A ”quarkyonic matter” like phase and inverse magnetic catalysis at high isospin density

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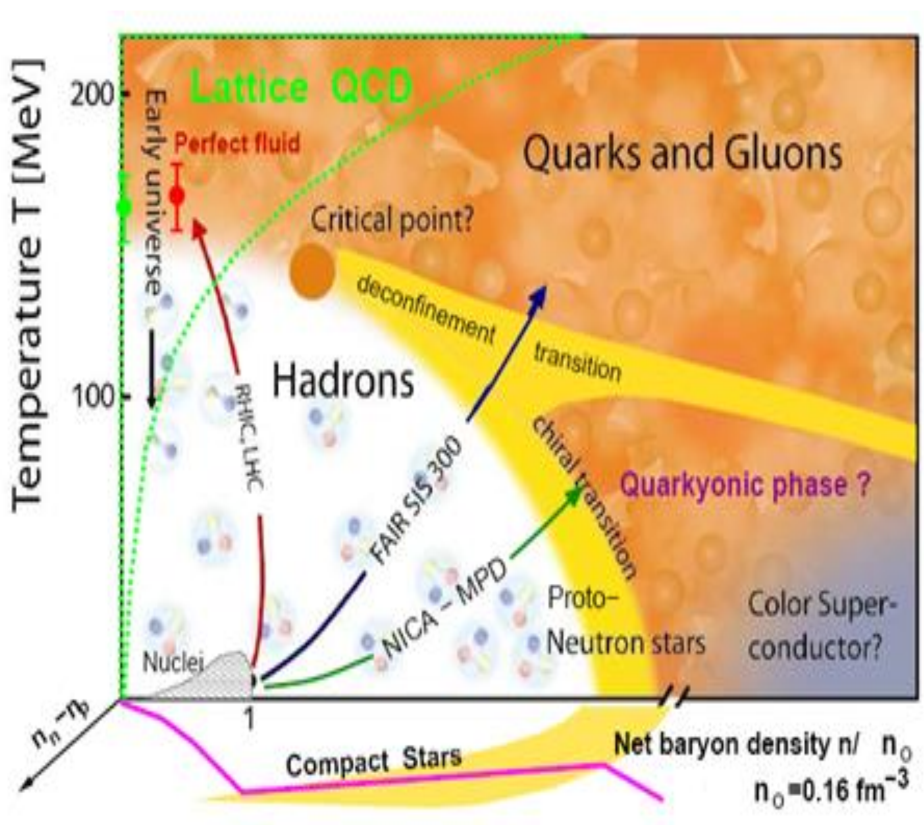
arXiv:1610.06438, PhysRevD.92.105030

Outline

- QCD phase diagrams at finite μ_B and μ_I
- Quarksonic matter VS quarkyonic matter
- Large N_c limit and phase diagram
- Perturbative calculations
- QCD phase diagrams at finite B
- Inverse magnetic catalysis in pion superfluidity
- Conclusions

$T - \mu_B$ phase diagram of QCD

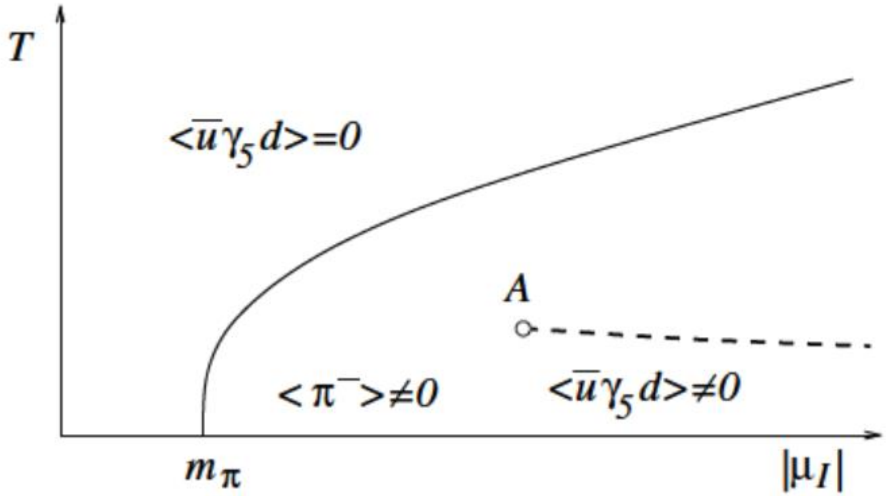


Crossover at low chemical potential--T_c~155MeV;

