Basics of Quantum Chromodynamics

(Two lectures)

Faisal Akram

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Outlines

- A brief introduction of QCD
  - Classical QCD Lagrangian
  - Quantization
  - Green functions of QCD and SDE’s
- Perturbative QCD
  - Perturbative calculation of QCD Green functions
  - Feynman Rules of QCD
  - Renormalization
  - Running of QCD coupling (Asymptotic freedom)
- Non-Perturbative QCD
  - Confinement
  - QCD phase transition
  - Dynamical breaking of chiral symmetry

Elementary Particle Physics Today

Elementary particles:

Quarks	Leptons	Gauge Bosons	Higgs Boson
(can interact through strong interaction)	(cannot interact through strong interaction)	(mediate interactions)	(impart mass to the elementary particles)

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}$ $\begin{pmatrix} t \\ b \end{pmatrix}$ → Hadrons:
- Meson: $s = 0,1,2\ \ldots$
- Baryons: $s = \frac{1}{2}, \frac{3}{2}, \ldots$

Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$ $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

Gauge Bosons: $\gamma,\ W^{\pm},\ Z^0$, and 8 gluons

Higgs Bosons: H (God/Mother particle)

Meson:

Baryon

399 Mesons, 574 Baryons have been discovered

- Strong interaction (Quantum Chromodynamics)
- Electro-weak interaction (Quantum electro-flavor dynamics)

} The Standard Model

Explaining the properties of the Hadrons in terms of QCD's fundamental degrees of freedom is the Problem laying at the forefront of Hadronic physics.

How these elementary particles and the SM is discovered?

1. Scattering Experiments
2. Decay Processes
3. Study of bound states

Cross sections, Decay Constants, Masses, Spin , Couplings, Form factors, etc.

Models

Particle Accelerators and detectors

Models

Experimental Particle Physicist

Particle Physics Phenomenologist

Theoretical Particle Physicist

Measured values of physical observables

$\approx$

Calculated values of physical observables

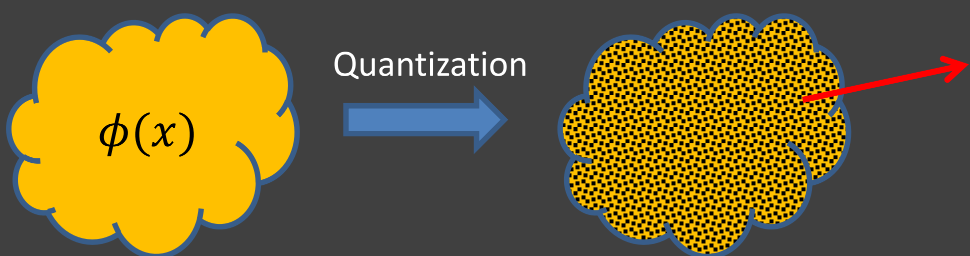
It doesn't matter how beautifull your theory is,
It doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong… R.P. Feynmann

It is more important to have beauty in one's equations than to have them fit experiment….. P.A.M Dirac

Decades of observation and calculations show that the Standard Model of particle physics can describe almost every thing which we have observed in the labs of high energy physics.

A Quick review of QFT's

- The standard model is a quantum field theory.
- In field theories, fields (defined by field functions) act as fundamental dof of the system.
- To every different kind of a particle we associate a field.
  (e.g., electron field, proton field, photon field etc)
- Particles appear as quanta of field.



Quanta of field has particle properties.

- Equation of motion of the fields

$(s = 0)$

Free Scalar Field:

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = 0$$

$$\mathcal{L}_0 = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$

$(s = 1/2)$

Free Dirac Field:

$$\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$$

$$\mathcal{L}_0 = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$(s = 1)$

Free abelian Vector Field:

$$\partial_\mu \partial^\mu A^\nu + \partial^\nu(\partial_\mu A^\mu) = 0$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$(s = 1)$

***Fee* Non-abelian Vector Field:**

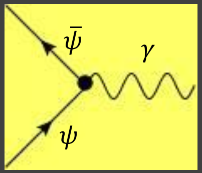
$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu a} F_a^{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_0 f_{abc} A_b^\mu A_c^\nu$$

A Quick review of QFT's

- Interaction is introduced by the coupling of fields .

  For example: $\mathcal{L}_I = e\bar{\psi}\gamma^\mu\psi A_\mu$ for QED



- Only **Lorentz invariant** couplings are allowed.

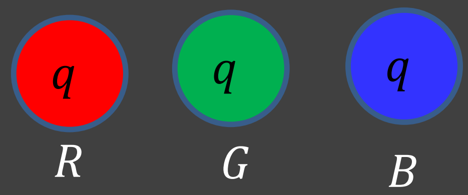
$$\mathcal{L}_I = \bar{\psi}\gamma^\mu\psi A_\mu;\ \bar{\psi}\gamma^\mu\gamma^5\psi A_\mu;\ \bar{\psi}\psi\phi; \bar{\psi}\gamma^5\psi\phi;\ \bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi; \ldots.$$

- In gauge theories coupling are further constrained by the **gauge symmetries**.

  i) $SU(2)_L \otimes U(1)_Y$ for electroweak interaction.
  ii) $SU(3)_c$ for strong interaction.

Quantum Chromo Dynamics (QCD)
(Foundations)

- QCD is the theory of strong interaction of quarks, which is based on $SU(3)_c$ color symmetry.
- It assumes each flavor of quark comes in three different *colors*.
- The color states of quarks are SU(3) triplets.



Quark Fields: ψ_C^f

$$f = 1,2,3 \ldots N_f$$
$$C = R, G, B$$

$N_f = 6$

Free Lagrangian of the Quarks:

$$\mathcal{L}_0 = \sum_{f=1}^{N_f} \sum_{C=R,G,B} \bar{\psi}_C^f (i\gamma^\mu \partial_\mu - m_0^f) \psi_C^f$$

The color states of quarks are $SU(3)_c$ triplets.

$$\psi^f = \begin{pmatrix} \psi_R^f \\ \psi_G^f \\ \psi_B^f \end{pmatrix}$$

$$\psi'^{\,f} = e^{i\theta_a t_a} \psi^f$$

Where t^a are Gell-Mann matrices.

$$[t_a, t_b] = i f_{abc} t_c$$

$$\mathcal{L}_0 = \sum_{f=1}^{N_f} \bar{\psi}^f (i\gamma^\mu \partial_\mu - m_0^f) \psi^f$$

This Lagrangian is invariant under global $SU(3)_c$ gauge transformation.

Free and Classical QCD Lagrangian

Quantum Chromo Dynamics (QCD)
(Foundations)

- Extending the global gauge symmetry to local symmetry.

Local gauge transformation:

$$\psi'^{f} = e^{i\theta_a(x)t_a}\psi^{f} \equiv U\psi^{f}$$

$\partial_\mu\psi^f \to U(\partial_\mu\psi^f)$ Global gauge transformation

$\partial_\mu\psi^f \to U(\partial_\mu\psi^f) + i(\partial_\mu\theta_a)t_a U\psi^f$ Local gauge transformation

Free Lagrangian does not possess local symmetry:

$$\mathcal{L}_0 = \bar{\psi}^f i\gamma^\mu\partial_\mu\psi^f + m_0^f\bar{\psi}^f\psi^f$$

$$\partial_\mu \to D_\mu = \partial_\mu - ig_0 t_a A_{a\mu}(x)$$

Such that $D_\mu\psi^f \to U(D_\mu\psi^f)$

$$\Rightarrow t_a A'_{a\mu} = U t_a A_{a\mu} U^{-1} - \frac{i}{g_0}(\partial_\mu U)U^{-1}$$

$$\mathcal{L} = \bar{\psi}^f i\gamma^\mu D_\mu\psi^f + m_0^f\bar{\psi}^f\psi^f$$

$$\mathcal{L} = \bar{\psi}^f i\gamma^\mu\partial_\mu\psi^f + m_0^f\bar{\psi}^f\psi^f + g_0\bar{\psi}^f\gamma^\mu t_a\psi^f A_{a\mu}$$

$$[t_a, t_b] = if_{abc}t_c$$

Including kinetic energy term of gauge fields

$$\mathcal{L} = \bar{\psi}^f i\gamma^\mu\partial_\mu\psi^f + m_0^f\bar{\psi}^f\psi^f + g_0\bar{\psi}^f\gamma^\mu t_a\psi^f A_{a\mu} - \frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu}$$

where, $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_0 f_{abc}A_b^\mu A_c^\nu$, $(a = 1, 2, \ldots, 8)$

26 fields = (6 flavors x 3 Color)+(8 gauge bosons); 7 parameter = 6 masses + 1 coupling

This complete Lagrangian is invariant under local $SU(3)_c$ gauge transformation.

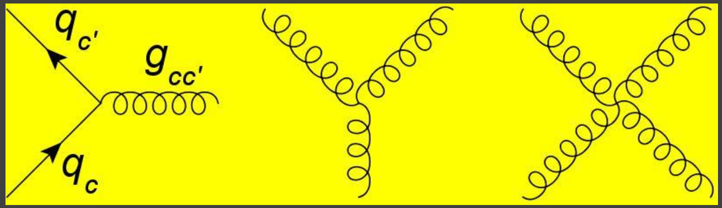
Quantum Chromo Dynamics (QCD)
(Foundations)

Interacting Classical QCD Lagrangian:

$$\mathcal{L} = \bar{\psi}^f i\gamma^\mu \partial_\mu \psi^f + m_0^f \bar{\psi}^f \psi^f + g_0 \bar{\psi}^f \gamma^\mu t_a \psi^f A_{a\mu} - \frac{1}{4} F_{\mu\nu a} F_a^{\mu\nu}$$

where, $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_0 f_{abc} A_b^\mu A_c^\nu,\ (a = 1, 2, \ldots, 8)$

$$-\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} = -\frac{1}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu})(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu) - \frac{1}{2} g_0 f_{abc}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu}) A_b^\mu A_c^\nu$$
$$-\frac{g_0^2}{4} f_{abc} f_{ab'c'} A_{b\mu} A_{c\nu} A_{b'}^\mu A_{c'}^\nu.$$



- Quark-gluon interaction
- 3-point gluon interaction
- 4-point gluon interaction

Quantum Chromo Dynamics (QCD)
(Quantization)

Green Functions of QFT

$$G(x_1, x_2, \ldots, x_n) \equiv \langle \Omega | T\{\phi(x_1)\phi(x_2)\ldots\phi(x_n)\} | \Omega \rangle$$

- Cross sections
- Decay constants
- Bound State masses
- Couplings
- Form factors

$$G(x_1, x_2, \ldots, x_n) = \frac{1}{Z(0)} \frac{-i\delta}{\delta J(x_1)} \frac{-i\delta}{\delta J(x_2)} \cdot \cdot \cdot \frac{-i\delta}{\delta J(x_n)} Z(J)\big|_{J=0}$$

Generating function: $Z(J) = \lim_{t\to\infty(1-i\epsilon)} \int [d\phi]\, e^{i\int_{-T}^{T} dt(\mathcal{L}+J(x)\phi(x))}$

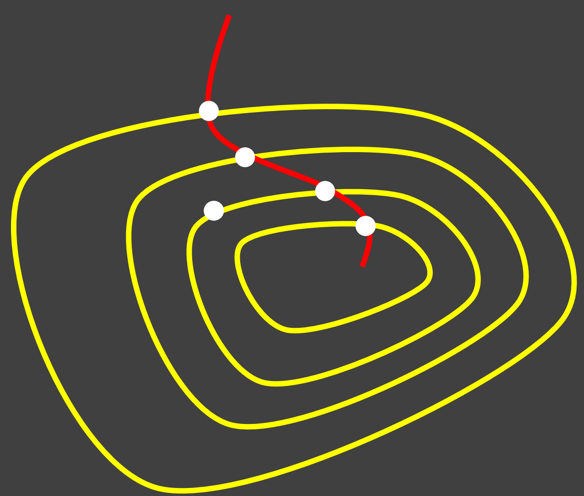
Generating function of QCD:

$$Z(\bar{\eta}, \eta, J_\mu) = \int d[\bar{\psi}\psi A_\mu] e^{i\int dx^4 (L_{QCD} + \bar{\psi}^f \eta^f + \bar{\eta}^f \psi^f + J_a^\mu A_{\mu a})}$$

The path integration over gauge fields diverges due to integrating over gauge equivalent configurations.

$$A'^a_\mu(x) = A^a_\mu(x) - \frac{1}{g_0} D^{ab}_\mu \theta^b(x)$$

Fixing the gauge means choosing one configuration out of gauge equivalent configurations.



Quantum Chromo Dynamics (QCD)
(Quantization)

Gauge fixing implies

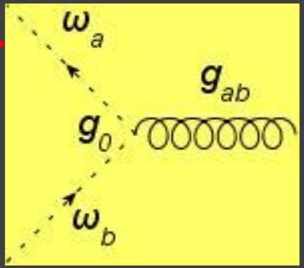
$$Z(\overline{\eta},\eta,J_{\mu},\bar{\varepsilon},\varepsilon)=\int d[\overline{\psi}\psi A_{\mu}\overline{\omega}\omega]e^{i\int dx^{4}(L_{QCD}\boxed{+L_{GF}+L_{FPG}}+\overline{\psi}^{f}\eta^{f}+\overline{\eta}^{f}\psi+A_{\mu a}J_{a}^{\mu}+\overline{\omega}_{a}\varepsilon_{a}+\bar{\varepsilon}_{a}\omega_{a})}$$

$$L_{GF}=-\frac{1}{2\xi_{0}}(\partial_{\mu}A_{a}^{\mu})^{2};\qquad L_{FPG}=(\partial^{\mu}\overline{\omega}^{a})\omega^{a}\boxed{-g_{0}f^{abc}(\partial^{\mu}\overline{\omega}^{a})\omega^{b}A_{\mu}^{c}}$$

Not gauge invariant due to gauge fixing.

The generating function *Z* must be gauge invariant because gauge transformation amounts to redefine the variables integration.

This trivial requirement of Gauge invariance of the generating function translates into non-trivial identities which all Green functions carrying gauge field(s) must satisfy.



