ABC- of the Standard Model

These lectures are based on the lectures by:

G. Fogli (INFN- Bari) Main Source

Useful Text Books

- F. Halzen & Martin
  "Quarks and leptons"
- For Non- Abelian Gauge theories: Book by
  Cheng & Lee
- A Modern Introduction to Particle Physics
  by
  Fayyazuddin & Riazuddin

Introduction to SM :

Electromagnetic Int.

Scalar field

$$(\partial^\mu \partial_\mu + m^2)\varphi(x) = 0 \quad \text{KG equation.}$$

This is nothing but E-momentum relation

$$E^2 - \vec{P}^2 = m^2$$

$$p^\mu = +i\hbar \frac{\partial}{\partial x_\mu}.$$

classically $p^\mu \to p^\mu + eA^\mu \xrightarrow{\text{QM}} i\partial^\mu \to i\partial^\mu + eA^\mu$

$$(\partial^\mu \partial_\mu \varphi + m^2)\varphi(x) = -V\varphi(x) \quad \text{with}$$

$$V = -ie(\partial_\mu A^\mu + A_\mu \partial_\mu) - e^2 A^2$$

related to Couplings.

In non-rel. theory (PT) the Scattering amplitude for a spinless particle can be written as

$$T_{fi} = -i\int \varphi_f^*(x) V(x) \varphi_i(x) d^4x = i\int \varphi_f^*(x)\, ie\, (A_\mu \partial_\mu + \partial_\mu A^\mu)\, \varphi_i(x)\, d^4x - e^2\int \varphi_f^* A^2 \varphi_i(x) d^4x$$

self int.

Take $\int \varphi_f^*(x)\partial_\mu (A^\mu \varphi_i(x)) d^4x = -\int \partial_\mu(\varphi_f^*) A_\mu \varphi_i(x)$

Therefore, we have obtained

$$T_{fi} = -i\int J_\mu^{fi}(x) A^\mu(x) d^4x$$

$$J_\mu^{fi} = -ie(\varphi_f^* \partial_\mu \varphi_i - (\partial_\mu \varphi_f^*)\varphi_i)$$

This is electromagnetic Current.

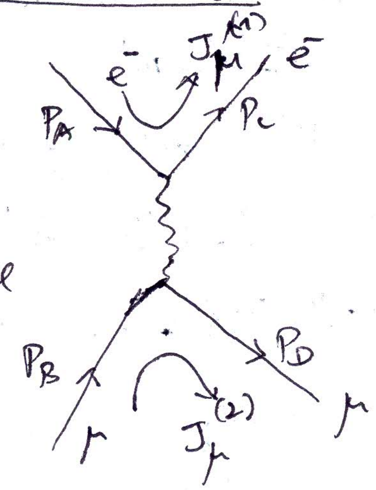
If incoming Scalar electron has mom P_i & outgoing has mom P_f.

$$\varphi_i = N_i e^{-iP_i x}, \quad \varphi_f = N_f e^{-iP_f \cdot x}$$

$$J_\mu^{fi} = -e N_i N_f (P_i + P_f)_\mu e^{i(P_f - P_i)\cdot x}$$

Spinless electron Scattering

Calculation: we have to identify A_μ with its source. This is performed through ~~e.m.~~ Maxwell equations.



$$\Box A_\mu = J_\mu^{(2)}$$

$$J_\mu^{(2)} = -e N_B N_D (P_D + P_B)_\mu e^{i(P_D - P_B)\cdot x}$$

$$\Box e^{iq\cdot x} = -q^2 e^{iq\cdot x}$$

$$A_\mu = -\frac{1}{q^2} J_\mu^{(2)} \; ; \quad q = P_D - P_B.$$

In Conclusion.

$$T_{fi} = -i \int J_\mu^{(1)} \left[-\frac{1}{q^2}\right] J^{\mu(2)}(x)\, d^4x$$

$$T_{fi} = -i N_A N_B N_C N_D (2\pi)^4 \delta^4(P_D + P_C - P_A - P_B) \mathcal{M}$$

$$\mathcal{M} = \left[ie(P_A + P_C)_\mu \left(-i\frac{g^{\mu\nu}}{q^2}\right)\left(ie(P_B + P_D)_\nu\right)\right]$$

This is invariant amplitude

(3) ▷ In $\mathcal{M}$: $(-ig^{\mu\nu}/q^2)$ is a propagator of photon exchanged b/w electrons and muon.

▷ The photon is virtual or "off-mass-shell": This charge particle carries the charge of γ but not the mass.

▷ each vertex contains a factor of e. & four vector index of current.

<u>Electromag. of spin-$\frac{1}{2}$ particle.</u>

e^- is $\frac{1}{2}$.

$$\psi(x) = u(p)e^{ip\cdot x} \quad : \text{Spinor}$$

This satisfies the DE. $(\gamma^\mu p_\mu - m)\psi = 0$

Again $p^\mu \to p^\mu + eA^\mu$.

Describes electron in the electromagnetic field

$$(\gamma^\mu p_\mu - m)\psi(x) = \gamma^0 V\psi(x) \quad ; \quad \gamma^0 V = -e\gamma^\mu A_\mu$$

This is used to make eq. relativistic invariant.

Again Scattering amplitude gives

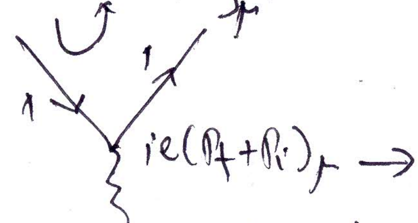
$$T_{fi} = -i\int \psi_f^\dagger(x)V(x)\psi_i(x)\,d^4x = ie\int \psi_f^\dagger(x)\gamma^0\gamma_\mu A^\mu \psi_i(x)\,d^4x$$

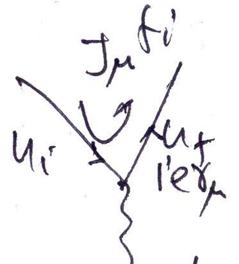
$$= -i\int J_\mu^{fi} A^\mu d^4x$$

$$J_\mu^{fi} = -e\bar{\psi}_f(x)\gamma_\mu\psi_i(x) = -e\bar{u}_f\gamma_\mu u_i e^{i(p_f-p_i)\cdot x}$$

$$\boxed{\bar{\psi} = \psi^\dagger\gamma^0}$$

(4) The vertex factor now is 4×4-matrix J_μ^{fi}





Gordon-decomposition of current shows that spin $\frac{1}{2}$ electron interacts via both its charge & mag mom

$$\bar{u}_f \gamma_\mu u_i = \frac{1}{2m} \bar{u}_f \left[(P_f+P_i)_\mu + i\sigma_{\mu\nu}(P_f-P_i)^\nu \right] u_i$$

$$\sigma_{\mu\nu} = \tfrac{1}{2}[\gamma_\mu, \gamma_\nu]$$

$$e^-\mu^- \to e^-\mu^-$$

$$T_{fi} = -i \int J_\mu^{(1)}(x) \left(-\frac{1}{q^2}\right) J^{\mu(2)}(x)\, d^4x$$

$$T_{fi} = -i(-e\bar{u}_C\gamma_\mu u_A)\left(-\frac{1}{q^2}\right)(-e\bar{u}_D\gamma_\mu u_B)(2\pi)^4\delta^4(P_A+P_B-P_C-P_D)$$