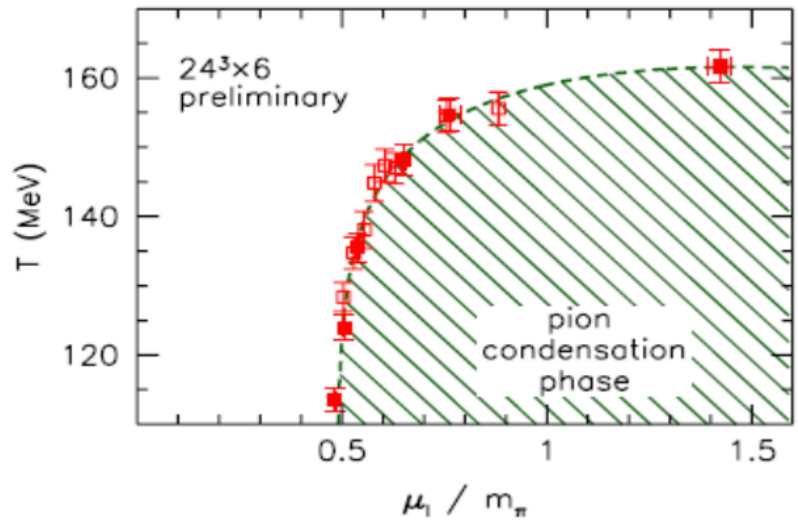
First-order transition at moderate chemical potential to quarkyonic matter--might be covered by inhomogeneous LOFF state;

Color superconductor at high chemical potential– crystalline structure.

$T - \mu_I$ phase diagram



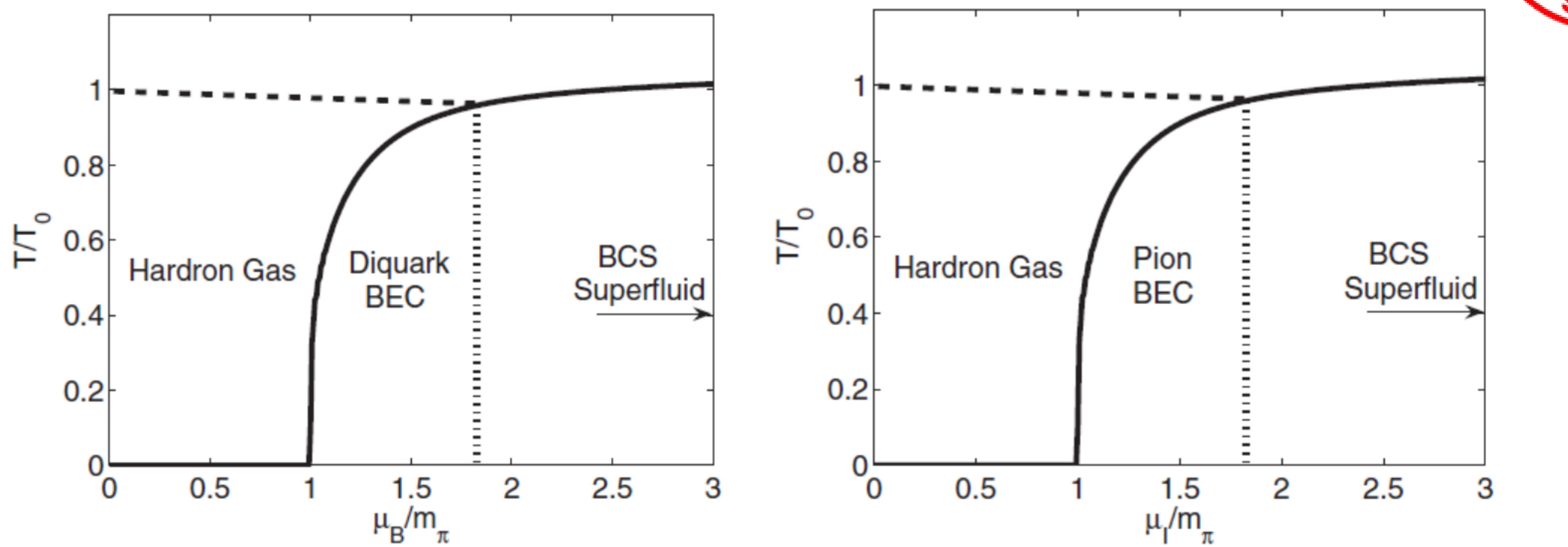
D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **86**, 592 (2001).



Gergely Endr˝odi’s talk at Goethe University of Frankfurt

1. Pion superfluidity for $|\mu_I| \geq m_\pi$;
2. Well defined second order transition;
3. Critical point for deconfinement;
4. Massless Goldstone mode with I_3 breaking.

