Decay rates and Cross section

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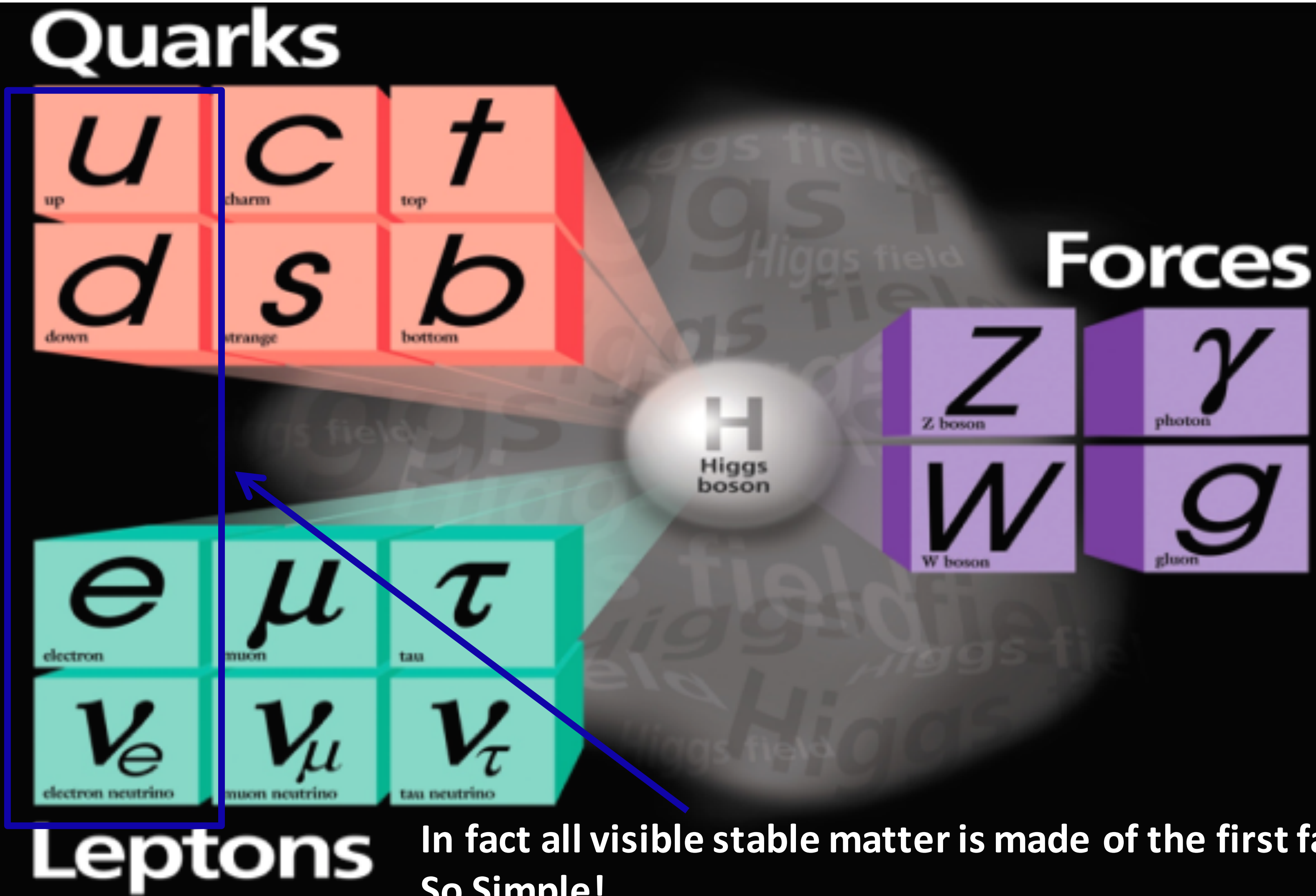
Outlines

- Introduction
- Basics variables used in Exp. HEP Analysis
- Decay rates and Cross section calculations and measurements(tqZ C&C analysis as example)
- Summary

Standard Model

With these particles we can explain the entire matter, from atoms to galaxies



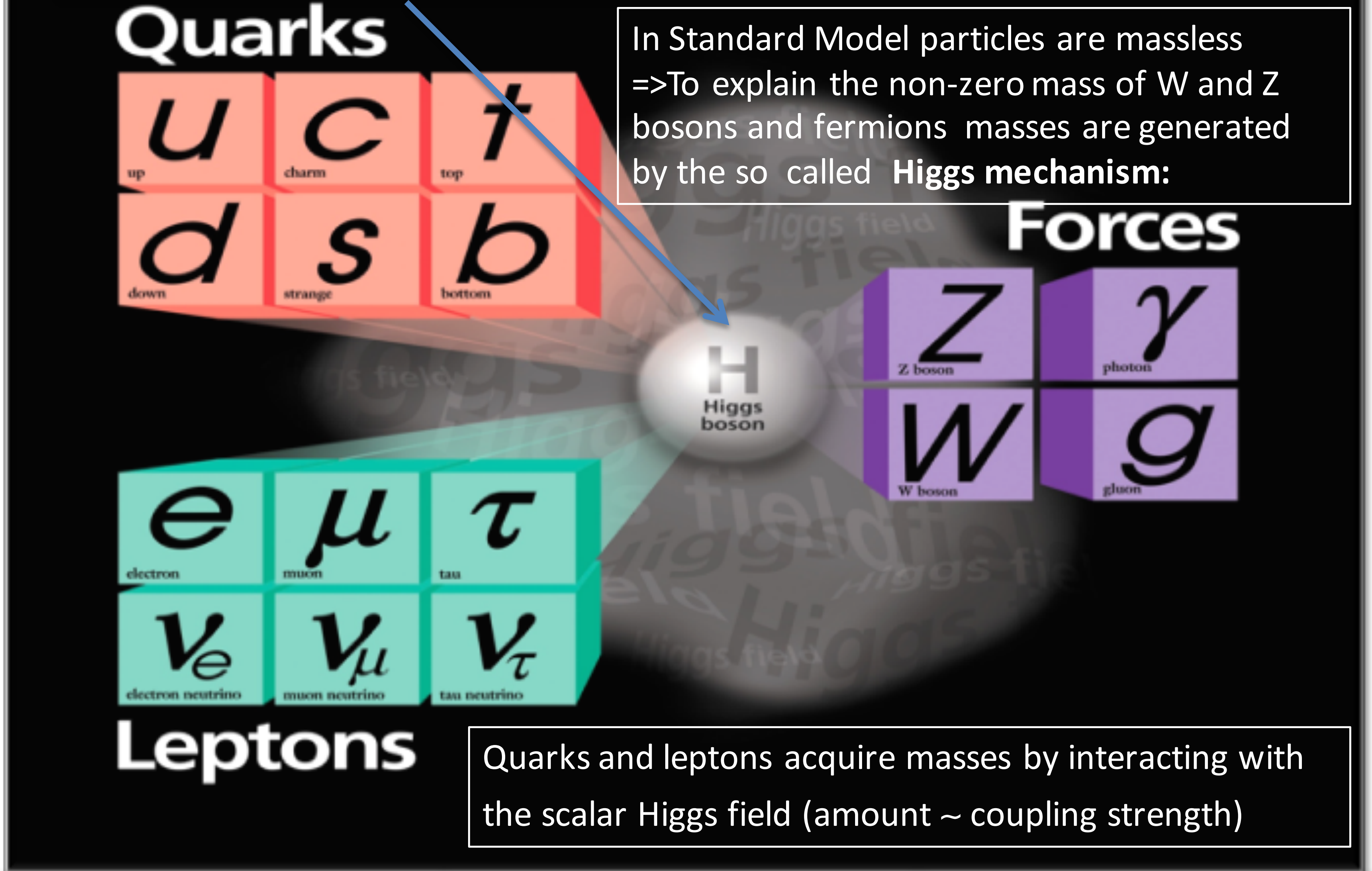
In fact all visible stable matter is made of the first family, So Simple!

Many Nobel prizes have been awarded (both theory/Exp. side)

Standard Model

Why Higgs Particle, the only missing piece until July 2012?

In Standard Model particles are massless
=>To explain the non-zero mass of W and Z bosons and fermions masses are generated by the so called **Higgs mechanism:**



Quarks and leptons acquire masses by interacting with the scalar Higgs field (amount ~ coupling strength)