QED:
Wards-Green-Takahashi identities (WGTI)

$$q_{\mu}D_{\mu\nu}^{-1}=\frac{q^{2}}{\xi_{0}}q_{\nu}$$

$$iq_{\mu}\Gamma_{\mu}(k,p)=S^{-1}(k)-S^{-1}(p)$$

QCD:
Slavnov Taylor identities (STI)

$$q_{\mu}D_{\mu\nu}^{-1}=\frac{q^{2}}{\xi_{0}}q_{\nu}$$

$$iq_{\mu}\Gamma_{\mu}(k,p)(1+b(k^{2}))=(1-B(k,p))S^{-1}(k)-S^{-1}(p)(1-B(k,p))$$

Green Functions of QCD

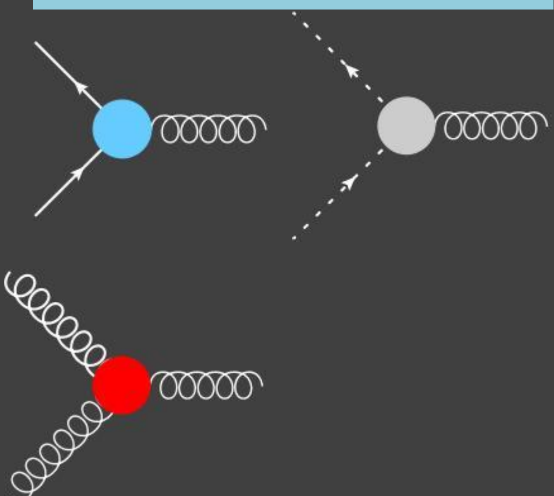
2-point Green functions:

$\langle \Omega | T\{\psi(x_1), \bar{\psi}(x_2)\} | \Omega \rangle \equiv$ 

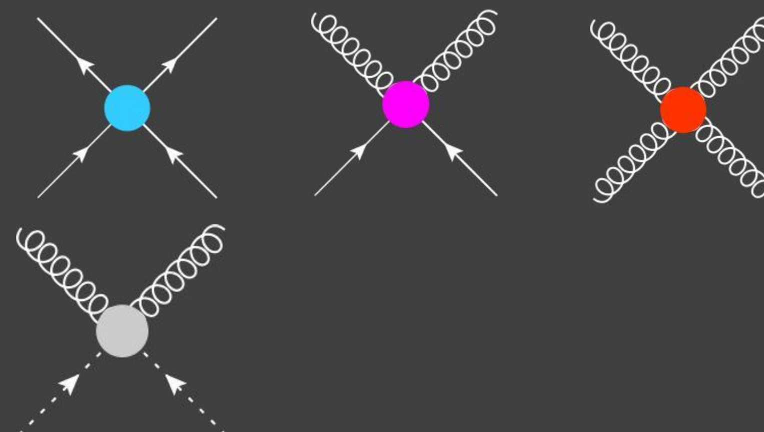
$\langle \Omega | T\{\ (A^a_\mu(x_1), A^b_\nu(x_2)\} | \Omega \rangle \equiv$ 

$\langle \Omega | T\{\ (c^a\,(x_1), c^b\,(x_2)\} | \Omega \rangle \equiv$ 

3-Point Green functions:



4-Point Green functions:

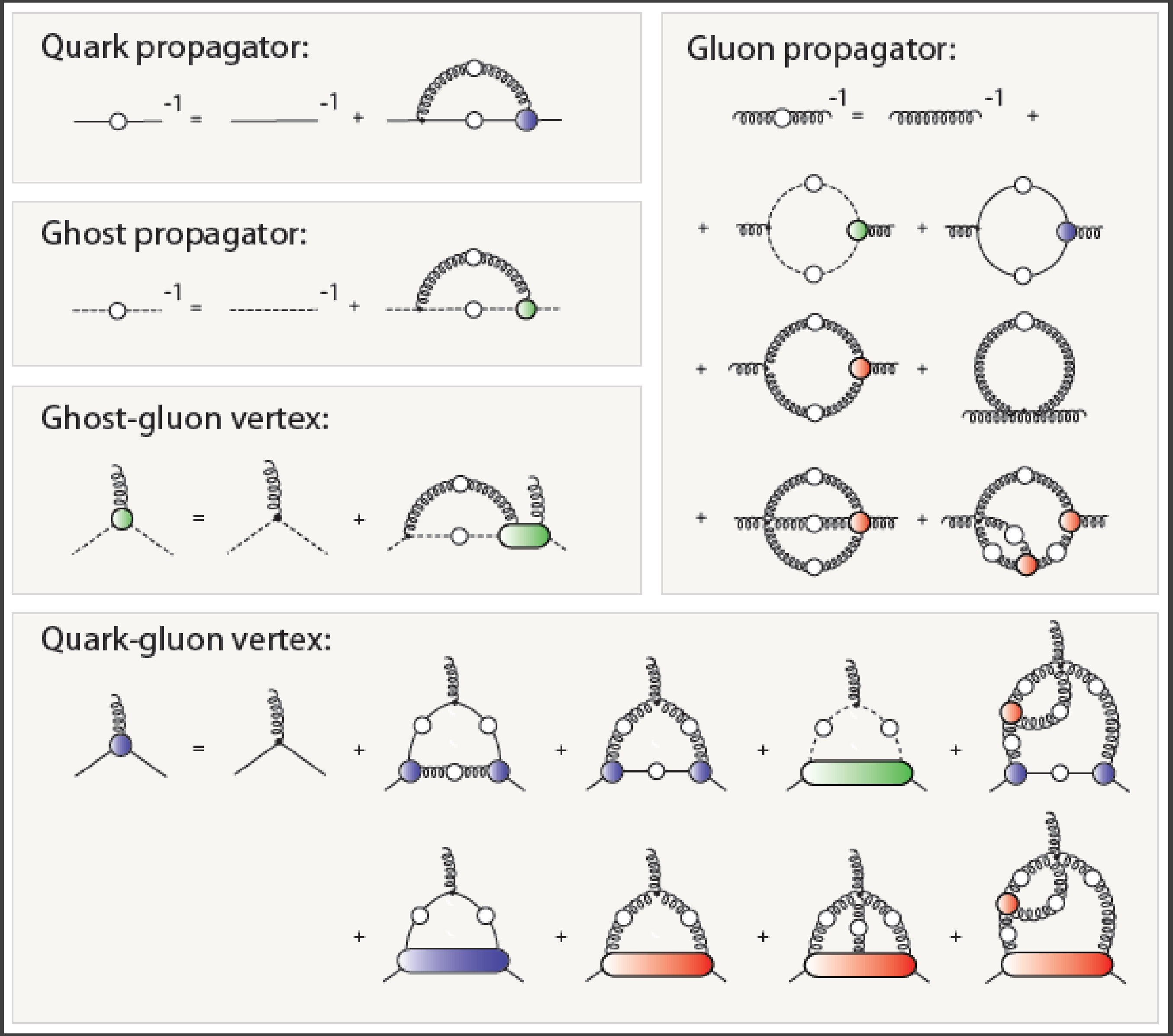


Schwinger-Dyson Equations (SDEs) of QCD

- In QFT the Green functions satisfy a set of exact mathematical equations called SDEs.

- Corresponding to each green function we have a SDE.

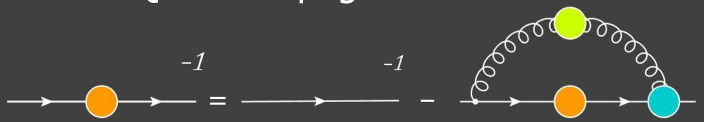
- These are exact equations and can be used to find Green functions perturbatively as well as non-perturbatively.



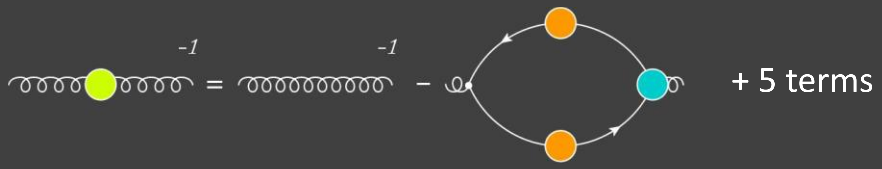
Quark propagator:
-1
=
-1
+
Ghost propagator:
-1
=
-1
+
Ghost-gluon vertex:
=
+
Gluon propagator:
-1
=
-1
+
Quark-gluon vertex:
=
+

Schwinger-Dyson Equations of QCD

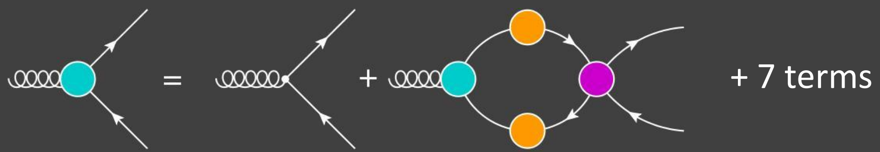
SDE of Quark Propagator:



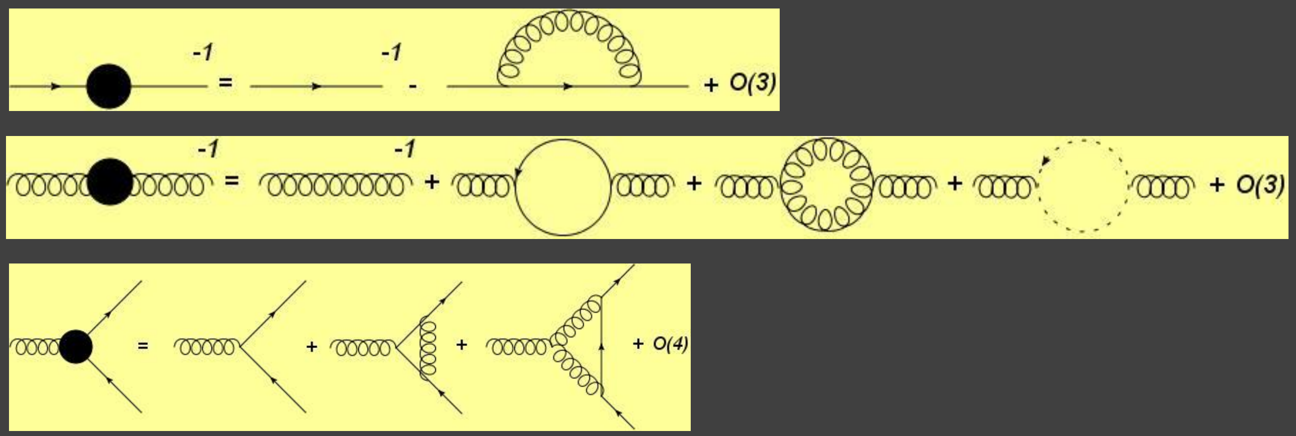
SDE of Gluon Propagator:



SDE of quark-gluon vertex:



Perturbative Truncation:



Non Perturbative Truncation:

One or more Green functions are modeled subjecting to some general field theoretical constraints.

Or

Using the results obtained from lQCD.

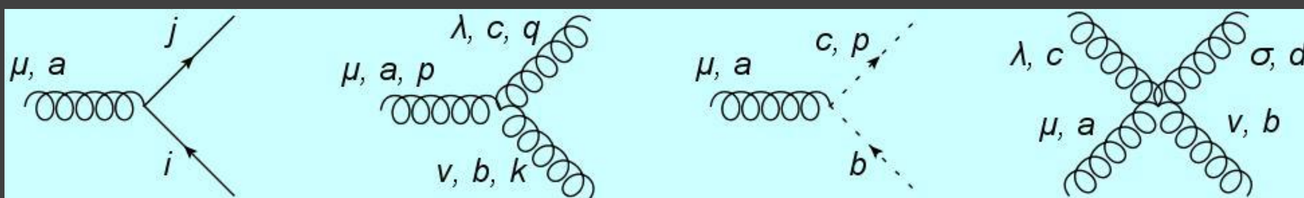
Feynman Rules of QCD

QCD full Lagrangian:

$$L_B = i\overline{\psi}^f\gamma^\mu\partial_\mu\psi^f - m^f\overline{\psi}^f\psi^f + g_0\overline{\psi}^f\gamma^\mu t_a\psi^f A_{a\mu}(x) - \frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} - \frac{1}{2\xi_0}(\partial^\mu A_\mu^a)^2$$
$$+\partial^\mu\overline{\omega}_0^a\partial_\mu\omega_0^a - g_0 f^{abc}\partial^\mu\overline{\omega}_0^a\omega_0^b A_{0\mu}^c$$

Where,
$$-\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} = -\frac{1}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu})(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu) - \frac{1}{2}g_0 f_{abc}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu})A_b^\mu A_c^\nu$$
$$-\frac{g_0^2}{4} f_{abc} f_{ab'c'} A_{b\mu} A_{c\nu} A_{b'}^\mu A_{c'}^\nu .$$

Vertices:



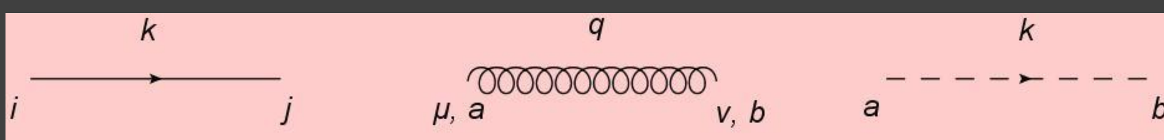
$ig_0\gamma^\mu t_{ij}^a$

$-g_0 f^{abc}[(p-q)_\nu g_{\lambda\mu} + (q-k)_\lambda g_{\mu\nu} + (k-p)_\mu g_{\lambda\nu}]$

$-g_0 f^{cab} p_\mu$

$$ig_0^2\{f^{abe}f^{cde}(g_{\lambda\sigma}g_{\mu\nu} - g_{\lambda\nu}g_{\mu\sigma}) + f^{abe}f^{cde}(g_{\lambda\sigma}g_{\mu\nu} - g_{\lambda\nu}g_{\mu\sigma}) + f^{abe}f^{cde}(g_{\lambda\sigma}g_{\mu\nu} - g_{\lambda\nu}g_{\mu\sigma})\}$$

Propgators:

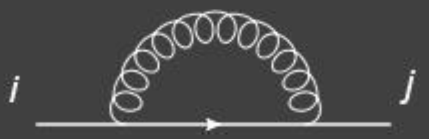


Free Quark propagator : $S_{ij}(k) = \delta_{ij}\dfrac{1}{\not{k} - m + i\varepsilon}$

Free gluon propagator : $D_{\mu\nu}^{ab}(q) = -\delta_{ab}\dfrac{1}{q^2 + i\varepsilon}\left(g_{\mu\nu} - \dfrac{q^\mu q^\nu}{q^2}\right) + \dfrac{\xi_0}{q^2 + i\varepsilon}\dfrac{q^\mu q^\nu}{q^2}$

Free ghost propagator : $G_{ab}(k) = \delta_{ab}\dfrac{1}{k^2 + i\varepsilon}$

Color Algebra:



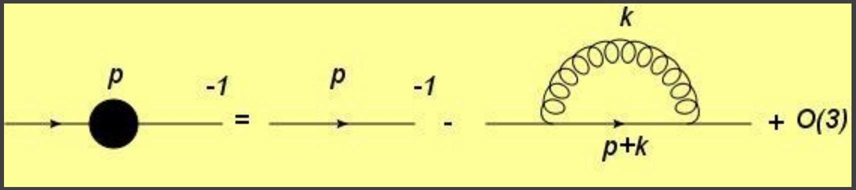
$$t^a_{ik}t^a_{kj} = C_F\delta_{ij}, \qquad C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$$

$$Tr(t^a t^b) = T_F\delta^{ab}, \quad T_F = \frac{1}{2}$$

$$f^{acd} f^{bcd} = C_A\delta^{ab}, \qquad C_A = N_c = 3$$

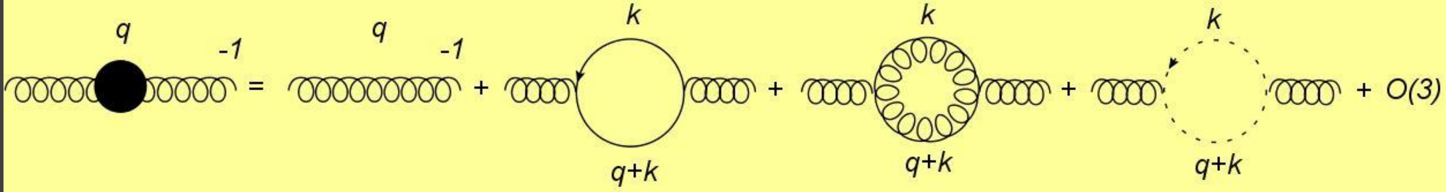
Leading order QCD green functions

Quark propagator:



$$S^{-1}(p) = (\gamma\cdot p - m_0) - iC_F\int\frac{dk^4}{(2\pi)^4}g_0^2 D_{0\mu\nu}(k)\gamma_\mu\frac{1}{\not p+\not k - m_0 + i0}\gamma_\nu \xrightarrow[(k\to\infty)]{} \propto \int_0^\infty \frac{k^3\,dk}{k^3}$$ (Linearly divergent)

Gluon propagator:



$$D^{-1}_{\mu\nu}(q) = D^{-1}_{0\mu\nu}(q) + \Pi^{(\mathrm{quark})}_{\mu\nu} + \Pi^{(\mathrm{gluon})}_{\mu\nu} + \Pi^{(\mathrm{ghost})}_{\mu\nu}$$

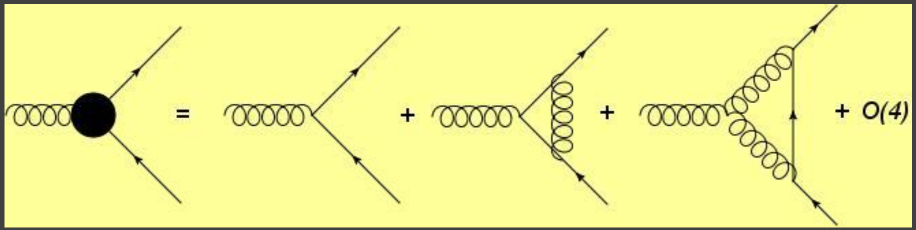
$$\Pi^{\mathrm{quark}}_{\mu\nu} = iT_F\sum_f\int\frac{dk^D\mu^{4-D}}{(2\pi)^D}g_0^2 Tr\left(\gamma_\mu\frac{1}{\not k - m_0^f}\gamma_\nu\frac{1}{\not q+\not k - m_0^f}\right),$$

$$\Pi^{\mathrm{gluon}}_{\mu\nu} = i\frac{C_A}{2}\int\frac{dk^4}{(2\pi)^4}g_0^2 D_{0\mu\alpha}(k)D_{0\alpha\nu}(q+k)\left\{\begin{array}{l}\left[-(2k+q)_\mu g_{\alpha\beta} + (k-q)_\beta g^\beta_\alpha + (2q+k)_\alpha g^\mu_\beta\right] \\ \quad\times\left[-(2k+q)_\nu g^{\alpha\beta} + (k-q)^\beta g^\alpha_\nu + (2q+k)^\alpha g^\beta_\nu\right]\end{array}\right\}$$

$$\Pi^{\mathrm{ghost}}_{\mu\nu} = -iC_A\int\frac{dk^4}{(2\pi)^4}g_0^2\frac{k_\mu(k+q)_\nu}{k^2(k+q)^2}$$

$$\propto \int_0^\infty k\,dk$$ (Quadratically divergent)

Quark gluon vertex (1PI):



$\propto \int_0^\infty dk\,/k$ (Logarithm divergent)

If theory is renormalizable then at any order of perturbation, all the divergences can be absorbed by redefinitions of the coupling and fields.

Redefinitions of coupling and fields are made through multiplicative constants called renormalization constants.

$$\psi_{0c}^{f}(x) = Z_F^{1/2}\psi_c^{f}(x), \quad \overline{\psi}_{0c}^{f}(x) = Z_F^{1/2}\overline{\psi}_c^{f}(x),$$
$$A_{0\mu}^{a}(x) = Z_B^{1/2}A_{\mu}^{a}(x),$$
$$\omega_{0a}(x) = Z_\omega^{1/2}\omega_a(x), \quad \overline{\omega}_{0a}(x) = Z_\omega^{1/2}\overline{\omega}_a(x),$$
$$g_0 = Z_g g, \quad m_0^f = Z_{m,f}m^f, \quad \frac{1}{\xi_0} = \frac{Z_\xi}{\xi}$$

Gauge invariance which is expressed in terms of STI's enforce that

i) All quarks are renormalized by same constant Z_F.
ii) All gluons are renormalized by same constant Z_B.
iii) All ghost are also renormalized by same constant Z_ω.

End of Lecture 1

Summery:

- In QCD we have 5 different interactions
  - i) Quark-Gluon interaction
  - ii) 3 Gluon interaction
  - iii) 4 Gluon interaction
  - iv) Ghost-Gluon interaction

- QCD green functions satisfy SDE's which are exact but unsolvable unless truncated.

- In perturbative truncation the amplitude of QCD green functions can be written directly from the Feynman diagram using Feynman rules.

- Naïve analysis show that these Green function carry ultraviolet divergences.

- Since QCD in renormalizable, therefore, the divergences can be absorbed by redefinition coupling and field of QCD.

Next Lecture:

- We will continue renormalization program in QCD using dimensional regularization.
- Discuss some non-perturbative aspects of QCD.

Procedure of Renormalization:

- Regularize the field theory.

(i.e., to modify it temporarily in such way that all divergent integrals become finite)

i) Dimensional regularization

$$4 \to D \equiv 4 - \varepsilon$$

$$\int \frac{dk^4}{(2\pi)^4} \to \int \frac{dk^D}{(2\pi)^D} \to \int \frac{dk^D v_0^{4-D}}{(2\pi)^D}$$

(breaks only scale invariance)

ii) Pauli-Villars regularization

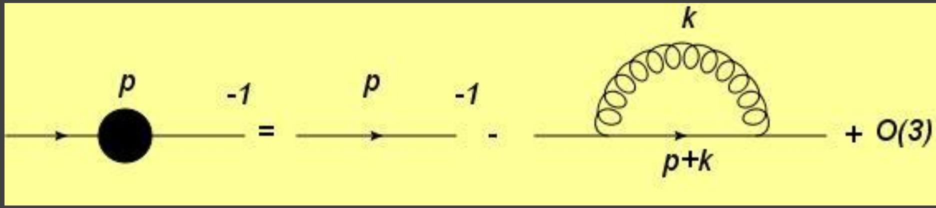
$$\frac{1}{k^2 + i0} \to \frac{1}{k^2 + i0} - \frac{1}{k^2 - \Lambda^2 + i0}$$

(breaks Hermicity and gauge invariance in non-Abelian gauge theories)

iii) Lattice regularization

continuous space-time is replace by discrete Lattice of space-time.
Lattice spacing acts as regularizing parameter.

(breaks Poincaré invariance)



$$S^{-1}(p) = (\gamma \cdot p - m_0) - \Sigma(p)$$

$$\Sigma(p) = iC_F \int \frac{dk^D v_0^{4-D}}{(2\pi)^D} g_0^2 D_{0\mu\nu}(k)\gamma_\mu \frac{1}{\not{p} + \not{k} - m_0 + i0}\gamma_\nu$$

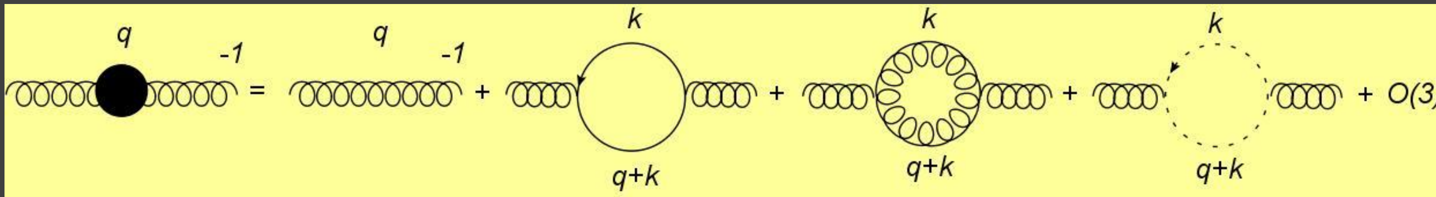
$$\Sigma(p) = (\not{p} - m_0^f)A(p^2) + m_0^f B(p^2)$$

$$A(p^2) = C_F \frac{g_0^2}{16\pi^2}\left\{-\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + 1 + 2I_1(p^2)\right\}$$

$$B(p^2) = -C_F \frac{g_0^2}{16\pi^2}\left\{-3\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + 1 + 2I_2(p^2)\right\}$$

$$I_1(p^2) = \int_0^1 (1-x)\log\left[\frac{xm^2 - x(1-x)p^2}{v_0^2}\right]$$

$$I_2(p^2) = \int_0^1 (1+x)\log\left[\frac{xm^2 - x(1-x)p^2}{v_0^2}\right]$$



$$D^{-1}_{\mu\nu}(q) = D^{-1}_{0\,\mu\nu}(q) + \Pi^{(\text{quark})}_{\mu\nu} + \Pi^{(\text{gluon})}_{\mu\nu} + \Pi^{(\text{ghost})}_{\mu\nu}$$

$$\Pi^{\text{quark}}_{\mu\nu}(q) = 2T_F \frac{g_0^2}{16\pi^2}(-g_{\mu\nu}q^2 + q_\mu q_\nu)\left\{\frac{2N_f}{3}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) - 4I_3(q^2)\right\}$$

$$\Pi^{\text{gluon}}_{\mu\nu} = -C_A \frac{g_0^2}{16\pi^2}\left\{\left[\frac{19}{6}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) - \frac{1}{2} - I_4(q^2)\right]g_{\mu\nu}q^2 - \left[\frac{11}{3}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + \frac{2}{3} - I_5(q^2)\right]q_\mu q_\nu\right\}$$

$$\Pi^{\text{ghost}}_{\mu\nu} = -C_A \frac{g_0^2}{32\pi^2}\left\{\left[\frac{1}{6}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + \frac{1}{6} - I_6(q^2)\right]g_{\mu\nu}q^2 - \left[-\frac{1}{3}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + 2I_6(q^2)\right]q_\mu q_\nu\right\}$$

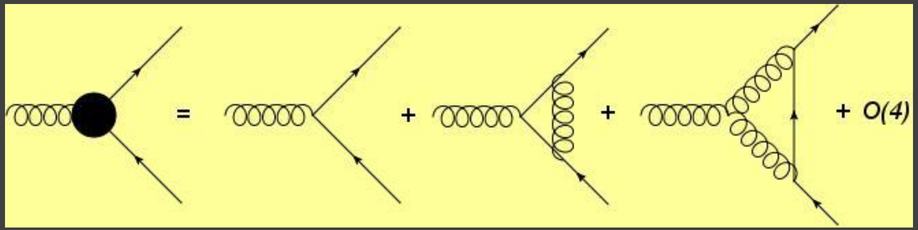
$$q^\mu D^{-1}_{\mu\nu} = \frac{q^2}{\xi_0} q_\nu \quad \Rightarrow \quad q^\mu \Pi_{\mu\nu} = 0$$

The $\Pi^{\text{gluon}}_{\mu\nu}$ and $\Pi^{\text{ghost}}_{\mu\nu}$ do not satisfy the condition of transversality.

