

Top (pair) cross section calculations for the LHC

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Work with:

Michael Czakon

arXiv:0812.0353

arXiv:0811.4119

in progress ...

George Sterman and Ilmo Sung

arXiv:0903.3241

in progress ...

Current status

The state of the art is NLO QCD corrections.

Original results derived long ago (**20 years**):

Nason, Dawson, Ellis (1988-90)

Beenakker, Kuijf, van Neerven, Smith (1989)

Beenakker, van Neerven, Meng, Schuler, Smith (91)

Mangano, Nason, Ridolfi (1992)

Bernreuther et al. (2004)

New calculations/results (**3-4 months ago**):

M. Czakon, A.M. (2008)

➤ Various observables:

a) Differential:

single particle inclusive,
pair-invariant mass distribution,
etc.

b) Fully inclusive (until few months ago – numerical; now analytic)

➤ Relevance of the differential vs the total cross section:

For not too strong cuts, the NLO effect is on normalization, not shapes !

Current status

Second source: NLL soft gluon (threshold) resummation.

The only source of new information in top production in the last > 10 years

➤ Various observables:

a) Differential:

single particle inclusive,
pair-invariant mass distribution,
etc.

Developed: Sterman et al mid-90's
Applied: Kidonakis, Laenen, Moch, Vogt

b) Fully inclusive

Developed (NLL): Bonciani, Catani, Mangano, Nason '98
Applied: Cacciari et al, Moch Uwer, Czakon AM

First step in promoting these to NNLL:

A.M., Sterman, Sung '09

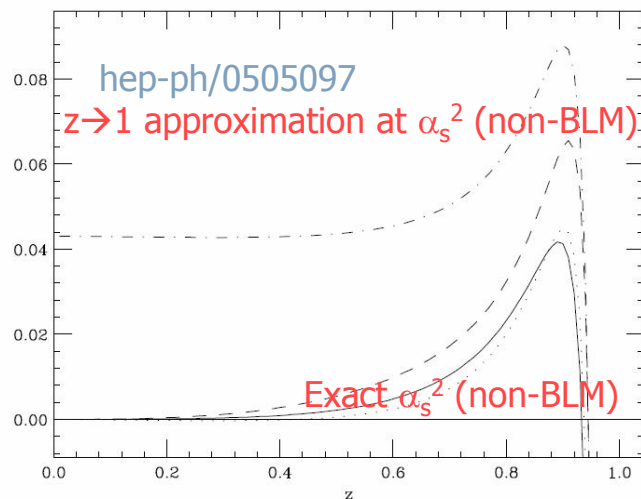
The relation between the two pictures is still unclear !

Top quark: what do we need/want to know?

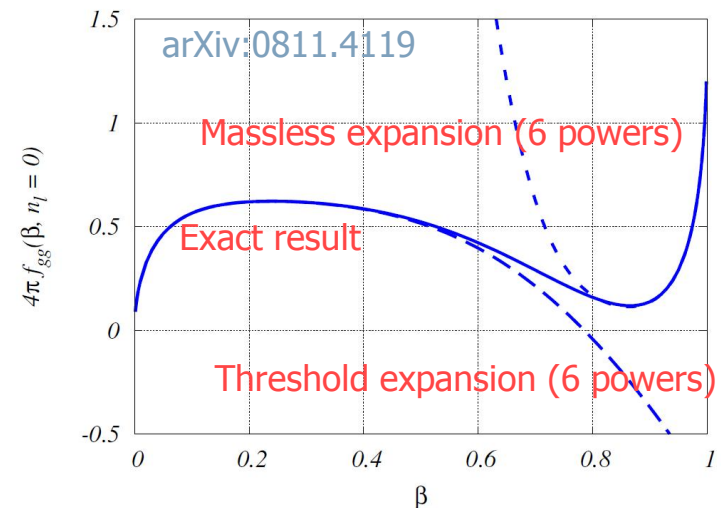
- ❖ Understanding true (scale) uncertainty requires full NNLO calculation !
- ❖ The appropriate observable is the total inclusive cross-section.
- ❖ Some NNLO terms can be obtained by truncating all-order resummation.
 - is this a systematic approximation?