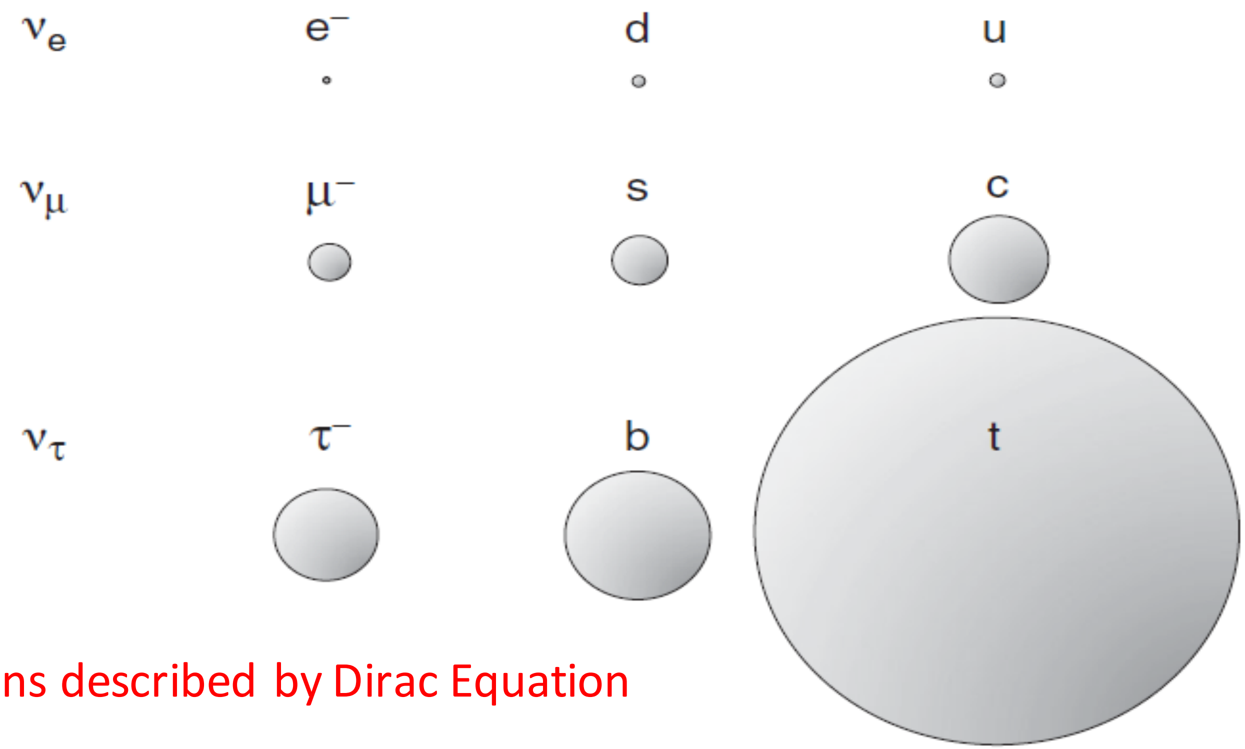
Fundamental Fermions

Leptons				Quarks			
Particle		Q	mass/GeV	Particle		Q	mass/GeV
electron	(e^-)	−1	0.0005	down	(d)	−1/3	0.003
neutrino	(ν_e)	0	$< 10^{-9}$	up	(u)	+2/3	0.005
muon	(μ^-)	−1	0.106	strange	(s)	−1/3	0.1
neutrino	(ν_μ)	0	$< 10^{-9}$	charm	(c)	+2/3	1.3
tau	(τ^-)	−1	1.78	bottom	(b)	−1/3	4.5
neutrino	(ν_τ)	0	$< 10^{-9}$	top	(t)	+2/3	174

1st generation

2nd generation

3rd generation



Dynamics of fermions described by Dirac Equation

Experiment and Theory

- *It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.*

Richard P. Feynman

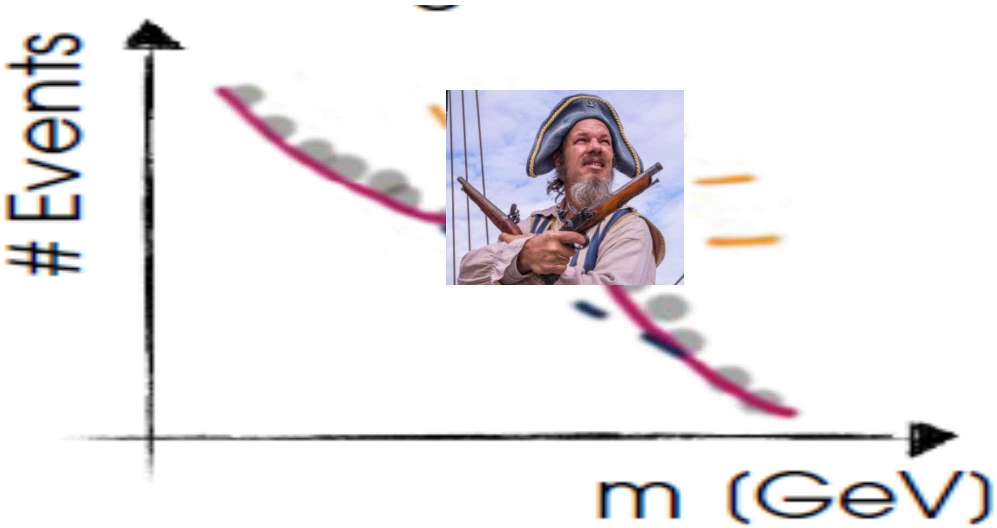
- *A theory is something nobody believes except the person who made it,*

  An experiment is something everybody believes except the person who made it.

Albert Einstein

Experiment and Theory

Experiment is something which creates hype!!!



Theory is something which makes you jumping into the train/bike☺

Five hundred papers and
citation about fluke at 750GeV

Some Basics

Mandelstam Variables

- In a two body scattering process of the form $1 + 2 \rightarrow 3 + 4$, there are 4 four-vectors involved, namely p_i (i =1,2,3,4) = (E_i, $\mathbf{p_i}$)
- Three Lorentz Invariant variables namely s, t and u are defined. These are equivalent to the four-momentum squared q^2 of the exchanged boson in the respective Feynman diagrams

$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$ Square of total CoM energy

$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$ Square of four momentum transfer between 1&3

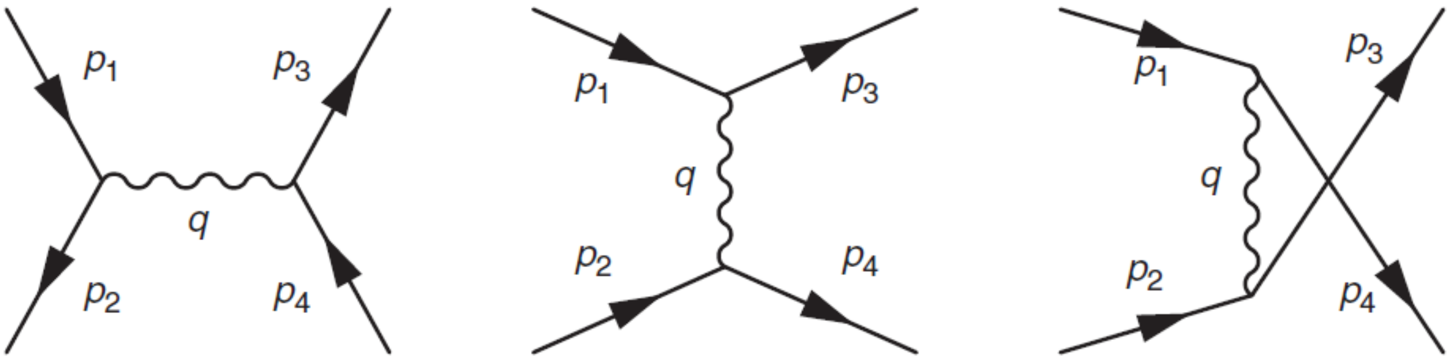
$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$ Square of four momentum transfer between 1&4

$$\boxed{s + u + t = m_1^2 + m_2^2 + m_3^2 + m_4^2}$$

where

For identical final state particles, distinction between t- and u-channel is important

s-channel t-channel u-channel



Rapidity/Pseudorapidity in Hadron Collider (1/2)

- In Hadron collider, angles of jets w.r.t beam axis are well measured
  - But jets are not produced at rest but are boosted along the of beam direction
  - The boost is because collision take place in the CoM frame of the pp system, which is not the CoM frame of the colliding partons
    - The net longitudinal momentum of colliding parton-parton system is $(x_1 - x_2)\, E_p$, where E_p is the energy of proton

- Therefore jet angles are usually expressed in terms of rapidity y defined by

$$y = \frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right)$$

where E = energy of jet
p_Z = jet momentum Z-component

- Advantages:
  - Rapidity differences are invariant under boost along the beam direction (hence cross section can be measured in rapidity bins)

$$y' = \frac{1}{2}\ln\left[\frac{E' + p'_z}{E' - p'_z}\right] = \frac{1}{2}\ln\left[\frac{\gamma(E - \beta p_z) + \gamma(p_z - \beta E)}{\gamma(E - \beta p_z) - \gamma(p_z - \beta E)}\right] = \frac{1}{2}\ln\left[\frac{(1-\beta)(E + p_z)}{(1+\beta)(E - p_z)}\right]$$

$$= y + \frac{1}{2}\ln\left(\frac{1-\beta}{1+\beta}\right).$$

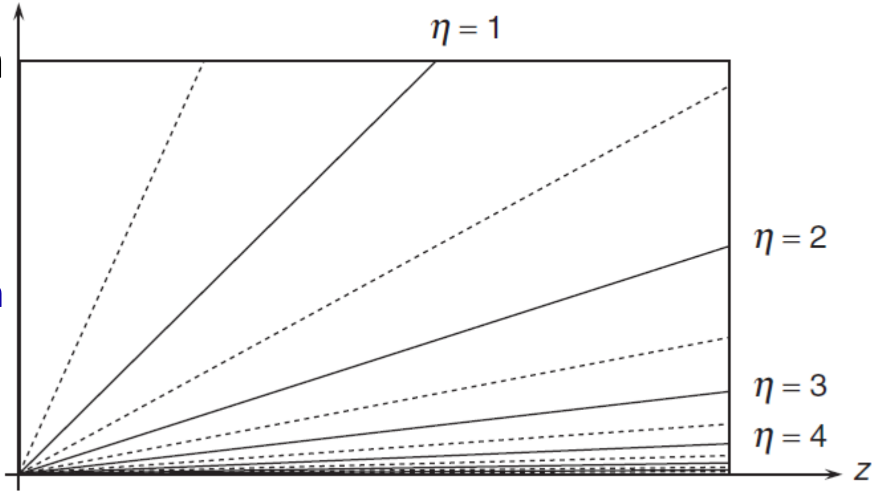
=> $\Delta y' = \Delta y$ Hence the unknown boost has no impact on Δy

Rapidity/Pseudorapidity in Hadron Collider (2/2)

