Chiral-scale effective theory including a dilatonic meson

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Outline

I. The scalar meson conundrum

II. The lightest scalar meson $f_0(500)$ as a Numbu-Goldstone boson

III. The chiral-scale perturbation theory

IV. Some discussions

I. The scalar meson conundrum

Scalar meson with $m_\sigma \cong 600$ MeV, very important in nuclear physic.
Known long time ago.

- PDG 2016 $f_0(500)$: $m = 400 \sim 550$ MeV, $\Gamma = 400 \sim 700$ MeV

But, a local bosonic field work;
e.g., Bonn boson exchange potential;
RMF

- QCD structure:
  - $\bar{q}q$, $(qq)\overline{(qq)}$, G^2, mixing ?
  - In LσM, $\bar{q}q$, $m \geq 1\ GeV$, irrelevant for physics below $4\pi f_\pi$, 4th component of chiral four-vector
  - Walecka model, chiral singlet

Chiral perturbation theory has been widely used in particle and nuclear physics.

$SU(N_f)_L \times SU(N_f)_R$

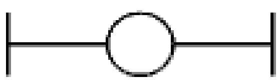
Absence of parity partner

$SU(N_f)_V + ({N_f}^2 - 1)$ NGBs
$U(x) = e^{i\pi/f_\pi}$

Chiral perturbation theory: power counting, derivative expansion

Weinberg, 79; Gasser and Leutwyler, 84

$U(x) = \xi_L \quad h(x)\, h(x)^\dagger \quad {\xi_R}^\dagger$



$K = 1: \quad \pi, \rho$

Son & Stephanov, 04

Effective theory of vector mesons, Hidden local symmetry

Bando, *et al* 89; Harada & Yamawaki, 03

- Chiral effective theory of baryon, extract the baryon mass in chiral limit, HHChPT
- Chiral effective theory of heavy meson.

?. How about the scalar meson, especially the lightest one $f_0(500)$?
If a NGB, can be incorporated to ChPT, power counting, derivative expansion.

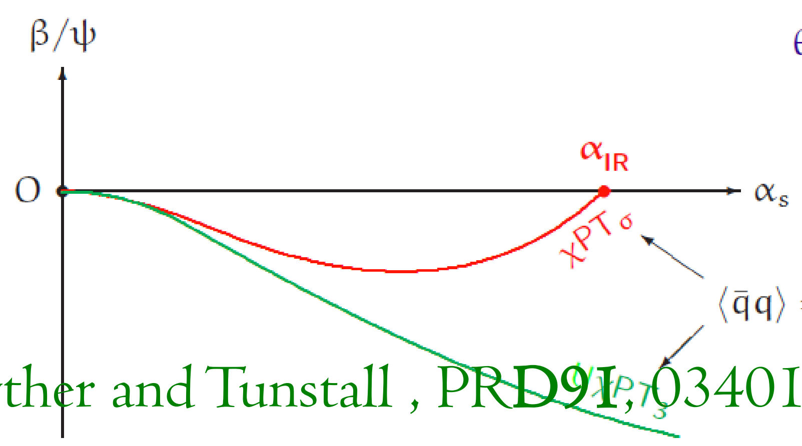
II. The lightest scalar meson as a NGB

$f_0(500)$ is a pNGB arising from (noted $m_{f_0} \cong m_K$)

The SB of scale invariance associated + an explicit breaking of SI.

EB of SI: Departure of α_s from IRFP + current quark mass.

Assumption: There is an IR fixed point in the running QCD coupling constant α_s.



$$\theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^2 + (1+\gamma_m) \sum_{q=u,d,s} m_q \bar{q} q$$

IR fixed point: $\beta(\alpha_{IR}) = 0$

$\chi PT_\sigma \Rightarrow$ expand in θ^μ_μ

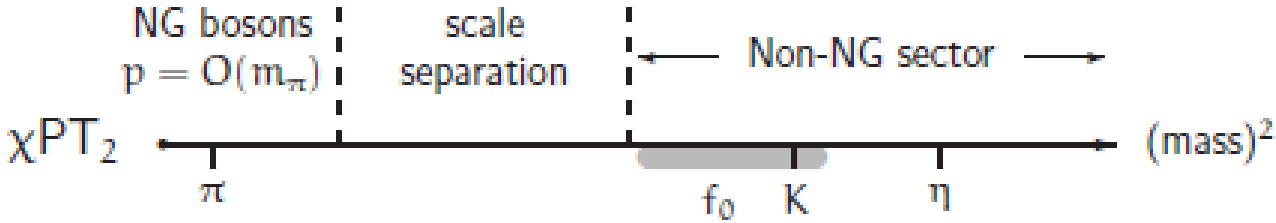
$\alpha_s \lesssim \alpha_{IR}$, $m_{u,d,s} \sim 0$

about scale-*dependent* $|\text{vac}\rangle$

$\Rightarrow$ NG bosons π, K, η, σ.

