

First phase diagram of hadronic matter

Consider phase transition of hadronic matter at nonzero:

T = temperature and μ = quark chemical potential (= 1/3 baryon chem. pot.)

Cabibbo and Parisi '75: Exponential (Hagedorn) spectrum limiting temperature, or transition to new, "unconfined" phase.

Assume "semi-circle" in plane of T and μ . Today: there is *no* semi-circle

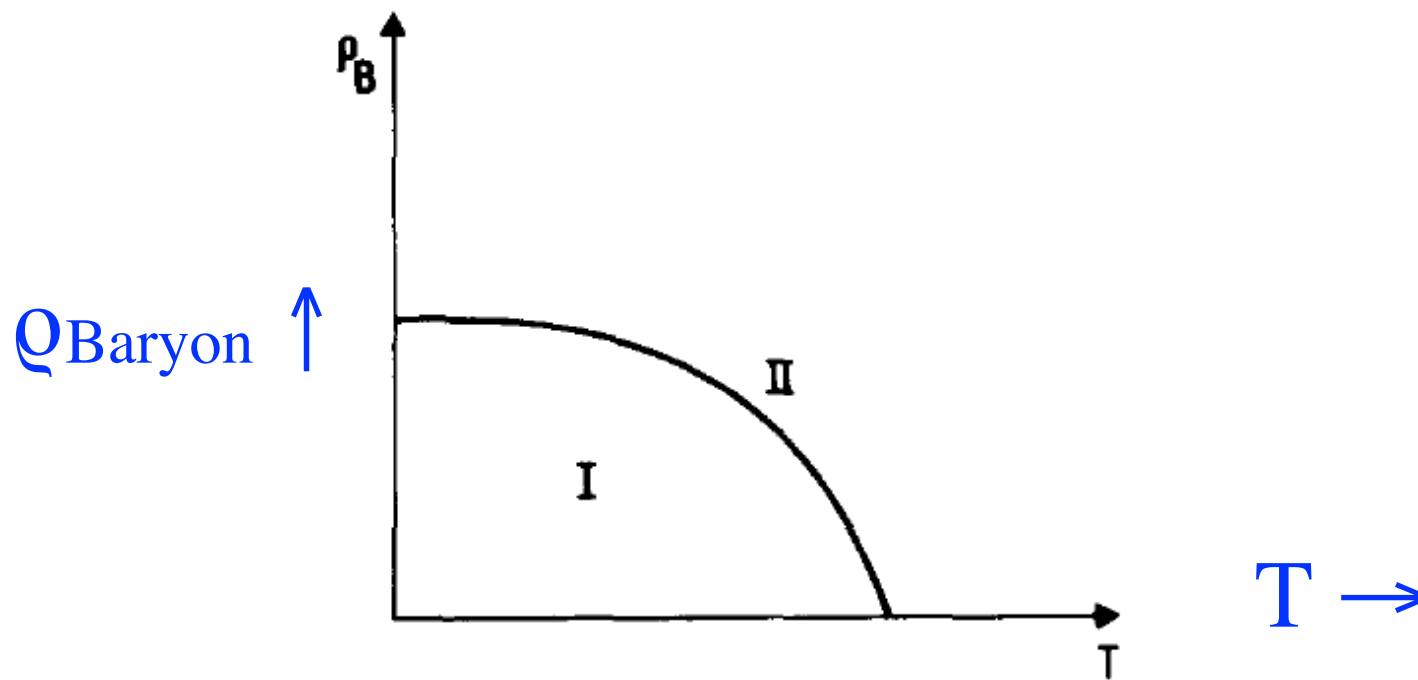


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

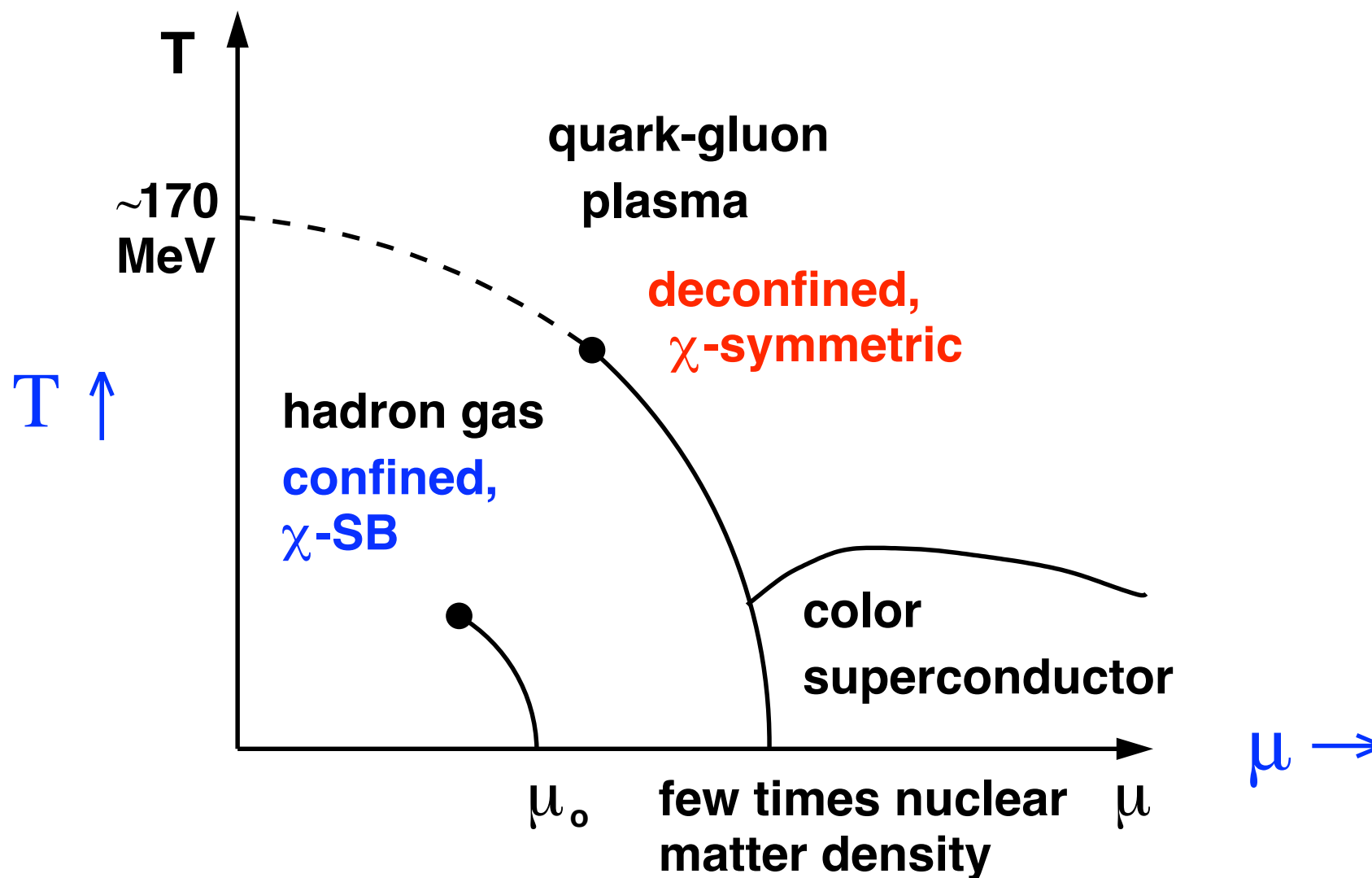
Phase diagram, ~ '06

Lattice, $T \neq 0$, $\mu = 0$: two possible transitions; one crossover, same T . Karsch '06

Remains crossover for $\mu \neq 0$? Stephanov, Rajagopal, & Shuryak '98:

Critical end point where crossover turns into first order transition

But still semi-circle in T and μ



Cold, dense quark matter at large N_c

Consider *large* number of colors: $N_c \rightarrow \infty$, with $g^2 \sim 1/N_c$ and *small* $N_f \sim 1$.
Standard 't Hooft limit. In general: any # gluon loops, but quarks only to 1 loop.

Simple but general example: Debye mass at leading order:

$$m_{Debye}^2 = g^2 \left(\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + N_f \frac{\mu^2}{2\pi^2} \right)$$

At $T \neq 0$ and any μ , *only* gluons contribute:

$$m_{Debye}^2 \sim g^2 N_c T^2 \sim N_c^0 T^2$$

So trivially, the Debye mass is independent of μ .

Conversely, at $T = 0$ and $\mu \neq 0$, the Debye mass is suppressed by $1/N_c$:

$$m_{Debye}^2 \sim (g^2 N_c) \frac{N_f}{N_c} \mu^2 \sim \frac{1}{N_c} \mu^2$$

Conclude: cold, dense quark matter is confined until $\mu \sim N_c^{1/2}$

Quarkyonic Matter

Assume μ is large, but not “too” large: $\mu \ll N^{1/2}$, so there is confinement.
However, also assume that $\mu \gg \Lambda_{\text{QCD}}$ = renormalization mass scale in QCD.
Consider the pressure, computed in perturbation theory:

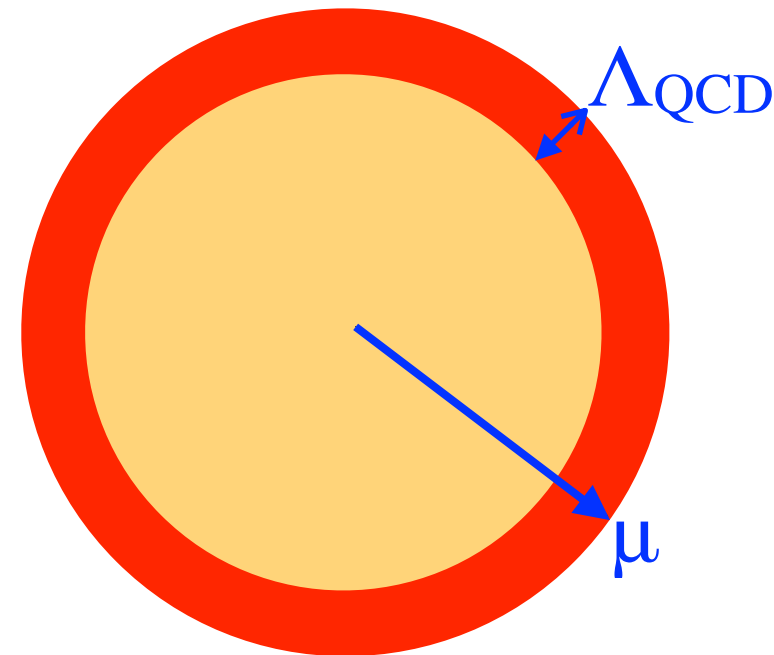
$$p(\mu) = \# \mu^4 (1 + c_2 g^2 + c_4 g^4 \log + c_6 g^6 (\log^2 + c'_6 \log + c''_6)) + \dots$$

Actually two kinds of log's: $\log(\mu/\Lambda_{\text{QCD}})$, and $\log(\mu/m_{\text{Debye}}) \sim \log(1/g)$. Suggestive.
Since $\mu \gg \Lambda_{\text{QCD}}$, we can compute $p(\mu)$ perturbatively.

But isn't the theory confined?