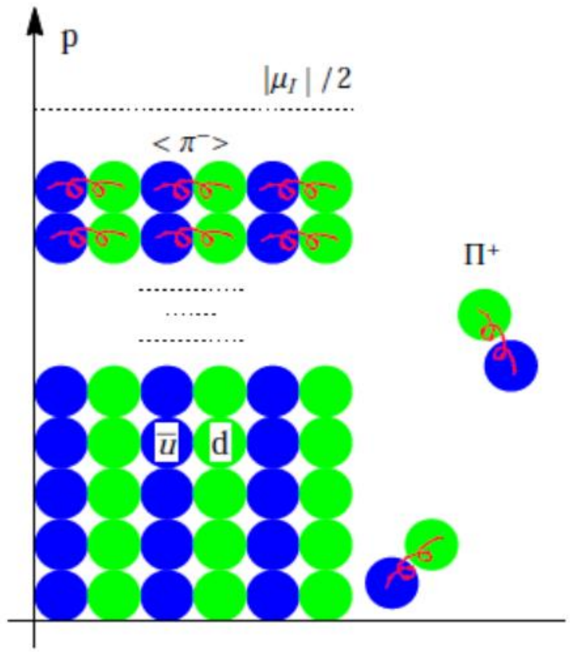
BEC-BCS crossover

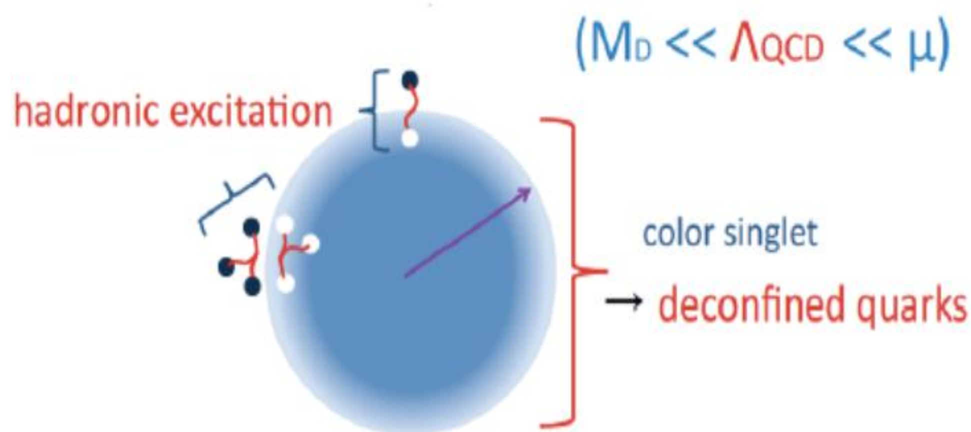


G. Sun, L. He and P. Zhuang, Phys. Rev. D 75, 096004 (2007).

1. **The two-color QCD is almost the same as isospin QCD;**
2. **Dilute diquark/pion BEC after μ^c to quark (antiquark) BCS crossover;**
3. **For $m_q(|\mu|) \geq |\mu|$, unpaired quark (antiquark) components increases.**

Quarksonic matter VS quarkyonic matter

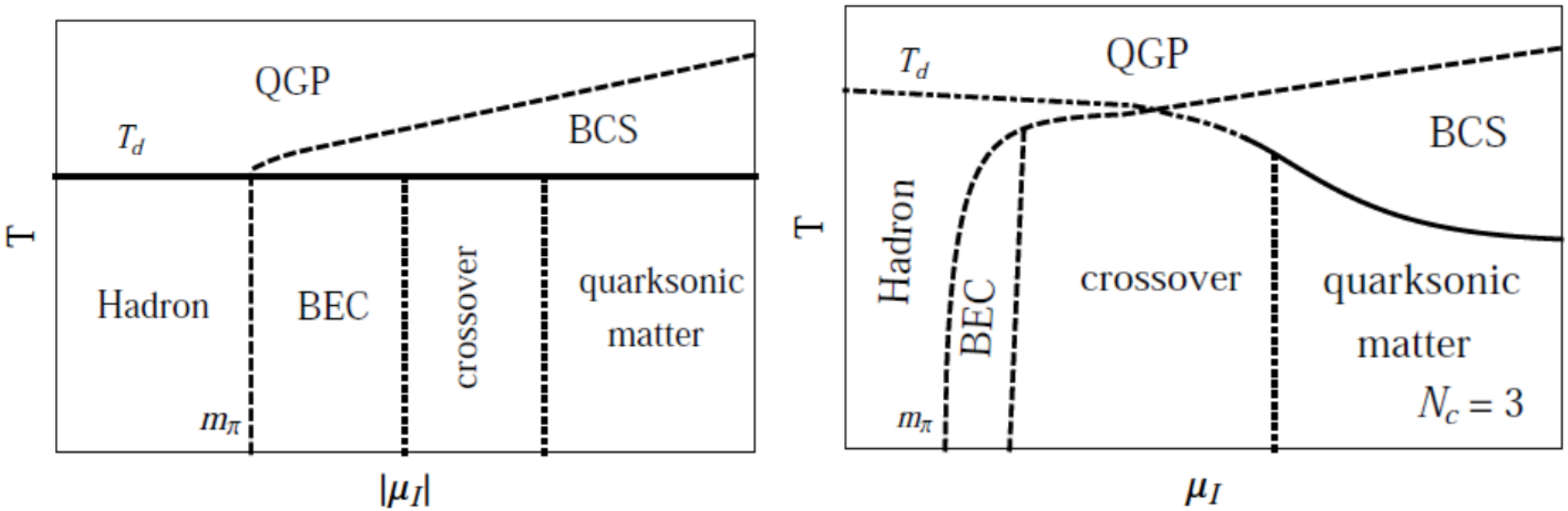




1. A free quark (antiquark) fermi sea with mesonic excitations;
2. Can be verified in LQCD;
3. Pairing, Goldstone mode and gluodynamics at large $|\mu_I|$.

1. A free quark fermi sea with baryonic excitations;
2. Proved in large N_c limit and two-color LQCD;
3. Quarkyonic Chiral Spiral.