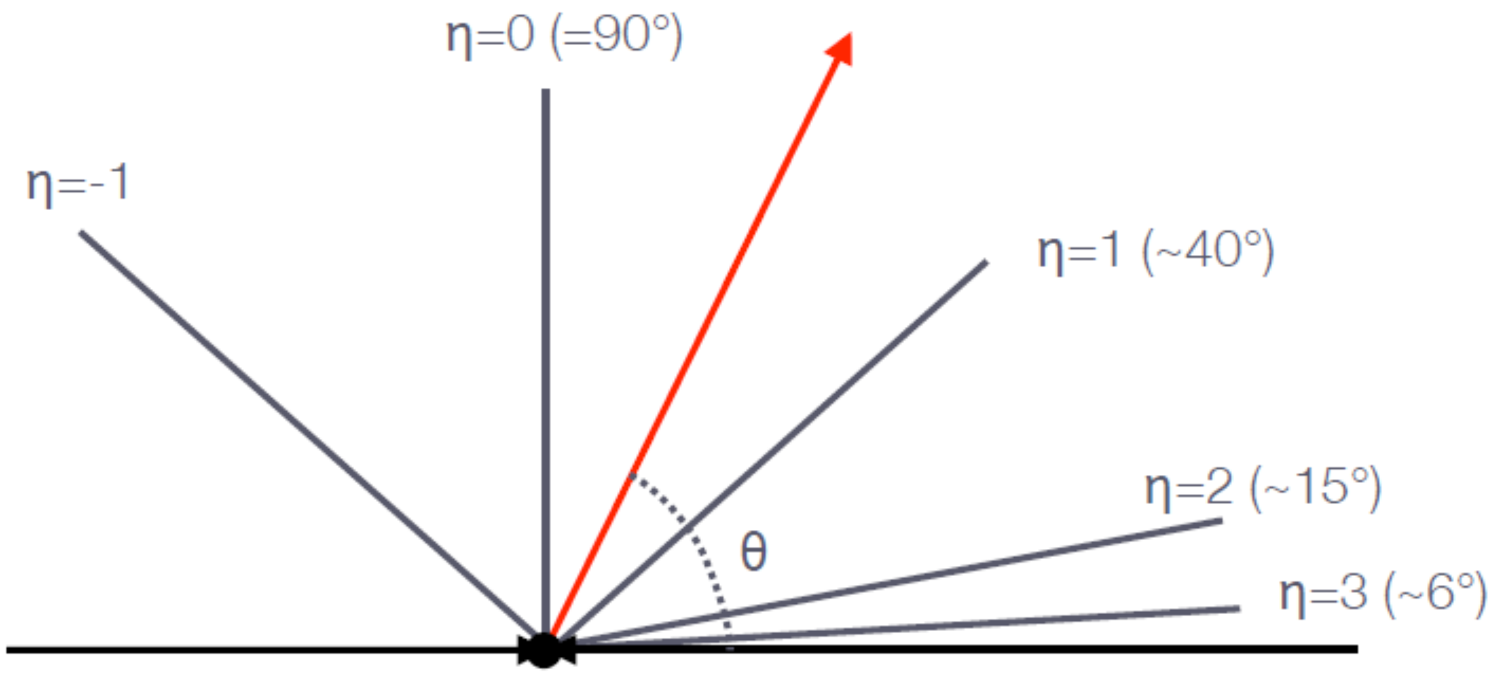
- Jet being a collection of particles, its mass is the invariant mass of its constituent particles, mainly produced during hadronisation process
  - Mass is not the same as the mass of primary parton
- For high energy jets, jet mass is usually small as compared to jet energy, hence for jet making angle θ with beam axis $p_Z \approx E \cos\theta$

=> $$y \approx \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{1}{2}\ln\left(\cot^2\frac{\theta}{2}\right)$$

- Hence we can define another variable called pseudorapidity(η) which can be used instead of y when jet mass can be neglected $\eta \equiv -\ln\left(\tan\frac{\theta}{2}\right)$
- In hadron colliders differential cross section for jet production is roughly uniform in pseudorapidity.
  - That means equal number of jets are produced in equal intervals of η
  - Hence reflecting forward nature of jet product in pp collisions



Pseudorapidity

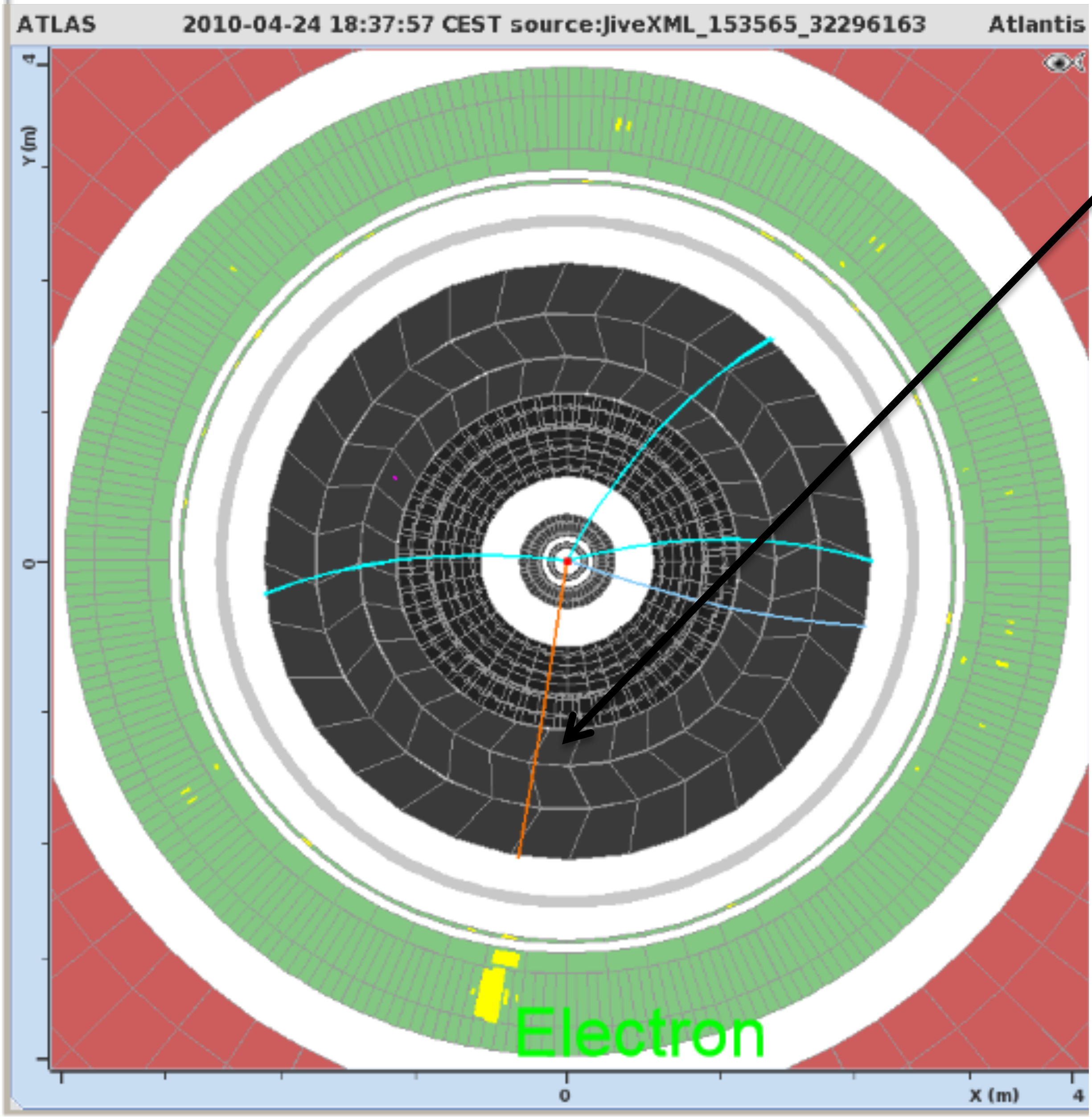


Missing Transverse Momentum/Energy (MET)

- Some of the particles produced in colliders leave no signal in the detector (no track or energy deposits in tracker or calorimeter)
  - For example neutrinos in SM, SUSY LSP and many hypothetical particles in extensions to the SM

- Their presence can be inferred indirectly through an imbalance in the total energy in the event

Event display of W→eν decay



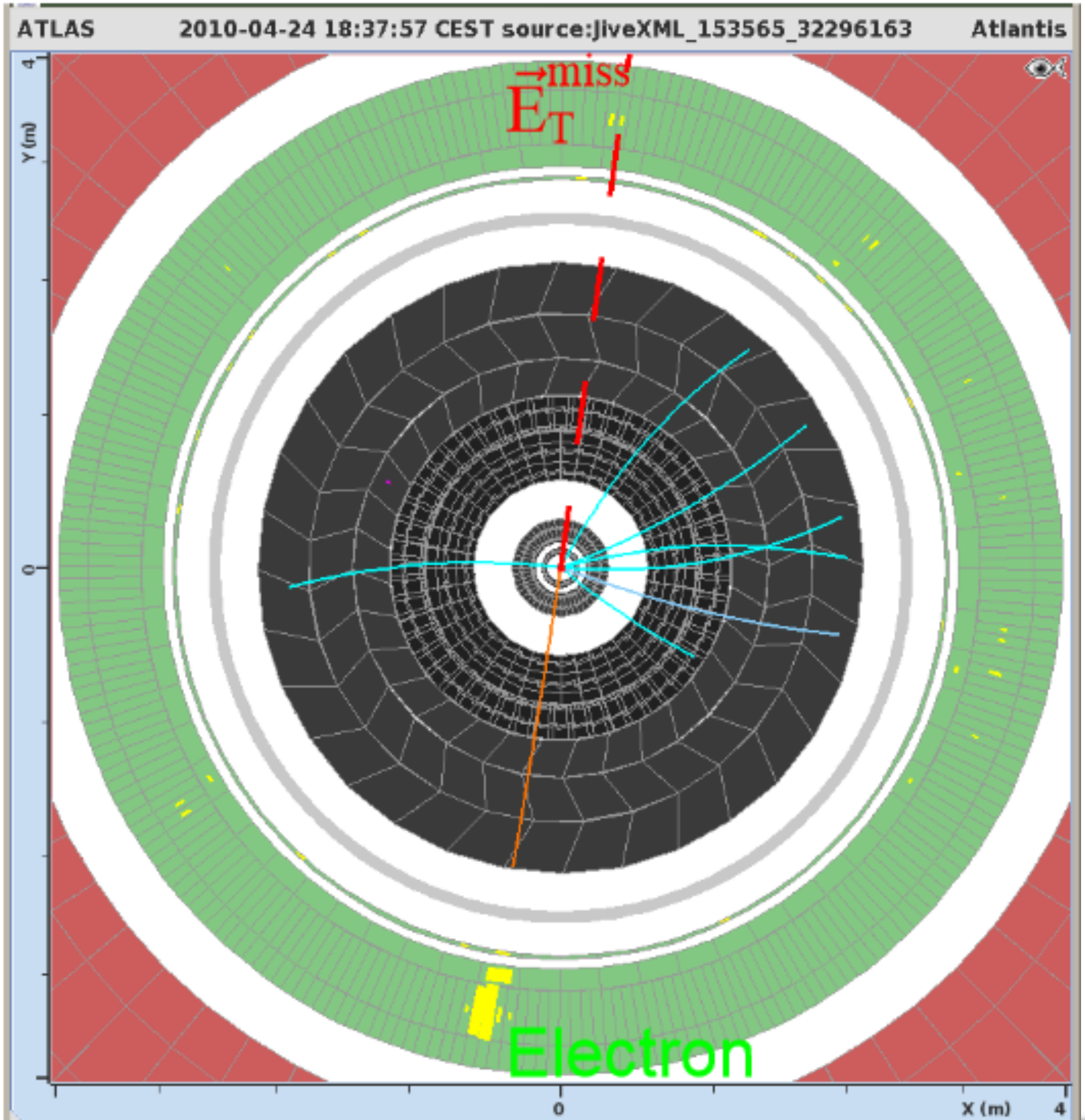
⇒ hight pt electron (pt = 29 GeV) with several low pt tracks (1 GeV)

