Ratios of Cross Sections

Overview of theory uncertainty estimations

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Outline

Intro / literature

Measurements at the LHC

Estimating theory uncertainties

Cross Section Ratios in pQCD at Hadron Colliders

We will talk about ratios of cross sections

$$R = \frac{d\sigma(pp \to X)}{d\sigma(pp \to Y)}$$

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$$= \left(\alpha_s^{N-D}\right) \left[\left(\frac{n^{(0)}}{d^{(0)}}\right) + \left(\frac{n^{(1)} - \frac{n^{(0)} d^{(1)}}{d^{(0)}}}{d^{(0)}}\right) \alpha_s \right]$$

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$$\left. + \left(\frac{n^{(2)} - \frac{n^{(0)} d^{(2)}}{d^{(0)}} - \left(n^{(1)} - \frac{n^{(0)} d^{(1)})}{d^{(0)}}\right) \frac{d^{(1)}}{d^{(0)}}}{d^{(0)}}\right) \alpha_s^2 + \cdots \right]$$

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In all cases considered here $N - D$ is either zero or one

Ratios of generic observables, like *n*jettiness, planar flow, other event shapes, will not be covered here. Recent literature is [Soyez, Salam, Kim, Dutta, Cacciari arXiv:1211.2811] and [Larkoski, Thaler arXiv:1307.1699].

Taming Systematics with Ratios

Cross-section ratios have a direct experimental interest as it helps to reduce uncertainties coming from:

- Jet energy scale
- Lepton efficiencies and acceptance
- Luminosity

Also theory predictions benefit:

- Milder dependence on parametric uncertainties (PDFs, α_s, masses, etc)
- Possible reduction in the ratio of unkown higher-order terms
- Reduced sensitivity to modelling of non-perturbative effects

Ratios as Pheno Tools

Many applications for cross-section ratios:

- Jet ratios for studying universal properties of QCD
- Reduction of theoretical uncertainties when involved processes share kinematical/dynamical properties
- Constraining parton distribution functions
- Energy ratios to reduce uncertainties from higher-order terms of the perturbative expansions
- Estimation of backgrounds through data-driven methods
- Extrapolations possible to high-multiplicity processes

Quick review of literature

1985: [Ellis, Kleiss, Stirling] vector boson ratios to inclusive cross sections. First jet-scaling study (LO up to 2 jets) and discussion of corresponding reduction of theory/experimental uncertainties. Early comparison to data.

1989: [Berends, Giele, Kleiss, Kuijf, Stirling] Extended previous study to 3-jet processes. Explore cross section properties as function of $p_T^{\min}$.

1991: [Berends, Giele, Kuijf, Tausk] First explicit use of jet ratios and study vector-boson production with up to 4 jets. Study scaling properties, and use that as a test on the absolute normalization of their Monte Carlo program.

2003: [Abouzaid, Frisch] First $(W + n$-jet$)/(Z + n$-jet$)$ ratio study as a tool for precision measurements at hadron collider. Also present *'less robust'* $(V + n$-jet$)/(V + (n + 1)$-jet$)$ studies. NLO correction included for up to 2-jet processes. First systematic estimation of PDF and scale sensitivity.

Quick review of literature (cont.)

2010: [Kom, Stirling] $(W^+ + n\text{-jet})/(W^- + n\text{-jet})$ ratios as a tool for BSM constraints through precision measurements. NNLO QCD for $n = 0$, and discuss sub-percent uncertainties.

2011: [Englert, Plehn, Schichtel, Schumann] Characterize *staircase* scaling for jet ratios without 'rigid' cuts. Show that also applies for pure QCD processes.

2011: [Bern, Diana, Dixon, FFC, Höche, et al.] $(Z\text{+jets})/(\gamma\text{+jets})$ for estimating missing transverse energy signals with jets. Estimated higher-order contributions not from scale variation, but by NLO fixed order vs. ME+PS comparisons.

2011: [Ask, Parker, Sandoval, Shea, Stirling] also studied Z to γ ratios in association with jets.

2012: [Gerwick, Plehn, Schumann, Schichtel] General study of scaling patterns in QCD for jet production. Distinguish Poisson and staircase scaling and associate them to hierarchical and democratic types of jet cuts respectively.

Quick review of literature (cont.)

2012: [Mangano, Rojo] Correlations of higher-order corrections at different energies exploited for producing precision energy-ratio observables. Theory systematics analyzed, constraining BSM.