Large N_c limit and phase diagram



1. **According to "large N_c volume independence", the phase transitions are independent of T below T_d;**
2. **Beyond m_π, the isospin density is of order N_c;**
3. **Quarksonic matter is realized only when large enough fermi sea is established.**
4. **Pion superfluidity happens at higher temperature than T_d.**

Perturbative calculations--b_π

The perturbative form of pion condensate:

$$\Delta = b_\pi |\mu_I| |g|^{-5} e^{-3\pi^2/(2g)}$$

The exact value of b_π is very important for correct results.

Considering $\pi^-(K) \neq 0$, the quark propagator takes

$$G_{uu/dd}(K) = \left([G^0_{uu/dd}(K)]^{-1} + \gamma_5 \pi^\pm [G^0_{dd/uu}]^{-1} \gamma_5 \pi^\mp \right)^{-1}$$

$$G_{ud/du}(K) = i G^0_{uu/dd}(K) \gamma_5 \pi^\pm(K) G_{dd/uu}(K),$$

$$G^0_{uu/dd}(K) \equiv [\not{K} \pm \mu\gamma_0 + \Sigma_{uu/dd}(K)]^{-1}$$

The self-energy should be evaluated in order to get correct b_π.

Exact value of b_π

The self-energy is quite similar with that in μ_B

$$\Sigma_{uu/dd}(K) = \gamma_0 \bar{g}^2 (k_0 \ln \frac{M^2}{k_0^2} + i\pi|k_0|),$$

$$\bar{g} = g/(3\sqrt{2}\pi) \text{ and } M^2 = (3\pi/4)m_g^2$$

The effect is modifying the definition of energy:

$$G^0_{uu/dd}(K) = [\gamma_0 \tilde{k}_0 - \gamma \cdot \mathbf{k} \pm \mu\gamma_0]^{-1},$$

$$\tilde{k}_0 = k_0/Z(k_0) \text{ and } Z(k_0) \equiv [1 + \bar{g}^2 \ln(M^2/k_0^2)]^{-1}$$

Using energy projectors to write $\pi^- = \sum_{s=\pm} \Lambda_s \pi_s^-$ **, then the gap equation becomes**

$$\pi_k = \frac{2g^2}{3} \frac{T}{V} \sum_Q \Delta_{\mu\nu}(K-Q) \frac{\pi_q}{\tilde{q}_0^2 - (E_{\mathbf{q}}^+)^2} \mathrm{Tr}[\Lambda_-(\mathbf{k})\gamma^\mu \Lambda_+(\mathbf{q})\gamma^\nu]$$

Exact value of b_π

The gluon propagators

$$\Delta_{00}(P)=\Delta_l(P)+\xi_C\frac{p_0^2}{p^4},\quad \Delta_{0i}(P)=\xi_C\frac{p_0p_i}{p^4}$$

$$\Delta_{ij}(P)=(\delta_{ij}-\hat{p}_i\hat{p}_j)\Delta_t(P)+\xi_C\frac{\hat{p}_i\hat{p}_j}{p^2}$$

The gap equation is similar with that in color superconductivity

$$\phi_h^e(K)=\frac{2}{3}g^2\frac{T}{V}\sum_Q \Delta_{\mu\nu}(K-Q)\left\{\frac{\phi_h^e(Q)}{q_0^2-[\epsilon_q^e(\phi_h^e)]^2}\mathrm{Tr}[\mathcal{P}_h^e(\mathbf{k})\gamma^\mu\mathcal{P}_{-h}^{-e}(\mathbf{q})\gamma^\nu]\right.$$
$$\left.+\frac{\phi_h^{-e}(Q)}{q_0^2-[\epsilon_q^{-e}(\phi_h^{-e})]^2}\mathrm{Tr}[\mathcal{P}_h^e(\mathbf{k})\gamma^\mu\mathcal{P}_{-h}^e(\mathbf{q})\gamma^\nu]\right\}$$

R. D. Pisarski and D. H. Rischke, Phys. Rev. D 61, 074017 (2000).

Exact value of b_π

By comparing the gap equations, we have

$$\phi_k \simeq \bar{g}^2 \int_0^\delta \frac{d(q-\mu)}{\tilde{\epsilon}_q} Z^2(\tilde{\epsilon}_q) \tanh\left(\frac{\tilde{\epsilon}_q}{2T}\right) \frac{1}{2} \ln\left(\frac{\tilde{b}^2\mu^2}{|\tilde{\epsilon}_q^2 - \tilde{\epsilon}_k^2|}\right) \phi_q$$

$$\pi_k \simeq \bar{g}^2 \int_0^\delta \frac{\pi_q d(q+\mu)}{Z^{-2}(\tilde{E}_\mathbf{q}^+)\tilde{E}_\mathbf{q}^+} \tanh\left(\frac{\tilde{E}_\mathbf{q}^+}{2T}\right) \ln\left(\frac{\tilde{b}^2\mu^2}{|(\tilde{E}_\mathbf{q}^+)^2 - (\tilde{E}_\mathbf{k}^+)^2|}\right)$$

Then the gap with $k = 0$ can be derived as

$$\Delta = b_\pi |g|^{-5} |\mu_I| e^{-\frac{\pi}{2\sqrt{2}g}}, \; b_\pi = 256\pi^4 e^{-\frac{4+\pi^2}{16}}$$

