Study of dense SU(3) QCD through lattice simulation of SU(2) QCD with chemical potential

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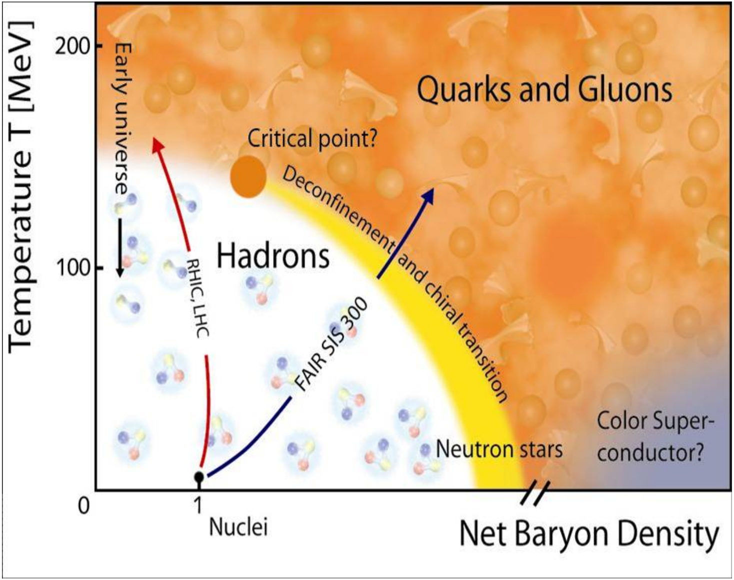
QCD phase diagram



SU(3) QCD

- $Z = \int DUD\bar{\psi}D\psi \;\exp\left(-S_G - \int d^4x\bar{\psi}(\hat{D}+m)\psi\right) =$
  $= \int DU \exp\left(-S_G\right) \times \det\left(\hat{D}+m\right)$
- Eigenvalues go in pairs $\hat{D}: \;\pm i\lambda \Rightarrow$
  $\det\left(\hat{D}+m\right) = \prod_\lambda(\lambda^2+m^2) > 0$
  i.e. one can use lattice simulation
- Introduce chemical potential: $\det\left(\hat{D}+m\right) \to \det\left(\hat{D}-\mu\gamma_4+m\right) \Rightarrow$
  the determinant becomes complex (sign problem)

SU(2) QCD

- $(\gamma_5 C\tau_2)\cdot D^* = D\cdot(\gamma_5 C\tau_2)$
- Eigenvalues go in pairs $\hat{D}-\mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D}-\mu\gamma_4+m\right) > 0 \;\Rightarrow$ free from sign problem

Differences between SU(3) and SU(2) QCD

- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons $(N_f = 2)$ $\pi^+, \pi^-, \pi^0, d, \bar{d}$

Similarities:

- There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of observables are equal up to few dozens percent:

Topological susceptibility (Nucl.Phys.B715(2005)461):

$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

Critical temperature (Phys.Lett.B712(2012)279):

$T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

Shear viscosity :

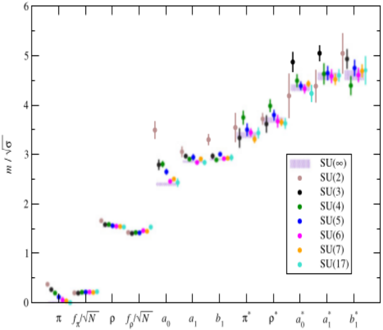
$\eta/s = 0.134(57)\ (SU(2)), \quad \eta/s = 0.102(56)\ (SU(3))$

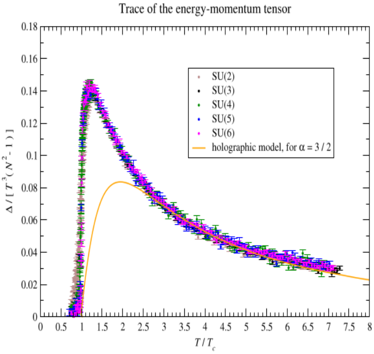
JHEP 1509(2015)082 Phys.Rev. D76(2007)101701

Similarities:

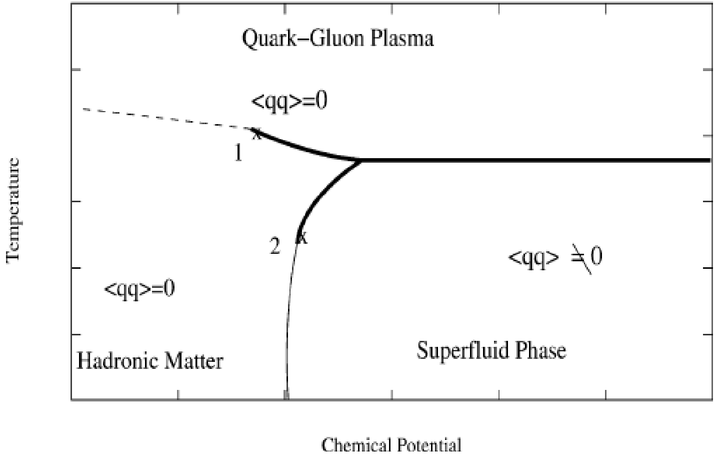
- Spectroscopy (Phys.Rep.529(2013)93)
- Thermodynamic properties (JHEP 1205(2012)135)
- Some properties of dense medium (Phys.Rev.D59(1999)094019):