Look at the Fermi sea: modes deep within are pert.
Confinement only matters for light modes, near the Fermi surface. Since these are a “skin”, they only contribute $\sim \mu^2 \Lambda_{\text{QCD}}^2$ to the pressure.

quark + baryonic = **quarkyonic**

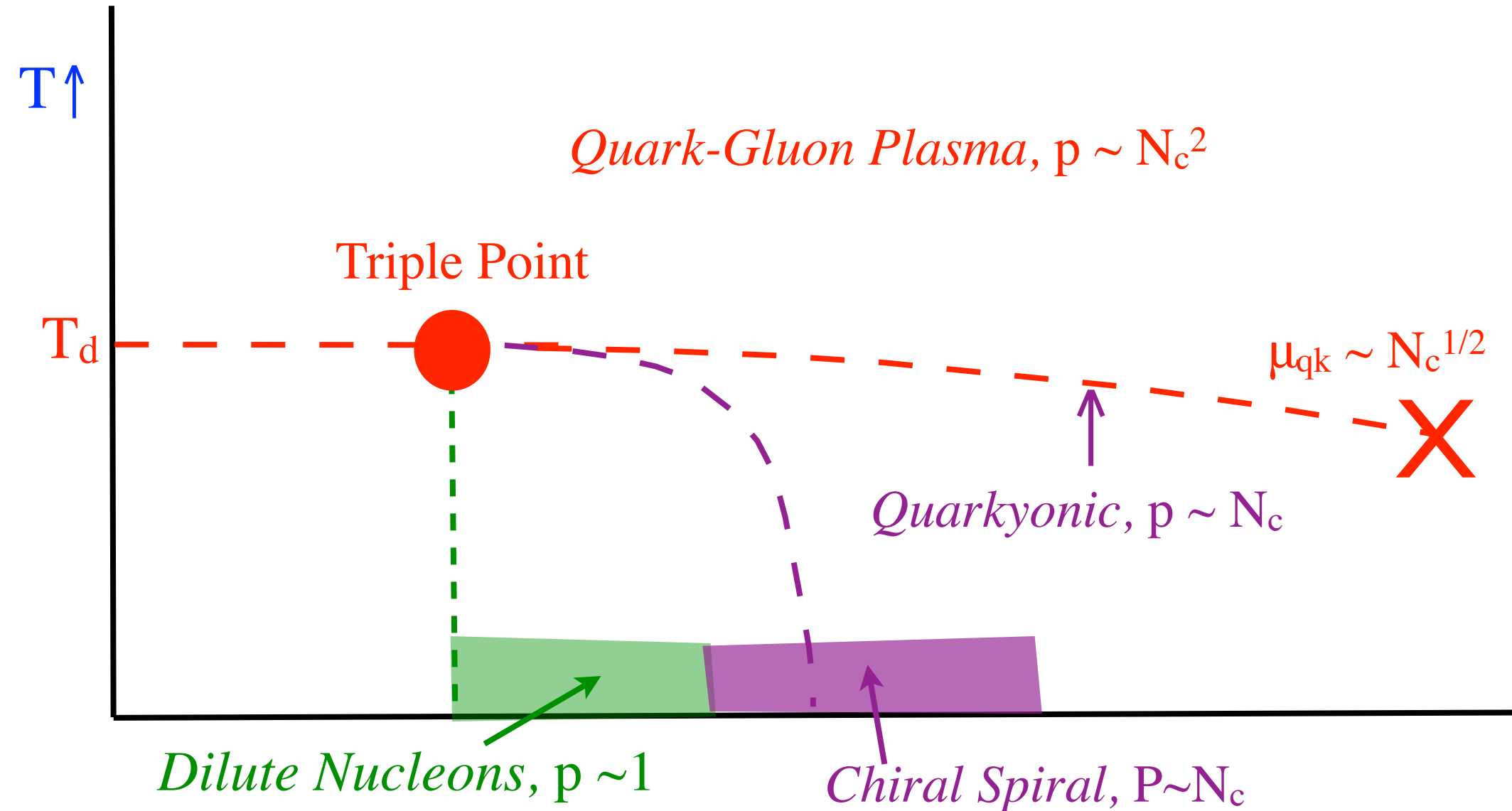


Phase diagram at large N_c

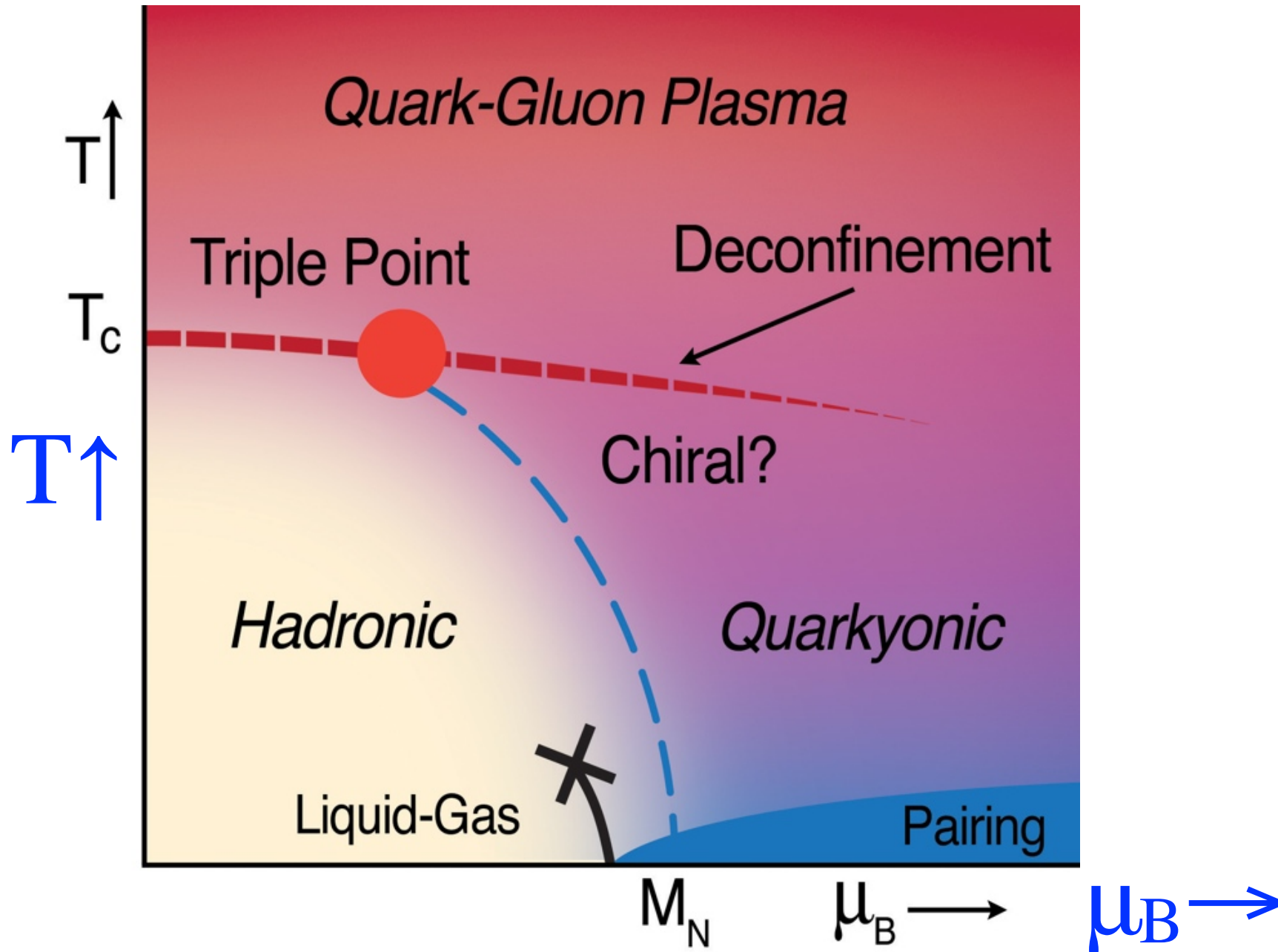
Below: cartoon of phase diagram at large N_c .

Clear separation of deconfining and chiral phase transitions. No semi-circle.

But what about $N_c = 3$?



New Phase Diagram of QCD



New Phase Diagram of QCD

“Semi”-QGP

Critical Endpoint = Triple Point?

Lin, RDP, Skokov

1301.7432

1312.3340

Lin, RDP + ...