For color superconductivity, the coefficient is

$$b_\pi = 256\pi^4 e^{-\frac{4+\pi^2}{8}}$$

Pure gluodynamics approximation

For large $|\mu_I|$, the thermodynamic potential has three parts:

$$\Omega = \mathcal{U}(\Phi) - \frac{\pi^2}{90}\frac{T^4}{v^3} + \Omega_q,$$

The pure gluon potential can be modified from LQCD by changing $T_0 = 270MeV$ to $\tilde{T}_0 = T_0(\tilde{\Lambda}/\Lambda_{QCD})$

$\tilde{\Lambda}$ determined by four loop α_s

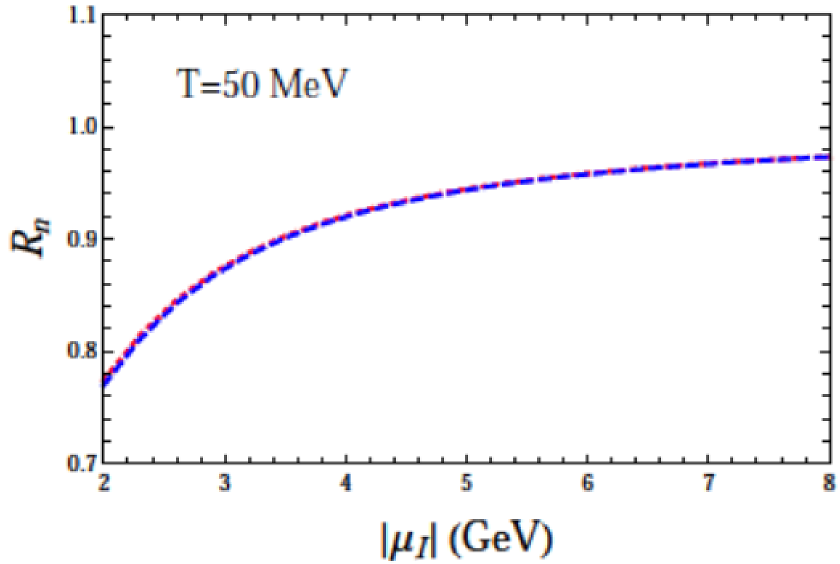
$$\frac{\mathcal{U}(\Phi)}{T^4} = -\frac{a(T)}{2}\Phi^2 + b(T)\log[1-6\Phi^2+8\Phi^3-3\Phi^4],$$

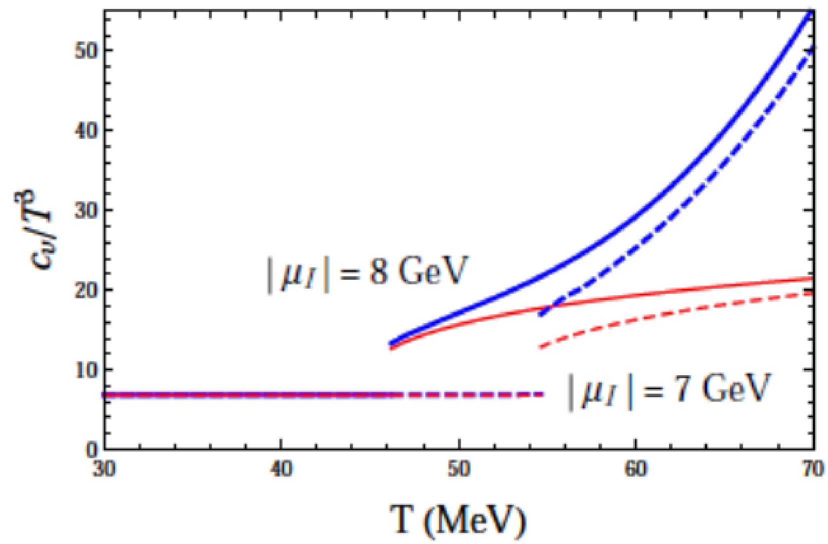
$$a(T) = a_0 + a_1\left(\frac{\tilde{T}_0}{T}\right) + a_2\left(\frac{\tilde{T}_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{\tilde{T}_0}{T}\right)^3,$$

The quark part is inspired from effective model with free qusi-quarks

$$\Omega_q = -2\int\frac{d^3p}{(2\pi)^3}\sum_{s=\pm}\left\{N_c E_{\mathbf{P}}^s + 2T\ln\left(1 + 3\Phi e^{-E_{\mathbf{P}}^s/T} + 3\Phi e^{-2E_{\mathbf{P}}^s/T} + e^{-3E_{\mathbf{P}}^s/T}\right)\right\},$$