$$\Delta \sim \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$





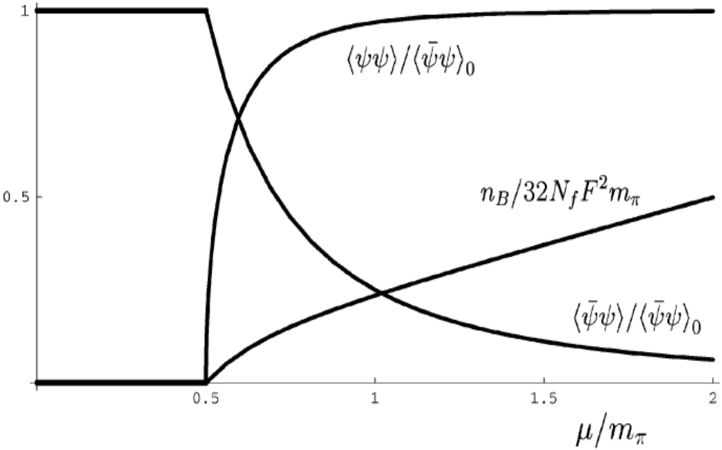
Staggered fermions $N_f = 4$



J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. B 642 (2002) 181–209

One can build CHPT for SU(2) QCD

Predictions of CHPT



Purpose of the work:

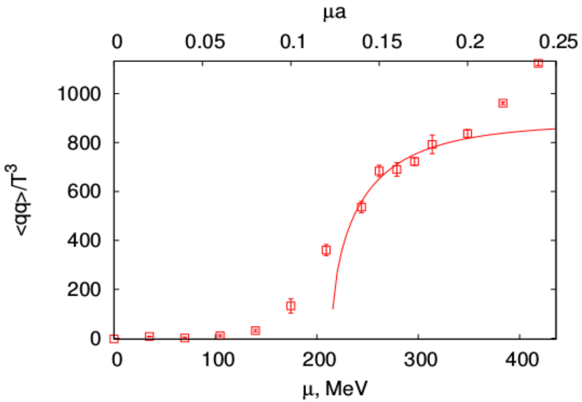
- Study of dense SU(2) QCD phase diagram
- Based on the results study properties of dense SU(3) QCD

Parameters of the calculation:

- In the calculation we use staggered fermions with rooting ($N_f = 2$)
- $a = 0.11$ fm, $16^3 \times 32$, $T = 55$ MeV, $m_\pi = 362(4)$ MeV

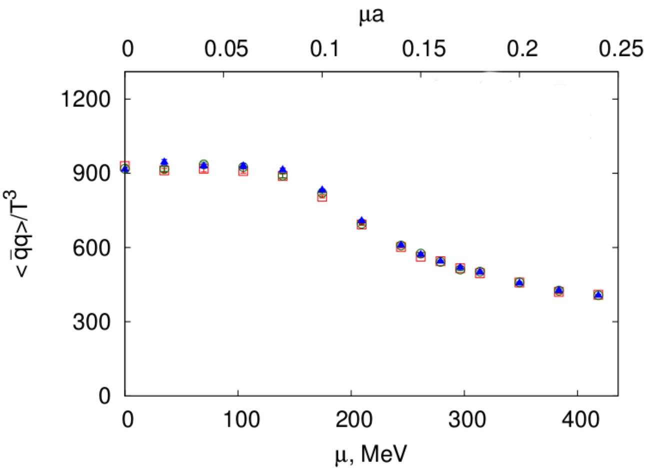
Small chemical potential $\mu < 350$ MeV

Diquark condensate



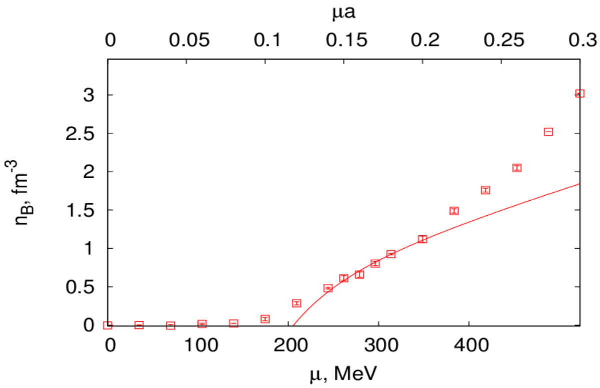
- Good agreement with CHPT $\langle\psi\psi\rangle/\langle\bar{\psi}\psi\rangle_0 = \sqrt{1 - \frac{m_\pi^4}{\mu^4}}$
- Phase transition at $\mu \sim m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200, 350)$ MeV