2013: [Czakon, Mangano, Mitov, Rojo] Among detailed study of uncertainties for $t\bar{t}$ total cross section, NNLO+NNLL energy ratio studied and potential for g-PDF improvements explored.

2014: [Bevilacqua, Worek] Study of the $(t\bar{t}b\bar{b})/(t\bar{t}jj)$ ratio as a tool for Yukawa coupling measurements. Dedicated correlated, uncorrelated and *relative* scale dependence uncertainties explored. Show kinematical similarities and differences between both processes.

2014: [Bern, Dixon, FFC, Höche, Kosower, Ita, Maître] Dedicated NLO QCD jet-ratio study in $V+n$-jet production ($n \leq 5$). From universal features, extrapolation of differential cross sections shown for $n=6$.

2016: [Schulze, Soreq] $(t\bar{t}\gamma)/(t\bar{t}Z)$ ratio employed for stringent test of anomalous dipole operators. Impose kinematical correlations to improve ratio stability.

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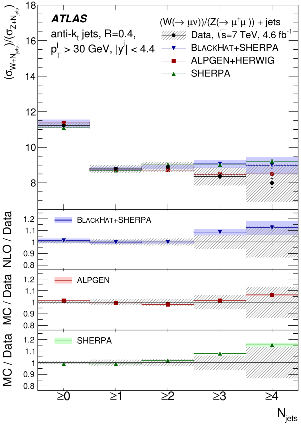
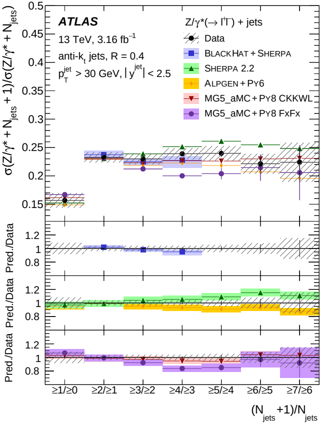
Vector Boson Production with Jets Ratios

arXiv:1408.6510 [hep-ex]

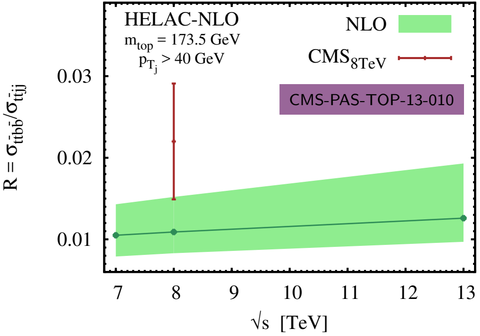


arXiv:1702.05725 [hep-ex]



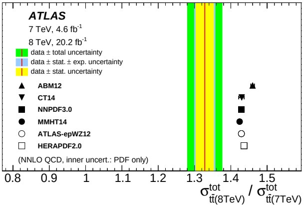
ATLAS and CMS have made extensive studies of multi-jet and V+jets processes at 7 TeV, including ratio studies.
13 TeV studies starting to appear (see for example [arXiv:1702.05725]).

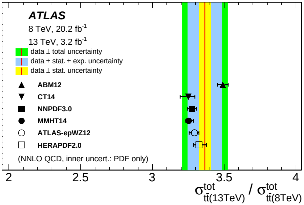
The $(t\bar{t}b\bar{b})/(t\bar{t}jj)$ Ratio



- From Bevilacqua and Worek [arXiv:1403.2046]
- Experimental result based on 19.6 fb^{-1} data set
- Systematic and statistical errors of same order
- Most of the systematic uncertainty related to mistag rate and b-tagging efficiency

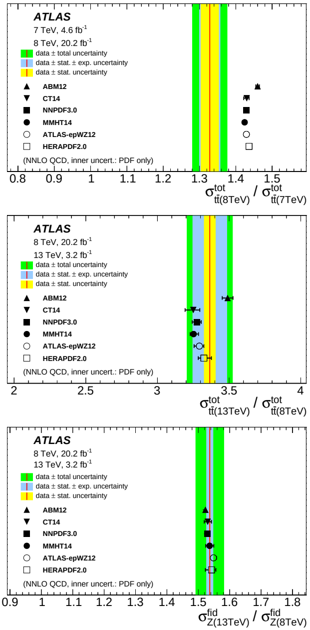
Energy Ratios at the LHC in $t\bar{t}$ Production





- Recent results by ATLAS [arXiv:1612.03636]
- The ratio 8TeV/7TeV shows a slight theory/data tension, a bit above 2σ
- Nevertheless, there is a good agreement for the 13TeV/8TeV ratio
- The PDF-set spread matches experimental uncertainty

