Supersymmetric AdS backgrounds and their moduli spaces

Jan Louis, University Hamburg

In collaboration with:

S.de Alwis, P.Karndumri, S.Lüst, L.McAllister, C.Muranaka, P.Rüter, H.Triendl, A.Westphal, M.Zagermann

[arXiv:1312.5659,1406.3363,1506.08040,1507.01623,1601.00482,1607.08249, 1611.02982, 1612.00301, and in progress]

Milano, Februrary 2017

Motivation

1. **AdS backgrounds often appear as intermediate step (before "uplifting") in models of string phenomenology**
2. **AdS backgrounds feature prominently in AdS/CFT correspondence**

In both cases their moduli space $\mathcal{C}$ also is of interest

1. **moduli stabilization/values of physical parameters**
2. **corresponds to conformal manifold of dual CFT**

$\Rightarrow$ study (supersymmetric) AdS backgrounds
and their moduli spaces in supergravity
in arbitrary space-time dimensions
with arbitrary number of supercharges

$D = 4, N = 1$ – basic formuli

chiral multiplet: $(\phi, \chi)^{\rm i}$, gravitational multiplet: $(g_{\mu\nu}, \psi_\mu)$

$$\mathcal{L}_{\rm bosonic} = \tfrac{1}{2}R - \mathbf{G_{i\bar{j}}}\,\partial_\mu\phi^{\rm i}\partial_\mu\bar{\phi}^{\rm \bar{j}} - \mathbf{V} \ , \qquad {\rm i}, {\rm \bar{j}} = 1, \ldots, {\rm n_c} \ ,$$

where

$$\mathbf{G_{i\bar{j}}} = \partial_{\mathbf{i}}\partial_{\mathbf{\bar{j}}}\mathbf{K} \ , \qquad \mathbf{V} = |\mathbf{A_{1i}}|^2 - 3|\mathbf{A_0}|^2 \ ,$$

$$\mathbf{A_0} = \mathbf{e^{\frac{1}{2}K}W}(\phi) \ , \qquad \mathbf{A_{1i}} = \mathbf{D_i A_0} \ , \quad \mathbf{D_i A_0} = \partial_{\mathbf{i}}\mathbf{A_0} + \tfrac{\mathbf{1}}{\mathbf{2}}(\partial_{\mathbf{i}}\mathbf{K})\mathbf{A_0}$$

Supersymmetry transformation of the fermions

$$\delta\chi_i = \mathbf{A_{1i}}\,\epsilon + \ldots \ , \qquad \delta\psi_\mu = D_\mu\epsilon + \mathbf{A_0}\gamma_\mu\epsilon + \ldots$$

Supersymmetric AdS backgrounds: $\langle\delta\psi_\mu\rangle = \langle\delta\chi_i\rangle = 0$

$$\Rightarrow \langle\mathbf{A_{1i}}\rangle = 0 \ , \quad \langle\mathbf{A_0}\rangle \neq \mathbf{0} \ , \quad \langle\mathbf{A_0}\rangle \sim \text{cos. const. / AdS-radius}$$

Classification for $N = 1$ difficult/impossible (?)

$D = 4, N = 1$ – **supersymmetric moduli space**

1. Minkowski background ($\langle \mathbf{A_0} \rangle = \mathbf{0}$)

   generic W: no moduli space

   special W: holomorphic $W(\phi)$ implies:
   moduli space $\mathcal{C}$ is Kähler submanifold
   of original Kähler field space $\mathcal{M}$
   (and protected by supersymmetry)

2. AdS background $\langle \mathbf{A_0} \rangle \neq 0$

   - $\bar{\phi}$ cannot drop out of $\mathbf{A_{1\phi}} = \partial_\phi \mathbf{A_0} + \frac{1}{2}(\partial_\phi K)\mathbf{A_0}$ and $\langle V \rangle$
   - no protection against quantum corrections
   - moduli space $\mathcal{C}$ is real submanifold of original Kähler manifold $\mathcal{M}$, at best with $\dim(\mathcal{C}) = \frac{1}{2}\dim(\mathcal{M})$