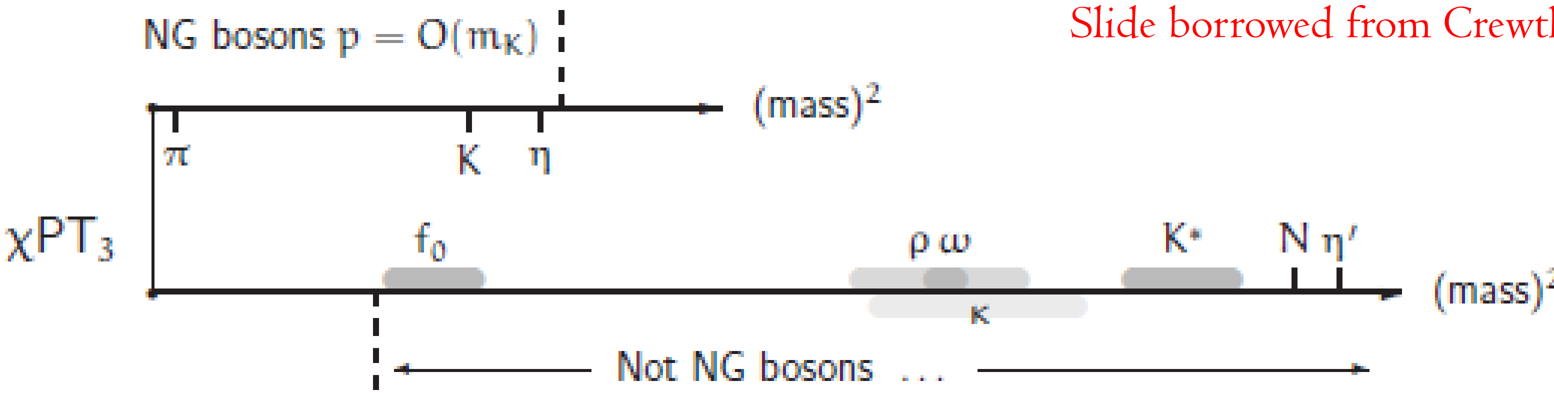
Crewther and Tunstall , PRD91, 034016

Different types of χPT



Standard χPT_2 OK: $m_s \neq 0$ stops θ^μ_μ from vanishing. Consistent with either χPT_3 or χPT_σ.

Slide borrowed from Crewther



Let d denote the scaling dimension of operators used to construct an effective chiral-scale Lagrangian.

① L_{inv}: Scale invariant term

② L_{mass}: Simulate explicit breaking of chiral symmetry by the quark mass term

$$d_{\text{mass}} = 3 - \gamma_m(\alpha_{\text{IR}}), \qquad 1 \leq d_{\text{mass}} < 4$$

① L_{anom}: Account for gluonic interactions responsible for the strong trace anomaly

$$d_{\text{anom}} = 4 + \beta'(\alpha_{\text{IR}}) > 4.$$

$$\mathcal{L}_{\chi\text{PT}_\sigma} = :\mathcal{L}_{\text{inv}}^{d=4} + \mathcal{L}_{\text{anom}}^{d>4} + \mathcal{L}_{\text{mass}}^{d<4}:.$$

III. Chiral-scale perturbation theory

The procedure of the construction of χPT_σ:

- Since χPT_σ, the NGBs are π, K and **σ,** one first writes down all possible derivative terms acting on these particles and counting each derivative as chiral-scale order $O(p)$.
- In the same way as in the standard χPT, the current quark mass is counted as chiral-scale order $O(p^2)$.
- Moreover, since the theory is constructed for α_s below but near the IR fixed point, one should expand $\beta(\alpha_s)$ and the quark mass anomalous dimension $\gamma(\alpha_s)$ around the IR fixed point $\alpha_s(IR)$ and counting $\Delta\alpha_s = \alpha_s - \alpha_{IR}$ as chiral-scale order $O(p^2)$ since it is proportional to m_σ^2 .