⇒transverse momentum is a tool for selecting events in which a W boson has occurred

⇒the total transverse momentum vector is not balanced

→Missing energy

Event display of W→eν decay



=> Missing transverse energy is defined as,

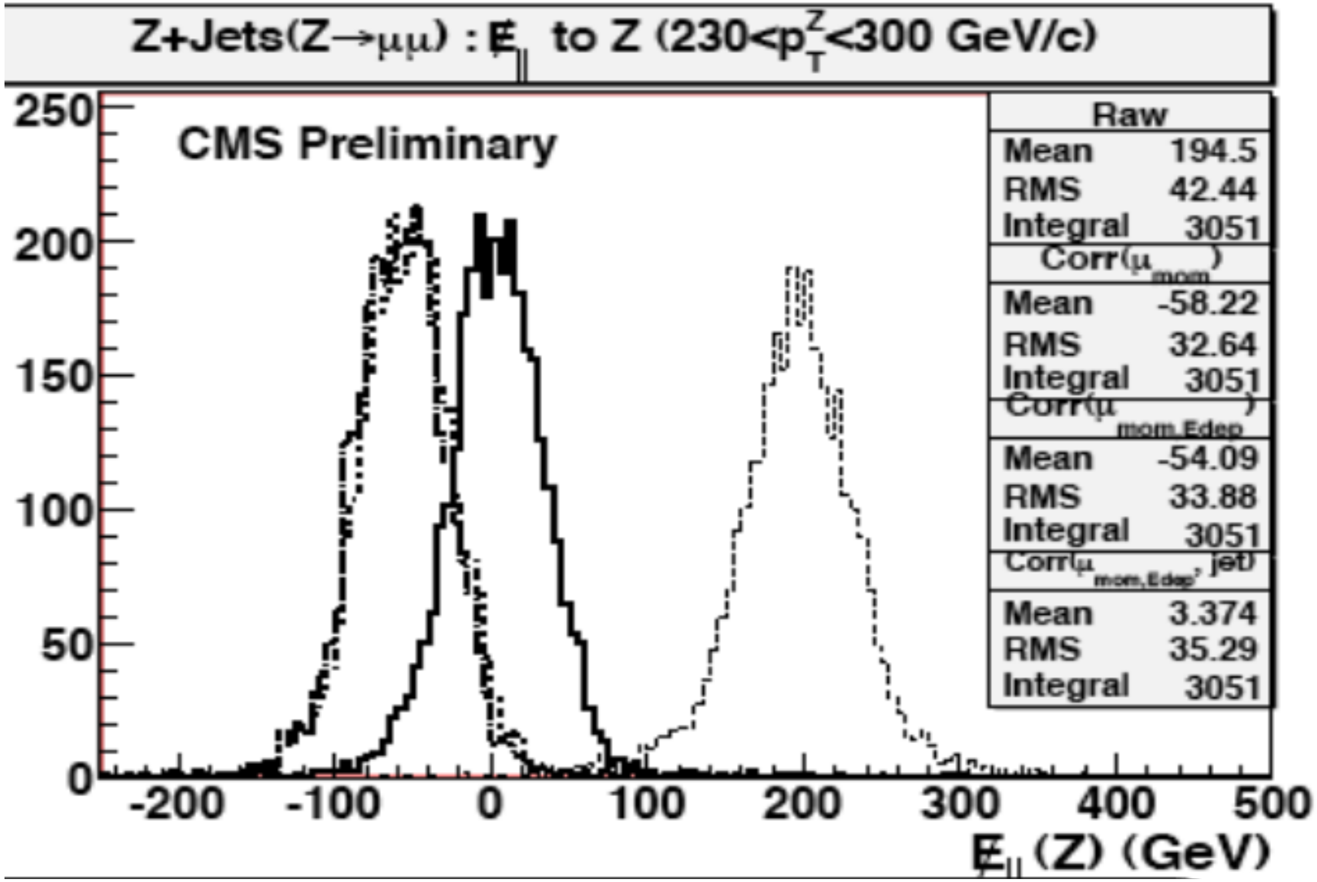
$$\vec{E}_T^{Miss} = -\sum_i E_T^i \hat{n}_i$$

=> Summation runs over all calorimeter cells and unit vector in the x-y plane is pointing from the beam axis to the *ith* cell

MET Corrections

- Total transverse energy has to be corrected for the non-neutrino contribution to the imbalance
- Has to be corrected for muons which deposit small amount of energy in calorimeter (~2-5 GeV)
  - i.e difference of calorimeter deposit and track momentum is added back into the sum
- Also other corrections like jet energy scale, electron scale, tau, pileup corrections etc
- Reconstruction of MET is very sensitive to particle momentum mismeasurements, particle misidentification, detector malfunctions, particles impinging on poorly instrumented regions of the detector, cosmic-ray particles etc
  - These may result in artificial MET (fake MET)

Muon Corrections in Z+ jets events

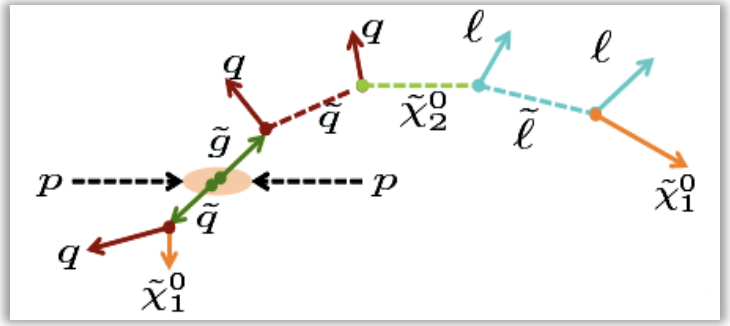


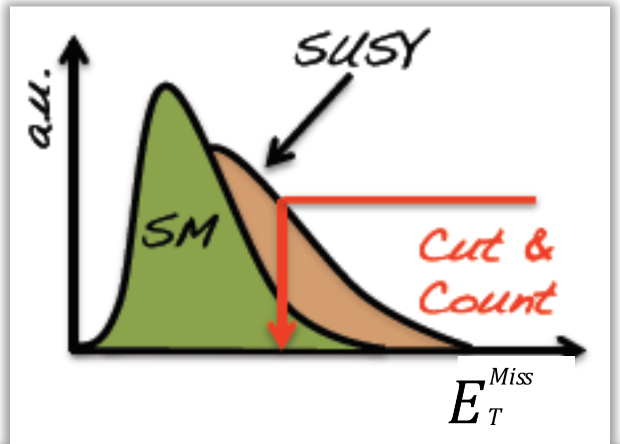
- Before corrections, <MET> ~200 GeV
- After muon corrections, <MET> ~ -54 GeV
- After type 1 corrections, <MET> ~ 3 GeV

Use of MET in Physics Analysis

- ❑ An important observable for discriminating leptonic decays of W bosons and top quarks from background events (multijet and Drell–Yan)
  - ➢ W and top mass measurements

- ❑ Very important for BSM searches such as
- ❑ R-Parity Conserving(RPC) SUSY models:
  - ➢ Sparticles produced in pairs
  - ➢ Decay chains terminating with stable and neutral LSP(neutralino or gravitino)
  - ➢ LSP leaves the detector unseen
    - ❖ Give rise to Missing Transverse Energy(MET)
  - ➢ No mass peak, signal in tails