$D=4, N=1$ – supersymmetric moduli space

necessary condition:

$$\delta\langle \mathbf{A_0}\rangle = \langle \mathbf{A_{1i}}\rangle \delta\phi^{i} + \langle \mathbf{\bar{A}_{1\bar{i}}}\rangle \delta\bar{\phi}^{\bar{i}} = 0\ , \quad \delta\langle \mathbf{\bar{A}_0}\rangle = \ldots = 0$$

$$\delta\langle \mathbf{A_{1i}}\rangle = \langle \partial_{\mathbf{j}} \mathbf{A_{1i}}\rangle \delta\phi^{j} + \tfrac{1}{2}\langle K_{i\bar{j}} \mathbf{A_0}\rangle \delta\bar{\phi}^{\bar{j}} = 0\ , \quad \delta\langle \mathbf{\bar{A}_{1\bar{i}}}\rangle = \ldots = 0$$

(1st eq. identically satisfied in supersymmetric backgrounds)

Rewrite 2nd condition:

$$\mathbb{M} \begin{pmatrix} \delta\phi^{j} \\ \delta\bar{\phi}^{\bar{j}} \end{pmatrix} = 0\ , \quad \text{with} \quad \mathbb{M} = \begin{pmatrix} \mathbf{m_{ij}} & \langle \mathbf{K_{i\bar{j}} A_0}\rangle \\ \langle \mathbf{K_{\bar{i}j} \bar{A}_0}\rangle & \mathbf{\bar{m}_{\bar{i}\bar{j}}} \end{pmatrix}\ ,$$

where $\mathbf{m_{ij}} = \langle \nabla_{\mathbf{i}} \mathbf{A_{1j}}\rangle$ = fermionic mass matrix

- ➪ generically $\mathbb{M}$ has full rank $\Rightarrow$ $\delta\phi^{j} = 0$ $\Rightarrow$ no moduli space
- ➪ for special (tuned) $\mathbf{W}/\mathbf{A_0}$: moduli space possible **[DLMTW, ...]**

Maximally supersymmetric backgrounds in arbitrary dim. D

Distinguish two cases: backgrounds **with**/**without** fluxes

1. **backgrounds without fluxes**

Maximally supersymmetric backgrounds in arbitrary dim. D

Distinguish two cases: backgrounds **with**/**without** fluxes

1. **backgrounds without fluxes**

Supersymmetry transformation of fermions in gauged supergravities

$$\delta\psi_\mu^I = D_\mu \epsilon^I + \mathbf{A_{0J}^I}(\phi)\gamma_\mu \epsilon^J + \dots\ , \qquad \delta\chi^i = \mathbf{A_{1J}^i}(\phi)\epsilon^J + \dots$$

$$\text{with} \qquad V \sim |\mathbf{A_1}|^2 - c(d,q)|\mathbf{A_0}|^2$$

Supersymmetric backgrounds preserving all q supercharges:

$$\langle D_\mu \epsilon^I + \mathbf{A_{0J}^I}(\phi)\gamma_\mu \epsilon^J \rangle = 0\ , \qquad \langle \mathbf{A_1} \rangle = 0\ , \quad \forall\, I, J$$

Consistency condition: $R_{\mu\nu\rho\sigma} \sim \mathrm{tr}\mathbf{A_0^2}\,(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$

Solutions: AdS_D or $M_d \times T^{(D-d)}\ , \quad 1 \leq d \leq D$

(note: $\mathrm{AdS}_d \times Y^{(D-d)}$ not possible)

Supersymmetric moduli spaces $\mathcal{C}$ of AdS_D

- supersymmetric moduli spaces $\mathcal{C} \subset \mathcal{M}$
- necessary condition: $\delta_\phi \langle \mathbf{A_0} \rangle = 0 = \delta_\phi \langle \mathbf{A_1} \rangle$
- Gradient-flow equations: [d'Auria, Ferrara]

$$\delta \langle \mathbf{A_0} \rangle = \langle \mathbf{A_1} \rangle \, \delta \vec{\phi} = 0 \,, \qquad \delta \langle \mathbf{A_1} \rangle = \mathbb{M} \, \delta \vec{\phi} \,, \qquad \mathbb{M} := \mathbf{m_f} + \langle \mathbf{A_0} \rangle$$

- necessary condition for moduli space: $\mathbb{M}$ has null eigenspace

$$\langle \mathbf{A_0} \rangle \quad \begin{cases} = 0 & \mathcal{C} \subset \mathcal{M} \text{ has } \underline{\text{same}}^* \text{ geometry as } \mathcal{M} \\ \neq 0 & \mathcal{C} \subset \mathcal{M} \text{ has } \underline{\text{different}} \text{ geometry as } \mathcal{M} \end{cases}$$

- $q = 4$: can tune superpotential/$\mathbf{m_f}$
  but $\mathcal{C}$ generically destroyed by quantum corrections
- $q = 8, 16, 32$: no superpotential, can choose spectrum and
  gauged isometries of $\mathcal{M}$, supersymmetric protection