$$\Pi^{\text{gluon}}_{\mu\nu} + \Pi^{\text{ghost}}_{\mu\nu} = -C_A \frac{g_0^2}{32\pi^2}(-g_{\mu\nu}q^2 + q_\mu q_\nu)\left\{-\frac{10}{3}\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) - \frac{62}{9} + \frac{10}{3}\log(q^2)\right\}$$

$$\Pi^{\text{all}}_{\mu\nu} = (-g_{\mu\nu}q^2 + q_\mu q_\nu)\Pi^{\text{all}}(q^2)$$

$$\Pi^{\text{all}}(q^2) = \frac{g_0^2}{16\pi^2}\left\{\left(T_F\frac{4N_f}{3} + C_A\frac{10}{6}\right)\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right) + C_A\left(\frac{31}{9} - \frac{5}{3}\log q^2\right) - 8T_F I_3(q^2)\right\}$$



$$ig_0\Gamma^a_\mu(p,p') = ig_0 t^a\gamma_\mu + ig_0\Lambda^{(1)a}_\mu(p,p') + ig_0\Lambda^{(2)a}_\mu(p,p')$$

$$ig_0\Lambda^{(1)a}_\mu = 3\left\{i\frac{3}{2}C_A t^a\right\}\frac{g_0^3}{16\pi^2}\gamma_\mu\left(\frac{2}{\varepsilon}-\gamma+\log 4\pi\right)+(\text{Finit Part})$$

$$ig_0\Lambda^{(2)a}_\mu = \left\{\left(-C_A/2+C_F\right)t^a\right\}i\frac{g_0^3}{16\pi^2}\gamma_\mu\left(\frac{2}{\varepsilon}-\gamma+\log 4\pi\right)+(\text{Finit Part})$$

$$ig_0\Lambda^a_\mu = (C_A+C_F)\,i\frac{g_0^3}{16\pi^2}\gamma_\mu\left(\frac{2}{\varepsilon}-\gamma+\log 4\pi\right)+(\text{Finit Part})$$

Renormalization:

$$\psi_{0c}^{f}(x) = Z_F^{1/2}\psi_c^{f}(x), \quad \overline{\psi}_{0c}^{f}(x) = Z_F^{1/2}\overline{\psi}_c^{f}(x),$$

$$A_{0\mu}^{a}(x) = Z_B^{1/2}A_\mu^{a}(x),$$

$$\omega_{0a}(x) = Z_\omega^{1/2}\omega_a(x), \quad \overline{\omega}_{0a}(x) = Z_\omega^{1/2}\overline{\omega}_a(x),$$

$$g_0 = Z_g g, \quad m_0^f = Z_{m,f} m^f, \quad \frac{1}{\xi_0} = \frac{Z_\xi}{\xi}$$

1. Multiplicative Renormalization Strategy:

$$\widetilde{S}(p) = Z_F^{-1}S(p)$$

$$\widetilde{D}_{\mu\nu}(q) = Z_B^{-1}D_{\mu\nu}(q)$$

$$\widetilde{\Gamma}_\mu(p,p') = \frac{Z_g}{Z_F Z_B^{1/2}}\Gamma_\mu(p,p')$$

2. Counter terms Strategy:

$$L_B = i\overline{\psi}_0^f \gamma^\mu \partial_\mu \psi_0^f - m_0^f \overline{\psi}_0^f \psi_0^f + g_0 \overline{\psi}_0^f \gamma^\mu t_a \psi_0^f A_{0a\mu}(x) - \frac{1}{4} F_{0a\mu\nu} F_{0a}^{\mu\nu} - \frac{1}{2\xi_0}(\partial^\mu A_\mu^a)^2$$
$$+ \partial^\mu \overline{\omega}_0^a \partial_\mu \omega_0^a - g_0 f^{abc} \partial^\mu \overline{\omega}_0^a \omega_0^b A_{0\mu}^c$$

$$L_B = Z_F i\overline{\psi}^f \gamma^\mu \partial_\mu \psi^f - m^f Z_{m,f} Z_F \overline{\psi}^f \psi^f + g Z_g Z_F Z_B^{1/2} \overline{\psi}^f \gamma^\mu t_a \psi^f A_{a\mu}(x) - \frac{Z_B}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{Z_B Z_\xi}{2\xi}(\partial^\mu A_\mu^a)^2$$
$$+ Z_\omega \partial^\mu \overline{\omega}^a \partial_\mu \omega^a - g\ Z_g Z_\omega Z_A^{1/2} f^{abc} \partial^\mu \overline{\omega}^a \omega^b A_\mu^c$$

$$Z_F = 1 + \Delta Z_F, \quad Z_B = 1 + \Delta Z_B, \quad Z_g = 1 + \Delta Z_g$$

$$L_B = i\overline{\psi}^f \gamma^\mu \partial_\mu \psi^f - m^f \overline{\psi}^f \psi^f + g \overline{\psi}^f \gamma^\mu t_a \psi^f A_{a\mu}(x) - \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2$$
$$+ \partial^\mu \overline{\omega}^a \partial_\mu \omega^a - g\ f^{abc} \partial^\mu \overline{\omega}^a \omega^b A_\mu^c$$
$$+ \Delta Z_F i\overline{\psi}^f \gamma^\mu \partial_\mu \psi^f - (Z_F m_0^f - m^f)\overline{\psi}^f \psi^f - (Z_g Z_F Z_B^{1/2} - 1) g \overline{\psi}^f \gamma^\mu t_a \psi^f A_{a\mu}(x) + \frac{\Delta Z_B}{4} F_{a\mu\nu} F_a^{\mu\nu}$$
$$+ \Delta Z_\omega \partial^\mu \overline{\omega}^a \partial_\mu \omega^a + -(Z_g Z_\omega Z_A^{1/2} - 1) g\ f^{abc} \partial^\mu \overline{\omega}^a \omega^b A_\mu^c$$

Renormalization Schemes:

$$\tilde{S}(p) = Z_F^{-1} S(p)$$
$$\tilde{D}_{\mu\nu}(q) = Z_B^{-1} D_{\mu\nu}(q)$$
$$\tilde{\Gamma}_\mu(p, p') = \frac{Z_g}{Z_F Z_B^{1/2}} \Gamma_\mu(p, p')$$

A renormalization constant Z:

$$Z = (\text{divergent part}) + (\text{finte part})$$

- The divergent part is chosen in such a way that it cancels the pole in un-renormalized Green function.
- Each different way of choosing the finite part defines a different renormalization scheme.

Minimal subtraction (MS) scheme:

The renormalization constants only cancel the divergent parts of the Green functions without affecting their finite part.

Modified Minimal subtraction ($\overline{\mathrm{MS}}$) scheme:

The renormalization constant also cancel a finite part in the Green function

$$N_\varepsilon = \frac{2}{\varepsilon} - \gamma + \log 4\pi$$

These renormalization schemes are called mass independent renormalization schemes.

The results of perturbative calculations are scheme dependent.

Renormalization constants of QCD in $\overline{\mathrm{MS}}$ at 1 loop order:

$$Z_F = 1 - C_F \frac{g^2}{16\pi^2} N_\varepsilon$$

$$Z_m = 1 - 3C_F \frac{g^2}{16\pi^2} N_\varepsilon$$

$$Z_B = 1 + \frac{g^2}{16\pi^2}\left(\frac{5}{3}C_A + \frac{4}{3}T_R N_f\right)N_\varepsilon$$

$$Z_g = 1 - \frac{g^2}{16\pi^2}\left(\frac{11}{6}C_A - \frac{2}{3}T_R N_f\right)N_\varepsilon$$

Renormalized gluon propagator at 1 loop order:

$$D^{-1}_{\mu\nu}(q) = D^{-1}_{0\mu\nu}(q) + \Pi^{(\mathrm{all})}_{\mu\nu}$$

$$D^{-1}_{0\mu\nu}(q) = -q^2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - \frac{q^2}{\xi}\frac{q_\mu q_\nu}{q^2}$$

$$\Pi^{\mathrm{all}}_{\mu\nu}(q) = \left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\Pi^{\mathrm{all}}(q^2)$$

$$\Pi^{\mathrm{all}}(q^2) = \frac{g_0^2}{16\pi^2}\left\{C_A\left(\frac{31}{9} - \frac{5}{3}\log q^2\right) - 8T_F I_3(q^2)\right\}$$

$$D_{\mu\nu}(q) = -\frac{1}{q^2\left(1+\Pi^{\mathrm{all}}(q^2)\right)}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - \frac{\xi}{q^2}\frac{q_\mu q_\nu}{q^2}$$

Renormalized quark propagator at 1 loop order:

$$S^{-1}(p) = (\not{p} - m^f) - \Sigma(p)$$

$$\Sigma(p) = (\not{p} - m^f)A(p^2) + m^f B(p^2)$$

$$A(p^2) = C_F \frac{g^2}{16\pi^2}\left\{1 + 2I_1(p^2)\right\}$$

$$B(p^2) = -C_F \frac{g^2}{16\pi^2}\left\{1 + 2I_2(p^2)\right\}$$

$$S(p) = \frac{1 + A(p^2)}{\not{p} - m^f\left(1 + B(p^2)\right)}$$

Scale dependence in QCD:

- Dimensional regularization necessitate the introduction of a scale parameter ν_0.

$$S_D(p,\nu_0),\ \ D_{D\mu\nu}(q,\nu_0),\ \ \Gamma_{D\mu}(p,p',\nu_0)$$

- This dependence on ν_0 can easily be removed by redefining the renormalization constants Z's.

$$\int \frac{dk^4}{(2\pi)^4} \to \int \frac{dk^D \nu_0^{\varepsilon}}{(2\pi)^D} \qquad Z(\nu) = \left(\frac{\nu}{\nu_0}\right)^{\varepsilon} Z \qquad N_\varepsilon = \frac{2}{\varepsilon} - \gamma + \log 4\pi + \log\left(\frac{\nu_0^2}{\nu^2}\right)$$

$$\tilde{S}(p,\nu) = Z_F^{-1}(\nu,\nu_0) S_D(p,\nu_0)$$
$$\tilde{D}_{\mu\nu}(q,\nu) = Z_B^{-1}(\nu,\nu_0) D_{D\mu\nu}(q,\nu_0)$$
$$\tilde{\Gamma}_{\mu}(p,p',\nu) = \frac{Z_g(\nu,\nu_0)}{Z_F(\nu,\nu_0) Z_B^{1/2}(\nu,\nu_0)} \Gamma_{D\mu}(p,p',\nu_0)$$

- Renormalized green functions are now function of arbitrary scale parameter ν.

- However, the physical observables must not depend upon the choice of ν. This is ensured by a correct ν dependence of fundament constants (m's and g's) of the field theory.

$$A(obs.) = A\big(m(\nu),\ \ g(\nu),\ \ \text{gre. fun.}\,(\nu)\big)$$

The running of QCD coupling:

$$Z_g(v)g(v) = g_0$$

Taking the derivative w.r.t $\log(v)$.

$$\frac{\partial Z_g}{\partial \log v} g(v) + Z_g \frac{\partial g}{\partial \log v} = 0$$

$$\frac{1}{g}\frac{\partial g}{\partial \log v} \equiv \beta(g)$$

$$\beta(g) = -Z_g^{-1} \frac{\partial Z_g}{\partial \log v}$$

$$Z_g(v) = 1 - \frac{g^2}{16\pi^2}\left(\frac{11}{6}C_A - \frac{2}{3}T_R N_f\right)N_\varepsilon$$

$$N_\varepsilon = \frac{2}{\varepsilon} - \gamma + \log 4\pi + \log\left(\frac{v_0^2}{v^2}\right)$$

$$\beta(g) = -\frac{g^2}{16\pi^2}\left\{\frac{11}{3}C_A - \frac{4}{3}T_F N_f\right\}$$

$$\beta_0 \equiv \frac{11}{3}C_A - \frac{4}{3}T_F N_f$$

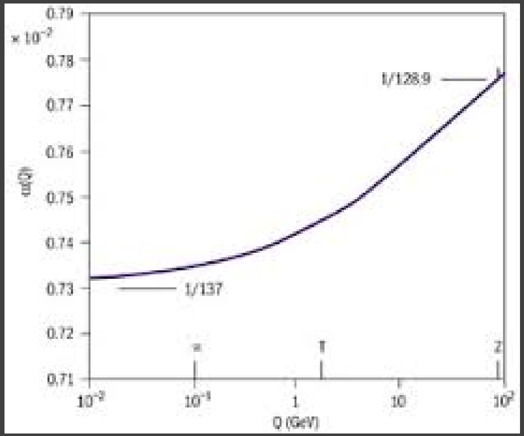
$$\frac{1}{g}\frac{\partial g}{\partial \log v} = -\frac{g^2}{16\pi^2}\beta_0$$

$$\alpha_s(v^2) \equiv \frac{g(v)^2}{4\pi}$$

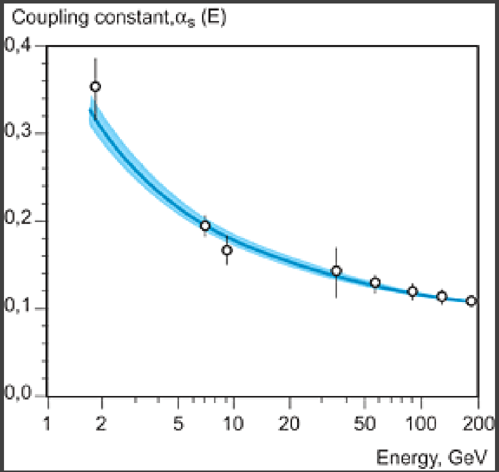
$$\frac{\partial \alpha_s}{\partial \log v} = -\frac{\alpha_s}{2\pi}\beta_0$$

$$\alpha_s(Q^2) = \frac{\alpha_s(v^2)}{1 + \alpha_s(v^2)\beta_0 \log(Q^2/v^2)/(4\pi)}$$

Running coupling of QED



QCD is asymptotically free



END of Lecture 1

Lecture 2 (non-perturbative QCD)

Quantum Chromodynamics

(Non-perturbative aspects)

Lecture 2

Faisal Akram

International Symposium on Physics Beyond the Standard Model

NCP Islamabad

2015

Outlines

- Characteristics of QCD
  - Asymptotic freedom
  - Confinement
  - QCD phase transition
  - Dynamical breaking of chiral symmetry
- Non-Perturbative truncation of Schwinger-Dyson equations
- Models of quarks-gluon vertex and gluon propagator
- Numerical Solution of SDE of quarks propagator
- Results and comparison with the experiments