1409.4778

Hidaka, Lin,

RDP, & Satow

1504.01770

Andronic...McLerran, RDP + ... 0911.4806

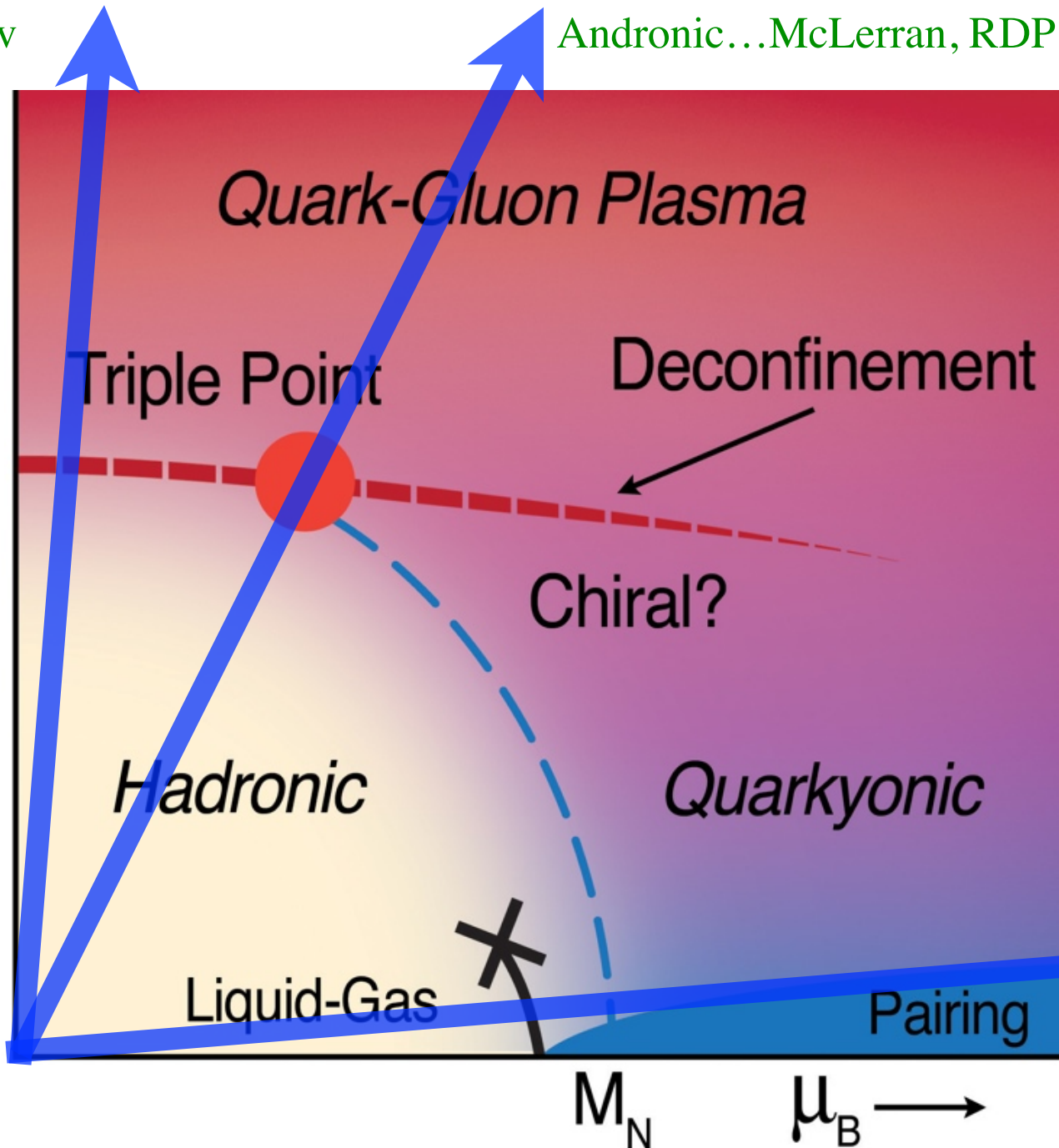
Kojo, Hidaka,
McLerran & RDP
0912.3800

Kojo, RDP &
Tsvetik,
1007.0248

Kojo, Hidaka,
Fukushima,
McLerran, & RDP
1107.2124

Quarkyonic
Chiral Spirals

$T \uparrow$



M_N

$\mu_B \rightarrow$

$\mu_B \rightarrow$

Two colors on lattice: where is Quarkyonic?

Braguta, Ilgenfritz, Kotov, Molochkov, & Nikolaev, 1605.04090 (earlier: Hands, Skellerud + ...)

Lattice: $N_c = 2$ (no sign problem!), $N_f = 2$ staggered quarks

$m_\pi \sim 400$ MeV, fixed $T \sim 50$ MeV, vary μ . Find four “phases”:

$0 \leq \mu < m_\pi / 2 \sim 200$ MeV. Hadronic phase: confined, no condensates

