Optimal fact^n scales for jet prod^n

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(i) First, a simpler example: open $b\bar{b}$ production at low x

1610.06034 with Emmanuel de Oliveira

(ii) Then: **inclusive high-p_T jet production 1702.01663**

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$b\bar{b}$ production at NLO at fac^n scale μ_f

μ_f dep. of $C^{(1)}$ occurs, -- need to subtract part of NLO diag generated by LO evolution

$$\sigma^{(0)}(\mu_f) + \sigma^{(1)}(\mu_f) = \alpha_s^2 \left[\mathrm{PDF}(\mu_f) \otimes C^{(0)} \otimes \mathrm{PDF}(\mu_f) + \mathrm{PDF}(\mu_f) \otimes \alpha_s C^{(1)}(\mu_f) \otimes \mathrm{PDF}(\mu_f)\right]$$

Free to evaluate LO term at diff.scale μ_F since it can be compensated by changes in $C^{(1)}$

$$\sigma^{(0)}(\mu_f)+\sigma^{(1)}(\mu_f) = \alpha_s^2 \left[\mathrm{PDF}(\mu_F) \otimes C^{(0)} \otimes \mathrm{PDF}(\mu_F) + \mathrm{PDF}(\mu_f) \otimes \alpha_s C^{(1)}_{\mathrm{rem}}(\mu_F) \otimes \mathrm{PDF}(\mu_f)\right]$$

$$\alpha_s^2\, \mathrm{PDF}(\mu_f) \otimes \left(C^{(0)} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{\mu_f^2}\right) (P_{\mathrm{left}} \otimes C^{(0)} + C^{(0)} \otimes P_{\mathrm{right}}) \right) \otimes \mathrm{PDF}(\mu_f)$$

LO DGLAP evolution

The idea is to use known NLO coeff fn $C^{(1)}(\mu_f)$ to choose μ_F so that $C_{\mathrm{rem}}^{(1)}(\mu_F)$ is a minimum

$$C^{(1)}_{\mathrm{rem}}(\mu_F) = C^{(1)}(\mu_f) - \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{\mu_f^2}\right) (P_{\mathrm{left}} \otimes C^{(0)} + C^{(0)} \otimes P_{\mathrm{right}})$$

change due to compensation

full NLO coeff.fn

part moved to LO PDFs

$$\sigma^{(0)}(\mu_f) + \sigma^{(1)}(\mu_f) = \alpha_s^2 \left[\mathrm{PDF}(\mu_f) \otimes C^{(0)} \otimes \mathrm{PDF}(\mu_f) + \mathrm{PDF}(\mu_f) \otimes \alpha_s C^{(1)}(\mu_f) \otimes \mathrm{PDF}(\mu_f)\right]$$

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$$\alpha_s^2\, \mathrm{PDF}(\mu_f) \otimes \left(C^{(0)} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{\mu_f^2}\right) \left(P_{\mathrm{left}} \otimes C^{(0)} + C^{(0)} \otimes P_{\mathrm{right}}\right) \right) \otimes \mathrm{PDF}(\mu_f)$$

LO DGLAP evolution

The idea is to use known $C^{(1)}(\mu_f)$ to choose μ_F so that $C_{rem}^{(1)}(\mu_F)$ is a minimum

$$C^{(1)}_{\mathrm{rem}}(\mu_F) = C^{(1)}(\mu_f) - \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{\mu_f^2}\right) \left(P_{\mathrm{left}} \otimes C^{(0)} + C^{(0)} \otimes P_{\mathrm{right}}\right)$$

change due to compensation

That is, μ_F is fixed so that the residual NLO coeff. f^n excludes $\alpha_s \log\mu_F^2 \log(1/x)$ contrib^{ns} moving them to the LO PDFs where they are resummed to all orders by DGLAP evolution

The residual fac^n scale, μ_f, dependence is optimized (reduced) for low x. Note that $C^{(1)}_{rem}(\mu_F)$ does not depend on μ_f --- the μ_f dependence occurs only in the PDFs in the NLO correction