Characteristics of QCD

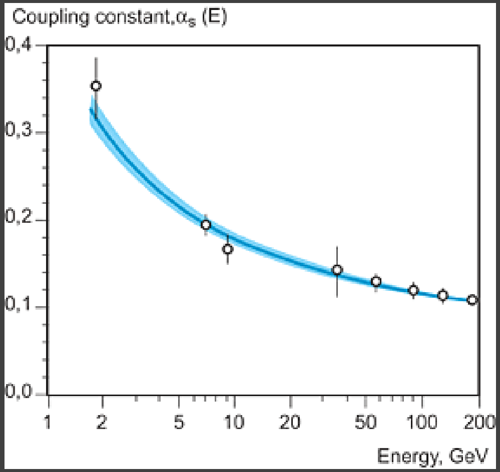
- Asymptotic freedom
- Quarks Confinement
- QCD phase transition
- Dynamical Breaking of Chiral Symmetry

$$Q^2 \frac{\partial \alpha_s}{\partial \log Q^2} = \beta(\alpha_s)$$

$$\beta(\alpha_s) = -\frac{\alpha_s}{4\pi}\beta_0 + O(2)$$

where $\beta_0 = (33 - 2N_f)/3$

$N_f < 16.5$ (Asymptotic freedom)



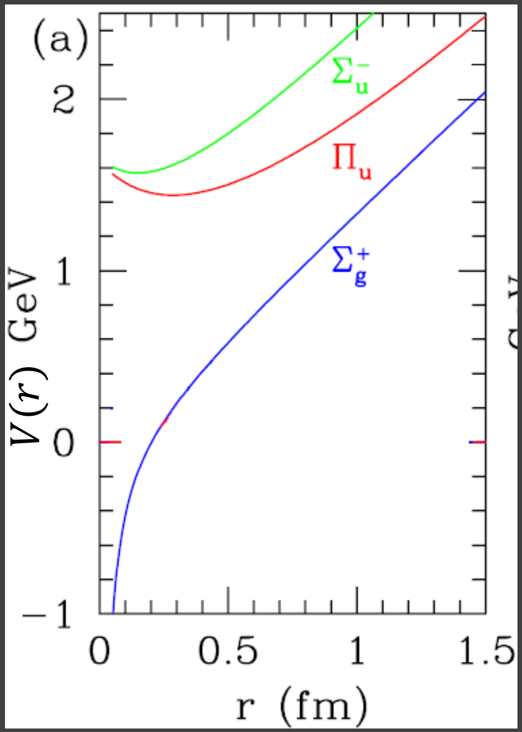
* D. J Cross and F. Wilczek, PRL. 30, 1343 (1973); H. D. Politzer, PRL 30, 1346 (1973).

Characteristics of QCD

- Asymptotic freedom
- Quarks confinement
- QCD phase transition
- Dynamical Breaking of Chiral Symmetry

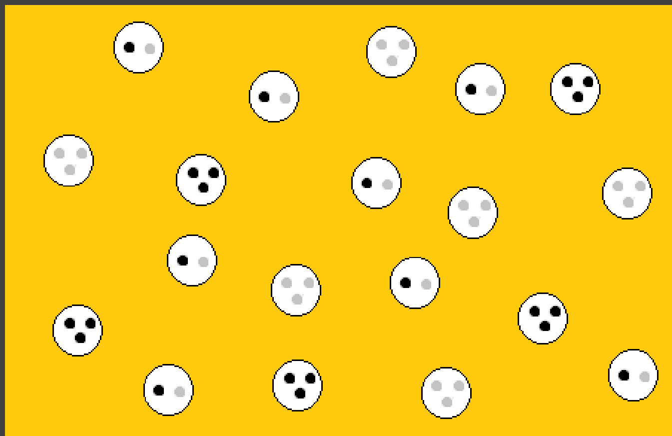
$$V(r) \approx br$$

$$F = - -\frac{\partial V}{\partial r} \approx -b$$

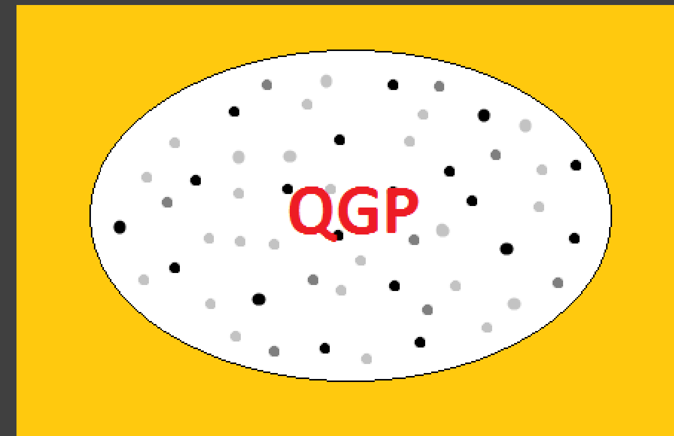


* K.J. Juge, J. Kuti, and C. Morningstar, Phys. Rev. Lett. 90, 161601 (2003).

QCD phase transition



Confined State of Matter

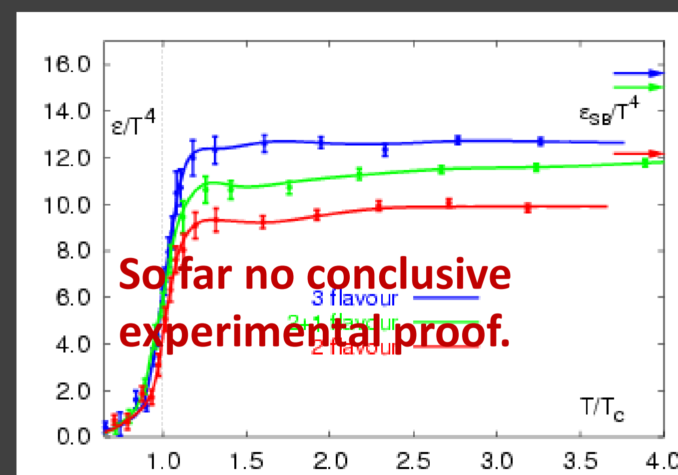


De-Confined State of Matter

QGP is expected to produce at $T_c \approx$ 170 MeV ≈ 2 x 10^{12} K ≈ 130,000 $T_{☼}$ and ♉ ≈ 0

Evidence of QGP?

Theoretical Evidence



So far no conclusive experimental proof.

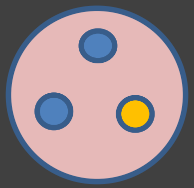
$$\varepsilon_{HG} = 3_I \frac{\pi^2}{30} T^4$$

$$\varepsilon_{QGP} = \left(2_f \cdot 2_s \cdot 2_q \cdot 3_c \frac{7}{8} + 2_s \cdot 8_c\right) \frac{\pi^2}{30} T^4 = 37 \frac{\pi^2}{30} T^4$$

Quantum Chromo-dynamics

- Asymptotic Freedom
- Quarks confinement
- QCD phase transition
- Dynamical Breaking of Chiral Symmetry

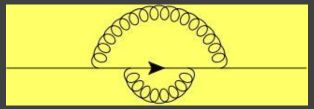
(2008 Nobel Prize)



Current mass: $m_{u/d} = 3 - 6$ MeV

$M_p = 939$ MeV

Self interaction of quarks



Constituent mass: $m_{u/d} \approx 350$ MeV

	u	d	s	c	b
Current mass (MeV)	3	6	100	1100	4200
Constituent mass (MeV)	350	350	530	1500	4600

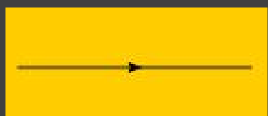
Dynamical Breaking of Chiral Symmetry in QCD

Fermionic part of QCD Lagrangian for light quarks:

$$L = i\bar{q}_i \not{D} q_i + \cancel{m_i \bar{q}_i q_i} \qquad \forall i = u, d, s$$

This Lagrangian is invariant under $SU(3)_L \otimes SU(3)_R$ symmetry if the current masses of light quarks are zero.

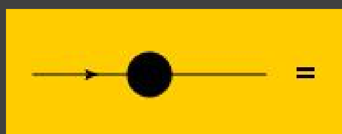
Free quark propagator:



$$= \frac{1}{i\not{p} + m}$$

Pole mass
$m_{u/d} = 3.7\text{MeV}$

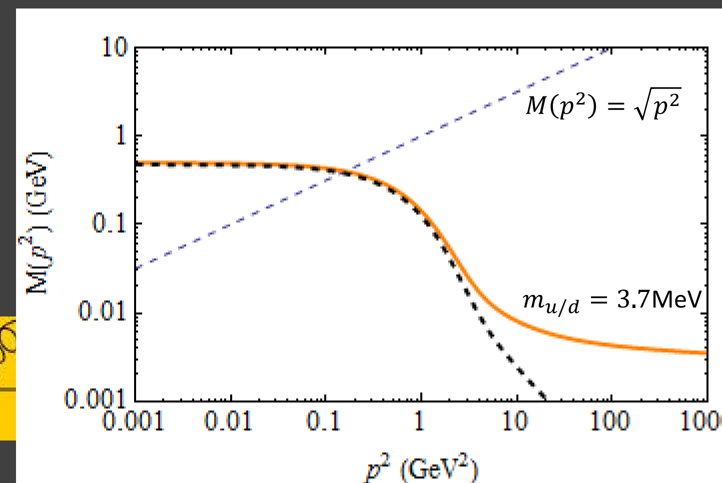
Full propagator:



$$= \frac{F(p^2)}{i\not{p} + M(p^2)}$$

Mass function
(dynamical mass)

Pole mass,
$M_{u/d} = 390\text{MeV}$



Perturbative expansion

$$M(p^2) \approx \frac{m}{\left(\frac{1}{2}\ln\left[p^2/\Lambda^2_{QCD}\right]\right)^{\gamma_m}}$$

If the current mass m = 0 then $M(p^2) = 0$

Where, $p^2 > \Lambda^2_{QCD} = 0.2$ GeV

What if, $p^2 < \Lambda^2_{QCD}$??

$$\gamma_m = 4/\beta_0$$

$$\beta_0 = \frac{11N_c - 2N_f}{3}$$

Schwinger-Dyson equations (A non-perturbative technique)

$$M(p^2) \approx \frac{2\pi^2\gamma_m}{3} \frac{-<\bar{q}q>^0}{p^2\left(\frac{1}{2}\ln\left[p^2/\Lambda^2_{QCD}\right]\right)^{1-\gamma_m}}$$

Renormalization point independent
Quark condensate

$$-<\bar{q}q>^0 = (0.241)^3 GeV^3$$

Non-zero value of $M(p^2)$ even when m = 0 means chiral symmetry is dynamically broken.

Green Functions of QCD

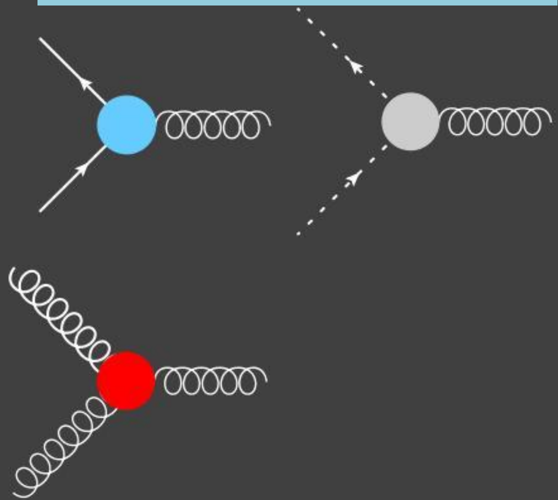
2-point Green functions:

$\langle \Omega | T\{\psi(x_1), \bar{\psi}(x_2)\} | \Omega \rangle \equiv$ 

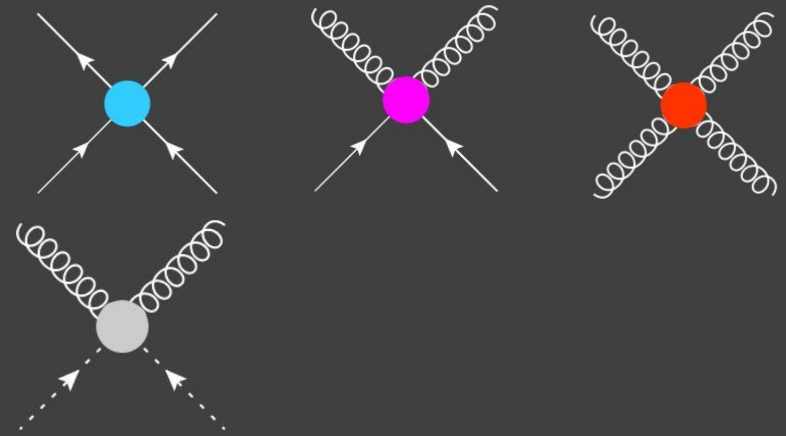
$\langle \Omega | T\{\ (A^a_\mu(x_1), A^b_\upsilon(x_2)\} | \Omega \rangle \equiv$ 

$\langle \Omega | T\{\ (c^a\,(x_1), c^b\,(x_2)\} | \Omega \rangle \equiv$ 

3-Point Green functions:



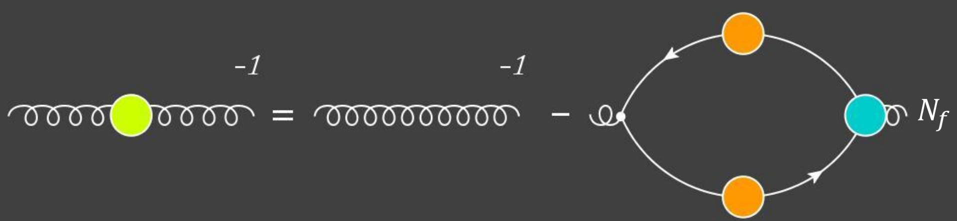
4-Point Green functions:



Schwinger-Dyson Equations of QCD

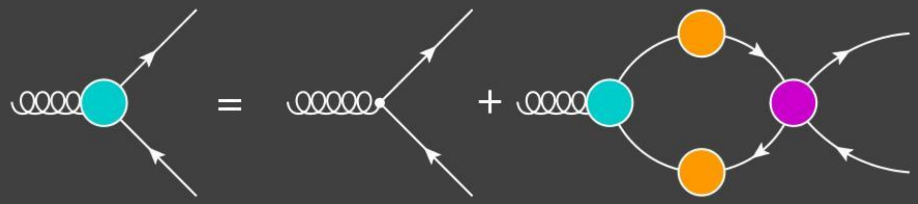
SDE of Quark Propagator:

SDE of Gluon Propagator:



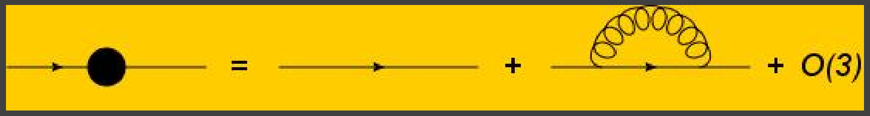
+ 5 terms

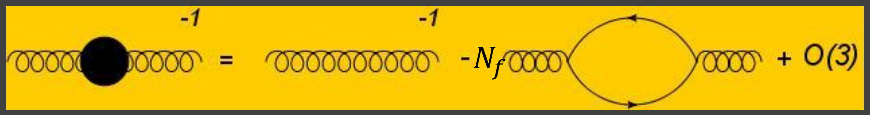
SDE of quark-gluon vertex:



+ 7 terms

Perturbative Truncation:





Non Perturbative Truncation:

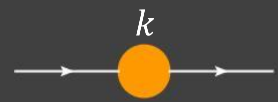
One or more green functions are modeled subjecting to some general field theoretical constraints.