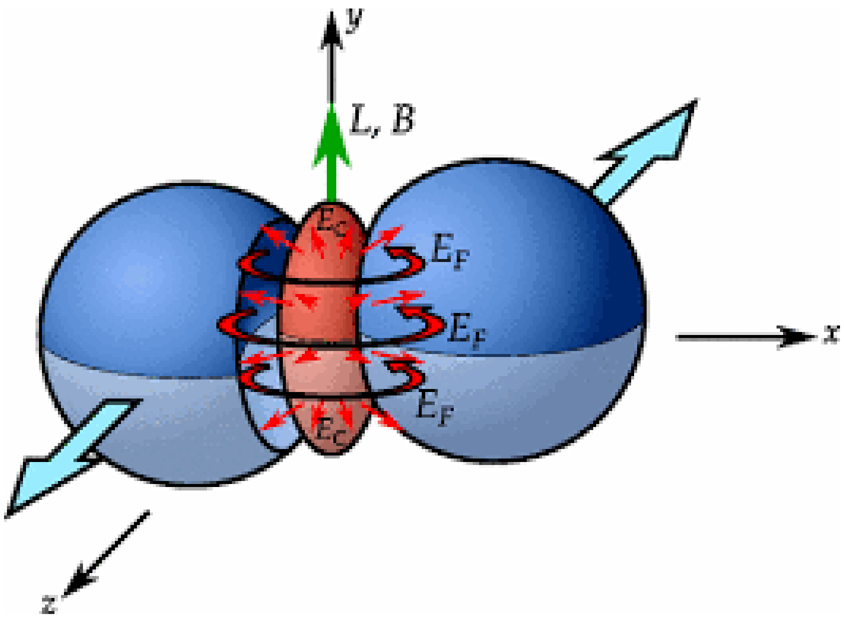
Numerical results

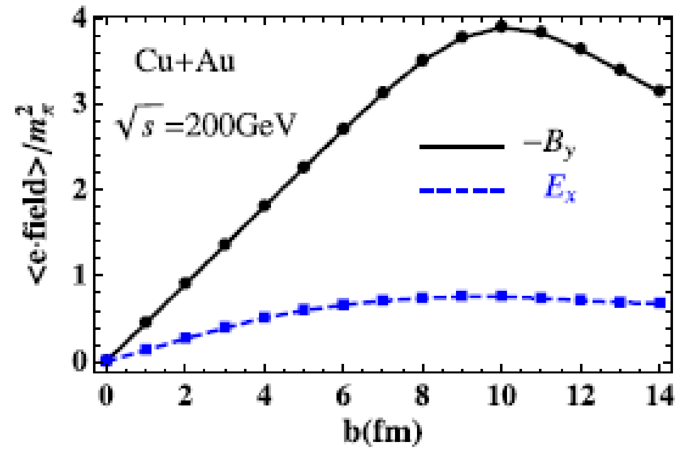


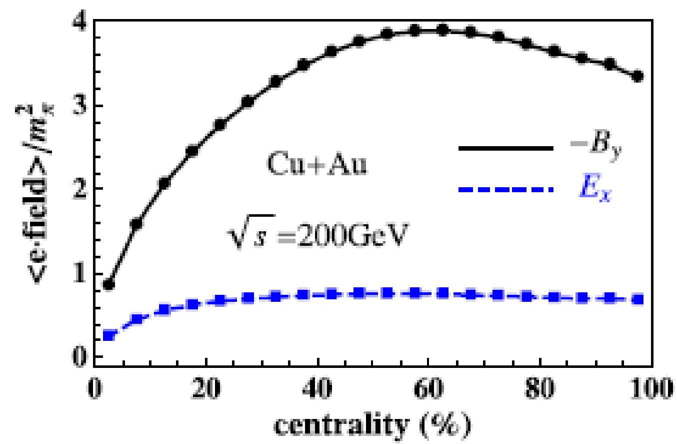


1. From R_n, large free fermi sea is established for large $|\mu_I|$--quarksonic matter verified;
2. From heat capacity c_v, the main thermal excitation is Goldstone modes at low temperature, gluons contribute immediately after deconfinement and quarks domain at higher temperature.