Therefore, the invariant amplitude is

$$-iM = ie(\bar{u}_C\gamma_\mu u_A)\left(\frac{-ig^{\mu\nu}}{q^2}\right)(ie\bar{u}_D\gamma_\nu u_B)$$

As e is spin $\frac{1}{2}$ particle, therefore, we have to estimate $|M|^2$ for all possible spin configuration

$$|M|^2 \Rightarrow |\bar{M}|^2 = \frac{1}{(2S_A+1)}\frac{1}{(2S_B+1)}\sum_{\text{spin}} |M|^2$$

$$M = -e^2\, \bar{u}(k')\gamma_\mu u(k)\, \frac{1}{q^2}\, \bar{u}(p')\gamma^\mu u(p)$$

$$|M|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} L^{m}_{\mu\nu}$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{S}(\bar{u}(k')\gamma^\mu u(k))(\bar{u}(k')\gamma^\nu u(k))^\dagger$$

(5)

$$\left(\bar{u}(k')\gamma^\nu u(k)\right)^\dagger = \bar{u}(k)\gamma^\nu u(k')$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{ss'} \underbrace{\bar{u}_\alpha^{s'}(k')\gamma_{\alpha\beta}}_{(\not{k}'+m)_{\delta\alpha}} \underbrace{\sum_s u_\beta^s \bar{u}_\gamma^s(k)}_{(\not{k}+m)_{\beta\gamma}} \gamma^\nu_{\gamma\delta} u_\delta^{s'}(k')$$

$$L_e^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu\right]$$

$$L_e^{\mu\nu} = 2\left[k'^\mu k^\nu + k'^\nu k^\mu - (k'\cdot k - m^2)g^{\mu\nu}\right]$$

$\downarrow$ $\underline{P', P, M}$

$$L^{\text{muon}}_{\mu\nu} = 2\left[P'_\mu P_\nu + P'_\nu P_\mu - (P'\cdot P - M^2)g_{\mu\nu}\right]$$

$\boxed{|M|^2, \quad \text{Product}}$

(5)

The V-A structure of weak int

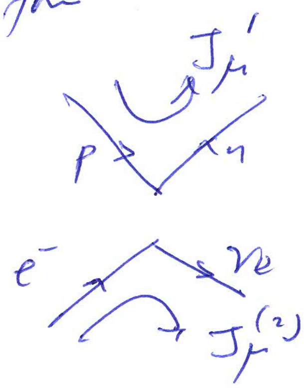
β-decay: $n \to p + e^- + \bar{\nu}_e$ T_{mean}: 880 Sec

μ-decay: $\mu \to e + \bar{\nu}_e + \nu_\mu$ $\tau = 2.2 \times 10^{-6}\,s$

π^--decay: $\pi^- \to \mu^- + \bar{\nu}_\mu$ $\tau = 2.6 \times 10^{-8}$ Sec

Fermi eq. of β-decay in (1932) is inspired by the structure of the e-mag. int. β-decay in crossed form

$$p + e^- \to n + \nu_e.$$



$$M = G\,(\bar{u}_n \gamma_\mu u_p)(\bar{u}_\nu \gamma^\mu u_e)$$

Fermi Coll.   Charged currents

<u>Parity violation</u>: Parity is not conserved in the weak interaction.

LH- neutrinos ν_L
RH. Antineutrino $\bar{\nu}_R$
Absence of ν_R & $\bar{\nu}_L$ shows parity is viol..

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) \quad \text{P-violation}$$

Not only parity is maximally violated in WI but also charge conjugation.

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to e^- \bar{\nu}_L)$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \bar{\nu}_R) \quad \text{CP- invariance}$$

$$f_L = \gamma^\mu \tfrac{1}{2}(1-\gamma_5)$$

$$f_R = \gamma^\mu \tfrac{1}{2}(1+\gamma_5)$$

⑦ $\mathcal{M}(p \to n + e^{+} + \nu_e) = \frac{G}{\sqrt{2}}\left(\bar{u}_n \gamma_\mu (1-\gamma_5) u_p\right)\left(\bar{u}_{\nu_e} \gamma^\mu (1-\gamma_5) u_e\right)$

$$\mathcal{M}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G}{\sqrt{2}}\left[\bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma_5) u_\mu\right]\left[\bar{u}_e \gamma^\mu (1-\gamma_5) u_e\right]$$

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} J^\mu J^\dagger_\mu, \quad J^\mu = \bar{u}_{\nu_e} \gamma^\mu \tfrac{1}{2}(1-\gamma_5) u_e$$

$$J^{\dagger\mu} = \bar{u}_e \gamma^\mu \tfrac{1}{2}(1-\gamma_5) u_{\nu_e}$$

Charge raising

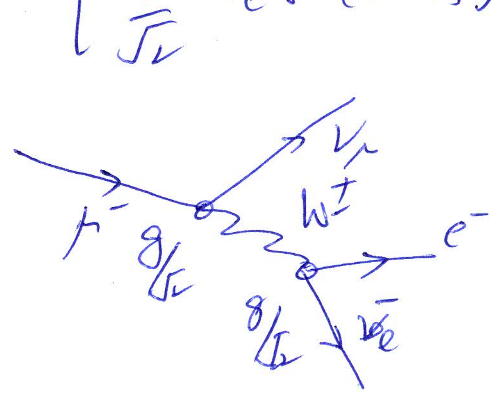
Charge lowering current

Interp. of G: a dimensional argument

electromag. & weak interaction comparison shows that ~~G~~ G has to be of dimension e^2/q^2. $\Rightarrow$ G has GeV^{-2}.

$$\mathcal{M}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{g}{\sqrt{2}}\left[\bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma_5) u_\mu\right] \frac{1}{M_W^2 - q^2} \left[\frac{g}{\sqrt{2}} \bar{u}_e \gamma^\mu (1-\gamma_5) u_{\nu_e}\right]$$

This is IVB theory



$q^2 \ll M_W^2$

$$\boxed{\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

Prop. disappears & int. is just point like

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Weak Neutral currents

$$\left.\begin{array}{l}\bar{\nu}_\mu \bar{e} \to \bar{\nu}_\mu \bar{e} \\ \nu_\mu N \to \nu_\mu N \\ \bar{\nu}_\mu N \to \bar{\nu}_\mu N\end{array}\right\}$$ Neutrino's detection through these three decays

These all are neutral currents.

$$\mathcal{M} = \frac{G_N}{\sqrt{2}} [\bar{u}_\nu \gamma_\mu (1-\gamma_5) u_\nu][\bar{u}_q \gamma^\mu (C_V - C_A \gamma_5) u_q]$$

G_N, C_V & C_A are new parameters.

$$J_\mu^{NC}(\nu) = \frac{1}{2}[\bar{u}_\nu \gamma^\mu \tfrac{1}{2}(1-\gamma_5) u_\nu]$$

$$J_\mu^{NC}(q) = [\bar{u}_q \gamma^\mu \tfrac{1}{2}(C_V^q - C_A^q \gamma_5) u_q]$$

Unlike charged currents, these are not purely V-A, ($C_V \neq C_A$). They have right-handed components. However neutrino's are LH

$C_V = C_A = \frac{1}{2}$.