- Its convenient to us conformal compensator χ

$$x_\mu \to \lambda^{-1} x_\mu; \quad \chi(x) \to \lambda(\lambda^{-1}x); \quad \chi = f_\sigma e^{\sigma/f_\sigma};$$ chiral singlet

$$\sigma(x) \to \sigma(\lambda^{-1}x) + f_\sigma \ln \lambda$$ Nonlinear realization of Scale symmetry

- Chiral field $U(x) = e^{i\pi/f_\pi}$

Chiral transformation: $U(x) \to g_L U(x) g_R^\dagger$

Scale transformation: $U(x) \to U(\lambda^{-1}x)$

II.I. χPT_σ at leading order

$$\mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma} = \mathcal{L}^{d=4}_{\chi{\rm PT}_\sigma;{\rm inv}} + \mathcal{L}^{d>4}_{\chi{\rm PT}_\sigma;{\rm anom}} + \mathcal{L}^{d<4}_{\chi{\rm PT}_\sigma;{\rm mass}},$$

with

$$\mathcal{L}^{d=4}_{\rm inv,LO} = c_1 \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}c_2 \partial_\mu\chi\partial^\mu\chi + c_3\left(\frac{\chi}{f_\sigma}\right)^4,$$

$$\mathcal{L}^{d>4}_{\rm anom,LO} = (1-c_1)\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}(1-c_2)\left(\frac{\chi}{f_\sigma}\right)^{\beta'} \partial_\mu\chi\partial^\mu\chi + c_4\left(\frac{\chi}{f_\sigma}\right)^{4+\beta'},$$

$$\mathcal{L}^{d<4}_{\rm mass,LO} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} {\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right),$$

$$4c_3 + (4+\beta')c_4 = -(3-\gamma_m)\langle {\rm Tr}(MU^\dagger + UM^\dagger)\rangle_{\rm vac} = -(3-\gamma_m)F_\pi^2\left(m_K^2 + \frac{1}{2}m_\pi^2\right).$$

c_3, c_4 are at $O(p^2)$

$\beta' \to 0$, turn off the explicit breaking of scale symmetry.

Both γ_m and β' are evaluated at α_{IR}

$$m_\sigma^2 F_\sigma^2 = F_\pi^2\left(m_K^2 + \frac{1}{2}m_\pi^2\right)(3-\gamma_m)(1+\gamma_m) - \beta'(4+\beta')c_4,$$

Analog to GMOR relation for pion

➢ Consider chiral limit. Minima of the dilaton potential:

$$4c_3 + (4+\beta')c_4 = 0 \quad \rightarrow c_3 = (4+\beta')c; \; c_4 = -4c$$

$$V(\chi) = -(4+\beta')c\left(\frac{\chi}{f_\sigma}\right)^4 + 4c\left(\frac{\chi}{f_\sigma}\right)^{4+\beta'},$$

$\beta' \neq 0$, NG mode
$\beta' = 0$, No SB

- ✓ The SB of scale symmetry and EB of scale symmetry are correlated and the SB is triggered by EB which agrees with that unlike chiral symmetry, SB of scale symmetry cannot take place in the absence of EB (Freund & Nambu,69).
- ✓ This implies that it is not possible to ``sit on'' the IR fixed point with both scale and chiral symmetries spontaneously broken. This is analogous to that one cannot ``sit'' on the VM fixed point in HLS theory (Harada & Yamawaki , 03).

- $\beta'(\alpha_{IR})$ is a small quantity, i.e., $\beta'(\alpha_{IR}) \ll 1$:

$$V(\chi) = \frac{m_\sigma^2 f_\sigma^2}{4} \left(\frac{\chi}{f_\sigma}\right)^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right],$$

Coleman-Winberg type potential used in the literature, Goldberger, et al, 08

- Infinitesimal scale transformation:

$$\langle \theta_\mu^\mu \rangle = \langle \partial_\mu D^\mu \rangle = 4\beta' c = \frac{m_\sigma^2 f_\sigma^2}{4},$$

PCDC

- A future application to nuclear physics: In a medium of baryonic matter, the vacuum is modified by density. Hence we expect that the decay constant of σ deviates from its vacuum value, i.e., $f_\sigma^* \neq f_\sigma = \langle \chi \rangle$. Since $f_\pi^* = f_\pi \langle \chi \rangle$ (Lee, Paeng & Rho, 15), chiral symmetry breaking is locked to scale symmetry $f_\pi \approx f_\sigma$.