Or

Using the results obtained from lQCD.

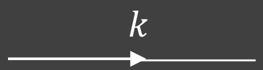
Green Functions of QCD (continued)

Quark Propagator:



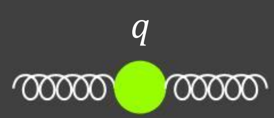
$$S(k)=\frac{F(k^2)}{i\not{k}+M(k^2)}=\frac{1}{i\not{k}A(k^2)+B(k^2)}$$

Free Quark Propagator:



$$S_0(k)=\frac{1}{i\not{k}+m}$$

Gluon Propagator:



$$D_{\mu\nu}(q)=D^T(q^2)\frac{1}{q^2}\left(\delta_{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)+D^L(q^2)\frac{1}{q^2}\frac{q^\mu q^\nu}{q^2}$$

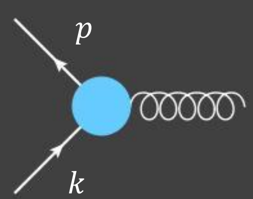
Free Gluon Propagator:

$$D_{\mu\nu}(q)=\frac{1}{q^2}\left(\delta_{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)+\frac{\xi_0}{q^2}\frac{q^\mu q^\nu}{q^2}$$

$$q^\mu D^{-1}_{\mu\nu}=\frac{q^2}{\xi_0}q_\nu$$

WGTI of gluon propagator implies $D^L(q^2)=\xi_0$

Quark-Gluon Vertex:



$$ig_0\frac{\lambda^a}{2}\Gamma_\mu(k,p)$$

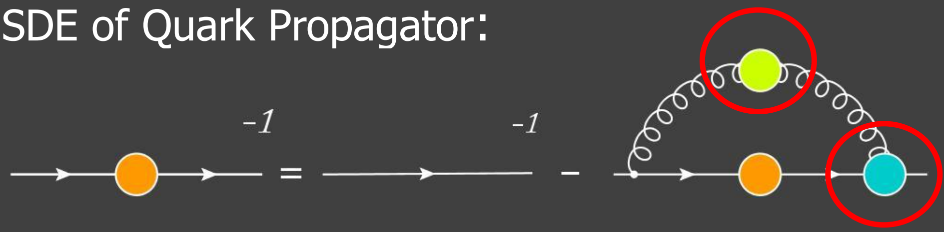
$$\Gamma_\mu(k,p)=\sum_{i=1}^{12}P_i(k^2,p^2,k\cdot p)V_{i\mu}(k,p)$$

$$iq_\mu\Gamma_\mu \;=S^{-1}(k)-S^{-1}(p)$$

WGTI of quark gluon vertex

- In covariant gauge Green functions are function of arbitrary gauge fixing parameter ξ.
  $S(k,\xi),\quad D_{\mu\nu}(q,\xi),\quad \Gamma_\mu(k,p;\xi)$
- A necessary condition that physical observable will not depend upon the choice of ξ is that all WGTI/STI's should be satisfied.

SDE of Quark Propagator:



$$S^{-1}(p) = (i\gamma\cdot p + m_0) - \int \frac{dk^4}{(2\pi)^4} g_0^2 D_{\mu\nu}(q) \frac{\lambda^a}{2} \gamma_\mu S(k) \frac{\lambda^a}{2} \Gamma_\nu^a(k,p)$$

To solve the SDE of quark propagator we require the knowledge of full gluon propagator and quark-gluon vertex.

We model gluon propagator and quark-gluon vertex subjecting to some general field theoretical constraints or using results from lQCD.

What necessary and sufficient knowledge of gluon propagator and quark-gluon vertex is required in order to describe the complete and correct behavior of quark propagator?

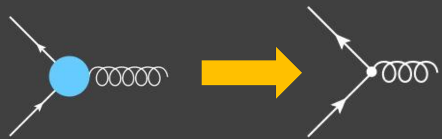
Field Theoretical Constraints:

- Gauge Invariance (WGTI's or STI's).
- Gauge covariant relations.
- Multiplicative renormalizability.
- Agreement with perturbation theory in weak coupling limit.

- The model should also agree with lQCD in construction as well as in its implications.
- Phenomenological agreement is an ultimate condition.

Models of Quark-gluon Vertex

1. Replacing by free vertex



$$\Gamma_\mu(k,p) = \gamma_\mu$$

Rainbow Truncation

This model breaks WGTI/STI of quark gluon vertex.

$$iq_\mu \Gamma_\mu = S^{-1}(k) - S^{-1}(p)$$

Constraint on the vertex due to WGTI:

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p) = \sum_{i=1}^{12} P_i(k^2, p^2, k\cdot p) V_{i\mu}(k,p)$$

$$iq_\mu \Gamma^L_\mu = S^{-1}(k) - S^{-1}(p)$$

$$q_\mu \Gamma^T_\mu = 0$$

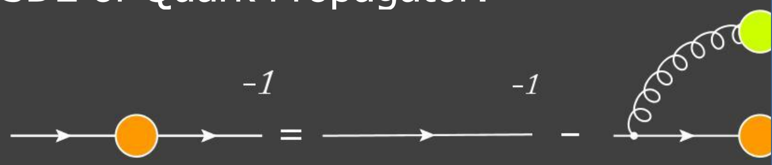
$$\Gamma^L_\mu = \left(\frac{A(k^2)+A(p^2)}{2}\right)\gamma_\mu + \left(\frac{A(k^2)-A(p^2)}{k^2-p^2}\right)\frac{(k+p)_\mu}{2}(\not{k}+\not{p}) + \left(\frac{B(k^2)-B(p^2)}{k^2-p^2}\right)(k+p)_\mu$$

Longitudinal part is called Ball Chiu (BC) part.

$$\Gamma^T_\mu = \sum_{i=1}^{8} \tau_i(k^2, p^2, k\cdot p) T_{i\mu}$$

Multiplicative renormalizability of SDE can be used to constrain $\Gamma^T_\mu(k,p)$

SDE of Quark Propagator:



$$S^{-1}(p)=(i\gamma\cdot p+m_0)-\int\frac{dk^4}{(2\pi)^4}g_0^2D_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\mu S(k)$$

The MR requires that all Green functions should be renormalizable by using finite number of multiplicative renormalization constant.

The condition of MR cannot be satisfied by any arbitrary quark-gluon vertex ansatz in SDEs.

How to do renormalization:

Regularize the theory by applying hard momentum cutoff.

$$S(p;\Lambda) \quad D_{\mu\nu}(q;\Lambda) \quad \Gamma_\mu(k,p;\Lambda)$$

$$\psi=Z_2^{-1/2}\psi_0$$
$$A_\mu^a=Z_3^{-1/2}A_{0\mu}^a$$
$$g=\frac{Z_2Z_3^{1/2}}{Z_1}g_0$$

$$\tilde{S}(p;\mu)=Z_2^{-1}(\mu,\Lambda)S(p;\Lambda)$$
$$\tilde{D}_{\mu\nu}(q;\mu)=Z_3^{-1}(\mu,\Lambda)D_{\mu\nu}(q;\Lambda)$$
$$\tilde{\Gamma}_\mu(k,p;\mu)=Z_1(\mu,\Lambda)\Gamma_\mu(k,p;\Lambda)$$

- Renormalized green functions are function of arbitrary scale parameter μ.
- However, the physical observables must not depend upon the choice of μ. This is ensured by a correct μ dependence of fundament constants (m's and g's) of the field theory.

$$A(obs.)=A\big(m(\mu),\ \ g(\mu),\ \ \text{gre. fun.}\,(\mu)\big)$$

$$\tilde{S}^{-1}(p)=Z_2(i\gamma\cdot p+m_0)-Z_1\int\frac{dk^4}{(2\pi)^4}g^2\tilde{D}_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\mu\tilde{S}(k)\tilde{\Gamma}_\nu^a(k,p)$$

Multiplicative Renormalizability

The condition of MR cannot be satisfied by any arbitrary quark-gluon vertex ansatz in SDEs.

(An example)
Quark renormalization function in leading-logarithm approximation:

$$S(k) = \frac{F(k^2)}{i\not{k} + M(k^2)}$$

$$F(p^2) = 1 + \alpha A_{1,1} \ln(p^2/\Lambda^2) + \alpha^2 A_{2,2} \ln^2(p^2/\Lambda^2) + \cdots\cdots$$

MR requires that $A_{2,2} = A_{1,1}^2/1!$

If we use bare approximation of quark-gluon vertex in quark propagator SDE we get

$$\frac{1}{F(p^2)} = 1 + \frac{\alpha\xi}{4\pi p^2}\left(\int_0^{p^2} dk^2 \frac{k^2}{p^2} F(k^2) + \int_{p^2}^{\Lambda^2} dk^2 \frac{p^2}{k^2} F(k^2)\right)$$

(where $M(p^2)$ is taken zero as an approximation)

$$F(p^2) = 1 + \left(\frac{\alpha\xi}{4\pi}\right)\ln(p^2/\Lambda^2) + \frac{3}{2}\left(\frac{\alpha\xi}{4\pi}\right)^2 \ln^2(p^2/\Lambda^2) + \cdots\cdots$$

In which $A_{2,2} = \frac{3}{2} A_{1,1}^2/1!$

MR can be used to constrain $\Gamma_\mu^T(k,p)$ as the BC vertex itself doesn't satisfy the condition of MR.

Models of Quark Gluon Vertices

2. Curtis Pennington (CP) Vertex:

$$\Gamma_\mu(k,p)=\Gamma_\mu^{BC}(k,p)+\frac{1}{2}\left(A(k^2)-A(p^2)\right)\frac{T_{\mu 6}}{d(k,p)}$$

CP vertex satisfies the condition of MR in quench approximation for zero as well as non-zero quark mass.

3. Kizilersu Pennington (KP) Vertex:

$$\Gamma_\mu(k,p)=\Gamma_\mu^{BC}(k,p)+\sum_{i=2,3,6,8}\tau_i T_{\mu i}$$

$$\tau_2=-\frac{4}{3}\frac{1}{(k^4-p^4)}\left(A(k^2)-A(p^2)\right)-\frac{1}{3}\frac{1}{(k^2+p^2)^2}\left(A(k^2)+A(p^2)\right)\ln\left(\frac{A(k^2)A(p^2)}{A(q^2)}\right),$$

$$\tau_3=-\frac{5}{12}\frac{1}{(k^2-p^2)}\left(A(k^2)-A(p^2)\right)-\frac{1}{6}\frac{1}{(k^2+p^2)}\left(A(k^2)+A(p^2)\right)\ln\left(\frac{A(k^2)A(p^2)}{A(q^2)}\right),$$

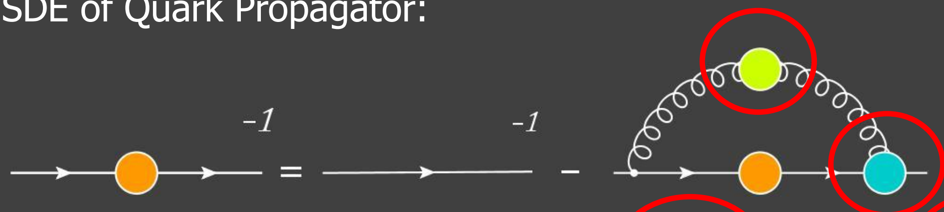
$$\tau_6=\frac{1}{4}\frac{1}{(k^2+p^2)}\left(A(k^2)-A(p^2)\right),$$

$$\tau_8=0.$$

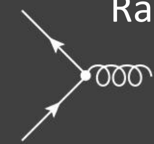
KP vertex satisfies the condition of MR in un-quenched approximation only for zero quark mass.

Models of gluon propagator

SDE of Quark Propagator:



Rainbow Truncation



$$S^{-1}(p) = Z_2(i\gamma\cdot p + m_0) - Z_1\int\frac{dk^4}{(2\pi)^4} g^2 D_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\mu S(k)\Gamma_\nu^a(k,p)$$

$$\frac{\lambda^a}{2}\gamma_\mu$$

Maris-Tandy Model: It is based on Rainbow truncation of vertex.