Luminosity

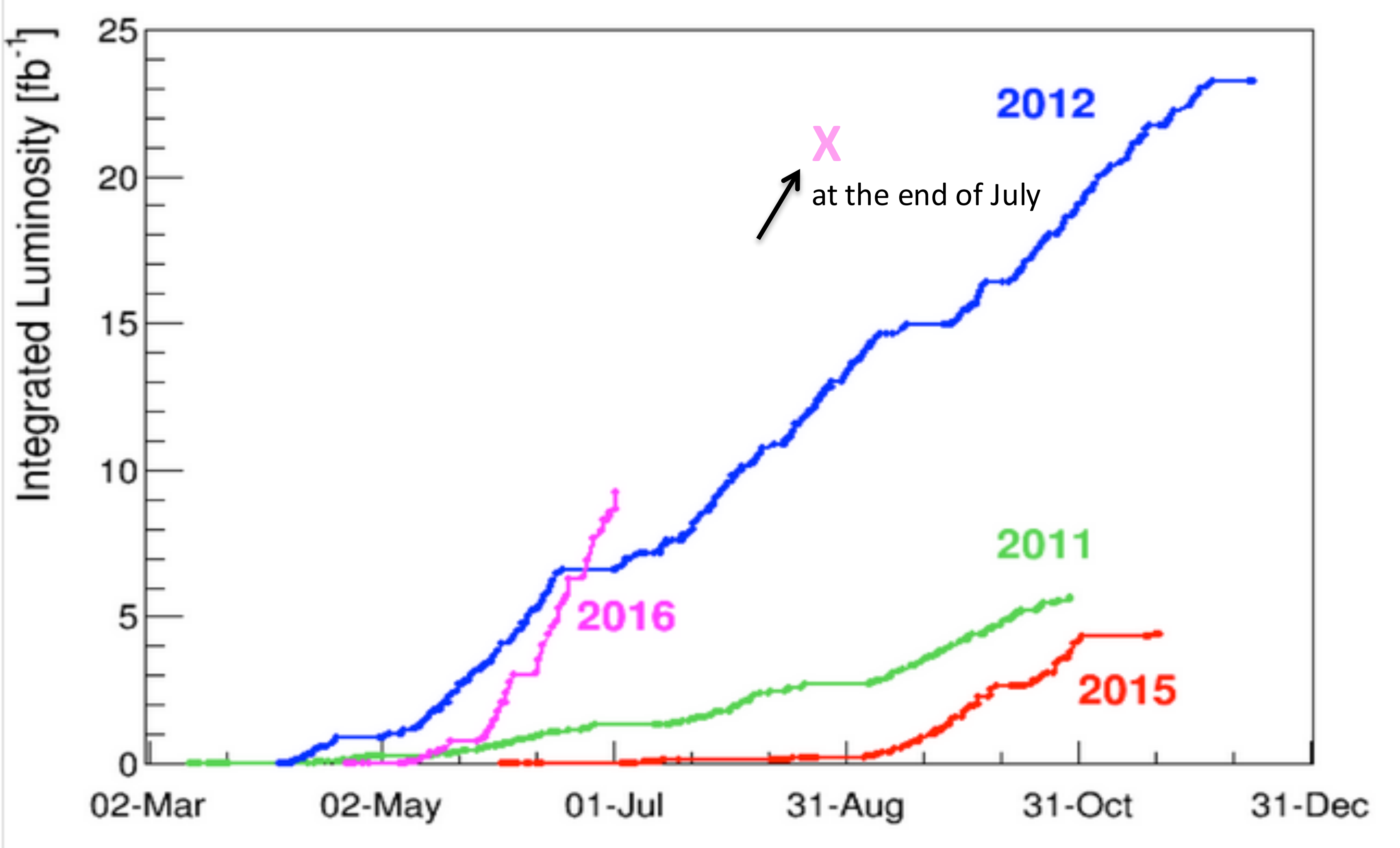
- Luminosity of collider can be defined as

$$\mathcal{L} = f\,\frac{n_1 n_2}{a}$$

  - n_1 and n_2 are the number of particles in the colliding bunches
  - f is the frequency of colliding bunches where $t = 1/f$
    - For LHC f = 40 MHZ (25 ns)
  - a = beam transverse profile (area of the beam)
  - Luminosity is measured in "# particles/cm^2/s"
  - at LHC luminosity ~ 10^{34} cm^{-2} sec^{-1} = 10 nb^{-1} sec^{-1}
- Luminosity determines event rate. The number of interaction for a given process is the product of the integrated luminosity and cross section for that process

$$N = \sigma \int \mathcal{L}(t)\,\mathrm{d}t$$

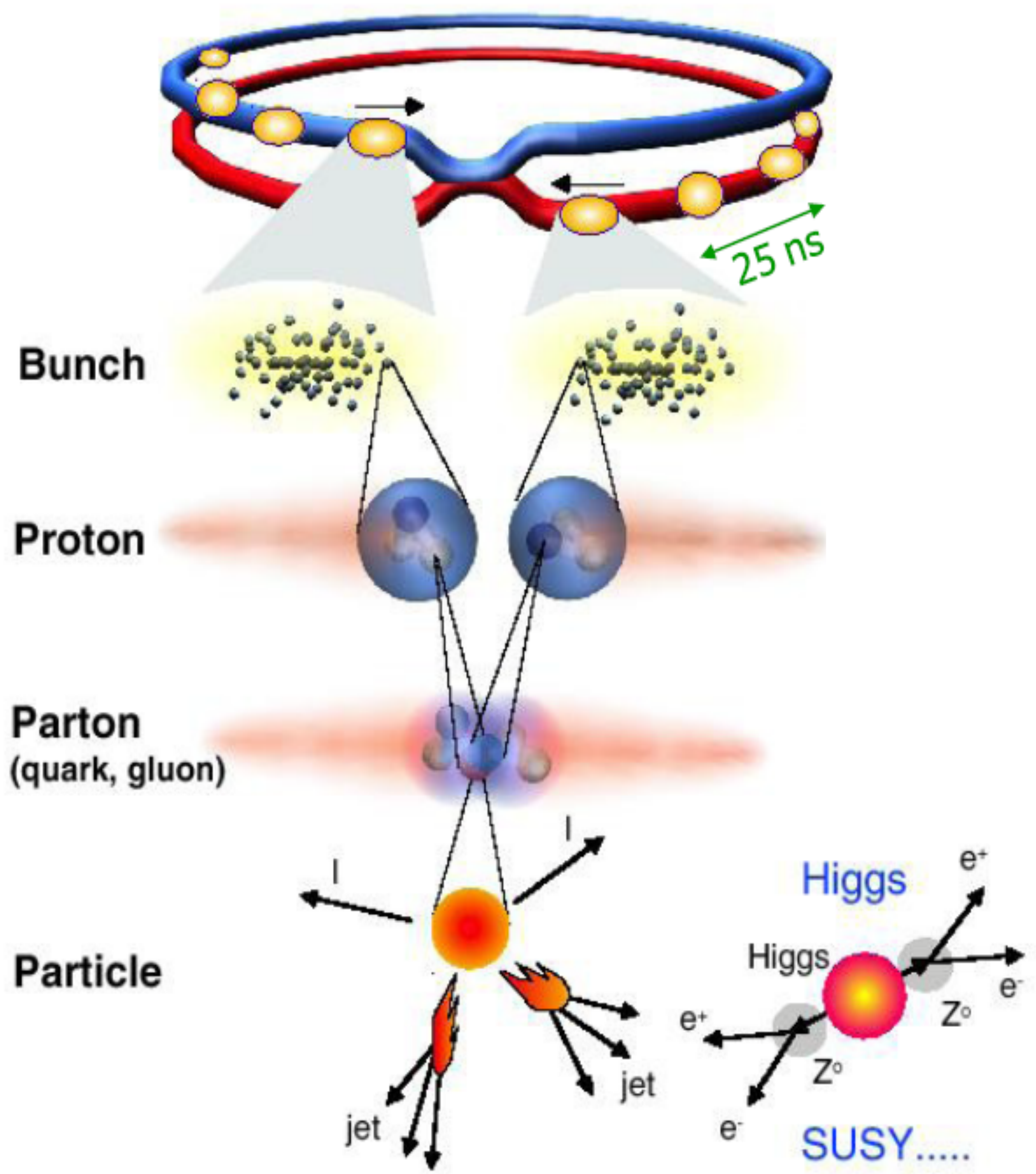
LHC Luminosity



This graph shows the integrated luminosity delivered by the LHC to the ATLAS and CMS experiments in 2011, 2012, 2015 and 2016. The integrated luminosity indicates the amount of data delivered to the experiments and is measured in inverse femtobarns. One inverse femtobarn corresponds to around 80 million million collisions.

Collision at LHC

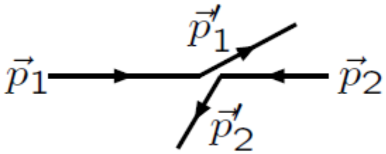
- Proton-Proton collisions
- Number of bunch = 2808(nom),
  = 1380 in 2011
  = 4 for LEP
- Proton/bunch ~ 10^{11}
- Beam Energy = 7 TeV (nom)
  = 3.5 TeV (2011)
  = 4 TeV (2012)
- Luminosity(L) = 10^{34} cm^{-2} sec^{-1}
  = 3.3 x10^{33} cm^{-2} sec^{-1}
  (in 2011)
- Bunch Spacing = 25 ns (~7m)
  or 40MHz frequency
- Collisions ~ 10^7 - 10^9 Hz
- Mostly "soft" collisions
- Interesting events are much rarer, few per second or less
- Event rate = L x σ, where σ is cross section



Luminosity at different particle accelerators

Collider	Laboratory	Type	Date	$\sqrt{s}$/GeV	Luminosity/cm^{-2}s^{-1}
PEP-II	SLAC	e^+e^-	1999–2008	10.5	1.2×10^{34}
KEKB	KEK	e^+e^-	1999–2010	10.6	2.1×10^{34}
LEP	CERN	e^+e^-	1989–2000	90–209	10^{32}
HERA	DESY	e^-p/ e^+p	1992–2007	320	8×10^{31}
Tevatron	Fermilab	$p\bar{p}$	1987–2012	1960	4×10^{32}
LHC	CERN	pp	2009–	14 000	10^{34}

- Two important features of an accelerator
  - Centre-of-mass energy which determines the type of particles that can be produced/studied
  - Luminosity, which determines event rate



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{p}_2 = 0 \end{cases}$$

Decay rates and Cross Section

Decay rates and Cross Section

- In particle physics we are mostly concerned with two main experimental observables namely particle interaction and decays
  - describe transition between states

- Collisions are the most important processes used to study structure in subatomic physics
  - behavior of a collision is usually expressed in terms of a cross section

- Better understanding of the cross section is needed not only to understand SM process but also for BSM physics
  - For example good understanding of QCD cross sections are crucial for observing new physics as deviations from the SM

Fermi Golden Rule (# 1)

- The transition rate or transition probability per unit time from initial state $|i\rangle$ to final state $\langle f|$ is given by,