Physical understanding of optimal scale

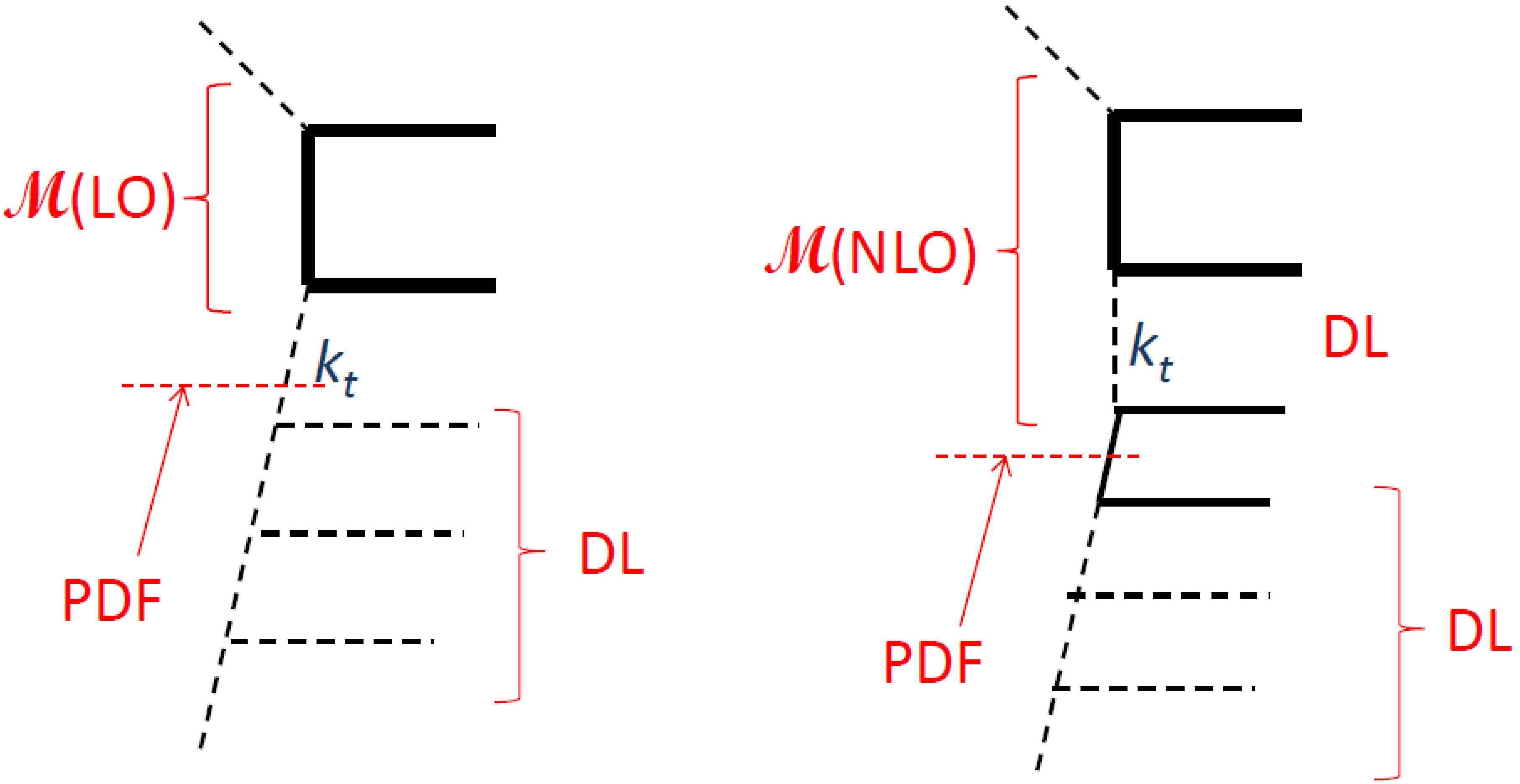
The log integration $\int \frac{dk^2}{k^2}$ over incoming parton virtuality k^2, hidden in DGLAP, does not extend to ∞, and is limited by off-shell matrix element $\mathcal{M}^{\mathrm{LO}}(k^2)$

