compared to earlier literature

Exponentiation in Mellin space: (1) $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho$ $\rho = 4m^2/s$ (2)

(3) $\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N)$ (4) $\sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\ \sigma_{ij,\mathbf{I}}^{\mathrm{H}}\ \Delta_{ij,\mathbf{I}}(N)$

We were the first to point out σ^H depend on the color state of the heavy quark pair. We calculated the two coefficients.

Change in the gg Sudakov resummed X-section: compare to Bonciani et al '98

$$\sigma_{gg}^{\mathrm{H\ (BCMN)}} = 1 + \frac{\alpha_s}{\pi}\, 14.39 + o(\alpha_s^2),$$

$$\sigma_{gg}^{\mathrm{H\ (BCMN)}}|_{C_3\ \mathrm{exact}} = 1 + \frac{\alpha_s}{\pi}\, 12.04 + o(\alpha_s^2),$$

$$\sigma_{gg,\mathbf{1}}^{\mathrm{H}} = 1 + \frac{\alpha_s}{\pi}\, 9.16 + o(\alpha_s^2),$$

$$\sigma_{gg,\mathbf{8}}^{\mathrm{H}} = 1 + \frac{\alpha_s}{\pi}\, 13.19 + o(\alpha_s^2),$$

C_3 numerics: -5%,
color singlet channel: -12%,
color octet channel: -3%,

Resummation - summary

These large corrections are partially offset:

$$\sigma_{RESUM} = \sigma_{FO} + \sigma_{SUDAKOV} - \sigma_{OVERLAP}$$

Their implications to previous studies:

- ✓ Formally these effects are beyond NLL; yet significant numerically
- ✓ Incorrect beyond NLL (only one such study Moch, Uwer '08)

The numbers follow:

Top quark: "the numbers"

The central values:

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4%
- New NLO effects / FO NLO: 1-1.5% Czakon, AM
- Beyond NLL effects / FO NLO: 0.8% Moch, Uwer

Current theory error estimate (NLO/NLL): ~ 10%

Uncertainty =/= just scale variation !!!

Important: No genuine NNLO term is known (could easily give 5%) !

Top quark: state of the art

Comparison of central values for:

- m_{top}=172.4 GeV
- μ=m
- correct exact hard matching coefficients
- Coulombic effects not elaborated upon.

$\alpha_s(M_Z)$:

CTEQ 6.6: 0.118
MRST 2006 nnlo: 0.119
MSTW 2008 nnlo: 0.117
MSTW 2008 nlo: 0.120

MRST 2006 nnlo

NLO = 890 pb
NLO+NLL = 918 pb

5.4 %

CTEQ 6.6

NLO = 844 pb
NLO+NLL = 871 pb

MSTW 2008 nnlo

NLO = 857 pb
NLO+NLL = 885 pb

1.6 %

MSTW 2008 nlo

NLO = 906 pb
NLO+NLL = 935 pb

7.3 %

Czakon, AM in progress

Conclusions

The summary with the new analytic calculation/updated resummation:

NLO/NLL exact and complete only now — M. Czakon, A.M. (2008); In progress

Conclusion #1: the earlier FO NLO calculations are of high quality 1%

Conclusion #2: the NLL resummation affected by our work (25-30% effect):

qq → tt unchanged at NLO/NLL (but likely at NNLO/NNLL)

Question: The uncertainty/how to = ? (it is close to 10%, not 3%)

Conclusion #3: Now we have good understanding of the "A-constant":

Bonciani, Catani, Mangano, Nason '98

Sub-leading powers are non-negligible; in fact are equally important!

The new set MSTW 2008 NNLO is (much) closer to CTEQ6.6 (for top-pair)