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f)$$

Easy to prove in QM using S.E

-where T_{fi} is transition matrix element

$$T_{fi} = \langle f|\hat{H}|i\rangle + \sum_{j\neq i} \frac{\langle f|\hat{H}|j\rangle\langle j|\hat{H}|i\rangle}{E_i - E_j} + \dots$$

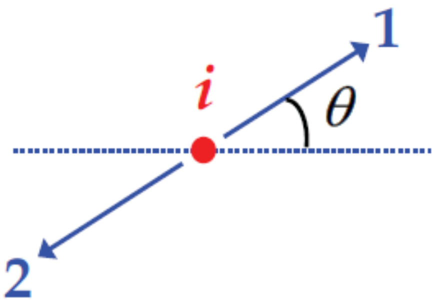
- $\hat{H}$ is the interaction Hamiltonian

- $\rho(E_f)$ is density of final states

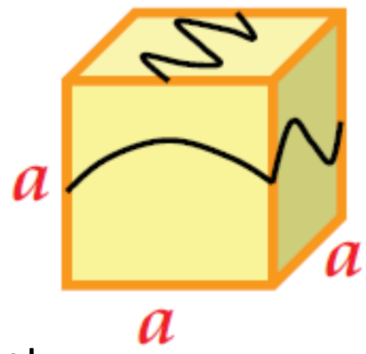
- Hence transition rate depends on "Matrix Element" and "Density of States"
- Matrix Element contains the fundamental particle physics
- Density of States carries kinematical information
- Γ_{fi} is not Lorentz Invariant

Lorentz Invariant terms for Decay rates

- Need to know the following for decay rates calculation
  - Wave function normalization
  - Transition Matrix element from perturbation theory
  - Expression for density of states



- Wave function normalization
  - Consider the particle being inside a cube of side *a*
  - Calculate decay rate in first order perturbation theory using plane-wave descriptions of the particles (Born approximation):



$$\psi = Ne^{i(\vec{p}.\vec{r}-Et)} = Ne^{-ip.x}$$

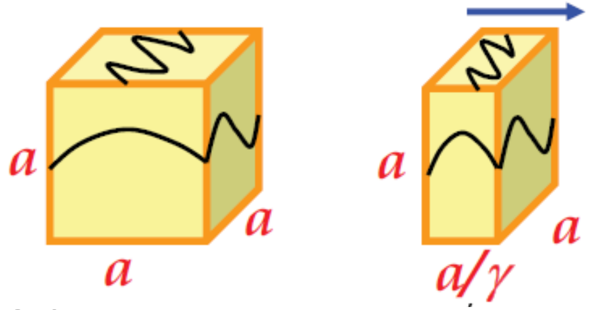
where N is a normalization constant and $p.x = p^{\mu}x_{\mu}$

$$\int \psi\psi^* \mathrm{d}V = N^2a^3 = 1 \quad \Rightarrow N^2 = 1/a^3$$

- Non-relativistic normalization to one particle in a cubic volume of side *a*

Lorentz Invariant Normalization

- Non-relativistic normalisation is 1/V
- By including relativistic effects volume contracts by $\gamma = E/m$
  - Hence probability increases by $\gamma = E/m$



- This demands a Lorentz Invariant normalization
- To cancel out the factor of "*E*", a relativistic invariant wave-function normalisation needs to be proportional to *E particles per unit volume*
- Convention is to normalize to 2*E* particles per unit volume i,e ψ' is normalized to 2*E particles per unit volume*

  => hence relativistic normalization is $\int \psi'^* \psi' \mathrm{d}V = 2E$

  where non-relativistic is $\int \psi^* \psi \mathrm{d}V = 1$

- Hence the two wave functions are related as $\psi' = (2E)^{1/2}\psi$

Lorentz Invariant Matrix Element

- For the decay process $a \rightarrow 1 + 2$, transition matrix element is given by,

$$T_{fi} = \langle \psi_1 \psi_2 | \hat{H}' | \psi_a \rangle$$
$$= \int_V \psi_1^* \psi_2^* \hat{H}' \psi_a \mathrm{d}^3\mathbf{x}$$

This is not Lorentz Invariant

- A generalized Lorentz Invariant Matrix Element (M_{fi}) is obtained by using Lorentz Invariant wave-functions

$$M_{fi} = \langle \psi_1' . \psi_2' ... | \hat{H} | ... \psi_{n-1}' \psi_n' \rangle = (2E_1 . 2E_2 . 2E_3 .... 2E_n)^{1/2} T_{fi}$$

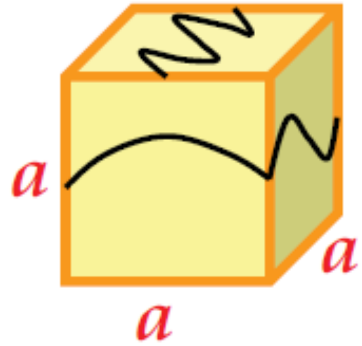
- For the above decay process, Lorentz Invariant Matrix Element becomes

$$\begin{aligned} M_{fi} &= \langle \psi_1' \psi_2' | \hat{H}' | \psi_a' \rangle \\ &= (2E_a . 2E_1 . 2E_2)^{1/2} \langle \psi_1 \psi_2 | \hat{H}' | \psi_a \rangle \\ &= (2E_a . 2E_1 . 2E_2)^{1/2} T_{fi} \end{aligned}$$

Non-relativistic Phase Space

- Normalization in a box of volume a^3 means that wave-function satisfy periodic boundary condition i,e

$$\psi(x+a,y,z)=\psi(x,y,z),$$

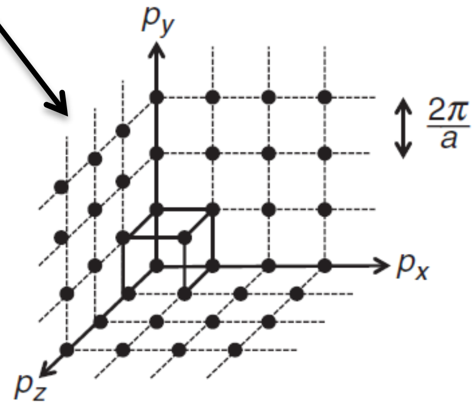


=>components of particle momentum are quantized

$$(p_x,p_y,p_z)=(n_x,n_y,n_z)\frac{2\pi}{a}$$

- Volume of a single cell in momentum space is given

$$d^3\mathbf{p}=dp_x dp_y dp_z=\left(\frac{2\pi}{a}\right)^3=\frac{(2\pi)^3}{V}$$

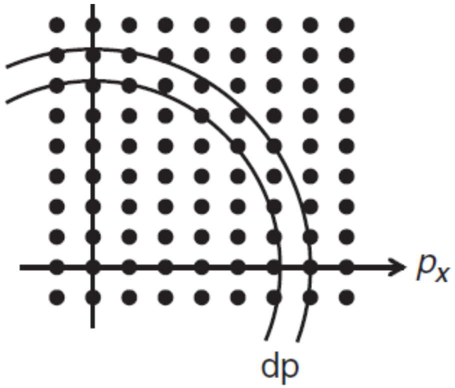


- The number of states dn in the range p → p + dp, is

$$dn=4\pi p^2 dp\times\frac{V}{(2\pi)^3} \quad \Rightarrow \quad \frac{dn}{dp}=\frac{4\pi p^2}{(2\pi)^3}V.$$

$$\rho(E)=\frac{dn}{dE}=\frac{dn}{dp}\left|\frac{dp}{dE}\right| \quad \Rightarrow \quad \boxed{\rho(E)=\frac{4\pi p^2}{(2\pi)^3}}$$

For V=1
and p=βE



density of states corresponds to the number of momentum states accessible to a particular decay and increases with the momentum of the final-state particle

Back to Fermi Golden Rule

- The density of final states can be re-written as

$$\rho(E_f) = \left|\frac{dn}{dE}\right|_{E_f} = \int \frac{dn}{dE}\delta(E - E_i)dE$$

- Delta function insures energy conservation of energy and integration is over all final state energies
- The Fermi Golden Rule $\Gamma_{fi} = 2\pi|T_{fi}|^2\rho(E_f)$ becomes $\Gamma_{fi} = 2\pi\int |T_{fi}|^2\delta(E_i - E)dn$
- As the number of independent states in the range p → p + dp are,

$$dn = \frac{d^3\mathbf{p}}{(2\pi)^3}.$$

For a particle decaying to N particles, this becomes $dn = \prod_{i=1}^{N-1} dn_i = \prod_{i=1}^{N-1}\frac{d^3\mathbf{p}_i}{(2\pi)^3}$

or $\boxed{dn = \prod_{i=1}^{N-1}\frac{d^3\mathbf{p}_i}{(2\pi)^3}\delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N}\mathbf{p}_i\right)d^3\mathbf{p}_N}$

- For the decay, $a \rightarrow 1 + 2$

$$\Gamma_{fi} = (2\pi)^4\int |T_{fi}|^2\delta(E_a - E_1 - E_2)\delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{(2\pi)^3}.$$

Matrix element | Energy conservation | Momentum conservation | Density of states

Is this expression Lorentz Invariant?

Transition Rate

- Replacing T_{fi} with Lorentz Invariant Matrix element, then decay rate becomes

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2)\delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}$$

Energy of the initial state particle

Lorentz Inv. Phase Space factor for each final state particle, Note factor of 2*E*, which appears due to wavefunction normalization.