III.2. χPT_{σ} at next to leading order

The NLO Lagrangian = the higher chiral-scale order corrections due to the current quark mass and derivatives on the NGBs + the leading terms in the expansion of γ_m and β' in $\Delta\alpha_s$.

$$\chi^{\gamma_m(\alpha_{\rm IR})} \to \chi^{\gamma_m(\alpha_{\rm IR})}\left[1+\sum_{n=1}^{\infty} C_n\left(\Delta\alpha_s\Sigma\right)^n\right],$$

$$\Delta\alpha_s \sim O(p^2).$$

$$\mathrm{Tr}\left(\partial_\nu U \partial^\nu U^\dagger\right);\ \ \partial_\nu\chi\partial^\nu\chi;\ \ \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right).$$

The same applies to $\chi^{\beta'(\alpha_{IR})}$.

$$\mathcal{L}^{\rm NLO}_{\chi{\rm PT}_\sigma} = \mathcal{L}^{{\rm LO}\times\Delta\alpha_s}_{\chi{\rm PT}_\sigma} + \mathcal{L}^{O(p^4)}_{\chi{\rm PT}_\sigma}.$$

$$\begin{aligned}
\mathcal{L}^{{\rm LO}\times\Delta\alpha_s}_{\chi{\rm PT}_\sigma} = & \left[(1-c_1)\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right) + \frac{1}{2}(1-c_2)\partial_\mu\chi\partial^\mu\chi + c_4\left(\frac{\chi}{f_\sigma}\right)^4\right]\left(\frac{\chi}{f_\sigma}\right)^{\beta'}\Sigma \\
& \times\left[D_1{\rm Tr}\left(\partial_\nu U\partial^\nu U^\dagger\right) + D_2\,\partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right) + D_3\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m}{\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger\mathcal{M}\right)\right] \\
& + {\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger\mathcal{M}\right)\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m}\Sigma \\
& \times\left[D_4{\rm Tr}\left(\partial_\nu U\partial^\nu U^\dagger\right) + D_5\,\partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right) + D_6\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m}{\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger\mathcal{M}\right)\right]
\end{aligned}$$

- Does not transform homogeneously under scale transformation. This Lagrangian serves as renormalization counter terms in the loop expansion of χPT_σ .
- Vanishes when the explicit breaking of scale symmetry is absent.

$$\mathcal{L}_{\chi\mathrm{PT}_\sigma}^{O(p^4)} = \mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{inv}}^{O(p^4);d=4} + \mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{anom}}^{O(p^4);d>4} + \mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{mass}}^{O(p^4);d<4},$$

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{inv}}^{O(p^4);d=4} = {} & L_1\left[\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2 + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right) \\
& + J_1 \partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right)\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + J_2 \partial_\mu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right)\mathrm{Tr}\left(\partial_\nu U \partial^\mu U^\dagger\right) \\
& + J_3 \partial_\mu\left(\frac{\chi}{f_\sigma}\right)\partial^\mu\left(\frac{\chi}{f_\sigma}\right)\partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{anom}}^{O(p^4);d>4} = {} & \Bigg\{ (1-L_1)\left[\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2 + (1-L_2)\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) \\
& + (1-L_3)\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right) \\
& + (1-J_1)\partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right)\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + (1-J_2)\partial_\mu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right)\mathrm{Tr}\left(\partial_\nu U \partial^\mu U^\dagger\right) \\
& + (1-J_3)\partial_\mu\left(\frac{\chi}{f_\sigma}\right)\partial^\mu\left(\frac{\chi}{f_\sigma}\right)\partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right)\Bigg\}\left(\frac{\chi}{f_\sigma}\right)^{\beta'}, \qquad (2
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}_\sigma;\mathrm{mass}}^{O(p^4);d<4} = {} & L_4\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right) + L_5\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right] \\
& + L_6\left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)}\left[\mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right]^2 + L_7\left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)}\left[\mathrm{Tr}\left(\mathcal{M}^\dagger U - U^\dagger \mathcal{M}\right)\right]^2 \\
& + L_8\left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)}\mathrm{Tr}\left(\mathcal{M}^\dagger U \mathcal{M}^\dagger U + U^\dagger \mathcal{M} U^\dagger \mathcal{M}\right) + H_2\left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)}\mathrm{Tr}\left(\mathcal{M}^\dagger \mathcal{M}\right) \\
& + J_4\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m}\partial_\mu\left(\frac{\chi}{f_\sigma}\right)\partial^\mu\left(\frac{\chi}{f_\sigma}\right)\mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right). \qquad (2
\end{aligned}$$