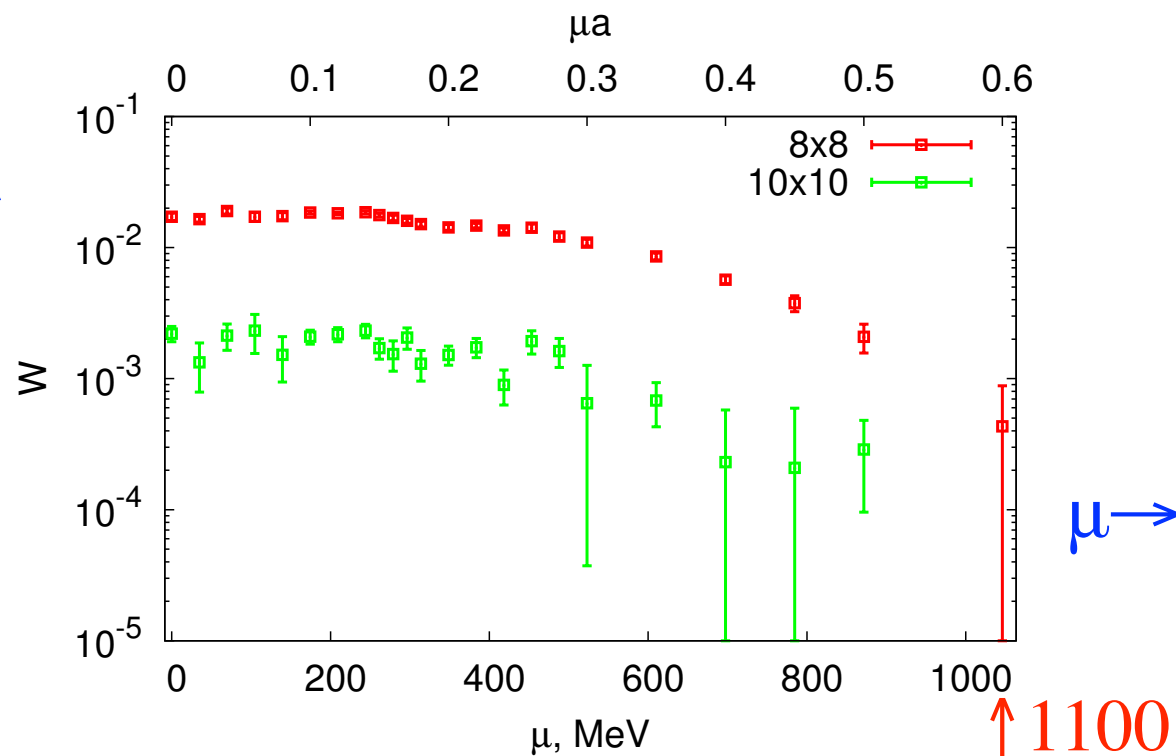
$200 < \mu < 350$: Bose-Einstein condensate (BEC) of diquarks, “dilute baryons”

$350 < \mu < 600$: BEC (\sim BCS), dense baryons (pressure \neq pert)

$600 < \mu < 1100$: *Quarkyonic*: pressure \approx pert., but *confined* (Wilson loop area law)

Quarkyonic up to *highest* $\mu > 1$ GeV. $N_c = 2$ is *not* large N_c

$\langle \text{Wilson loop} \rangle \uparrow$



When is perturbation theory valid? $T \neq 0, \mu = 0$

Consider first $T \neq 0$: gluon propagator $\Delta(p_0, p) \sim 1/(p_0^2 + p^2)$, $p_0 = 2\pi n T$

Braaten & Nieto, hep-ph/9501375: dominant $p \sim 2\pi T$

Laine & Schroder, hep-ph/0503061: by 2-loop calc. in effective theory, find for $N_c = 3$:

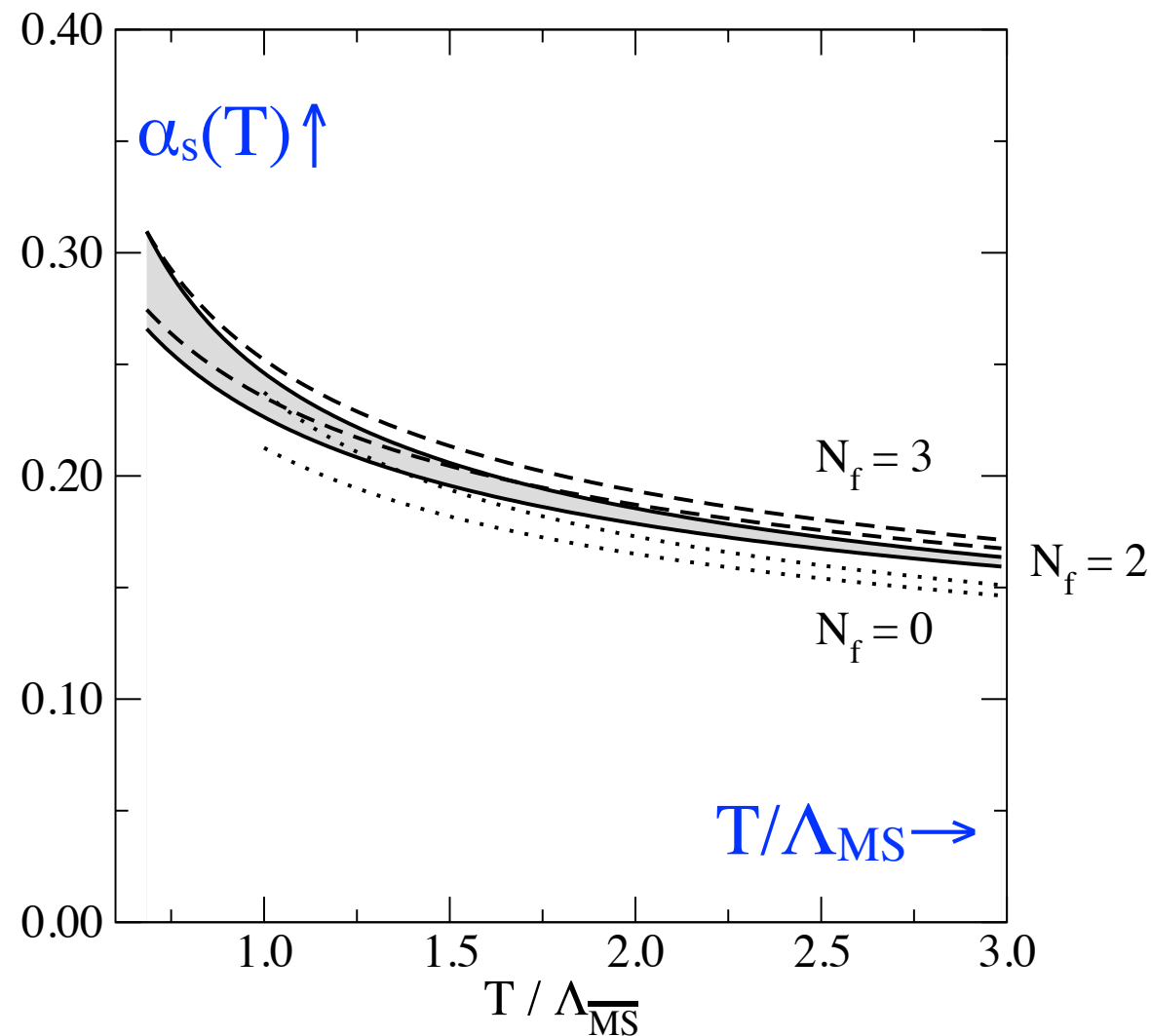
$$N_f = 0 : \Lambda_{pert} \sim 7T$$

$$N_f = 3 : \Lambda_{pert} \sim 9T$$

Band: change in effective $\alpha_s(T)$,
by varying Λ_{pert} by a factor of two.

Even down to $T \sim 150$ MeV,
 Λ_{pert} is still ~ 1 GeV.

N.B.: effective theory resums
modes with $p \sim T$, then $p \sim g T$,
then $p \sim g^2 T$.