In general, this is a poor approximation to fixed order calculations:

Photon spectrum in $B \rightarrow s + \gamma$:

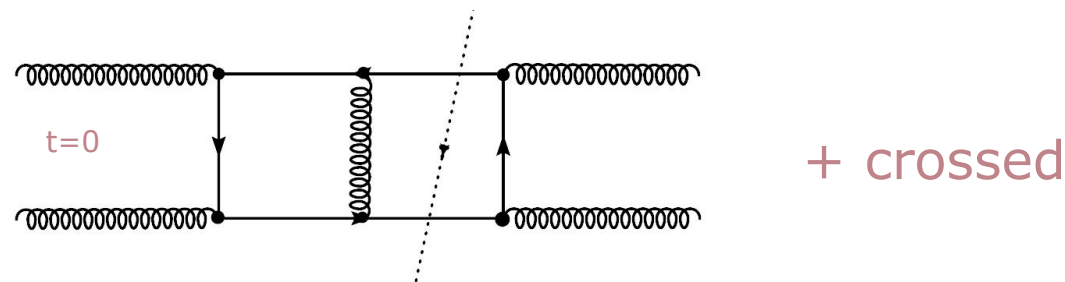
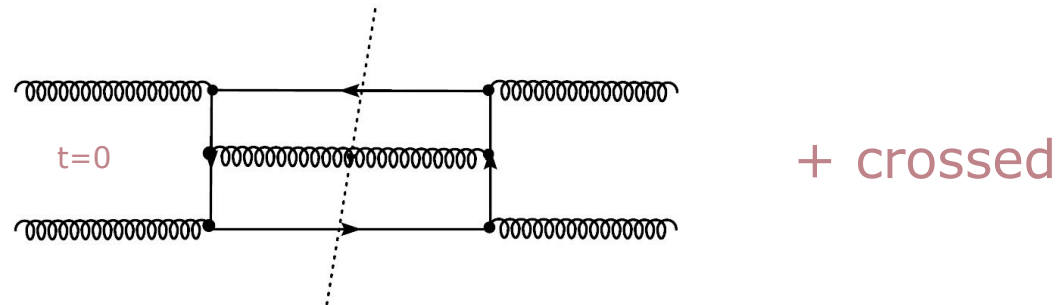


Top X-section: NLO correction



Top quark: How complicated is the NLO?

Here are few sample diagrams at NLO:

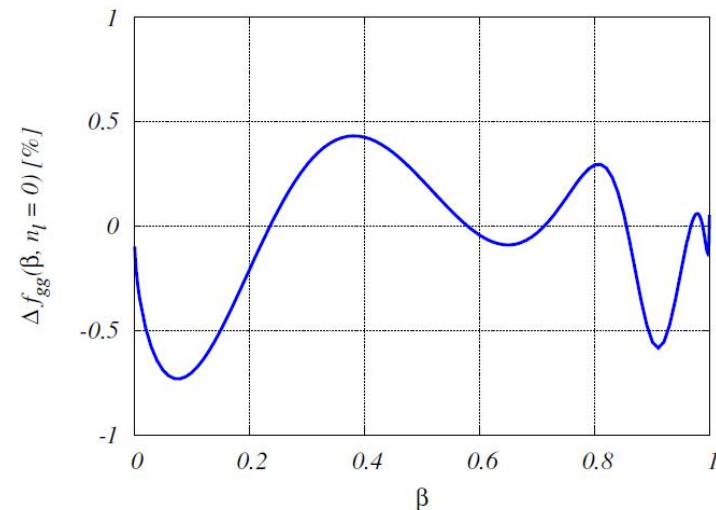


- Note: these are 2 loop (cut) boxes with masses. Not studied before.

Main details of the exact NLO calculation (1/3)

- ❖ For 20 years σ_{TOT} was known as a numerically derived fit
- ❖ Newly calculated analytical results (new techniques):
 - ❖ The whole problem is mapped into 37 masters (real+virtual)
 - ❖ We find that the cross-section develops new unphysical singularities!
 - ❖ Appearance of elliptic functions,
 - ❖ We confirm the high numerical accuracy of the earlier FO results ($< 1\%$)

Czakon, AM '08



Details of the exact NLO calculation (2/3)

Threshold expansion
 $\beta \rightarrow 0$:
 (i.e. $4m^2 \rightarrow s$)

$$f_{gg}^{(1)}(\beta) = \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta) \left(\left(C_F - \frac{(N^2 - 4)C_A}{2(N^2 - 2)} \right) \frac{\pi^2}{2\beta} + 2C_A \log^2(8\beta^2) - \frac{(9N^2 - 20)C_A}{N^2 - 2} \log(8\beta^2) \right. \\ \left. + C_A \left(\frac{21N^2 - 50}{N^2 - 2} - \frac{(17N^2 - 40)\pi^2}{24(N^2 - 2)} + \frac{(N^2 - 4)\log 2}{N^2 - 2} - 2\log^2 2 \right) \right. \\ \left. + C_F \left(-5 + \frac{\pi^2}{4} \right) + o(\beta) \right). \quad (27)$$