III. 3. Scale invariant hidden local symmetry HLS_σ

◎ Hidden Local Symmetry Theory ･･･ EFT for ρ and π

M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, PRL 54 1215 (1985)
M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988)
M.H. and K.Yamawaki, Physics Reports 381, 1 (2003

Based on chiral symmetry of QCD

ρ ･･･ gauge boson of the HLS

$m_\rho = g f_\rho$ $\cdots$ Higgs mechanism

g $\cdots$ gauge coupling of the HLS

We consider: $[SU(3)_L \times SU(3)_R]_{chiral} \times [SU(3)]_{HLS}$

☆ Hidden Local Symmetry

M.Bando, T.Kugo, S.Uehara, K.Yamawaki and T.Yanagida, PRL 54, 1215 (1985)

M.Bando, T.Kugo and K.Yamawaki, Phys. Rept. 164, 297 (1988)

$$[SU(N_f)_L \times SU(N_f)_R]_{global} \times [SU(N_f)_V]_{local} \rightarrow [SU(N_f)_V]_{global}$$

$$U = e^{2i\pi/F_\pi} = \xi_L^\dagger \xi_R$$

$$\xi_{L,R} = e^{i\sigma/F_\sigma} e^{\pm i\pi/F_\pi} \rightarrow h\, \xi_{L,R}\, g_{L,R}^\dagger \quad \begin{cases} h \in [SU(N_f)_V]_{local} \\ g_{L,R} \in [SU(N_f)_{L,R}]_{global} \end{cases}$$

F_π , F_σ ・・・ Decay constants of π and σ

- Particles

$V_\mu = V_\mu^a T_a$ ・・・ **HLS gauge boson**

$\pi = \pi^a T_a$ ・・・ **NG boson of $[SU(N_f)_L \times SU(N_f)_R]_{global}$ symmetry breaking**

$\sigma = \sigma^a T_a$ ・・・ **NG boson of $[SU(N_f)_V]_{local}$ symmetry breaking**

- Maurer-Cartan 1-forms

$$\hat{\alpha}^{\mu}_{\perp,\parallel} = \left(D^{\mu}\xi_{\mathrm{L}} \cdot \xi_{\mathrm{L}}^{\dagger} \mp D^{\mu}\xi_{\mathrm{R}} \cdot \xi_{\mathrm{R}}^{\dagger} \right) / (2i)$$

$$D_{\mu}\xi_{\mathrm{L}} = \partial_{\mu}\xi_{\mathrm{L}} - iV_{\mu}\xi_{\mathrm{L}} + i\xi_{\mathrm{L}}\mathcal{L}_{\mu}$$

$$D_{\mu}\xi_{\mathrm{R}} = \partial_{\mu}\xi_{\mathrm{R}} - iV_{\mu}\xi_{\mathrm{R}} + i\xi_{\mathrm{R}}\mathcal{R}_{\mu}$$

V_{μ} : HLS gauge field

$\mathcal{L}_{\mu}$, $\mathcal{R}_{\mu}$: gauge fields of $\mathrm{SU}(N_f)_{\mathrm{L,R}}$

Transformation: $\hat{\alpha}^{\mu}_{\perp,\parallel} \rightarrow h\,\hat{\alpha}^{\mu}_{\perp,\parallel}\,h^{\dagger}$

- Lagrangian

$$\mathcal{L}_{\mathrm{HLS}} = F_{\pi}^{2}\,\mathrm{tr}\left[\hat{\alpha}^{\mu}_{\perp}\hat{\alpha}_{\perp\mu}\right] + aF_{\pi}^{2}\,\mathrm{tr}\left[\hat{\alpha}^{\mu}_{\parallel}\hat{\alpha}_{\parallel\mu}\right] - \frac{1}{2g^{2}}\mathrm{tr}\left[V_{\mu\nu}V^{\mu\nu}\right]$$

◎ 3 parameters at the leading order

F_{π} ・・・ pion decay constant

g ・・・ gauge coupling of the HLS

$a = (F_{\sigma}/F_{\pi})^{2}$ ⇔ validity of the vector dominance

$$m_{\rho}^{2} = ag^{2}F_{\pi}^{2}$$

It is easy to incorporate the dilaton in HLS and obtain HLS_σ. As in the χPT_σ case, the leading-order HLS Lagrangian consists of three components:

$$\mathcal{L}^{\rm LO}_{{\rm HLS}_\sigma} = \mathcal{L}^{d=4}_{{\rm HLS}_\sigma;{\rm inv}} + \mathcal{L}^{d>4}_{{\rm HLS}_\sigma;{\rm anom}} + \mathcal{L}^{d<4}_{{\rm HLS}_\sigma;{\rm mass}},$$