→ In SM, C_V & C_A are all given in terms of $\sin^2\theta_W$

Weak Isospin and Hypercharge.

Charged Current

$$J_\mu = J_\mu^{(+)} = \bar{u}_{\nu e}\gamma^\mu\left(\tfrac{1}{2}\right)(1-\gamma_5)u_e = \bar{\nu}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)e$$
$$= \bar{\nu}_L\gamma^\mu e_L$$

$$J_\mu^\dagger = J_\mu^{(-)} = \bar{u}_e\gamma^\mu \tfrac{1}{2}(1-\gamma_5)u_{\nu e} = \bar{e}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\nu_e$$
$$= \bar{e}_L\gamma^\mu\nu_L.$$

Let's introduce a doublet

$$\chi_L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \qquad \tau^\pm = \tfrac{1}{2}(\tau_1 \pm i\tau_2)$$

stepup, down

$$J_\mu^{(+)} = \bar{\chi}_L\gamma_\mu\tau_+\chi_L, \quad J_\mu(-) = \bar{\chi}_L\gamma_\mu\tau_-\chi_L$$

Let's write a third current

$$J_\mu^{(3)} = \bar{\chi}_L\gamma_\mu\frac{\tau_3}{2}\chi_L = \tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L$$

Thus isospin triplet of weak currents can be written as

$$J_\mu^{(i)} = \bar{\chi}_L\gamma_\mu\tfrac{1}{2}\tau_i\chi_L$$

Corresponding charges are

$$T^i = \int J_0^i(x)\, d^3x \quad \text{Satisfies } SU(2)_L$$

algebra $[T^i, T^j] = i\varepsilon^{ijk}T^k$

T's are generators of new quantum #, the <u>weak isospin</u>

electromagnetic current contains both the left and right handed components

$$J_\mu^{em}(x) = -\bar{e}\gamma_\mu e = -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R$$

Current corresponding to e-m. int. can be written as

$$J_\mu = e\, J_\mu^{em} = e\,\bar{\psi}\gamma_\mu Q\psi.$$

Q is electric charge generator. Its value is -1 for $\bar{e}$. Q is a generator of $U(1)_{em}$ symmetry group of e-m. interactions.

<u>Gell-Mann - Nishijima relation</u>

$$Q = T^{(3)} + \frac{1}{2}Y$$

$T^{(3)}$: 3rd component of w. isospin generator

Y: new generator, the weak hyper-charge

$$J_\mu^{em} = J_\mu^{(3)} + \frac{1}{2}J_\mu^{(Y)}$$

The Y can be taken as a generator of new abelian group $U(1)_Y$, so that the complete symmetry group is $SU(2)_L \otimes U(1)_Y$ and $U(1)_{em}$ appears as a sub <u>group of it</u>.

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$$J_\mu^Y = 2J_\mu^{em} + 2J_\mu^{(3)}$$

$$= -2(\bar{e}_R \gamma_\mu e_R + \bar{e}_L \gamma_\mu e_L) - (\bar{\nu}_L \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L)$$

$$= -2\bar{e}_R \gamma_\mu e_R - 1\, \bar{\chi}_L \gamma_\mu \chi_L$$

$\Rightarrow$ Left handed doublet has hypercharge -1

& isospin singlet has hypercharge $= -2$.

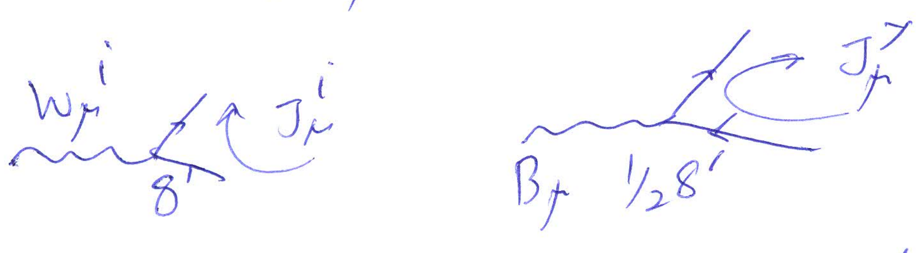
The basic e-m interaction

Let's develop QED from basic e-m interactions

$$\underbrace{-ie}_{\text{coupling}}\ \underbrace{j_\mu^{em}}_{\text{current}}\ \underbrace{A^\mu}_{\text{vector boson}}$$

Let's introduce isotriplet of vector bosons $W_\mu^{(i)}$ & singlet vector boson B_μ with coupling g & g' respectively, to describe $SU(2)_L \otimes U(1)_Y$.

$$-ig J_\mu^{(i)} W^{\mu(i)} - i g' \tfrac{1}{2} j_\mu^{em} J_\mu^Y B^\mu$$



$W_\mu^{(3)}$ & B_μ both correspond to kind of neutral currents

(12)

$$A_\mu = B_\mu \cos\theta_W + W_\mu^{(3)} \sin\theta_W \quad \text{massless}$$

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^{(3)} \cos\theta_W \quad \text{massive.}$$

weinberg angle.

This is a phenomenological parameter and can be determined from experiments

$$M^{-1}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} B_\mu \\ W_{3\mu} \end{pmatrix}$$

$$M = \begin{pmatrix} C\theta_W & S\theta_W \\ -S\theta_W & C\theta_W \end{pmatrix}$$

$$-ig J_\mu^{(3)} W^{\mu(3)} - ig' \tfrac{1}{2} J_\mu^{Y} B^\mu$$

$$= -i\left(g \sin\theta_W J_\mu^{(3)} + \tfrac{1}{2} g' \cos\theta_W J_\mu^{Y}\right) A^\mu$$

$$+ i\left(g \cos\theta_W J_\mu^{(3)} - \tfrac{1}{2} g' \sin\theta_W J_\mu^{Y}\right) Z^\mu$$

$$e J_\mu^{em} = e\left(J_\mu^3 + \tfrac{1}{2} J_\mu^{Y}\right)$$

$$g \sin\theta_W = g' \cos\theta_W = e$$

$$\tan\theta_W = g'/g$$

$$-i\left(g \cos\theta_W J_\mu^3 - \tfrac{1}{2} g' \sin\theta_W J_\mu^{Y}\right) Z^\mu$$