Strong EM field in HIC





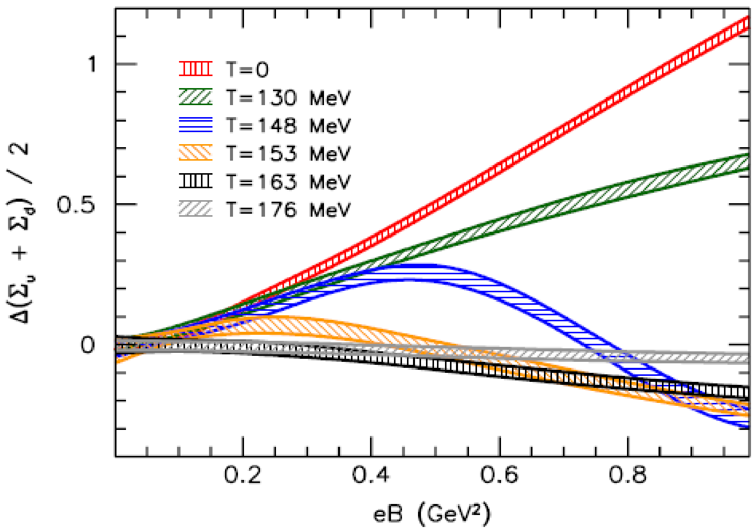


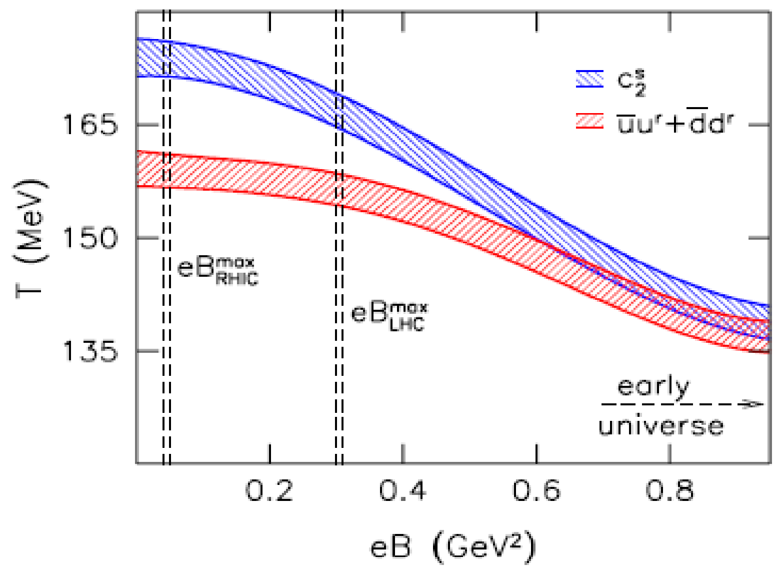
In LHC, magnetic field is even larger ——1 GeV^2.

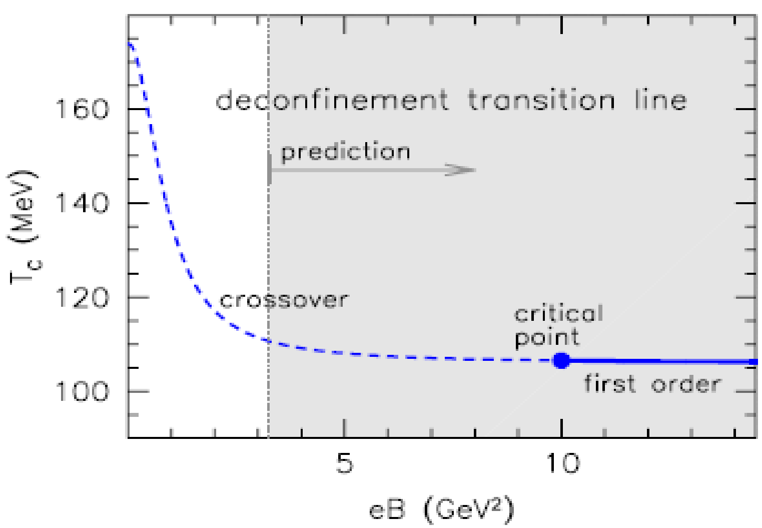
QCD phase diagram in strong EM field ?

W.-T. Deng, X.-G. Huang, PLB 742, 296 (2015).

Inverse magnetic catalysis (LQCD)







(1) At $T=0$, Σ increases with B; around T_c, Σ decreases with B;

(2) T_c monotonically decreases with B;

(3) T_c converges to a finite value at large enough B and the crossover becomes first order.

G. Bali, et al., JHEP 1202 (2012) 044;
Phys.Rev. D86 (2012) 071502(R);
G. Endrődi, JHEP 1507 (2015) 173 .

Pion superfluidity in magnetic field

The Lagrangian of Nambu-Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0 + \frac{\mu_I}{2}\gamma_0\tau_3\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right]$$

The mean field thermodynamic potential

$$\Omega = \frac{(m-m_0)^2 + \Delta^2}{4G} + \frac{i}{V_4}\mathrm{Tr}\ln\begin{pmatrix} (iG_u)^{-1} & -i\gamma_5\Delta \\ -i\gamma_5\Delta^* & (iG_d)^{-1} \end{pmatrix}$$

Can not be simply expressed by Landau levels of u and d quarks as Δ is charged

GL expansion around small Δ

$$\begin{aligned}\Omega &= \frac{\Delta^2}{4G} - \frac{i}{2V_4}\mathrm{Tr}G_{xy}G_{yx} - \frac{i}{4V_4}\mathrm{Tr}G_{xy_1}G_{y_1y_2}G_{y_2y_3}G_{y_3x} \\ &= \mathcal{A}\Delta^2 + \frac{\mathcal{B}}{2}\Delta^4\end{aligned}$$

$$G_{xy} = \begin{pmatrix} iG_u(x,y) & 0 \\ 0 & iG_d(x,y) \end{pmatrix}\begin{pmatrix} 0 & -i\gamma_5\Delta_y \\ -i\gamma_5\Delta_y^* & 0 \end{pmatrix}$$

Gap equations

Quark propagators in Schwinger representation:

$$
\begin{aligned}
G_f(x,y) &= e^{-iq_f\int_y^x \tilde{A}_f^{\mu} dx_\mu} S_f(x-y), \\
S_f(x) &= -i\int_0^\infty \frac{ds}{16(\pi s)^2} e^{-i\left[sm^2+\frac{1}{4s}\left(x_0^2-x_3^2-\mathbf{x}_\perp^2 B_f^s \cot B_f^s\right)\right]} B_f^s\left[\cot B_f^s-\gamma_1\gamma_2\right]\left[m+\frac{1}{2s}\left(\not{x}_0-\not{x}_3-B_f^s\left((\not{x}_1+\not{x}_2)\cot B_f^s+\not{x}_{21}-\not{x}_{12}\right)\right)\right]
\end{aligned}
$$

In order to get rid of gauge dependence, we should introduce Wilson line to Δ.