$$\mathcal{L}^{d=4}_{{\rm HLS}_\sigma;{\rm inv}} = f_\pi^2 c_1 \left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}[\hat{a}_{\perp\mu}\hat{a}_\perp^\mu] + a f_\pi^2 c_2 \left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}[\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu] - \frac{1}{2g^2} c_3 {\rm Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2} c_4 \partial_\mu\chi\partial^\mu\chi + c_5 \left(\frac{\chi}{f_\sigma}\right)^4$$

$$\mathcal{L}^{d>4}_{{\rm HLS}_\sigma;{\rm anom}} = f_\pi^2 (1-c_1) \left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}[\hat{a}_{\perp\mu}\hat{a}_\perp^\mu] + (1-c_2) a f_\pi^2 \left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}[\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu]$$
$$- \frac{1}{2g^2}(1-c_3)\left(\frac{\chi}{f_\sigma}\right)^{\beta'} {\rm Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2}(1-c_4)\left(\frac{\chi}{f_\sigma}\right)^{\beta'} \partial_\mu\chi\partial^\mu\chi + c_6 \left(\frac{\chi}{f_\sigma}\right)^{4+\beta'},$$

$$\mathcal{L}^{d<4}_{{\rm HLS}_\sigma;{\rm mass}} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} {\rm Tr}\left(\hat{\mathcal{M}} + \hat{\mathcal{M}}^\dagger\right),$$

NLO, operators account for $\Delta\alpha_s$:

$${\rm Tr}(\hat{a}_{\perp\mu}\hat{a}_\perp^\mu); \quad {\rm Tr}(\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu); \quad \frac{1}{2g^2}{\rm Tr}(V_{\mu\nu}V^{\mu\nu});$$
$$\partial_\mu\chi\partial^\mu\chi; \quad {\rm Tr}\left(\hat{\mathcal{M}} + \hat{\mathcal{M}}^\dagger\right).$$

III. 4. Scale invariant HLS with baryon octet: *bs*HLS

$$B(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$B(x) \to h(x)B(x)h^\dagger(x),$$

$$B(x) \to \lambda^{3/2}B(\lambda^{-1}x).$$

$$\mathcal{L}_{bs\mathrm{HLS}} = \mathcal{L}^{d=4}_{bs\mathrm{HLS;inv}} + \mathcal{L}^{d>4}_{bs\mathrm{HLS;anom}},$$

$$\begin{aligned}
\mathcal{L}^{\mathrm{LO};d=4}_{bs\mathrm{HLS;inv}} = {} & c_1 \mathrm{Tr}\left(\bar{B}i\gamma_\mu D^\mu B\right) - \tilde{\mathring{m}}_B \frac{\chi}{f_\sigma}\mathrm{Tr}\left(\bar{B}B\right) - \tilde{g}_{A_1}\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5\left\{\hat{\alpha}^\mu_\perp, B\right\}\right) \\
& - \tilde{g}_{A_2}\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5\left[\hat{\alpha}^\mu_\perp, B\right]\right) + \tilde{g}_{V_1}\mathrm{Tr}\left(\bar{B}\gamma_\mu\left\{\hat{\alpha}^\mu_\parallel, B\right\}\right) + \tilde{g}_{V_2}\mathrm{Tr}\left(\bar{B}\gamma_\mu\left[\hat{\alpha}^\mu_\parallel, B\right]\right), \\
\mathcal{L}^{\mathrm{LO};d>4}_{bs\mathrm{HLS;anom}} = {} & \Big[(1-c_1)\mathrm{Tr}\left(\bar{B}i\gamma_\mu D^\mu B\right) - (\mathring{m}_B - \tilde{\mathring{m}}_B)\frac{\chi}{f_\sigma}\mathrm{Tr}\left(\bar{B}B\right) - (g_{A_1} - \tilde{g}_{A_1})\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5\left\{\hat{\alpha}^\mu_\perp, B\right\}\right) \\
& - (g_{A_2} - \tilde{g}_{A_2})\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5\left[\hat{\alpha}^\mu_\perp, B\right]\right) + (g_{V_1} - \tilde{g}_{V_1})\mathrm{Tr}\left(\bar{B}\gamma_\mu\left\{\hat{\alpha}^\mu_\parallel, B\right\}\right) \\
& + (g_{V_2} - \tilde{g}_{V_2})\mathrm{Tr}\left(\bar{B}\gamma_\mu\left[\hat{\alpha}^\mu_\parallel, B\right]\right)\Big]\left(\frac{\chi}{f_\sigma}\right)^{\beta'},
\end{aligned}$$