$$= -i \frac{g}{\cos\theta_W}\left(J_\mu^{(3)} - \frac{1}{2}\frac{g'}{g} \sin\theta_W \cos\theta_W J_\mu^{Y}\right) Z^\mu$$

$$= -i\, g/\cos\theta_W \left(J_\mu^3 - \sin^2\theta_W J_\mu^{em}\right) Z^\mu$$

$$= -i \frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu$$

Thus we have identified

$$\boxed{J_\mu^{NC} = J_\mu^{(3)} - \sin^2\theta_W \, j_\mu^{em}}$$ Coupled to Z_μ

with Coupling $\boxed{\dfrac{g}{\cos\theta_W}}$

<u>Effective current - current Int.</u>

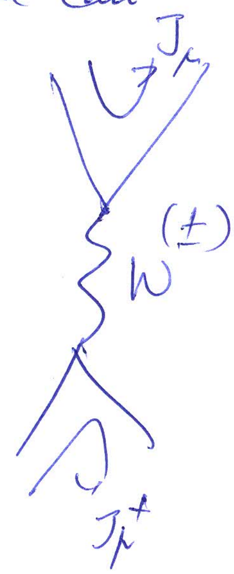
$$\mathcal{M}^{cc} = \frac{4G}{\sqrt{2}} J_\mu J^{\dagger\mu}$$

In isospin notation

$$J_\mu \equiv J_\mu^{(+)}(x) = \bar{\chi}_L \gamma_\mu \tau_+ \chi_L$$

Introducing the charge vector boson, we can write the interaction as

$$-i \frac{g}{\sqrt{2}} \left(J^\mu W_\mu^{(+)} + J_\mu^\dagger W_\mu^{(-)} \right]$$



This leads to

$$\mathcal{M}^{cc} = \left(\frac{g}{\sqrt{2}} J_\mu \right) \left(\frac{1}{M_W^2} \right) \left(\frac{g}{\sqrt{2}} J^{\dagger\mu} \right)$$

At low q^2

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

(14) Just like for NC

$$\mathcal{M}^{NC} = \left(\left(\frac{g}{\cos\theta_W}\right) J_\mu^{NC}\right) \frac{1}{M_Z^2} \left(\frac{g}{\cos\theta_W} J^{\mu\, NC}\right)$$

$$= \frac{4G}{\sqrt{2}}\, 2\rho\, J_\mu^{NC} J^{\mu\, NC}$$

$$\boxed{\rho \frac{G}{\sqrt{2}} = \frac{g^2}{8 M_Z^2 \cos^2\theta_W}}$$

Comparing expressions of charged & neutral currents, we will find

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$$

SM: predicts $\rho = 1$, Confirmed with small ~~error~~ errors in experiments.

(15) <u>Gauge Symmetries</u>

These will be discussed in Lag. framework.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \quad , \quad L = T - V$$

To extend it to Continuous System $\varphi = \varphi(x,t)$

$$L(q_i, \dot{q}_i, t) \rightarrow \mathcal{L}(\varphi, \partial_\mu \varphi, x^\mu) \quad ; \; L = \int \mathcal{L} \, d^3x$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial x_\mu}\right)}\right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

<u>Relation b/w Lag. & Feynman Rules</u>

3. step process.

1) Associate various terms in the Lag, a set of prop. & vertex factors

2) The prop. are determined by quadratic terms in fields i.e terms like φ^2, $\bar{\psi}\psi$, etc

3) The other terms are associated to interaction vertices.

<u>Start</u> $\mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$

$\psi \rightarrow e^{i\alpha}\psi(x)$: with real Const $\mathcal{L}' \rightarrow \mathcal{L}$.

Phase transformations $U(\alpha) = e^{i\alpha}$ forms a unitary abelian gauge.

<u>Noether</u>: To every symmetry, there is a conserved quantity. This $U(\alpha)$ leads to the Conservation of <u>'Generalized Charge'</u>

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$\partial \mathcal{L} = 0 \Rightarrow$
$\partial^\mu J_\mu = 0 \qquad J_\mu = ie\frac{1}{2}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\psi - \bar{\psi}\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})}\right)$

$$= -e\bar{\psi}\gamma_\mu \psi$$

$$Q = \int J^0(x)\, d^3x$$

This is Conserved electromag. charge.

<u>global phase transformation</u>

<u>U(1) local gauge transformations</u>

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

The Lag. is not invariant because of $\partial\psi$ term. In order to preserve the invariance, we have to re-write the derivative such that it Compensate the change.

$$\partial^\mu \to D^\mu = \partial^\mu - ieA^\mu$$

$$A^\mu \to A'^\mu = A^\mu + \frac{1}{e}\partial^\mu \alpha(x)$$

<u>Assignment</u> check That $D^\mu\psi \to e^{i\alpha(x)} D^\mu \psi(x)$

Therefore Lag. is invariant if expressed in terms of the Covariant derivatives.

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + e\bar{\psi}\gamma_\mu\psi A^\mu$$

<u>Lag. for QED</u>

Therefore, we can derive QED from the requirement of local gauge invariance.

(17) Regarding photon field, we have to add to the Lag. a term corresponding to k.E. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ s.t.

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + e\bar{\psi}\gamma_\mu\psi A^\mu - \left(\frac{1}{4}\right) F^{\mu\nu} F_{\mu\nu}$$

<u>observation</u> we can not add a term like $\frac{1}{2}M^2 A_\mu A^\mu$.

b/c it spoils the gauge invariance

<u>Non. Abelian gauge theory</u>

$$\mathcal{L}_0 = \bar{q}_i (i\gamma_\mu \partial^\mu - m) q_i \qquad i = 1,2,3$$

$$q(x) \to U q(x) = e^{i\alpha_a(x) T_a} q(x) \quad (a = 1, \dots, 8)$$

U is arbitrary 3x3 matrix. T_a's are the generators of $SU(3)$

$$[T_a, T_b] = i f_{abc} T_c$$

↳ structure functions of $SU(3)$

~~Let~~ Let's repeat steps of abelian case

$$q(x) \to [1 + i\alpha_a(x) T_a] q(x)$$

$$\partial^\mu q \to (1 + i\alpha_a T_a)\partial^\mu q(x) + i T_a q \partial^\mu \alpha_a$$

Again introduce a gauge field

$$G_a^\mu \to G_a^\mu - \frac{1}{g}\partial^\mu \alpha_a$$

$$D^\mu \to \partial^\mu + i \underbrace{g}_{\text{Coupling Const}} T_a G_a^\mu$$

$$\mathcal{L}_0 \to \mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m) q - g(\bar{q}\gamma_\mu T_a q) G_a^\mu$$

(18) Under non-Abelian gauge transformations. this is not gauge invariant Lag.