Extraction of the constant in the threshold limit:

$$C_A \left(\frac{21N^2 - 50}{N^2 - 2} - \frac{(17N^2 - 40)\pi^2}{24(N^2 - 2)} + \frac{(N^2 - 4)\log 2}{N^2 - 2} - 2\log^2 2 \right) + C_F \left(-5 + \frac{\pi^2}{4} \right) \\ = \frac{1111}{21} - \frac{283\pi^2}{168} + \frac{15\log 2}{7} - 6\log^2 2 \simeq 34.88,$$

Czakon, AM '08

Nason, Dawson, Ellis '89

$$\frac{768\pi}{7} \alpha_0^{gg} \simeq 37.25.$$

X-section better than 1%. But the constant in gg is 7% different.

Turns out, it is all consistent ...

Hagiwara, Sumino, Yokoya '08

Significant (and unexpected) effect for threshold resummation!

Details of the exact NLO calculation (3/3)

From resummation,
the following 2 loop
logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\text{Bom}}(\beta) + \frac{\alpha_s}{4\pi} \sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + o(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\text{Bom}}(\beta) (4608 \log^4 \beta + 1894.9 \log^3 \beta - 3.4811 \log^2 \beta + o(\log \beta))$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384C_3$

where: $C_3 = 37.23$ As extracted from NDE '89 and used in ALL resummation literature

$C_3 = 34.88$ The exact value just recently derived Czakon, AM '08

Therefore the coefficient of $\ln^2(\beta)$ becomes
-912.35

**Note: the reason is
pure numerics!**

i.e. a change by **a factor of 260 !**

More on resummation in top-pair

The changes discussed so far are purely due to numerics.

However: there is another modification compared to earlier literature

Exponentiation in Mellin space: (1) $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho$. $\rho = 4m^2/s$ (2)

$$(3) \sigma_{ij}^{\text{TOT}}(N) = \sigma_{ij,1}(N) + \sigma_{ij,8}(N) \quad (4) \sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\text{Born}}(N) \sigma_{ij,\mathbf{I}}^{\text{H}} \Delta_{ij,\mathbf{I}}(N)$$

We were the first to point out σ^{H} depend on the color state of the heavy quark pair. We calculated the two coefficients.

Change in the gg Sudakov resummed

X-section: compare to Bonciani et al '98

C_3 numerics: -5%,
 color singlet channel: -12%,
 color octet channel: -3%,

$$\begin{aligned} \sigma_{gg}^{\text{H (BCMN)}} &= 1 + \frac{\alpha_s}{\pi} 14.39 + o(\alpha_s^2), \\ \sigma_{gg}^{\text{H (BCMN)}}|_{C_3 \text{ exact}} &= 1 + \frac{\alpha_s}{\pi} 12.04 + o(\alpha_s^2), \\ \sigma_{gg,1}^{\text{H}} &= 1 + \frac{\alpha_s}{\pi} 9.16 + o(\alpha_s^2), \\ \sigma_{gg,8}^{\text{H}} &= 1 + \frac{\alpha_s}{\pi} 13.19 + o(\alpha_s^2), \end{aligned}$$

Resummation - summary

These large corrections are partially offset:

$$\sigma_{\text{RESUM}} = \sigma_{\text{FO}} + \sigma_{\text{SUDAKOV}} - \sigma_{\text{OVERLAP}}$$

Their implications to previous studies:

- ✓ Formally these effects are beyond NLL; yet significant numerically
- ✓ Incorrect beyond NLL (only one such study [Moch, Uwer '08](#))

The numbers follow:

Top quark: “the numbers”

The central values:

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4%
- New NLO effects / FO NLO: 1-1.5% Czakon, AM
- Beyond NLL effects / FO NLO: 0.8% Moch, Uwer

Current theory error estimate (NLO/NLL): $\sim 10\%$

Uncertainty \neq just scale variation !!!

Important: No genuine NNLO term is known (could easily give 5%) !