Optimal choice of μ_F is such that value of DGLAP integral up to μ_F equals the value of the convergent integral :

$$\int_{Q_0^2}^{\infty} \frac{dk^2}{k^2} |\mathcal{M}^{\mathrm{LO}}(k^2)|^2 = \int_{Q_0^2}^{\mu_F^2} \frac{dk^2}{k^2} |\mathcal{M}^{\mathrm{LO}}(k^2=0)|^2 = |\mathcal{M}^{\mathrm{LO}}(k^2=0)|^2 \ln\frac{\mu_F^2}{Q_0^2}$$

So part of NLO correction, which has same DGLAP structure, is moved to the LO term --- this minimizes the NLO correction

Physical understanding continued



Double Log (DL) integral in NLO matrix elem. is the same as in the gluon cell below the LO matrix elem.

Now consider jet production

Summary of NLO $b\bar{b}$ prod. to determine optimal fac. scale μ_F

$$\sigma^{(0)}(\mu_f)+\sigma^{(1)}(\mu_f) = \alpha_s^2\left[\mathrm{PDF}(\mu_F)\otimes C^{(0)}\otimes \mathrm{PDF}(\mu_F) + \mathrm{PDF}(\mu_f)\otimes \alpha_s C^{(1)}_{\mathrm{rem}}(\mu_F)\otimes \mathrm{PDF}(\mu_f)\right]$$

LO term

small residual NLO correction

$$\alpha_s^2\,\mathrm{PDF}(\mu_f)\otimes\left(C^{(0)}+\frac{\alpha_s}{2\pi}\ln\left(\frac{\mu_F^2}{\mu_f^2}\right)\left(P_{\mathrm{left}}\otimes C^{(0)}+C^{(0)}\otimes P_{\mathrm{right}}\right)\right)\otimes\mathrm{PDF}(\mu_f)$$

P's are LO DGLAP splitting fns.

Generalise to jet production – but now three scales μ_-, μ_+, μ_D

jet frag. fn.

$$\Bigg[\alpha_s^2\,\mathrm{PDF}(\mu_-)\otimes\left(C^{(0)}+\frac{\alpha_s}{2\pi}\left[\ln\left(\frac{\mu_F^2}{\mu_-^2}\right)P_{\mathrm{left}}\otimes C^{0)}+\ln\left(\frac{\mu_F^2}{\mu_+^2}\right)C^{(0)}\otimes P_{\mathrm{right}}\right]\right)\otimes\mathrm{PDF}(\mu_+)$$
$$\otimes\left(1+\frac{\alpha_s}{2\pi}\ln\left(\frac{\mu_F^2}{\mu_D^2}\right)P_D\right)\otimes D(z,\mu_D)\Bigg]$$