Example: $d=5, q=16$ supercharges [JL, Triendl, Zagermann]

➪ multiplets and scalar geometry

- gravity $(g_{\mu\nu}, \psi_\mu^i, A_\mu^{0,m}, \chi^i, \Sigma),\; m=1,\ldots,5 \Rightarrow$ 6 graviphotons
- vector $(A_\mu^a, \lambda^a, \phi^{am}),\; a=1,\ldots,\mathrm{n_v}$

scalar field space: $\mathcal{M}(\boldsymbol{\Sigma},\phi) = \mathbb{R}^+ \times \frac{\mathbf{SO(5,n_v)}}{\mathbf{SO(5)\times SO(n_v)}}$

➪ supersymmetric AdS backgrounds can be "classified"

gauge group:

$$\mathbf{G} = \mathbf{U(1)} \times \hat{\mathbf{G}} \subset \mathbf{SO(5,n_v)}$$

where $\hat{\mathbf{G}} \subset \mathbf{SO(3,n_v)} \to \mathbf{SO(3)} \times \mathbf{H}$, $\quad \mathbf{H} =$ compact subgroup

➪ moduli space possible: [Günaydin,Pilch,Warner,Zagermann]

$$\mathcal{C} = \frac{\mathbf{SU(1,p)}}{\mathbf{U(1)\times SU(p)}} \subset \frac{\mathbf{SO(2,2p)}}{\mathbf{SO(2)\times SO(2p)}} \subset \frac{\mathbf{SO(5,n_v)}}{\mathbf{SO(5)\times SO(n_v)}}$$

General story: AdS moduli space for $\mathcal{M} = G/H$

[JL, S. Lüst, Rüter]

- gauging isometries of $\mathcal{M}$ induces gauging of R-symmetry H_R
- Maximally supersymmetric AdS backgrounds:

  $H_R^g \subset H_R$ gauged only by (subset of) graviphotons A_μ
- A_0 and A_μ transform in specific rep. of H_R depending on D, q
- $\langle A_0 \rangle \neq 0$ implies $H_R \to H_R^l$ (little group)
- additional constraint:

  $H_R^g \subset H_R^l \subset H_R$ of AdS-vacuum has to have A_μ in its adj. rep.
- flat directions of V:

  H_R^g charged Goldstone bosons + H_R^g-singlets = moduli

Example: $d = 7, q = 32$ **supercharges** [JL, S. Lüst, P. Rüter]

- gravity multiplet: $(g_{\mu\nu}, \psi_\mu^I, A_\mu^{[MN]}, \chi^{IJK}, \phi)$

$$I, J, K = 1, \ldots, 4, \quad M, N = 1, \ldots, 5$$

- scalar field space: $\mathcal{M}(\phi) = \frac{\mathbf{SL(5)}}{\mathbf{SO(5)}}$

- $A_0 \in \mathbf{1} \oplus \mathbf{5}, \quad A_1 \in \mathbf{5} \oplus \mathbf{14} \oplus \mathbf{35}$ of $H_R = SO(5) \sim USp(4)$

$$\Rightarrow H_R = H_R^l = H_R^g = USp(4)$$

- $\delta\phi \in \mathbf{14} \Rightarrow$ no GB, no singlets $\Rightarrow$ no moduli

Results so far:

$\mathrm{AdS}_{(d,q)}$	$\mathcal{M}$	$\mathcal{C}$	[ ]
(4,4)	Kähler	Real	[DLMTW]
(4,8)	SK× QK	Real×Kähler	[DMLTW]
(5,8)	SR× QK	Kähler	[Tachikawa,LM]
(6,8)	$\frac{O(1,n_T)}{O(n_T)}$× QK	–	[?]

SK: Special Kähler

QK: Quanternionic Kähler

SR: Special Real

[DMLTW]=[de Alwis, JL, McAllister, Triendl, Westphal]

[LM]=[JL, Muranaka]

Results so far:

AdS$_{(d,q)}$	$\mathcal{M}$	H_R^g	$\mathcal{C}$	[ ]
(4,16)	$\frac{SO(6,6+n_v)}{SO(6)\times SO(6+n_v)} \times \frac{SU(1,1)}{U(1)}$	$SO(4)$	AX	[LT]
(5,16)	$\frac{SO(5,5+n_v)}{SO(5)\times SO(5+n_v)} \times \mathbb{R}^+$	$U(2)$	$\frac{SU(1,p)}{SU(p)\times U(1)}$	[GPWZ,LTZ]
(6,16)	$\frac{SO(4,4+n_v)}{SO(4)\times SO(4+n_v)} \times \mathbb{R}^+$	$SU(2)\times SU(2)$	AX	[KL]
(7,16)	$\frac{SO(3,3+n_v)}{SO(3)\times SO(3+n_v)} \times \mathbb{R}^+$	$USp(2)$	AX	[LL]
(4,32)	$\frac{E_{7,7}}{SU(8)}$	$SO(8)$	AX	[LLR]
(5,32)	$\frac{E_{6,6}}{USp(8)}$	$SU(4)$	$\frac{SU(1,1)}{U(1)}$	[LLR]
(6,32)	$\frac{SO(5,5)}{SO(5)\times SO(5)}$	XX	XX	[LLR]
(7,32)	$\frac{SL(5)}{SO(5)}$	$USp(4)$	AX	[LLR]

AX: AdS background exists and can be classified but no moduli space,

XX: AdS background does not exist

[LT]=[JL,Triendl] **[LLR]=[JL, Lüst, Rüter]** **[KL]=[Karndumri,JL]**

AdS/CFT correspondence

AdS$_d$ bulk with q supercharges

$\Leftrightarrow$

SCFT$_{d-1}$ on AdS$_d$ boundary with $\frac{q}{2}+\frac{q}{2}$ super+sconformal charges

AdS	$\Leftrightarrow$	**SCFT**
(broken) gauge group		**(anomalous) flavour group**
scalars with mass m		**couplings of gauge invariant operators**
moduli		**exactly marginal couplings**
moduli space $\mathcal{C}$		**conformal manifold $\mathcal{C}$**

$$S = S_0 + \sum_i \int \varphi^{\mathbf{i}} \mathbf{O_i}$$

$$g_{ij}(\varphi) = x^{2d} \langle \mathbf{O_i(x) O_j(0)} \rangle_S$$ **[Zamolodchikov]**

Compare AdS with SCFT-results

$\mathsf{AdS}_{(d,q)}$	$\mathcal{C}$	[ ]	$\mathsf{SCFT}_{(d-1,\frac{q}{2})}$	$\mathcal{C}_{\mathrm{SCFT}}$	[ ]
(4,4)	Real	[DLMTW]	(3,2)	Real[1]	[CDI]
(4,8)	Real×Kähler	[DMLTW]	(3,4)	Kähler	[GKSTW]
(5,8)	Kähler	[T,LM]	(4,4)	Kähler	[Ansin]
(6,8)	–	[?]	(5,4)	SXX	

SX: SCFT exists but no moduli space,

SXX: SCFT does not exist

[CDI]=[Cordova,Dumitrescu,Intriligator]

[GKSTW]=[Green,Komargodski,Seiberg,Tachikawa,Wecht]

[1] none known, no susy protection

Compare AdS with SCFT-results (Note: no SCFT in $d > 6$)

AdS$_{(d,q)}$	$\mathcal{C}$	[ ]	SCFT$_{(d-1,\frac{q}{2})}$	$\mathcal{C}_{\text{SCFT}}$	[ ]
(4,16)	AX	[LT]	(3,8)	SX	[CDI]
(5,16)	$\frac{SU(1,p)}{SU(p)\times U(1)}$	[GPWZ,LTZ]	(4,8)	Kähler (?)^2	[P,GGK]
(6,16)	AX	[KL]	(5,8)	SX	[CDI]
(7,16)	AX	[LL]	(6,8)	SX	[LL]
(4,32)	AX	[LLR]	(3,16)	SX	[CDI]
(5,32)	$\frac{SU(1,1)}{U(1)}$	[LLR]	(4,16)	$\frac{SU(1,1)}{U(1)}$	[?]
(6,32)	XX	[LLR]	(5,16)	SXX	[Nahm]
(7,32)	AX	[LLR]	(6,16)	SX	[LL]

[GPWZ]=[Günaydin,Pilch,Warner,Zagermann]

[P,GGK]= [Papadodimas; Gerchkovitz,Gomis,Komargodski]