Matrix Element is L.I, written in terms of relativistically normalized wave-functions

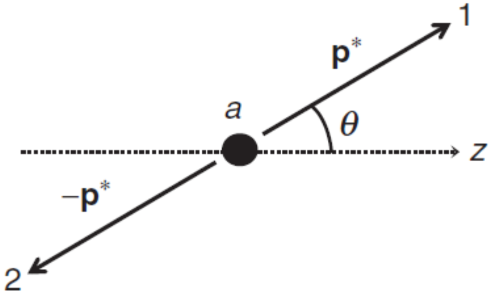
Delta functions insure conservation of energy and momentum

The integral is now Lorentz Invariant

Γ_{fi} is inversely proportional to the energy of decaying particle as expected from time dilation

Decay Rate Calculations for Two Body Decay

- Because the integral is Lorentz invariant (i.e. frame independent) it can be evaluated in any frame we choose
  - The C-o-M. frame is most convenient one



- In the C-o-M frame $E_a = m_a$ and $\mathbf{p}_a = \mathbf{0}$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |\mathcal{M}_{fi}|^2 \delta(m_a - E_1 - E_2)\delta^3(\mathbf{p}_1 + \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{2E_1}\frac{d^3\mathbf{p}_2}{2E_2}$$

- Integrating over $\mathbf{p}_2$ using δ-function, we get,

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |\mathcal{M}_{fi}|^2 \frac{1}{4E_1E_2}\delta(m_a - E_1 - E_2)\, d^3\mathbf{p}_1$$

where $E_2^2 = (m_2^2 + p_1^2)$

Using $d^3\mathbf{p}_1 = p_1^2 dp_1 \sin\theta\, d\theta\, d\phi = p_1^2\, dp_1 d\Omega$ gives,

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |\mathcal{M}_{fi}|^2 \delta\left(m_a - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2}\right)\frac{p_1^2}{4E_1E_2}\, dp_1 d\Omega$$

or

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |\mathcal{M}_{fi}|^2 g(p_1)\,\delta(f(p_1))\, dp_1 d\Omega$$

Using property of δ-function we get

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2}\int |\mathcal{M}_{fi}|^2\, d\Omega$$

$$p^* = \frac{1}{2m_a}\sqrt{\left[(m_a^2 - (m_1 + m_2)^2\right]\left[m_a^2 - (m_1 - m_2)^2\right]}$$

Interaction Cross Section

- For a beam of particles of type *a, with flux* ϕ_a, crossing a region of space in which there are n_b particles per unit volume of type *b*. The interaction rate per target particle r_b will be proportional to the incident particle flux and can be written

$$r_b = \sigma \phi_a$$

- Where proportionality constant σ has dimension of area and is known as interaction cross section

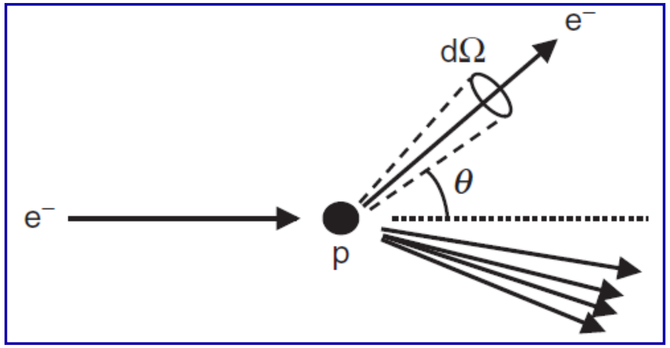
$$\sigma = \frac{\text{Number of interaction per unit time per target particle}}{\text{incident flux}}$$

incident flux = number of incoming particles per unit area per unit time

- σ can be thought as the *effective* cross sectional area associated with each target particle for the interaction to occur
  - This is true in some cases like neutron absorption by nucleus but in general it has nothing to do with physical x-sectional area of the target
  - Cross section is simply an expression for the underlying Quantum mechanical probability that an interaction will occur.

Differential Cross Section

- Diff. X-section is the distribution of x-section in bins of some kinematic variables
- In $e^- p \rightarrow e^- X$ scattering where proton breaks up, angular distribution of electron provides essential information about the fundamental physics of interaction



$$\frac{d\sigma}{d\Omega} = \frac{\text{number of particles scattered into } d\Omega \text{ per unit time per target particle}}{\text{incident flux}}$$

- In this case total x-section is obtained

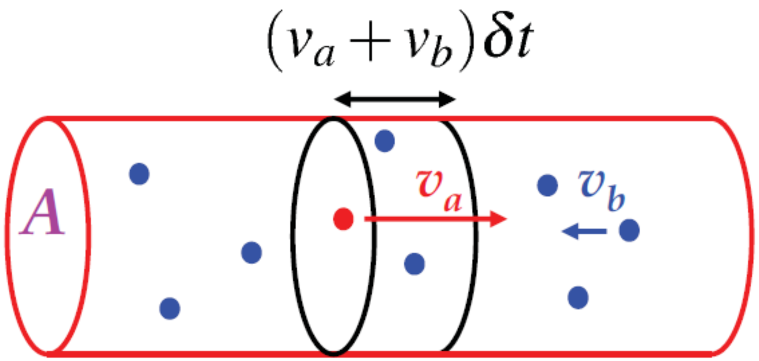
$$\boxed{\sigma = \int \frac{d\sigma}{d\Omega} d\Omega}$$

- If energy distribution of the scattered particle is sensitive to the underlying physics

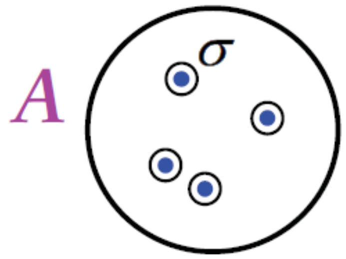
$$\frac{d\sigma}{dE} \quad \text{or} \quad \frac{d^2\sigma}{dE\, d\Omega}$$

Details…

- Consider a single particle of type *a* with velocity, v_a, traversing a region of area *A* containing n_b particles of type *b* per unit volume moving with velocity v_b in opposite direction
- In time δt , particle *a* crosses a region containing $\delta N = n_b (v_a + v_b) A\delta t$ particles of type *b*



- Interaction probability can be obtained from the effective total cross sectional area of the δN particles divided by the area *A*
  - The probability that incident particle passes through one of the regions of area σ drawn around each of the δN target particles



I.Prob. = $\delta P = \dfrac{\delta N\,\sigma}{A} = \dfrac{n_b(\mathrm{v}_a + \mathrm{v}_b)A\,\sigma\delta t}{A} = n_b \mathrm{v}\sigma\delta t.$ with $\mathrm{v} = \mathrm{v}_a + \mathrm{v}_b$

Interaction rate per particle of type *a* = $r_a = \dfrac{\mathrm{d}P}{\mathrm{d}t} = n_b \mathrm{v}\sigma$

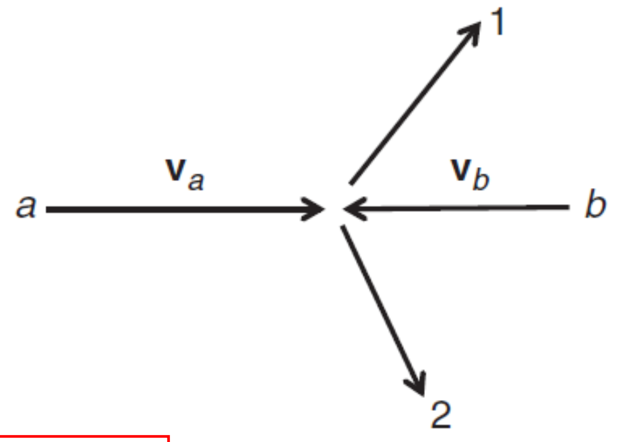
Total Interaction rate (considering volume V) = $r_a\, n_a V = (n_b \mathrm{v}\sigma)\, n_a V.$

or $\quad \text{rate} = (n_a \mathrm{v})(n_b V)\sigma = \phi\, N_b\, \sigma.$

Cross Section Calculation (1)

- Consider scattering process $a + b \rightarrow 1 + 2$
- Interaction rate in volume V is given by

$$\boxed{\text{rate} = \phi_a n_b V \sigma = (\mathrm{v}_a + \mathrm{v}_b) n_a n_b \sigma V,}$$



- Normalizing wavefunctions to one particle in a volume V => $n_a = n_b = 1/V$
- Hence interaction rate in volume V is $\boxed{\Gamma_{fi} = \frac{(\mathrm{v}_a + \mathrm{v}_b)}{V}\sigma}$
- As V will cancel out in the final expression therefore considering unit volume $\boxed{\sigma = \frac{\Gamma_{fi}}{(\mathrm{v}_a + \mathrm{v}_b)}}$
- Using Fermi's Golden Rule, we get

$$\sigma = \frac{(2\pi)^4}{(\mathrm{v}_a + \mathrm{v}_b)} \int |T_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2)\frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3}$$

Is it Lorentz Invariant?

Cross Section Calculation (2)

- To get Lorentz Invariant form, we need to use wavefunctions normalized to $2E$ particles per unit volume i,e $\psi' = (2E)^{1/2}\psi$
- Using Lorentz Invariant Matrix element $\mathcal{M}_{fi} = (2E_1\, 2E_2\, 2E_3\, 2E_4)^{1/2} T_{fi}$

$$\boxed{\sigma = \frac{(2\pi)^{-2}}{4\, E_a E_b (\mathrm{v}_a + \mathrm{v}_b)} \int |\mathcal{M}_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2) \frac{\mathrm{d}^3\mathbf{p}_1}{2E_1} \frac{\mathrm{d}^3\mathbf{p}_2}{2E_2}}$$

- Integral is Lorentz Invariant and $F = 4E_a E_b(\mathrm{v}_a + \mathrm{v}_b)$ is the L.I. flux factor. It can be written as product of four vectors i,e

$$\boxed{F = 4\left[(p_a \cdot p_b)^2 - m_a^2 m_b^2\right]^{\frac{1}{2}}}$$

Prove this

- In the C-o-M frame, flux factor becomes

$$\boxed{\begin{aligned} F &= 4E_a^* E_b^* (\mathrm{v}_a^* + \mathrm{v}_b^*) = 4E_a^* E_b^* \left(\frac{\mathrm{p}_i^*}{E_a^*} + \frac{\mathrm{p}_i^*}{E_b^*}\right) = 4\mathrm{p}_i^* (E_a^* + E_b^*) \\ &= 4\mathrm{p}_i^* \sqrt{s}. \end{aligned}}$$