Energy Ratios at the LHC in $t\bar{t}$ Production



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- The PDF-set spread matches experimental uncertainty
- To compare, we show the 13TeV/8TeV ratio for Z production

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PDF Uncertainties in Ratios

- PDF-uncertainty estimation in ratios has been performed consistently in the literature. Variations of sets in numerator and denominator should naturally be done in a correlated way.
- If processes involved are kinematically similar, important reductions in PDF dependence of the ratio is observed.
- Specifics of variations change. For example
  - Schulze and Soreq 2016 employ three different PDF sets.
  - FFC, Bern, et al 2014 employ only error set provided by MSTW.
  - Mangano and Rojo 2012 employ a 1000-replica set from NNPDF2.1.
  - Czakon et al 2013 systematically use all mayor PDF sets, together with the error sets they provide.
- It is important to notice the tensions that might appear from particular PDF sets, like for example observed in $t\bar{t}$ production with `ABM12`.

Scale Sensitivity in Ratios

- Regarding scales, the literature is much less consistent in the treatment of scale variations in ratios.
- How to handle central-scale choices of numerator and denominator?
- What variations considered in renormalization and factorization scales?
- Should uncertainties be propagated? Or do correlated/uncorrelated samples?
- More robust ways to estimate impact from higher-order corrections?

Scales in the $t\bar{t}b\bar{b}(\mu_1)/t\bar{t}jj(\mu_2)$ Ratio

- Same underlying basic $t\bar{t}$ process, but sizable jet-activity differences
- Bevilacqua and Worek 2014 explore stability under jet cuts

anti-k_T $R = 0.5$ jets with:

$$p_T^j > 40\ GeV\ ,\quad |y_j| < 2.5\ ,\quad \Delta R_{jj} > 0.5$$

and choosing $\mu_f = \mu_r = \mu_0$ with:

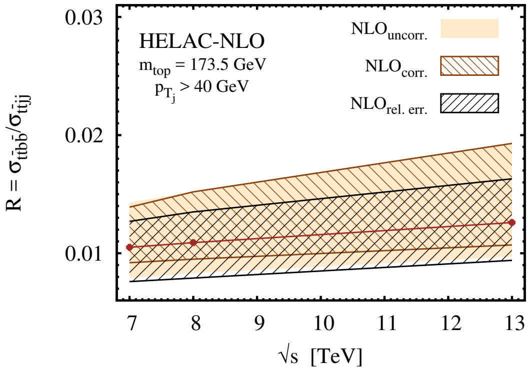
$$\mu_0^2(pp \to t\bar{t}b\bar{b}) = m_t\sqrt{p_T(b_1)p_T(b_2)}\ ,$$
$$\mu_0^2(pp \to t\bar{t}jj) = m_t^2\ .$$

Three approaches are explored for scale sensitivity of the ratio:

$$R(\mu_1, \mu_2) = \sigma^{\mathrm{NLO}}_{t\bar{t}b\bar{b}}(\mu_1)/\sigma^{\mathrm{NLO}}_{t\bar{t}jj}(\mu_2)$$

- *uncorrelated*: extremes from $(\mu_1/\mu_0, \mu_2/\mu_0) \in \{(2,2),(2,1),(2,0.5),(1,2),(1,1),(1,0.5),(0.5,2),(0.5,1),(0.5,0.5)\}$
- *correlated*: extremes from $(\mu_1/\mu_0, \mu_2/\mu_0) \in \{(2,2),(1,1),(0.5,0.5)\}$
- *relative error*: propagate independently numerator and denominator

Scales in the $t\bar{t}b\bar{b}(\mu_1)/t\bar{t}jj(\mu_2)$ Ratio



- Ratio computed at the 7, 8 and 13 TeV LHC
- Uncorrelated variations gives the most conservative estimate of uncertainty, chosen for exp. comparison
- In this case, the relative scale dependence on R is worse than the relative scale dependence of the cross sections.