Chiral condensate



Good agreement with CHPT

Baryon density



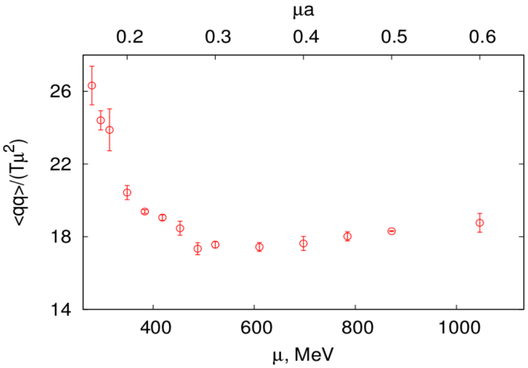
- Good agreement with CHPT $n \sim \mu - \frac{m_\pi^4}{\mu^3}$
- Phase transition at $\mu \sim m_\pi/2$
- Departure from CHPT prediction starts from $n \sim 1$ fm^{-3}

Large chemical potential $\mu > 350$ MeV

Phase diagram for $N_c \to \infty$

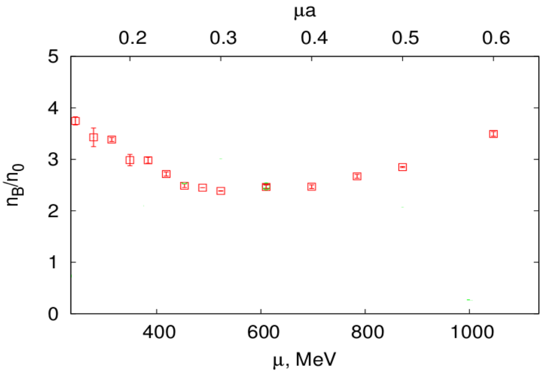
- Hadron phase $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
  - Degrees of freedom:
    - Baryons (on the surface)
    - Quarks (inside the Fermi sphere $|p| < \mu$)
  - No chiral symmetry breaking
  - The system is in confinement phase
- Deconfinement ($p \sim N_c^2$)

Diquark condensate



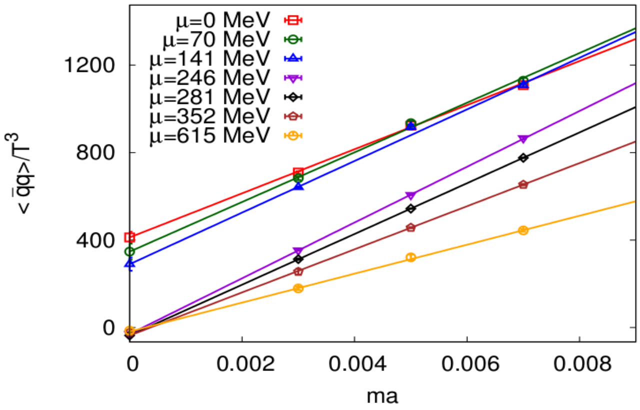
- Bardeen–Cooper–Schrieffer (BCS) phase $\mu > 500$ MeV, $\langle\psi\psi\rangle \sim \mu^2$
- **Baryons (on the surface)**

Baryon density



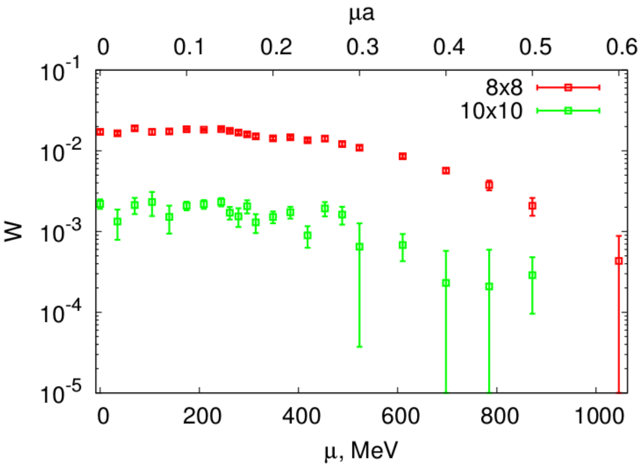
- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3} \theta(|p| - \mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$ $(n \sim (4-5)\times$nuclear density$)$

Chiral condensate (chiral limit $m \to 0$)



No chiral symmetry breaking

Wilson loop



The system is in confinement phase

Conclusion:

- We observe $\mu < m_\pi/2$ hadron phase
- Transition to superfluid phase $\mu \simeq m_\pi/2$ (BEC)
- $\mu > m_\pi/2, \mu < m_\pi/2 + 150$ MeV dilute baryon gas
- BCS phase $\mu \sim 500$ MeV ($n \sim (4-5)\times$nuclear density), transition BEC→BCS is smooth
- BCS phase is similar to quarkyonic phase

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Predictions for SU(3) (preliminary!):

- Quarkyonic phase starts from $n \sim (4-5)\times$nuclear density
- Restoration of chiral symmetry (($4-5)\times$nuclear density) $\Rightarrow$ can be seen in experiment

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Monte-Carlo simulation of SU(2) QCD is the best approach to study properties of SU(3) QCD at large baryon density