$$(\bar{q}\gamma_\mu T_a q) \to \bar{q}\gamma_\mu T_a q + i\alpha_b \bar{q}\gamma_\mu (T_a T_b - T_b T_a) q$$
$$\to \bar{q}\gamma_\mu T_a q + f_{abc}\alpha_b \bar{q}\gamma_\mu T_c q$$

In order to have gauge invariance, we have to re-write the field as

$$G_a^\mu \to G_a^\mu - \frac{1}{g}\partial^\mu \alpha_a - f_{abc}\alpha_b G_c^\mu$$

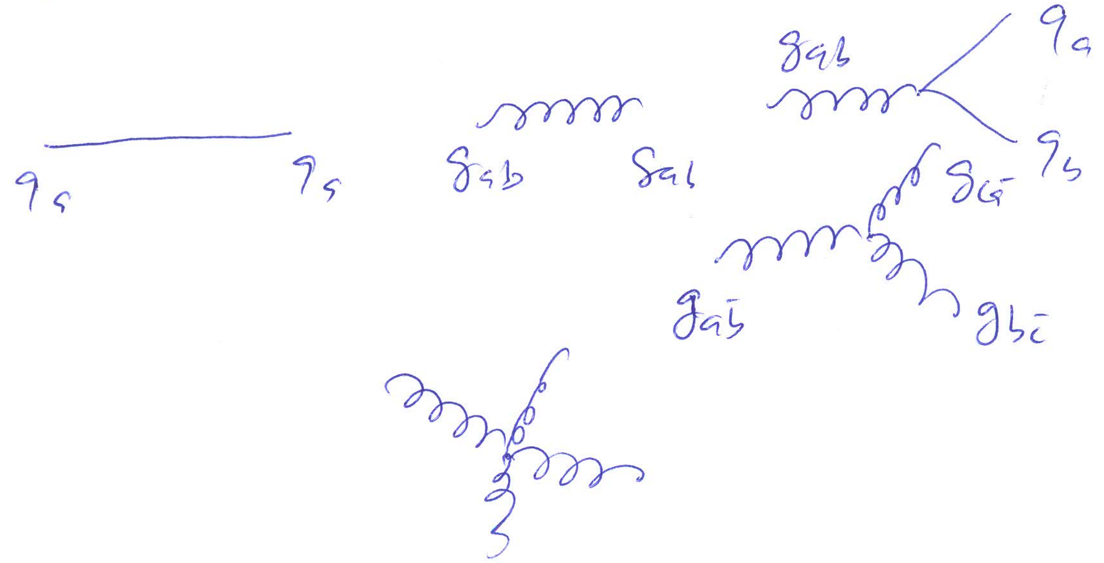
Adding to $\mathcal{L}$ the gauge invariant k.E term, we have $\mathcal{L} = \bar{q}(i\gamma_\mu \partial^\mu - m)q - g(\bar{q}\gamma_\mu T_a q)G_\mu^a - G_{\mu\nu}^a G_a^{\mu\nu}$

where

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f_{abc} G_\mu^b G_\nu^c$$

Colored quark q, with gauge boson G_μ & Coupling is G.

$$\mathcal{L} = \text{"}qq\text{"} + \text{"}G^2\text{"} + g\,\text{"}qqG\text{"} + g\,\text{"}G^3\text{"} + g^2\,\text{"}G^4\text{"}$$



(19) Spontaneous symmetry breaking

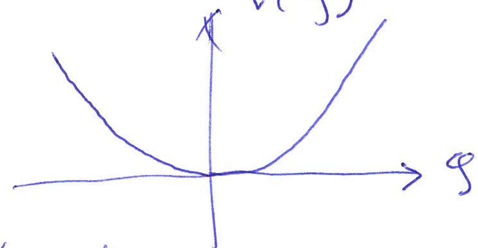
Consider a Lag. of a self interacting scalar field

$$\mathcal{L} = T - V = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \left(\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4\right)$$

$\lambda > 0$

This has discrete reflection symmetry $\varphi \to -\varphi$.

Two possibilities $\mu^2 > 0$



The Ground state is the one for which $\varphi = 0$. This solution satisfies the reflection symmetry.

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

$$V(\varphi) = 0 \Rightarrow \frac{1}{2}\varphi^2\left(\mu^2 + \frac{1}{2}\lambda\varphi^2\right) = 0$$

*

$$\Rightarrow \varphi = 0, \quad \varphi = \sqrt{-\frac{2\mu^2}{\lambda}}$$

Even $\frac{dV}{d\varphi} = 0 \Rightarrow$?

$$\frac{dV}{d\varphi} = \mu^2\varphi + \lambda\varphi^3, \quad \frac{d^2V}{d\varphi^2} = \mu^2 + 3\lambda\varphi^2$$

$$\varphi^2 = -\frac{\mu^2}{\lambda} \qquad = \mu^2 - 3\lambda\left(\frac{\mu^2}{\lambda}\right)$$

$\frac{d^2V}{d\varphi^2} < 0$, maxima

for $\varphi = 0$,

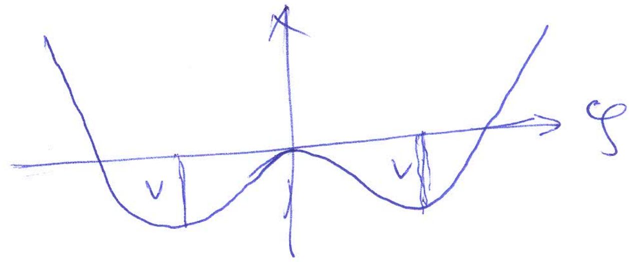
$\frac{d^2V}{d\varphi^2} > 0$, min.

(20) For $\mu^2 < 0$

$$\frac{\partial V}{\partial \varphi} = 0 \Rightarrow \varphi = 0$$

$$\varphi = \pm v$$

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$



Now $\varphi = 0$ is extremum but not minimum.

Again ~~than~~ there is a reflection symmetry and this does not lost if we choose any φ i,e $\varphi = +v$, or $-v$.

Let's choose $\varphi_0 = v$ & expand φ around this minimum

$$\varphi(x) = v + \eta(x)$$

$$\mathcal{L}' = \frac{1}{2}\partial_\mu \eta \, \partial^\mu \eta - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \text{Const}$$

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

Higher order η terms represents interaction with itself

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$\mathcal{L}$ and $\mathcal{L}'$ describes the same physics.

solving $\mathcal{L}'$ is the right choice. we apply the perturbative approach and calculate fluctuations around the vacuum.

<u>what about reflection sym.</u>

In $\mathcal{L}$ the symmetry is manifest, in $\mathcal{L}'$ it is hidden.

we find the same result for $\varphi = v$ & $\varphi = -v$.

spontaneous symmetry breaking has occured, which has generated the mass of the particle.