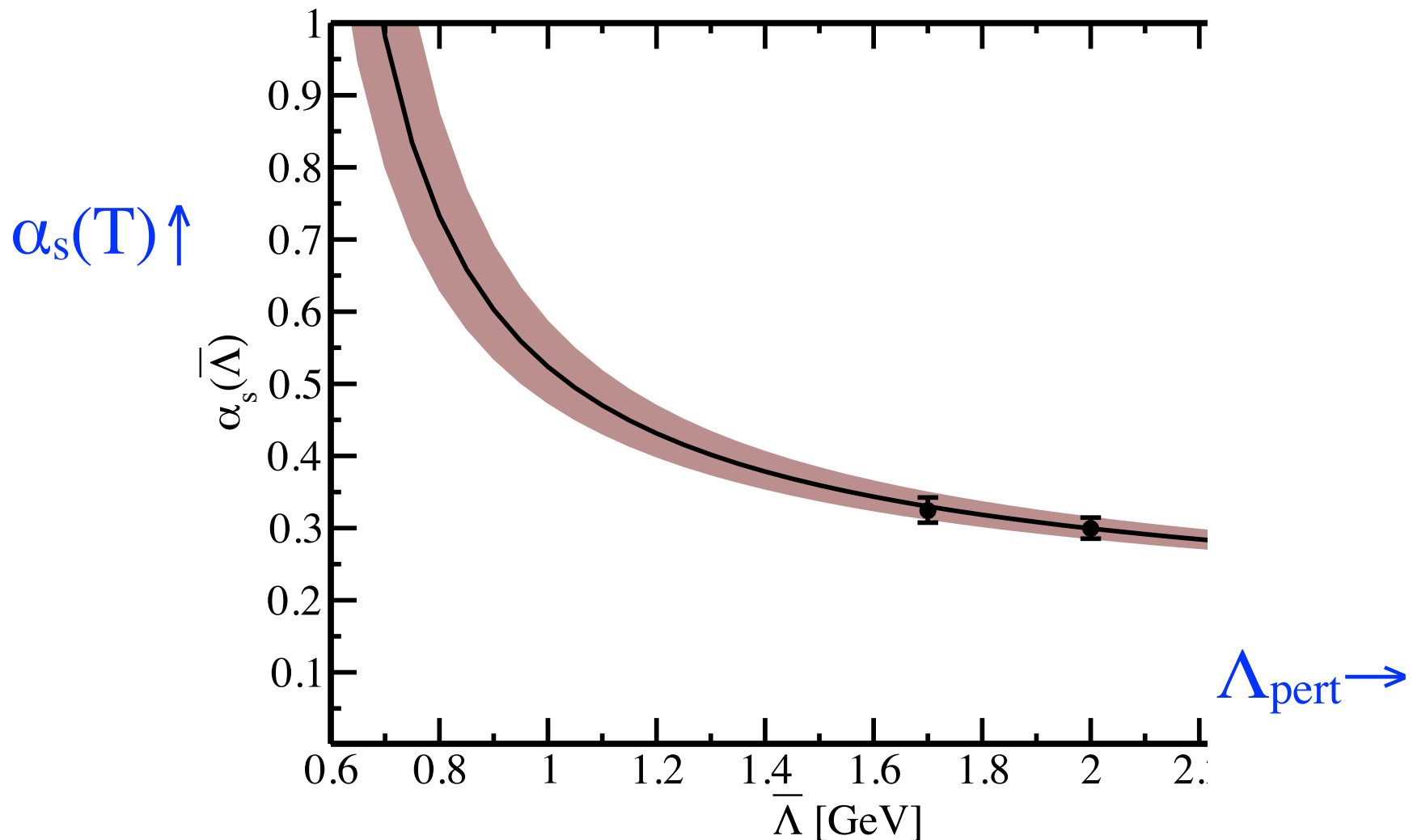


When is perturbation theory valid? $\mu \neq 0, T = 0$

Kurkela, Romatschke, & Vuorinen, 0912.1856: pressure to $\sim \alpha_s^2(T)$, 2+1 flavors ($m_s \neq 0$)