Consider the terms contributing to the baryon mass in chiral limit:

$$\mathcal{L}_{bsHLS}^{\text{mass}} = \left[\tilde{\mathring{m}}_B \frac{\chi}{f_\sigma} + (\mathring{m}_B - \tilde{\mathring{m}}_B)\frac{\chi}{f_\sigma}\left(\frac{\chi}{f_\sigma}\right)^{\beta'}\right] \mathrm{Tr}\,(\bar{B}B)$$

Mason fluctuation & small β' expansion

$$\mathcal{L}_{bsHLS}^{\text{mass}} = \mathring{m}_B \mathrm{Tr}\,(\bar{B}B) + \mathring{m}_B \sum_{m=1}^{\infty} \frac{1}{m!}\Sigma^m \mathrm{Tr}\,(\bar{B}B)$$
$$+ (\mathring{m}_B - \tilde{\mathring{m}}_B) \sum_{n=1}^{\infty} \frac{1}{n!}(\beta'\Sigma)^n$$
$$\times \left(1 + \sum_{m=1}^{\infty} \frac{1}{m!}\Sigma^m\right) \mathrm{Tr}\,(\bar{B}B)$$

Baryon mass in chiral limit

$\sigma^n BB$ coupling

E. g., for $n = 1$

$$g_{\sigma BB} = \mathring{m}_B \frac{1}{f_\sigma} + (\mathring{m}_B - \tilde{\mathring{m}}_B)\beta' \frac{1}{f_\sigma}$$

G-T type relation in the dilaton sector

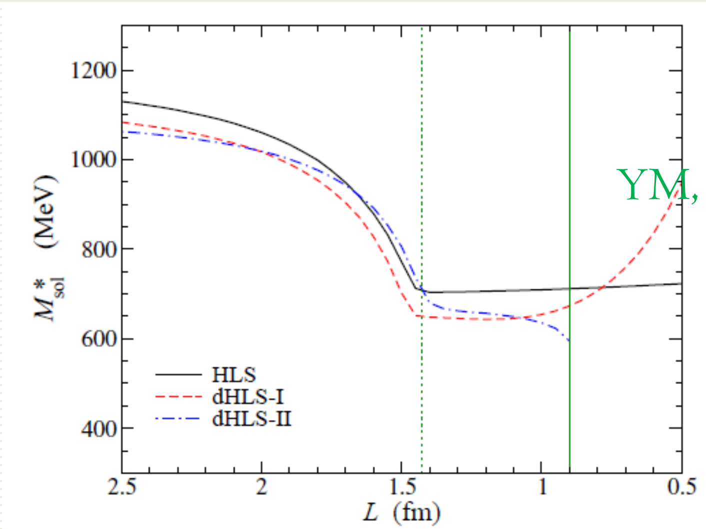
- To finalize the heavy-baryon expansion, we should set up the chiral-scale counting of the interaction terms. Since the dilaton couples to baryons nonderivatively, one cannot do the usual power counting as with the derivative in pion-nucleon coupling.
- In the absence of first-principle guidance, we establish the power counting using a numerical estimation.
- If we take the nucleon mass in the chiral limit $m_B \cong 900$MeV, by taking $f_\pi \approx f_\sigma$, we obtain $g_{\sigma BB} \approx 10$ which is close to $g_{\pi BB} \approx 13$. This suggests that the other terms could be considered as of chiral-scale order $O(p)$. That is, in terms of the compensator χ

$$\mathring{m}_B\left(\frac{\chi}{f_\sigma}-1\right)+(\mathring{m}_B-\tilde{\mathring{m}}_B)\left[\left(\frac{\chi}{f_\sigma}\right)^{\beta'}-1\right]\frac{\chi}{f_\sigma}\sim O(p),$$

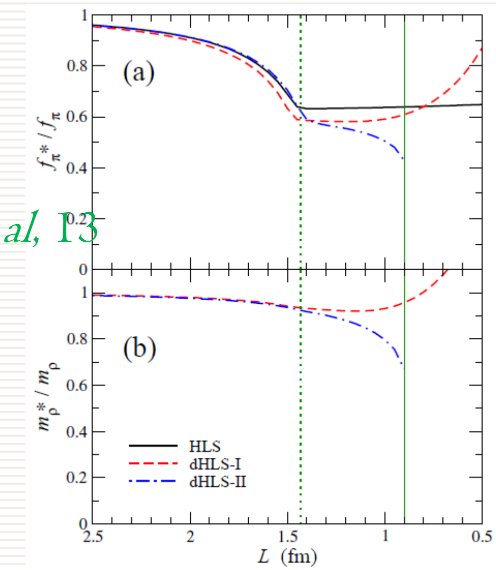
Then, the HBChPT including dilaton can be formulated in a straight forward way.