<u>spontaneous breaking of global gauge symmetry</u>

$$\mathcal{L} = (\partial_\mu \varphi)^*(\partial^\mu \varphi) - \mu^2 \varphi^* \varphi - \lambda(\varphi^*\varphi)^2$$

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 \pm i\varphi_2)$$

$$\mathcal{L}' = \mathcal{L}\ , \quad \varphi \to e^{i\alpha}\varphi$$

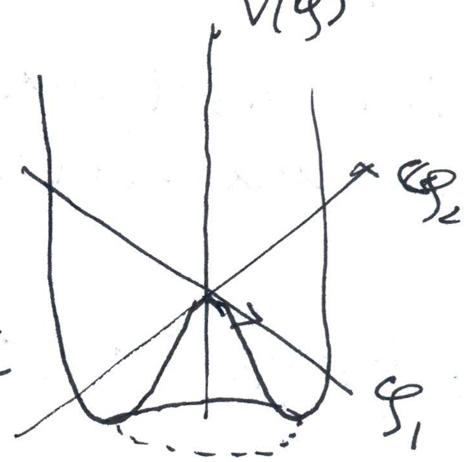
for case $\lambda > 0$, & $\mu^2 < 0$, we can rewrite $\mathcal{L}$ in terms of φ_1 & φ_2

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_1)^2 - \frac{1}{2}(\partial_\mu \varphi_2)^2 - \frac{1}{2}\mu^2(\varphi_1^2 + \varphi_2^2)$$
$$-\frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2$$

The pot. now is the function of φ_1 & φ_2

$$V = V(\varphi_1, \varphi_2)$$

For $\mu^2 < 0$, there is now a circle of minima $V(\varphi)$ in φ_1, φ_2 plane.



$$\varphi_1^2 + \varphi_2^2 = v^2$$

$$v^2 = -\mu^2/\lambda$$

without loss of generality, choose

$$\varphi_1 = v, \quad \varphi_2 = 0$$

as minima of $V(\varphi)$ & expand $\mathcal{L}$ around the vacuum in terms of $\eta(x)$ & $\xi(x)$ through substitution

$$\varphi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\xi(x)\right)$$

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 + \mu^2\eta^2 + \text{Const} + O(\eta^3) + O(\xi^3)$$

This has the form $-\frac{1}{2} m_\eta^2 \eta^2$

$$\Rightarrow \boxed{m_\eta = \sqrt{-2\mu^2}}$$

K.E. term for $\xi(x)$. There is no mass term corresponding to $\xi(x)$.

<u>Goldston theorem</u>

states that massless scalar field occure whenever a continuous symmetry of physical system is spontaneously broken.

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Higgs mechanism

Consider now the spontaneous breaking of local gauge symmetry.

$$\varphi(x) \to e^{i\alpha(x)}\varphi(x)$$

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 \pm i\varphi_2)$$

$$\mathcal{L} = (\partial_\mu \varphi^*)(\partial^\mu \varphi) - \mu^2(\varphi^*\varphi) - \lambda(\varphi^*\varphi)^2$$

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$$

$$A^\mu = A^\mu + \frac{1}{e}\partial^\mu \alpha(x)$$

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\varphi^*(\partial^\mu - ieA^\mu)\varphi - \mu^2|\varphi|^2 - \lambda|\varphi^2|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$\mu^2 > 0$ QED Lag. for charged scalar field of mass 'm' with addition of φ^4 term.

Take $\mu^2 < 0$ As we would like to generate mass through spontaneous symmetry breaking.

$$\varphi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

mass term

mass term

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A_\mu A^\mu$$

$$- evA_\mu\partial^\mu\xi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{Int. terms}$$

strange off-diagonal term.

The particle spectrum in $\mathcal{L}'$ contains

- massless Goldstone boson $m_\xi = 0$
- A massive scalar field $\eta(x)$: $m_\eta = \sqrt{-2\mu^2}$ $= \sqrt{2\lambda v^2}$
- A massive vector field $A_\mu(x)$ $m_A = ev.$

we have a massive vector field but we still have the occurance of GB.

The # of Dof of A_μ increases from 2 to 3. The Dof for φ remains the same.

$\mathcal{L}'$ has extra D.o.f. Which field in $\mathcal{L}'$ is unphysical. A more appropriate question is: Can we make use of gauge invariance to find a particular gauge transformation that eliminates one Dof from $\mathcal{L}$.

$$\varphi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \simeq \frac{1}{\sqrt{2}}[v+\eta(x)]e^{i\xi(x)/v}$$

This suggest that we use a different specific set of field in original Lagrangia

$$h(x),\ \theta(x),\ A_\mu(x)$$

<u>Assumption</u>

$$\varphi(x) \to \frac{1}{\sqrt{2}}(v + h(x) + i\theta(x)) \simeq \frac{1}{\sqrt{2}}(v+h(x))e^{i\theta(x)/v}$$

$$A^\mu(x) \to A^\mu(x) + \frac{1}{e}\partial^\mu \theta(x)$$

(25) This corresponds to specific choice of the gauge. chosen such a way that it makes $h(x)$ to be real.

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu - \lambda v h^3 - \frac{1}{4}\lambda v h^4 + \frac{1}{2} e^2 v^2 A_\mu A^\mu h^2 + e^2 v A_\mu A^\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

The Goldstone boson is dis appeared. The extra DoF is superious, it corresponds to the freedom of making a gauge transformation.

Two massive particles $h(x)$ & $A_\mu(x)$.

$h(x)$: Higgs field

Unwanted massless GB has been turned into the log. polarization of the massive vector bosons.

SSB of local SU(2) gauge symmetry

$$\mathcal{L} = (\partial_\mu \varphi^\dagger)(\partial^\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda(\varphi^\dagger \varphi)^2$$

$\varphi(x)$ is SU(2) doublet of complex fields

$$\varphi(x) = \begin{pmatrix} \varphi_\alpha \\ \varphi_\beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

$$\varphi \to \varphi' = e^{i\alpha_a \tau_a/2} \varphi$$

$$\partial_\mu \to D_\mu = \partial_\mu + i g \frac{\tau_a}{2} W_\mu^a$$

$$\varphi(x) \to \varphi'(x) = \left(1 + i\alpha_a(x) \tau_a/2\right) \varphi(x)$$

$$W_\mu \to W_\mu - \frac{1}{g}\partial_\mu \vec{\alpha} - \vec{\alpha} \times \vec{W}_\mu$$ ← Because of non-Abelian

$$\mathcal{L} = \left[\partial_\mu \varphi + i g \tfrac{1}{2} \vec{\tau}.\vec{W}_\mu \varphi\right]^\dagger \left[\partial^\mu \varphi + \tfrac{i}{g} i g \tfrac{1}{2} \vec{\tau}.W^\mu \varphi\right]$$

$$- V(\varphi) - \frac{1}{4}\vec{W}_{\mu\nu}\cdot \vec{W}^{\mu\nu}$$

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi - \lambda(\varphi^\dagger\varphi)^2$$

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g \vec{W}_\mu \times \vec{W}_\nu$$

↑ Non Abelian nature.