Gluon propagator:

$$g^2 D_{\mu\nu}(q) = g^2\frac{D(q^2)}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$ (in Landau gauge)

$$\lim_{q^2\leq\Lambda_{QCD}^2}\frac{g^2 D(q^2)}{q^2} = ??$$

$$\lim_{q^2\gg\Lambda_{QCD}^2}\frac{g^2 D(q^2)}{q^2} \approx 4\pi\frac{\gamma_m\pi}{\ln[q^2/\Lambda_{QCD}^2]}\frac{1}{q^2}$$ (Perturbative calculation)

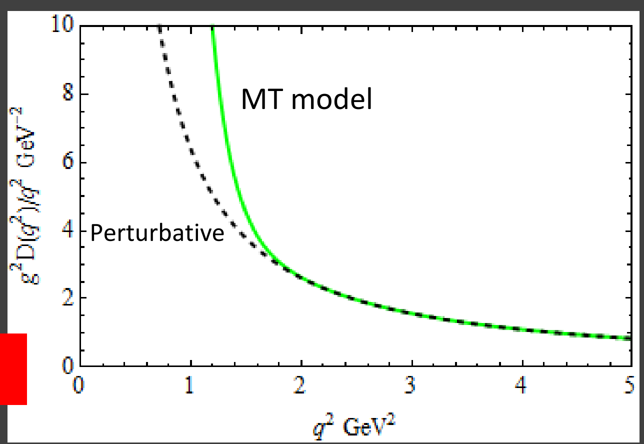
$$\frac{g^2 D(q^2)}{q^2} = \frac{4\pi^2}{\omega^6}Dq^2 e^{-q^2/\omega^2} + 4\pi\frac{\gamma_m\pi}{1/2\ln[\tau + (1+q^2/\Lambda_{QCD}^2)^2]}\frac{1-e^{-q^2/4m_t^2}}{q^2}$$

Maris-Tandy Model

Infrared dominant; A Model

UV dominant; Given by perturbative QCD

$\omega = 0.4$ so that the first term don't perturb with 2nd in UV region.
$D = 0.93$ GeV^2 is fitted to lQCD result of quark condensate ,
$-\langle\bar{q}q\rangle_{\mu=1\ GeV} = (0.241)^3$GeV^3.



This model does not include any N_f dependence in the infrared part.

Solution of SDE of quark propagator in Rainbow truncation + MT model

$$S^{-1}(p)=Z_2(i\gamma\cdot p+m_0)-\frac{4}{3}Z_1\int\frac{dk^4}{(2\pi)^4}\frac{g^2D(q^2)}{q^2}(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2})\frac{\lambda^a}{2}\gamma_\mu S(k)\gamma_\nu$$

- We need to first fix the renormalization constants Z_1 and Z_2.
  $Z_2=Z_1$ (WGTI of quark-gluon vertex)
- On-mass shell renormalization scheme.

Renormalization BC:

$$S(p)|_{p^2=\mu^2}=\frac{1}{i\gamma\cdot p+m_\mu}$$

$$S(p)=\frac{F(p^2)}{i\gamma\cdot p+M(p^2)}$$

$$F(p^2)_{p^2=\mu^2}=1,$$
$$M(p^2)_{p^2=\mu^2}=m_\mu$$

Renormalized current mass

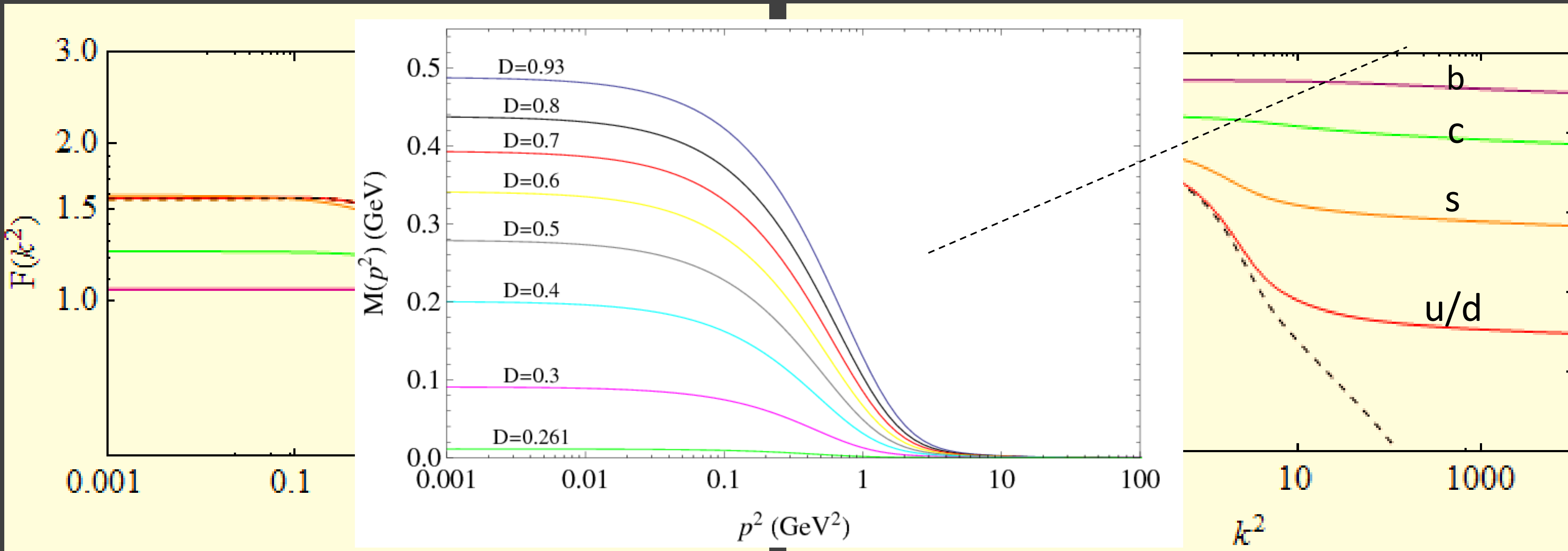
$$\frac{1}{F(p^2)}=Z_2\left(1+\frac{1}{12\pi^4p^2}\int dk^4\frac{g^2D(q^2)}{q^2}\frac{F(k^2)}{k^2+M^2(k^2)}\left[k\cdot p+\frac{2(k\cdot q)(p\cdot q)}{q^2}\right]\right)$$

$$\frac{M(p^2)}{F(p^2)}=Z_2\left\{m_0+\frac{1}{4\pi^4}\int_0^\infty dk^4\frac{g^2D(q^2)}{q^2}\frac{M(k^2)F(k^2)}{k^2+M^2(k^2)}\right\}$$

$$Z_2=\left(1+\frac{1}{12\pi^4p^2}\int dk^4\frac{g^2D(q^2)}{q^2}\frac{F(k^2)}{k^2+M^2(k^2)}\left[k\cdot p+\frac{2(k\cdot q)(p\cdot q)}{q^2}\right]\right)^{-1}_{p^2=\mu^2}$$

$$m_\mu=Z_2\left\{m_0+\frac{1}{4\pi^4}\int_0^\infty dk^4\frac{g^2D(q^2)}{q^2}\frac{M(k^2)F(k^2)}{k^2+M^2(k^2)}\right\}_{p^2=\mu^2}$$

Quark Propagator using MT model



$$
\left.\begin{aligned}
m_{u/d} &= 0.00374 \text{ GeV} \\
m_s &= 0.083 \text{ GeV} \\
m_c &= 0.88 \text{ GeV} \\
m_b &= 3.8 \text{ GeV}
\end{aligned}\right\} \mu = 19 \text{ GeV}
$$

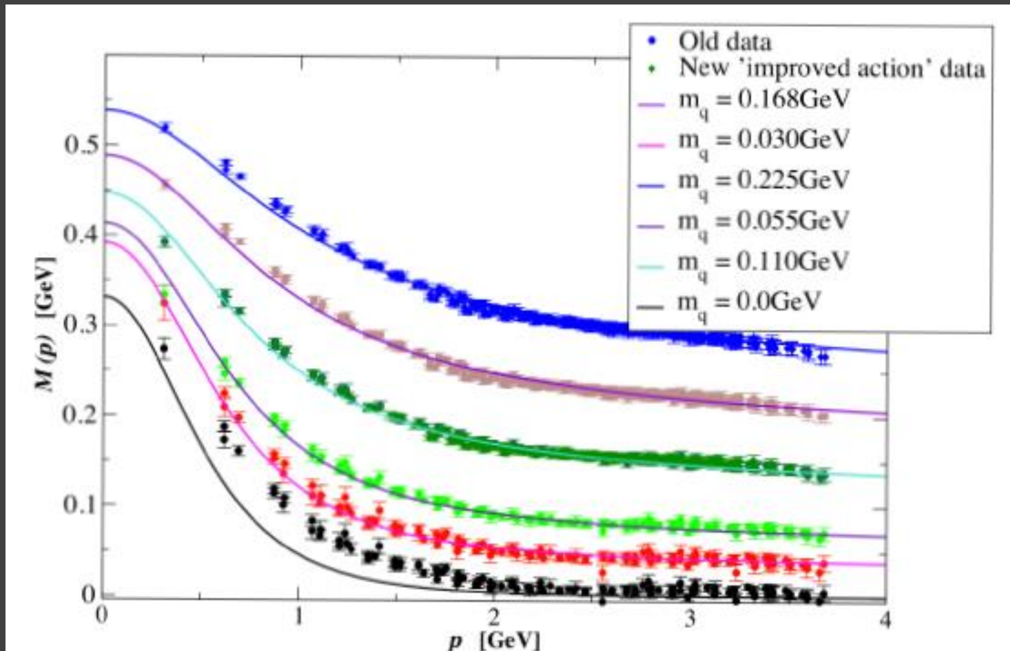
How the model generates Dynamical mass?

- The 1st term of the MT model enhance the coupling strength in IR region through the controlling parameter D.
- It is this enhancement which generates dynamical mass in IR region.
- Gradual decrease in parameter D shows that DCS is
- restored if D < 0.261.

MT model of gluon propagator function:

$$
\frac{g^2 D(q^2)}{q^2} = \frac{4\pi^2}{\omega^6} D q^2 e^{-q^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{1/2 \ln[\tau + (1 + q^2/\Lambda_{QCD}^2)^2]} \frac{1 - e^{-q^2/4m_t^2}}{q^2}
$$

Comparison with Qu-lQCD results:

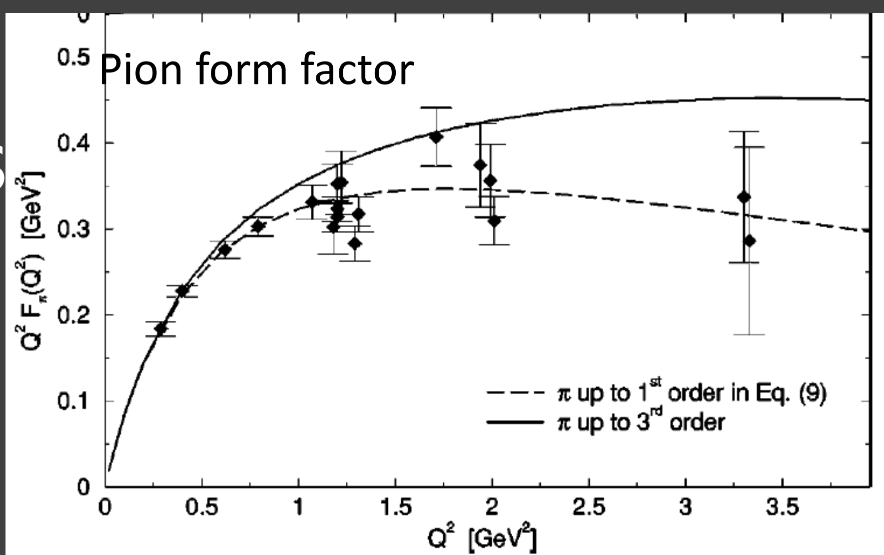


Pseudoscalar (PM, Roberts, PRC56, 3369)	expt.	calc.
$-\langle \bar{q}q\rangle_\mu^0$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Vector mesons (PM, Tandy, PRC60, 055214)		
$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)		
$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^* K\pi}$	4.60	4.1

hBS



Troubles with MT model:

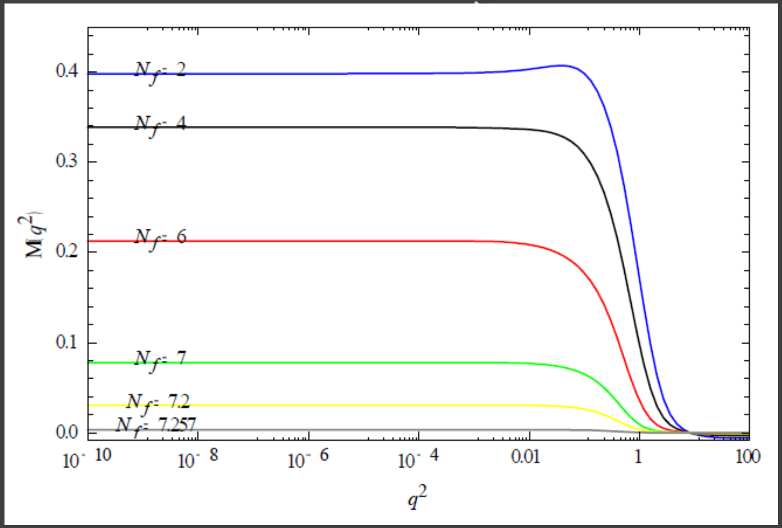
- Rainbow truncation breaks Gauge invariance as the WGTI/STI of quark-gluon is not satisfied.

As a result physical observable may develop dependence on gauge fixing parameter ξ.

- Violates multiplicative renormalizability.

As a result physical observable may develop dependence on arbitrary scale parameter μ.

Quark Mass functions using KP Vertex



Kizilersu Pennington (KP) Vertex:

$$\Gamma_\mu(k,p) = \Gamma_\mu^{BC}(k,p) + \sum_{i=2,3,6,8} \tau_i T_{\mu i}$$

$$\tau_2 = -\frac{4}{3}\frac{1}{(k^4-p^4)}\left(A(k^2)-A(p^2)\right) - \frac{1}{3}\frac{1}{(k^2+p^2)^2}\left(A(k^2)+A(p^2)\right)\ln\left(\frac{A(k^2)A(p^2)}{A(q^2)}\right),$$

$$\tau_3 = -\frac{5}{12}\frac{1}{(k^2-p^2)}\left(A(k^2)-A(p^2)\right) - \frac{1}{6}\frac{1}{(k^2+p^2)}\left(A(k^2)+A(p^2)\right)\ln\left(\frac{A(k^2)A(p^2)}{A(q^2)}\right),$$

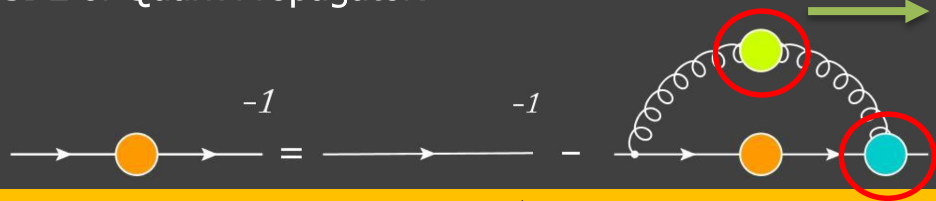
$$\tau_6 = \frac{1}{4}\frac{1}{(k^2+p^2)}\left(A(k^2)-A(p^2)\right),\ \tau_8 = 0.$$

KP vertex satisfies the condition of MR in un-quenched approximation

Critical number of quark flavors in QCD

- Asymptotic freedom requires that $N_f > 16.5$ (at 1 loop order).
- Dynamical breaking of chiral symmetry may also require a critical number of quark flavors.