LO term

$$+\ \ldots\ldots\ C^{(1)}_{\mathrm{rem}}(\mu_-,\mu_+,\mu_D)\ \ldots\ldots$$

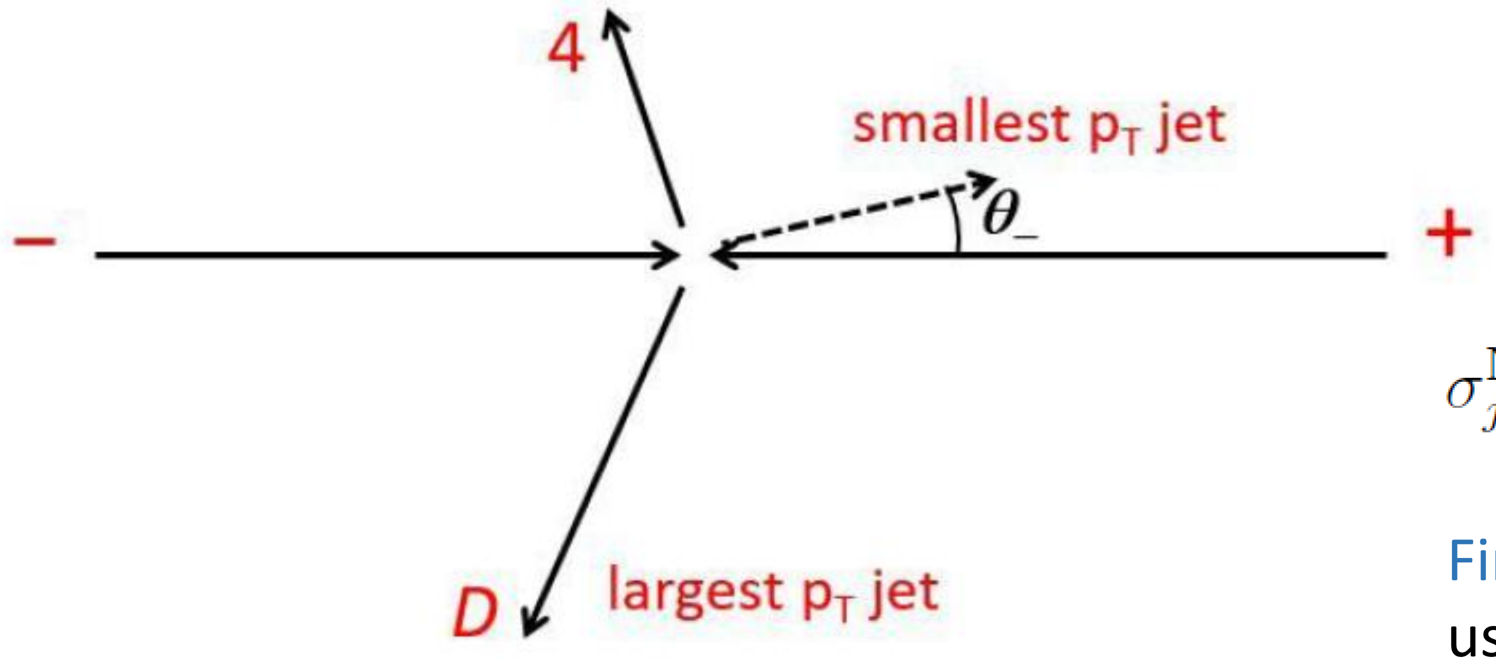
small residual NLO correction

The optimum value of μ_D is driven by the jet cone size ΔR

Since DGLAP evolution is written in terms of collinear factorization --- it is most convenient to order the contributions to the **jet process in terms of angles.**

To obtain the optimal LO description we study the 2→3 NLO subprocess.

First calculate the angles θ_i of the 4 partons relative to lowest p_T outgoing parton



Next, divide $\sigma_{2\to3}$ into 4 parts corresponding to the smallest θ_i's

$$\sigma^{\rm NLO}_{j=+,-,D,4} \;=\; \sigma_{2\to3}\;\Pi_{i\neq j}\;\Theta(\theta_i-\theta_j)$$

Finally, determine 3 optimal scales μ_j using 3 coupled eqs. each of the form :

j=+,-,D

$$\sigma^{\rm NLO}_j(\mu_0) \;=\; |\mathcal{M}^{\rm LO}(k^2=0)|^2 \otimes {\rm PDF}_{i\neq j}(\mu_0) \otimes D_{i\neq j}(\mu_0) \otimes {\rm PDF}_j(\mu_0) \otimes P^{\rm real}(z)\;\ln\frac{\mu_j^2}{\mu_0^2}$$

(μ_0 is a low dummy scale. Iterate with μ_0 replaced by new μ_j in turn)

see boxed eq.
3 slides previous

$\hat{P}^{+}_{\rm LO}$ means sp.fn acts on PDF(μ_+)

That is, the 3 optimal scales are fixed to make residual NLO coefficient function as small as possible ; namely

$$C^{(1)}_{\rm rem}(\mu_-,\mu_+,\mu_D) \;=\; C^{(1)}(\mu_f) - \sum_{j=+,-,D} C^{(0)}\,\frac{\alpha_s}{2\pi}\ln\left(\frac{\mu_j^2}{\mu_f^2}\right)\hat{P}^{j}_{\rm LO}\;,$$

NLO coeff.fn

part moved to LO PDFs, *D* fn.

by absorbing (and resumming) in the LO term as much as possible of the NLO correction. In this way the dependence on the residual factorization scale, μ_f, is reduced to a minimum.

The procedure can be extended in a straightforward way to NNLO. We fix the scales in the NLO correction by dividing up the real 2→4 NNLO contribution into 5 parts in terms of angles. Only three parts, those collinear to the +, - and *D* directions, are relevant.