Kinematical Correlations

Schulze and Soreq 2016 explored the ratios:

$$R_\gamma = \sigma^{\mathrm{NLO}}_{t\bar{t}\gamma} / \sigma^{\mathrm{NLO}}_{t\bar{t}} \quad \text{and} \quad R_Z = \sigma^{\mathrm{NLO}}_{t\bar{t}Z} / \sigma^{\mathrm{NLO}}_{t\bar{t}}$$

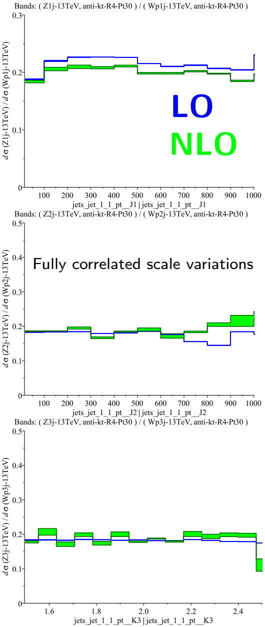
and argued that to ensure reduction of uncertainty special kinematical cuts should be imposed to the simpler denominator process $t\bar{t}$. This would ensure sampling similar $\langle \hat{E} \rangle$ in both numerator and denominator. They end up adding the cuts:

$$m_{t\bar{t}} > 470 \text{ GeV} \quad \text{for denominator in} \quad R_\gamma$$
$$m_{t\bar{t}} > 700 \text{ GeV} \quad \text{for denominator in} \quad R_Z$$

with this they estimate, using correlated scales:

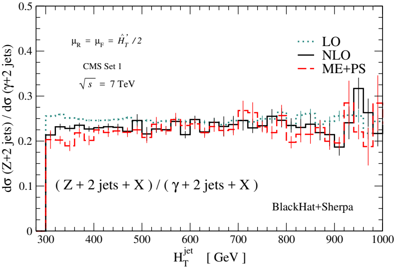
$$R_\gamma \times 10^{-3} = \begin{cases} 11.4^{-0.7\%}_{+0.7\%} & \text{at LO} \\ 12.6^{+3.1\%}_{-1.8\%} & \text{at NLO} \end{cases} \qquad R_Z \times 10^{-4} = \begin{cases} 2.27^{-1.7\%}_{+2.0\%} & \text{at LO} \\ 1.99^{-1.9\%}_{+2.8\%} & \text{at NLO} \end{cases}$$

Highly Correlated Processes



- Ratios $(Z + n \text{ jets})/(W^+ + n \text{ jets})$ $(n = 1, 2, 3)$
- Differential distributions in (log) of softest jet p_T
- Scale bands done in correlated fashion
- Percent-level sensitivity (tough to get numerically!)
- In any case, so far way tinier than achievable experimental uncertainties

The $Z(\to \nu\bar{\nu})$+jets/γ+jets Ratio



- Important tool for $\not{E}_T$+jet SM background estimation
- Ratio very stable over PS
- Scale sensitivity appears as tiny (few percent)
- CMS asks: Are theoretical uncertainties below 10%?

Bern, FFC, Höche et al 2011 by comparing fixed-order calculation to ME+PS results we gave a positive answer. Nevertheless, Ask, Stirling et al 2011 quoted indeed a 3% scale sensitivity in a similar calculation:

"we have used the same form of scale variation simultaneously in both the numerator and denominator cross sections... we argue that if we select Z and γ events for which the kinematics of the (colour-singlet) vector bosons and the jets are the same (with $p_T >> M_Z$), then the higher-order pQCD corrections to both cross sections should essentially be the same and should therefore largely cancel in the ratio."

Mangano and Rojo 2012 also argue for fully correlated scale variations for W/Z ratios, and addition in quadrature for other ratios involving less correlated processes.

Explicit Higher Orders

- Czakon et al 2013 look at energy ratios in $t\bar{t}$ with up to NNLO+NNLL QCD corrections.
- High correlation on higher-order terms expected between different energies.
- They add in quadrature parametric sources of errors and linearly the (correlated) scale sensitivity.
- Confirm the reliability of the scale-dependence uncertainty: NLO+NNLL result agrees well with NNLO+NNLL, while scale sensitivity in the ratio is decreased from 1% to about 0.3%!

Outlook

- Parametric uncertainties in cross-section ratios (PDFs, α_s, masses, etc) should be studied in correlated way and added in quadrature.
- Scale sensitivities in the ratios poses few riddles: (avoid all together? →See Stan's talk)
  - The choice of central scales for numerator and denominator (central topic of this Workshop) →See all talks!
  - After agreeing a grid for μ_f and μ_r variations, ratios involving closely associated processes should have correlated numerator-denominator scale variations, while more independent processes shall be treated by additions in quadrature
  - Scale choices with different functional forms should also be considered →See Nigel's talk #1
  - One might stabilize the ratio by inducing kinematical correlations (but don't overdo!)
  - Independent calculations, like based on merged parton showers, can give another estimate of higher-order uncertainties in ratios
  - NLO QCD results tend to be first numerically-reliable prediction. When feasible, one should explicitly compute higher-order corrections