SDE of Quark Propagator:

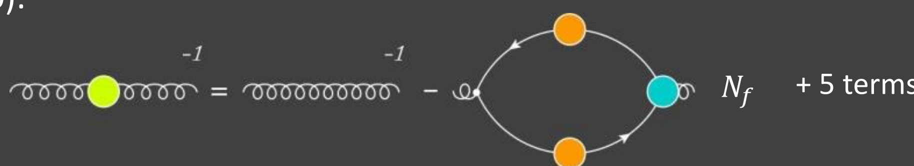


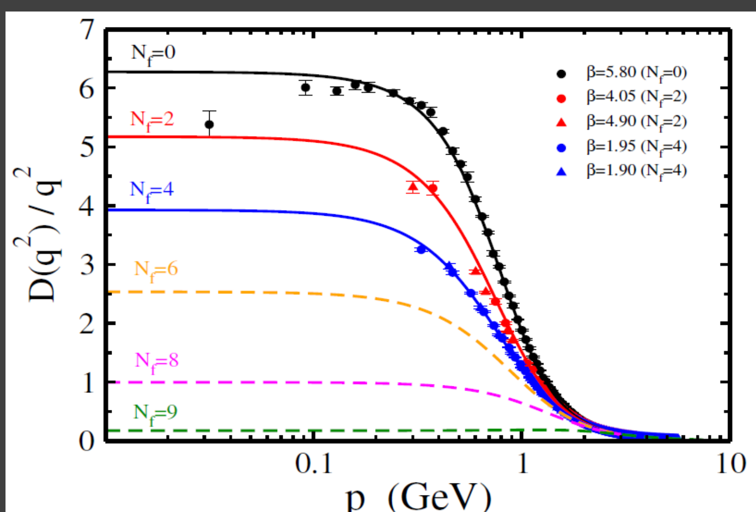
$$S^{-1}(p) = Z_2(i\gamma\cdot p + m_0) - Z_1\int\frac{dk^4}{(2\pi)^4}\, g^2 D_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\mu S(k)\Gamma_\nu^a(k,p)$$

MT model does not allow as to study the flavor dependence of quark propagator in the infrared region as the gluon propagator is itself taken Flavor independent.

Gluon propagator (in Landau gauge ξ=0):

$$D_{\mu\nu}(q,N_f) = \frac{D(q^2,N_f)}{q^2}\left(\delta_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$





Fit with Refined Gribov-Zwanziger (RGZ) form

$$D(q^2,N_f) = \frac{z(\mu^2)q^2(q^2+M^2)}{q^4 + q^2\left(M^2 - 13g^2\langle A^2\rangle/24\right) + M^2 m_0^2}$$

$$m_0(N_f) = 1.011(9.161 - N_f)^{-1/2}\,GeV$$

$$g^2\langle A^2\rangle(N_f) = 0.474(16.406 - N_f)\,GeV$$

$$M^2 = 4.85\,GeV$$

A. Ayala, A. Bashir, D. Binosi M. Cristofretti, and J. Rodriguez-Quintro, Phys. Rev. D 86, 074512 (2012)

Solution of SDE of the Quark Propagator

SDE of Quark Propagator:

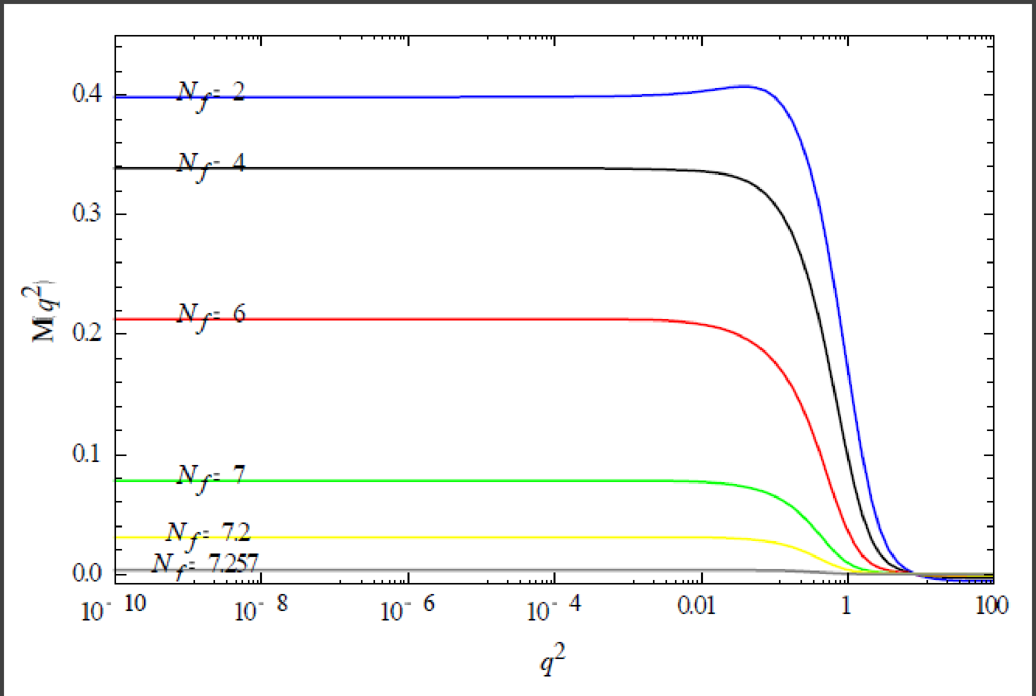
$$S^{-1}(p,N_f) = Z_2(i\gamma\cdot p + m_0) - Z_1\int \frac{dk^4}{(2\pi)^4} g^2 D_{\mu\nu}(q,N_f)\frac{\lambda^a}{2}\gamma_\mu S(k,N_f)\frac{\lambda^a}{2}\Gamma_\nu(k,p)$$

General form of quarks propagator:

$$S(p,N_f) = \frac{F(p^2,N_f)}{i\gamma\cdot p + M(p^2,N_f)}$$

Lattice QCD

KP Vertex



(Current mass $m = 0$)

Final remarks

- Asymptotic freedom in QCD requires that $N_f < 16.5$.
- Latest lattice results of gluon propagator when used in SDE of quark propagator truncated by KP vertex shows that $N_f < 7.2$ if QCD is to exhibit DCSB.

Extra slides

Elementary Particle Physics Today

Elementary particles:

Quarks
(can interact through strong interaction)

Leptons
(cannot interact through strong interaction)

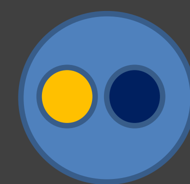
Gauge Bosons
(mediate interactions)

Higgs Boson
(impart mass to the elementary particles)

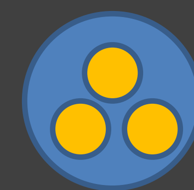
Quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$ $\longrightarrow$ Hadrons: $\begin{cases} \text{Meson: } s = 0,1,2 \ldots \\ \text{Baryons: } s = \frac{1}{2}, \frac{3}{2}, \ldots \end{cases}$

Leptons: $\begin{pmatrix} v_e \\ e \end{pmatrix} \quad \begin{pmatrix} v_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} v_\tau \\ \tau \end{pmatrix}$

Meson:



Baryon



Gauge Bosons: $\gamma,\ W^{\pm},\ Z^{0}$, and 8 gluons

Higgs Bosons: H (God/Mother particle)

399 Mesons, 574 Baryons have been discovered

- Strong interaction (Quantum Chromodynamics)
- Electro-weak interaction (Quantum electro-flavor dynamics)

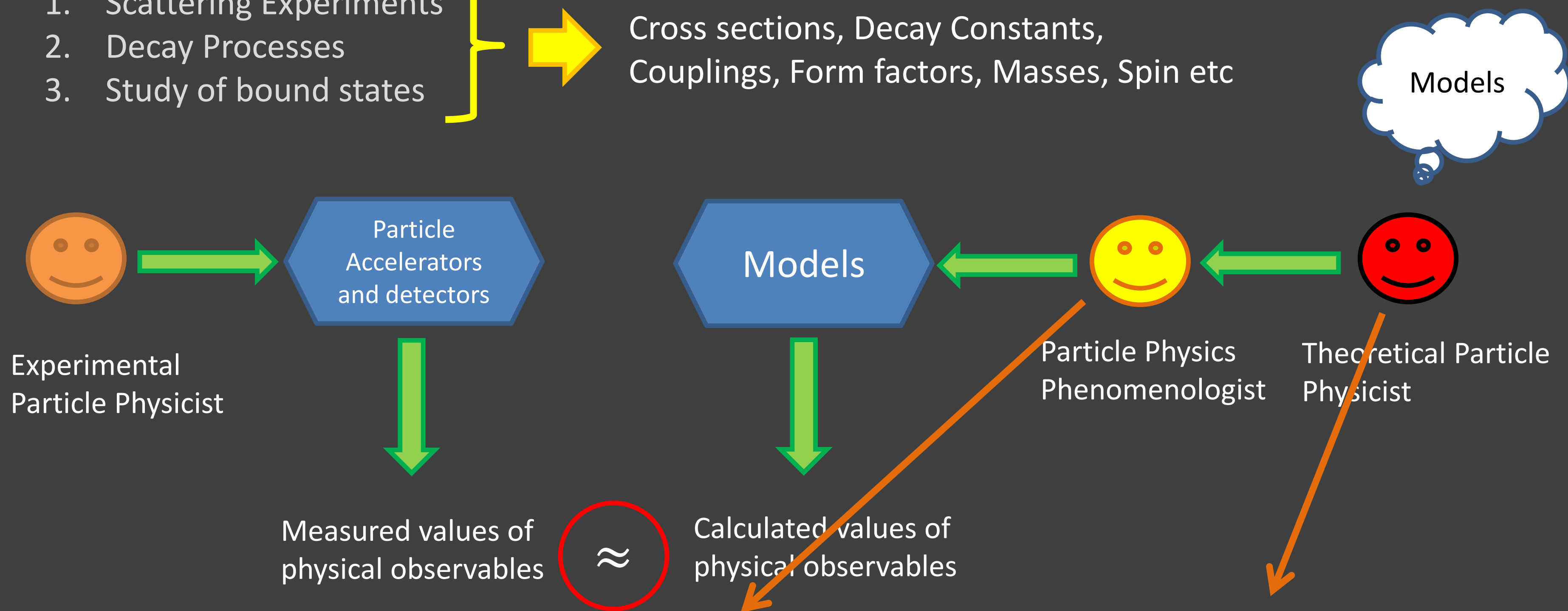
} The Standard Model

Explaining the properties of the Hadrons in terms of QCD's fundamental degrees of freedom is the Problem laying at the forefront of Hadronic physics.

Elementary Particle Physics

1. Scattering Experiments
2. Decay Processes
3. Study of bound states

→ Cross sections, Decay Constants, Couplings, Form factors, Masses, Spin etc



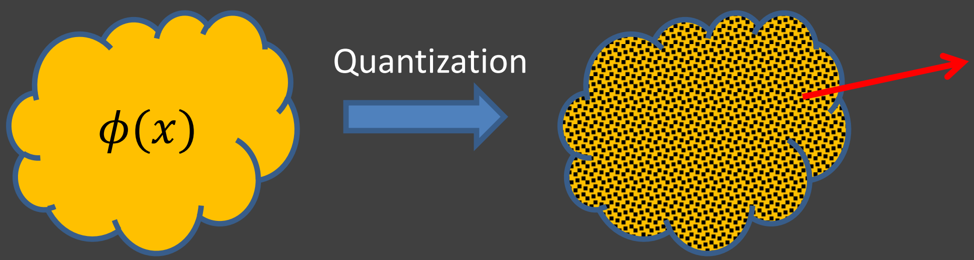
It doesn’t matter how beautifull your theory is, It doesn’t matter how smart you are. If it doesn’t agree with experiment, it’s wrong… R.P. Feynmann

It is more important to have beauty in one's equations than to have them fit experiment..... P.A.M Dirac

Decades of observation and calculations show that the Standard Model of particle physics can describe almost every thing which we have observed in the labs of high energy physics.

The Standard Model

- The standard model is a field theory.
- In field theories we associate a field to every different kind of a particle.
  (e.g., electron field, proton field, photon field etc)
- Particles appear as quanta of field.



Quanta of field has particle properties.

- Equation of motion of the fields

$(s = 0)$

Free Scalar Field:

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = 0$$

$$\mathcal{L}_0 = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$

$(s = 1/2)$

Free Dirac Field:

$$\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$$

$$\mathcal{L}_0 = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$(s = 1)$

Free Abelian Vector Field:

$$\partial_\mu \partial^\mu A^\nu + \partial^\nu(\partial_\mu A^\mu) = 0$$

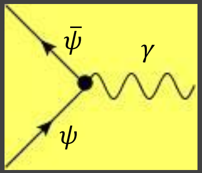
$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The Standard Model

- Interaction is introduced by the coupling of fields .

  For example: $\mathcal{L}_I = e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ for QED



- Only **Lorentz invariant** couplings are allowed.

$$\mathcal{L}_I = \bar{\psi}\gamma^{\mu}\psi A_{\mu};\ \bar{\psi}\gamma^{\mu}\gamma^{5}\psi A_{\mu};\ \bar{\psi}\psi\phi; \bar{\psi}\gamma^{5}\psi\phi;\ \bar{\psi}\gamma^{\mu}\psi\bar{\psi}\gamma_{\mu}\psi; \ldots .$$

- Possible coupling are further constrained by the **gauge symmetries**.

  i) $SU(2)_L \otimes U(1)_Y$ for electroweak interaction.
  ii) $SU(3)_c$ for strong interaction.