IV. Some dicussions

- The chiral-scalar effective theory discussed here can be used in the study of dense matter physics and going beyond the mean-field-based analysis. By using a chiral-scale Lagrangian with the log-type potential, the dilaton suppresses the baryon mass and restores the scale symmetry at high density with $f_\sigma^* \to 0$.



YM, *et al*, 13

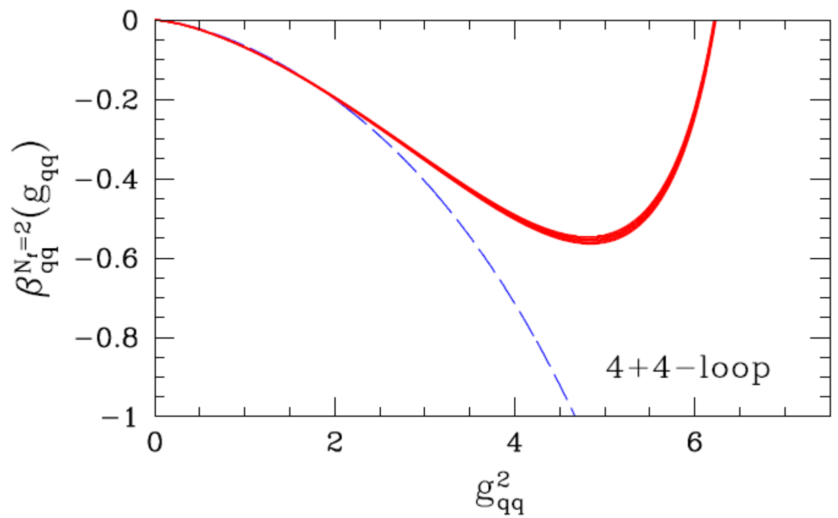


In the present construction, the explicit scale symmetry can be taken into account by the deviation from the IR fixed point α_{IR} which gives the Lagr. used in above at the leading order of small β' so we believe the present Lagr. can yield a result closer to nature.

- Since the present chiral-scale effective theory is constructed with three flavors, it can provide a systematic way to study effects of strangeness in nuclear matter which has hitherto not been feasible in a consistent way.

- In addition to nuclear dynamics under extreme conditions that we are primarily interested in, the presence of an IR fixed point with certain properties, such as low-energy theorems, analogous to what we encounter in dense baryonic matter plays an important role in strongly coupled systems which are intrinsically different from QCD matters, an intriguing recent development being certain technicolor theories purported to go beyond the Standard Model in particle physics (Yamawaki, 1609.03715 for a review).

- The IR fixed point exists for three-flavor QCD in the matter-free space? Not yet established and remains highly controversial.
  - On the one hand, there are no lattice indications or compelling theoretical arguments for the existence of IR fixed points for $N_f \leq 8$.
  - On the other hand, none of the presently available lattice calculations can be taken as an unambiguous no-go theorem. Although not quite compelling, there is even a tentative support from a stochastic numerical perturbation calculation in Pade approximant that indicates an IR fixed point for two-massless flavor quarks.



Horsley, *et al*, 14

Figure 5: The β function $\beta_{qq}^{N_f=2}$ against the coupling g_{qq}^2. The solid band shows the result of the Padé fit, including the error of the pure gauge part. The dashed curve shows the analytic 4 + 4-loop result.

- What interests us, which may not be crucially tied to the precise notion of IR fixed point in the real world of broken symmetries with masses, is the possibility that such an IR fixed point even if hidden in the vacuum can be generated as an emergent symmetry in dense matter.

- This is similar to the issue of restoring of $U_A(A)$ symmetry at high temperature even though the axial anomaly cannot be turned off. (Pisarski & Wilczek, 89) It is also similar to hidden local symmetry with the vector manifestation (VM) with the vanishing ρ mass. (Harada & Yamawaki, 03) The VM is not in QCD in the vacuum, but there is nothing to prevent it from emerging in dense/hot matter.

Thank you for your attention