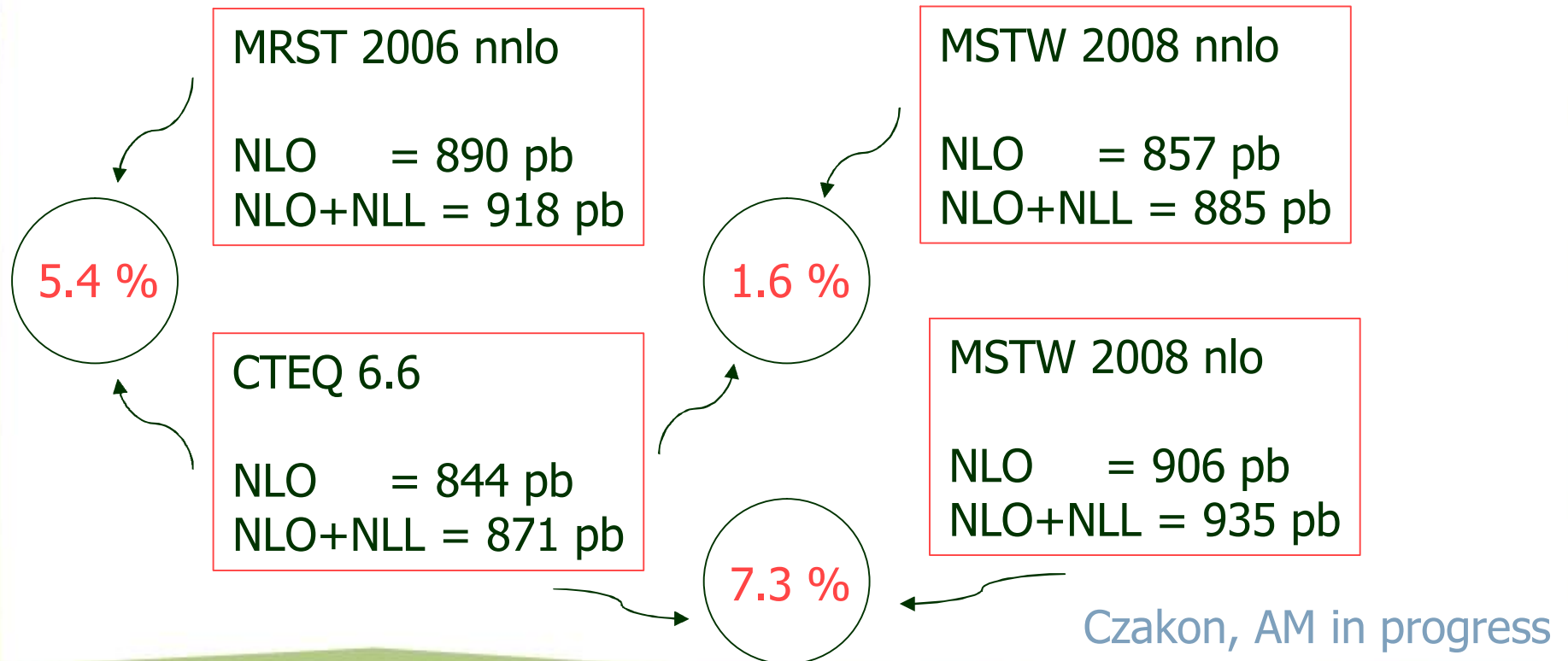
Top quark: state of the art

Comparison of central values for:

- $m_{\text{top}} = 172.4 \text{ GeV}$
- $\mu = m$
- correct exact hard matching coefficients
- Coulombic effects not elaborated upon.

$\alpha_s(M_Z)$:

CTEQ 6.6: 0.118
MRST 2006 nnlo: 0.119
MSTW 2008 nnlo: 0.117
MSTW 2008 nlo: 0.120



Conclusions

The summary with the new analytic calculation/updated resummation:

NLO/NLL exact and complete only now M. Czakon, A.M. (2008); In progress

Conclusion #1: the earlier FO NLO calculations are of high quality 1%

Conclusion #2: the NLL resummation affected by our work (25-30% effect):

qq \rightarrow tt unchanged at NLO/NLL (but likely at NNLO/NNLL)

Question: The uncertainty/how to = ? (it is close to 10%, not 3%)

Conclusion #3: Now we have good understanding of the "A-constant":

Bonciani, Catani, Mangano, Nason '98

Sub-leading powers are non-negligible; in fact are equally important!

The new set MSTW 2008 NNLO is (much) closer to CTEQ6.6 (for top-pair)