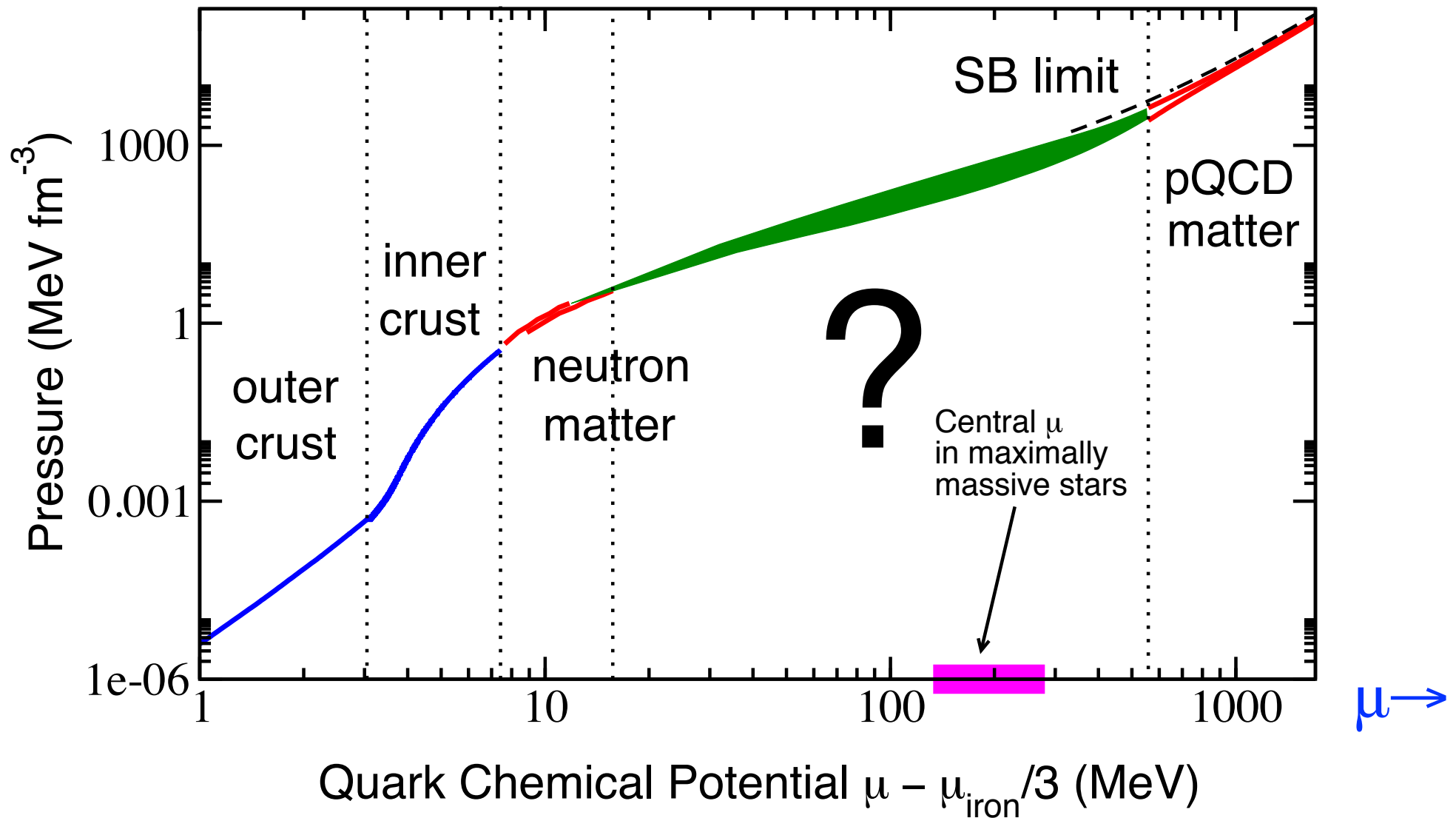
Take Λ_{pert} from Debye mass,

$$\Lambda_{\text{pert}} \sim m_{\text{Debye}}^2 / g^2 = \sqrt{(2\pi T)^2 + (2\mu)^2} = 2\mu, \quad T = 0$$



When is perturbation theory valid? $\mu \neq 0, T = 0$

Fraga, Kurkela, & Schaffner-Bielich, 1402.6618: using $\Lambda_{\text{pert}} \sim 2 \mu$:



When is cold quark matter Quarkyonic?

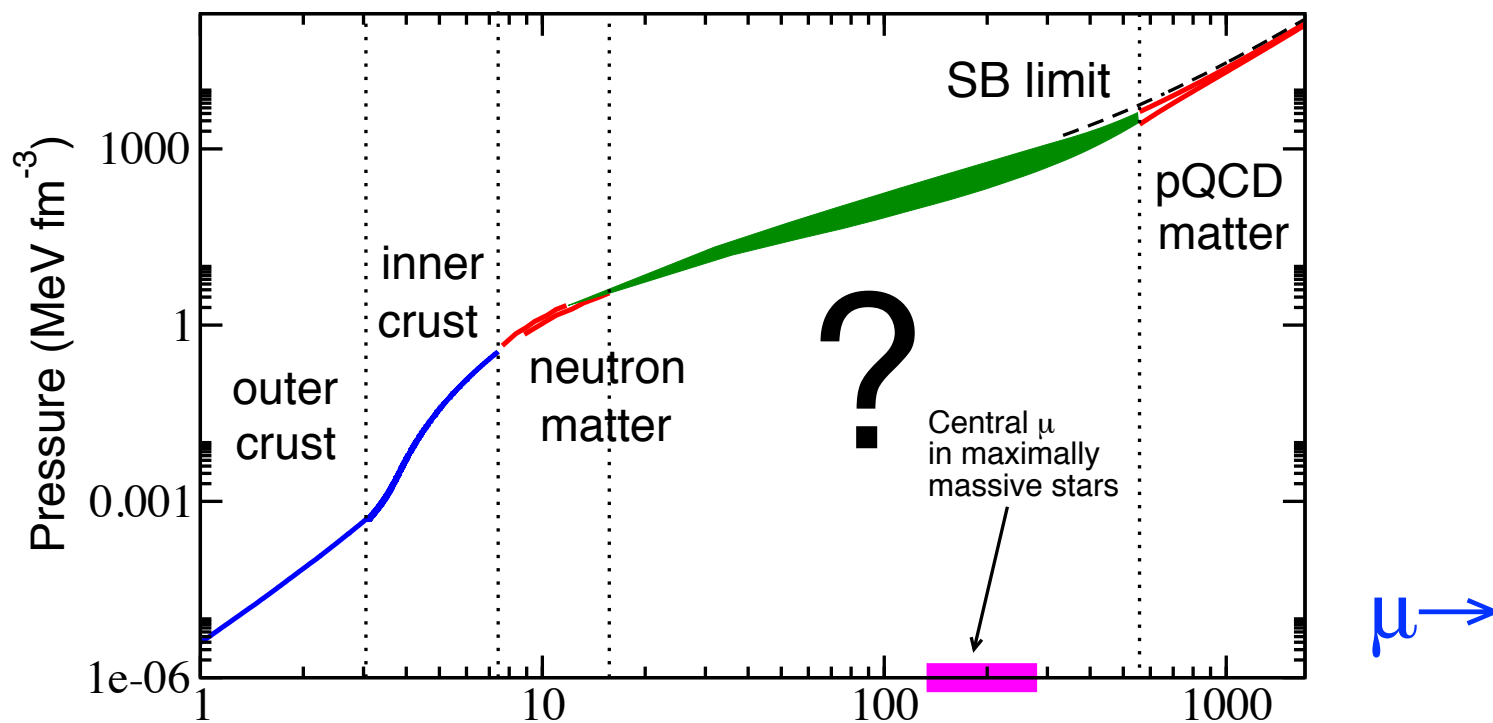
For perturbation theory in vacuum: valid for momenta $p > \Lambda_{vac} = 1 \text{ GeV}$

Suggest: Fermi momenta is a momentum. In *strict* analogy to vacuum:

If $\mu > 1 \text{ GeV}$, $\Lambda_{pert} \sim \mu$. For $\mu < 1 \text{ GeV}$, Quarkyonic or dense baryons.

Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, & Vuorinen, 1609.04339: $\text{pressure}(\mu) \sim g^6$.

Will be able to compute $\Lambda_{pert} = \# \mu$. $\# \sim 1$?



Pure conjecture:

Dense
Baryons

Quarkyonic

Perturbative