Again $\lambda > 0$, $\mu^2 < 0$.

$$\varphi^* \varphi = \frac{1}{2}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) = -\frac{\mu^2}{\lambda}$$

Special choice

$$\varphi_1 = \varphi_2 = \varphi_4 = 0$$

$$\varphi_3^2 = -\frac{\mu^2}{\lambda} = v^2$$

This choice introduces the spontaneous breaking of SU(2) symmetry. Let's expand around the specific vacuum.

$$\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Following same approach as used for U(1)

$$\varphi(x) = \frac{1}{\sqrt{2}} e^{i\vec{\tau}.\vec{\theta}(x)/v}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

(27)

The tree gauge bosons are now massive

$$\left| i g \frac{1}{2} \vec{T} \cdot W_\mu \varphi_0 \right|^2 = \frac{g^2}{8} \left| \begin{pmatrix} W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{g^2 v^2}{8} \sum_i \left(W_\mu^i \right)^2$$

we find their common mass to be

$$\boxed{M_W = \frac{1}{2} g v}$$

Theory characterized by

* $h(x)$ field
* 3 - massive vector fields.
* Goldstone boson has been eaten up by the gauge field when it became massive.

The Salam – Weinberg SM

Revisting EW – Interactions

- EM amplitude has been described as

$$i e J_\mu^{em} A^\mu = -ie(\bar{\psi}\gamma_\mu Q \psi) A^\mu$$

$\hookrightarrow$ Generator of $U(1)_{em}$.

Free field Lag. was $\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi$

$\downarrow$

under $U(1)_{em}$ gauge transformation.

$$\psi \to \psi' = e^{i\alpha(x) Q} \psi .$$

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi}_{\text{K.E \& mass of } \psi} - \underbrace{e(\bar{\psi}\gamma_\mu Q \psi) A^\mu}_{\text{Interaction}} - \underbrace{\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{K.E of } A_\mu}$$

In order to include electroweak Interactions, we have to introduce interactions due to $SU(2)_L \times U(1)_Y$.

$SU(2)_L$: Coupling of isotriplet of LH weak current $J_\mu^{(i)}$ with vector boson $W_\mu^{(i)}$.

$$-ig \vec{J}_\mu \cdot \vec{W}^\mu = -ig \bar{\chi}_L \gamma_\mu \vec{T} \cdot \vec{W}^\mu \chi_L$$

$U(1)_Y$: Hypercharge Current. Coupled to V.B. B_μ

$$-ig' \frac{1}{2} J_\mu^Y B^\mu = -ig' \bar{\psi} \gamma_\mu \frac{1}{2} Y \psi B^\mu$$

Transformation properties of LH & RH fields are then

$$\chi_L \to \chi_L' = e^{i\vec{\alpha}(x)\cdot\vec{T} + i\beta(x) Y} \chi_L ; \quad \psi_R \to \psi_R' = e^{i\beta(x) Y} \psi_R$$

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<u>Example</u> In case of $(\nu_e\ \bar{e})$ lepton pair, we have

$$\chi_L = \begin{pmatrix} \nu_e \\ \bar{e} \end{pmatrix}_L, \quad T = \frac{1}{2} \text{ (isodoublet) } Y = -1$$

$$\psi_R = e_R \qquad \text{isosinglet } T=0, \; Y=-2$$

Electromagnetic interaction is embeded in $SU(2)_L \otimes U(1)_Y$. being

$$Q = T^{(3)} + \frac{1}{2}Y \quad \text{and then} \quad j_\mu^{em} = J_\mu^{(3)} + \frac{1}{2} j_\mu^Y$$

The neutral current of $SU(2)_L \otimes U(1)_Y$ interaction can be re-written by introducing $W_\mu^{(3)}$ & B_μ interms of A_μ & Z_μ.

$$-ig J_\mu^3 W^{\mu,3} - ig' \frac{1}{2} j_\mu^Y B^\mu$$

$$= -i\left(g \sin\theta_W J_\mu^{(3)} + \frac{1}{2} g' \cos\theta_W j_\mu^Y\right) A^\mu$$

$$\cancel{-ig}\left(-ig \cos\theta_W J_\mu^{(3)} - \frac{1}{2} g' \sin\theta_W j_\mu^Y\right) Z^\mu$$

$$= -ie\, j_\mu^{em} A^\mu - i \frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu$$

$$\Rightarrow \quad J_\mu^{NC} = J_\mu^{(3)} - \sin^2\theta_W\, j_\mu^{em}$$

$$\text{and} \quad g \sin\theta_W = g' \cos\theta_W = e.$$

From the requirement of gauge invariance under $SU(2)_L \otimes U(1)_Y$ it is possible to derive the EW gauge invariant Lagrangian. In case of (ν_e $\bar{e}$) lepton pair

$$\mathcal{L}_1 = \bar{\chi}_L \gamma_\mu [i\partial^\mu - g\tfrac{1}{2}\vec{\tau}.\vec{W}^\mu - g'(-\tfrac{1}{2})B^\mu]\chi_L$$
$$+ \bar{e}_R \gamma_\mu [i\partial^\mu - g'(-1)B^\mu] e_R - \underbrace{\tfrac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu}}_{\text{K.E.s, self int. of W}} - \underbrace{\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{\text{K.E. of } B_\mu}$$

values of Hyperchage is explicitly inserted.

- $\mathcal{L}$ neutral: Gauge invariance reqs. the Lag. to be neutral i.e. it must transform as singlet under $U(1)_{em}$ in order to imply charge conservation.

- <u>Fermion mass</u>: $\mathcal{L}_1$ describes massless gauge boson. Mass term for the gauge boson are not. gauge invariant and can not be added. The same is true for fermion mass, e.g.,

$$-m_e\bar{e}e = -m_e\bar{e}[\tfrac{1}{2}(1-\gamma_5) + \tfrac{1}{2}(1+\gamma_5)]e$$
$$= -m_e(\bar{e}_R \underbrace{e_L}_{\text{member of isodoublet}} + \bar{e}_L \underbrace{e_R}_{\text{member of isosinglet}})$$

This violates gauge invariance.

Fermion Mass term is introduced by <u>SSB</u>..

(31)

2. Higgs Field

Introduce Higgs mechanism in such a way that $W^{\pm}, Z^{\circ}$ are massive but A_μ remains massless.

This is obtained by Adding $\mathcal{L}_2$ to $\mathcal{L}_1$ and $\mathcal{L}_2$ is $SU(2)_L \times U(1)_Y$ invariant

$$\mathcal{L}_2 = \left|\left(i\partial_\mu - g\vec{T}\cdot W_\mu - g'\tfrac{1}{2}YB_\mu\right)\varphi\right|^2 - V(\varphi)$$

(MSM Weinberg 1967)

Where

$$\varphi(x) = \begin{pmatrix} \varphi^+ \\ \varphi^\circ \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

is an isospin doublet with weak Hypercharge $Y = 1$

$V(\varphi)$ is usual potential.