- When target particle is at rest $\boxed{F = 4\, m_b\, p_a}$

Scattering in the Centre of Mass frame

- For any 2→2 scattering in the C-o-M frame, x-section can be calculated by previous L.I. formula
- In C-o-M frame $\vec{p}_a + \vec{p}_b = 0$ and using L.I. flux factor

$$\sigma = \frac{1}{(2\pi)^2}\frac{1}{4\mathrm{p}_i^*\sqrt{s}}\int |\mathcal{M}_{fi}|^2 \delta\left(\sqrt{s} - E_1 - E_2\right)\delta^3(\mathbf{p}_1 + \mathbf{p}_2)\frac{\mathrm{d}^3\mathbf{p}_1}{2E_1}\frac{\mathrm{d}^3\mathbf{p}_2}{2E_2}$$

- Integral here is the same as in the particle decay except m_a replaced with $\sqrt{s}$, after simplification, we get

$$\sigma = \frac{1}{16\pi^2\mathrm{p}_i^*\sqrt{s}} \times \frac{\mathrm{p}_f^*}{4\sqrt{s}}\int |\mathcal{M}_{fi}|^2 \mathrm{d}\Omega^*,$$

=> $$\sigma = \frac{1}{64\pi^2 s}\frac{\mathrm{p}_f^*}{\mathrm{p}_i^*}\int |\mathcal{M}_{fi}|^2 \mathrm{d}\Omega^*$$

Cross section for any 2 →2 process in C-o-M frame

Lorentz Invariant differential x-section

$$\boxed{\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\int |\mathcal{M}_{fi}|^2 d\Omega^*}$$

=>

$$\boxed{d\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2 d\Omega^*}$$

- This is differential x-section in C-o-M frame, valid for colliding beams
- Differential x-section in terms of the angles of one of the final state particles can be easily obtained
- In fixed target exp. where target is at rest one, this need to be expressed in terms of lab. frame quantities such as scattering angle of electron

$$\boxed{\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2}$$

- Need to express $d\Omega^*$ in terms of Mandelstam variable

$$t = (p_1^* - p_3^*)^2 = p_1^{*2} + p_3^{*2} - 2p_1^* \cdot p_3^*$$
$$= m_1^2 + m_3^2 - 2E_1^* E_3^* + 2\mathrm{p}_1^*\mathrm{p}_3^* \cos\theta^*.$$

=> $\boxed{dt = 2\mathrm{p}_1^*\mathrm{p}_3^*\, d(\cos\theta^*)}$ hence $d\Omega^* \equiv d(\cos\theta^*)\, d\phi^* = \dfrac{dt\, d\phi^*}{2\mathrm{p}_1^*\mathrm{p}_3^*}$

$$\boxed{\frac{d\sigma}{dt} = \frac{1}{64\pi s\, \mathrm{p}_i^{*2}}|\mathcal{M}_{fi}|^2}$$

where $\mathrm{p}_i^{*2} = \dfrac{1}{4s}[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]$

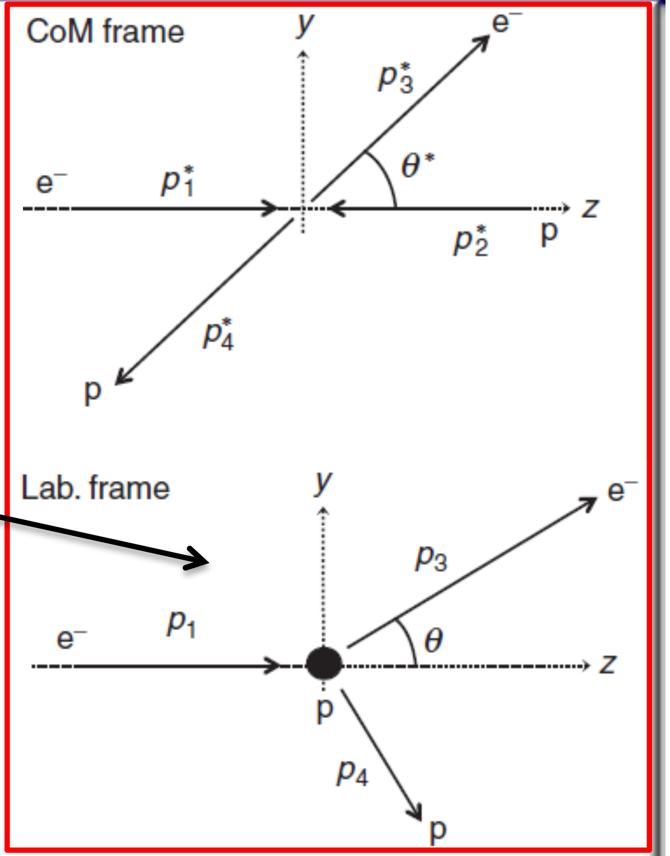
- All quantities in the above are L.I.

Application: Laboratory Frame Differential x-section

- $e^-p \rightarrow e^-p$ elastic scattering, electron energies are and its rest mass can be neglected
- Four momenta in lab frame are,

$$\boxed{\begin{aligned} p_1 &\approx (E_1, 0, 0, E_1), \\ p_2 &= (m_p, 0, 0, 0), \\ p_3 &\approx (E_3, 0, E_3 \sin\theta, E_3 \cos\theta) \\ p_4 &= (E_4, \mathbf{p}_4). \end{aligned}}$$

$$\boxed{p_i^{*2} \approx \frac{(s - m_p^2)^2}{4s}}$$



Summary

- Discussed some basic quantities useful for Exp. HEP analysis

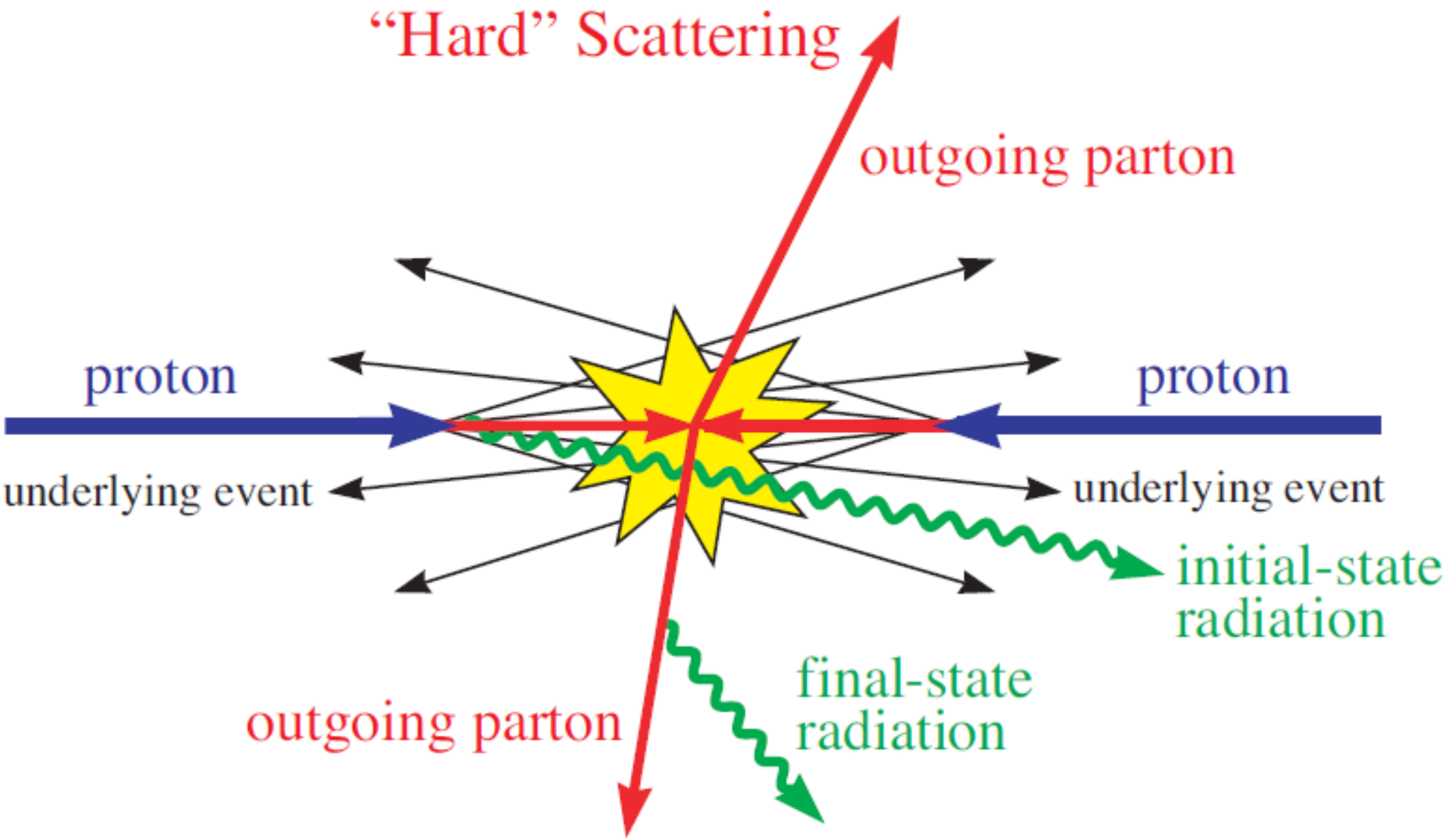
- Derived Decay rate, its given by

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2} \int |\mathcal{M}_{fi}|^2 \, d\Omega \quad \text{for} \quad p^* = \frac{1}{2m_a} \sqrt{\left[(m_a^2 - (m_1 + m_2)^2\right]\left[m_a^2 - (m_1 - m_2)^2\right]}$$

- 2→2 scattering cross-section in C-o-M frame

$$\boxed{\sigma = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} \int |\mathcal{M}_{fi}|^2 d\Omega^*}$$

Hadron Collider (proton-proton scattering)



References

- Mark Thomson Particle Physics Book
- Quarks and Leptons by Halzen and Martin