^{2}[GGK] only show Kähler, [P] shows tt^*-geometry

Supersymmetric backgrounds in arbitrary dimensions D

Distinguish two cases: backgrounds **with**/**without** fluxes

2. **backgrounds with fluxes**

Supersymmetric backgrounds in arbitrary dimensions D

Distinguish two cases: backgrounds **with**/**without** fluxes

2. **backgrounds with fluxes** **[JL, S. Lüst, ...]**

Supersymmetry transformation of the fermions

$$\delta\psi^I_\mu = D_\mu \epsilon^I + (\mathcal{F}_{\mathbf{0}\mu})^{\mathbf{I}}_{\mathbf{J}} \epsilon^J + A^I_{0\ J} \gamma_\mu \epsilon^J \ , \qquad \delta\chi^i = (\mathcal{F}_{\mathbf{1}})^{\mathbf{i}}_{\mathbf{J}} \epsilon^J + A^i_{1J}\, \epsilon^J \ ,$$
$$\delta\lambda^A = (\mathcal{F}_{\mathbf{2}})^{\mathbf{A}}_{\mathbf{J}} \epsilon^J + A^A_{2J}\, \epsilon^J \ ,$$

$\mathcal{F}_{\mathbf{0},\mathbf{1}/\mathbf{2}}$ = sum of fluxes in gravitational multiplet/other multiplets

supersymmetric backgrounds: $\mathcal{F}_{\mathbf{1}} = \mathcal{F}_{\mathbf{2}} = A_1 = A_2 = 0 \quad \Rightarrow \quad \mathcal{F}_{\mathbf{0}} = 0$

two exceptions:

1. chiral theories with selfdual fluxes
2. supergravities without χ's in gravitational multiplet

Supersymmetric backgrounds in arbitrary dimensions D

➪ candidate supergravities/fluxes:

dimension	supersymmetry	q	possible flux	classification of maximal supersymmetric solutions
$D = 11$	$N = 1$	32	$F^{(4)}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 10$	IIB	32	$F_+^{(5)}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 6$	$N = (2,0)$	16	$5 \times F_+^{(3)}$	[Chamseddine,Figueroa-O'Farrill,Sabra]
$D = 6$	$N = (1,0)$	8	$F_+^{(3)}$	[Gutowski,Martelli,Reall]
$D = 5$	$N = 2$	8	$F^{(2)}$	[Gauntlett,Gutowski,Hull,Pakis,Reall]
$D = 4$	$N = 2$	8	$F^{(2)}$	[Tod]

➪ study this list case by case to find

- $\langle A_0 \rangle = \langle A_1 \rangle = \langle A_2 \rangle = 0$
- no flux for R-symmetry possible

  [Hristov,Looyestijn,Vandoren;Gauntlett,Gutowski;Akyol,Papadopoulos]

➪ background solutions have to coincide with ungauged solutions
they are classified in ungauged case
(see refs. above and [Gran,Gutowski,Papadopoulos])

AdS backgrounds

dim.	SUSY	q	$AdS \times S$	
$D = 11$	$N = 1$	32	$\mathrm{AdS}_4 \times S^7$	[Freund,Rubin]
			$\mathrm{AdS}_7 \times S^4$	
$D = 10$	IIB	32	$\mathrm{AdS}_5 \times S^5$	[Schwarz,West]
$D = 6$	$N = (2,0)$	16	$\mathrm{AdS}_3 \times S^3$	[Gibbons,Horowitz,Townsend]
	$N = (1,0)$	8	$\mathrm{AdS}_3 \times S^3$	[Gibbons,Horowitz,Townsend]
$D = 5$	$N = 2$	8	$\mathrm{AdS}_2 \times S^3$	[Gibbons,Horowitz,Townsend]
			$\mathrm{AdS}_3 \times S^2$	[Chamseddine,Ferrara,Gibbons,Kallosh]
$D = 4$	$N = 2$	8	$\mathrm{AdS}_2 \times S^2$	[Bertotti,Robinson]

list is exhaustive (cf. [Alonso-Alberca,Ortin])

in addition:

- Hpp-wave solutions [Penrose; Kowalski-Glikman; Blau,Figueroa-O'Farrill,Hull,Papadopoulos]
- further special solutions in $D = 5$ [Cvetic,Larsen; Gauntlett,Meyers,Townsend; Gauntlett,Gutowski,Hull,Pakis,Reall]

Conclusion/Outlook

Studied fully supersymmetric AdS backgrounds in D-dimensional supergravities with q supercharges

1. without fluxes
   - classified AdS$_D$ backgrounds with $q = 16, 32$ supercharges and determined their classical moduli spaces
   - determined the geometry of classical moduli spaces for $q = 4, 8$
2. with fluxes
   - AdS-backgrounds only exist in
     - chiral supergravities
     - supergravities without graviphotinos
   - in both cases AdS-solution coincides with solution of corresponding ungauged supergravity which are classified

Results are consistent with dual SCFTs (not yet clear in (4,8))