$\lambda > 0$, $\mu^2 < 0$. Choosing vacuum expectation value

$$\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$T = \tfrac{1}{2},\ Y = 1,\ Q = 0$$

This breaks $T^{(3)}$ and Y, but leaves Q unbroken

$$Q\varphi_0 = 0 \Rightarrow \varphi_0 \Rightarrow \varphi_0' = e^{i\alpha(x)Q}\varphi_0 = \varphi_0$$

32 $\Rightarrow$ Since φ_0 is still left invariant by the subgroup of a gauge group, then the gauge boson associated with this group left massless. (Photon)

$\Rightarrow$ This is a case of $U(1)_{em}$ with the photon expected to be massless after the occurance of SSB.

$\Rightarrow$ The other three bosons associated with $SU(2)\times U(1)_Y$ will become massive and identified as $W^\pm$, Z°.

Masses of Gauge bosons

This comes from $\mathcal{L}_2$ when a usual shift is made

$$\varphi(x) \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} = \varphi_0 + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h(x) \end{pmatrix}$$

From terms Associated with φ_0

$$\left|\left(ig\frac{1}{2}\vec{\tau}.\vec{W}_\mu - ig'\frac{1}{2}B_\mu\right)\varphi_0\right|^2 = \frac{1}{8}\left|\begin{pmatrix} gW_\mu^{(3)}+g'B_\mu & g(W_\mu^{(1)}-iW_\mu^{(2)}) \\ g(W_\mu^1+iW_\mu^2) & -gW_\mu^{(3)}+g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

(22)

$$= \left(\tfrac{1}{2}vg\right)^2 W_\mu^{(+)} W^{\mu(-)} + \tfrac{1}{2}v^2 \underset{1\times 2}{\begin{pmatrix} W_\mu^{(3)} & B_\mu \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} B^\mu \\ W^{(3)\mu} \end{pmatrix}}$$

Comparing with

$$M_W^2 W_\mu^{(+)} W^{\mu(-)}$$

$$\Rightarrow \boxed{M_W = \tfrac{1}{2}vg}$$

$$\frac{1}{8v^2}\left(g^2 W_\mu^{(3)} W^{\mu(3)} - 2gg' W_\mu^{(3)} B^\mu + g'^2 B_\mu B^\mu\right)$$

$$= \underbrace{\frac{1}{8}}_{\text{eigen values}} v^2 \left(g W_\mu^{(3)} - g B_\mu\right)^2 + \underbrace{0}_{\text{eigen values}} \left(g W_\mu^{(3)} + B_\mu\right)^2$$

This shows that the eigen value of the the Combination $g W_\mu^{(3)} + B_\mu$ is zero. The orthogonal Combination $g W_\mu^{(3)} + B_\mu$ has eigen value non-zero. This is identified as Z-boson orthogonal to photon.

$$\tfrac{1}{2} M_Z^2 Z_\mu Z^\mu + \tfrac{1}{2} M_A^2 A_\mu A^\mu$$

Normalizing the fields

$$A_\mu = \frac{g' W_\mu^{(3)} + g B_\mu}{\sqrt{g^2 + g'^2}}$$

$$Z_\mu = g' W_\mu^{(3)} - g B_\mu \Big/ \sqrt{g^2 + g'^2}$$

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$M_A = 0,$

$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$

But $g \sin\theta_W = g' \cos\theta_W = e$

$\tan\theta_W = \frac{g'}{g}$

$A_\mu = B_\mu C_W + W_\mu^{(3)} S_W$

$Z_\mu = - B_\mu S_W + W_\mu^{(3)} C_W$

$\boxed{M_W = - C_W M_Z}$

The two vector bosons are different, and appears an the effect of mixing.

Also to have numerical value

$\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$

$\frac{1}{2v^2} = g^2 / 8M_W^2 = \frac{G}{\sqrt{2}}$

$\boxed{v = 246 \text{ GeV}}$

$M_W = \frac{37.3}{\sin\theta_W} \text{GeV} = \frac{74.6}{\sin 2\theta_W} \text{ GeV}$

35

Fermion masses

$-m\bar{\psi}\psi$ is not gauge invariant. But the term corresponding to fermion mass will come from $\mathcal{L}_3$ that couples to Higgs sector.

$$\mathcal{L}_3 = -G_e\left[(\bar{\nu}_e \;\; \bar{e})_L \begin{pmatrix}\varphi^{(+)}\\ \varphi_0\end{pmatrix} e_R + \bar{e}_R(\varphi^{+} \;\; \bar{\varphi}^{0})\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]$$

$$\varphi(x) \Rightarrow \frac{1}{\sqrt{2}}\varphi_0 + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h(x)\end{pmatrix}$$

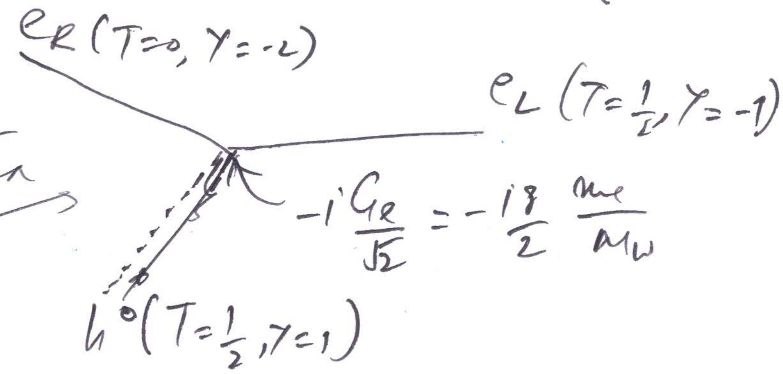
$$\mathcal{L}_3 = -\frac{1}{\sqrt{2}}v(\bar{e}_L e_R + \bar{e}_R e_L)G_e - \frac{1}{\sqrt{2}}G_e(\bar{e}_L e_R + \bar{e}_R e_L)h$$

$$= -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh$$

$$m_e = \frac{1}{\sqrt{2}}vG_e$$

this should be small

$he\bar{e}$ vertex



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Higgs Mass

This cannot be predicted from theory.

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi - \lambda(\varphi^\dagger \varphi)^2$$

First two terms give

$$m_h^2 = 2v^2\lambda$$

~~Since λ is large~~.

Since v is large, large value of m_h corresponds to large value of λ. A meaningful perturbative approach requires $\lambda < 1$, and so Higgs mass can not be larger than <u>few 100</u> GeV

Also it should be two small ~ 10 GeV, otherwise the corrections would washout minimum at $v \neq 0$.

<u>Thanks to LHC.</u>

~~126~~ 125.5 GeV

There may be some errors in signs, etc.

Please reproduce the things carefully.