Then A and B can be expressed in momentum space:

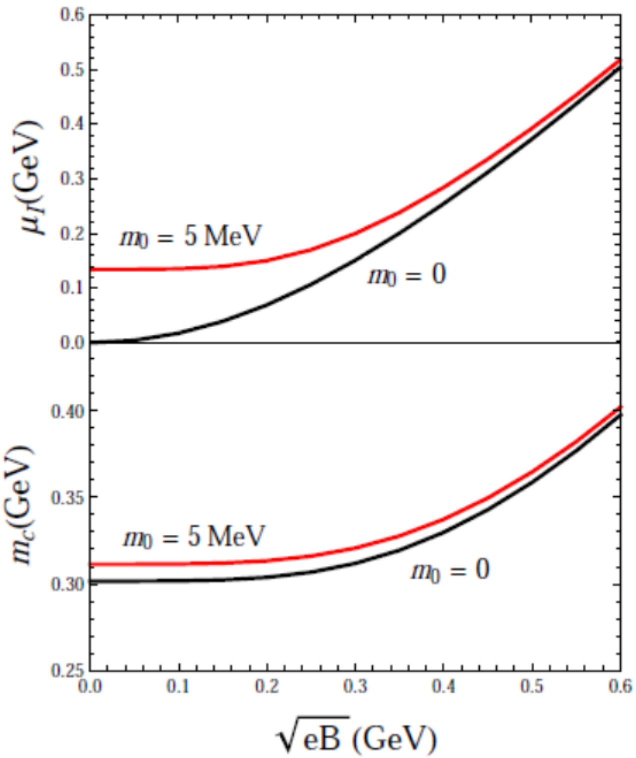
$$
\begin{aligned}
\mathcal{A} &= \frac{1}{4G}-N_cT\sum_n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\mathrm{Tr}\left[S_u^E\left(\omega_n+i\frac{\mu_I}{2},\mathbf{k}\right)i\gamma_5 S_d^E\left(\omega_n-i\frac{\mu_I}{2},\mathbf{k}\right)i\gamma_5\right] \\
&= \frac{1}{4G}-4N_cT\sum_n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int ds\int dt R(s,t,\omega_n,\mathbf{k})\left\{\left(m^2+\omega_n^2+\left(\frac{\mu_I}{2}\right)^2+k_3^2\right)(1+f_u(s)f_d(t))+\mathbf{k}_\perp^2\left(1-f_u^2(s)\right)\left(1-f_d^2(t)\right)\right\}
\end{aligned}
$$

The critical gap equations

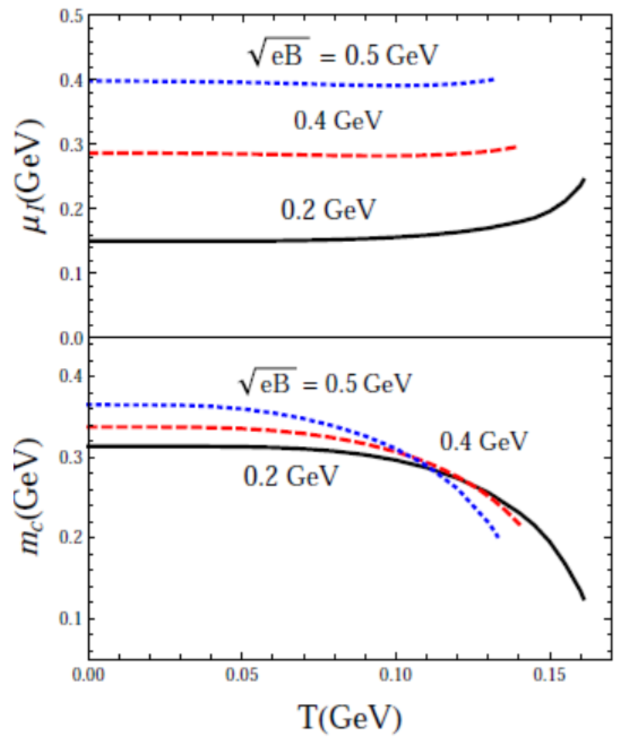
$$
\begin{aligned}
A &= 0 \\
0 &= \frac{m-m_0}{2G}-\frac{1}{\beta V}\sum_{f=u,d}\mathrm{Tr}G_f(x,y)
\end{aligned}
$$

Phase transition lines

$T = 0$



$T > 0$



The critical mass and chemical potential both increases with B --inverse magnetic catalysis

de Haas—van Alphan oscillation shows up in the critical mass

Conclusions

- Quarksonic matter at high isospin density is studied in both large N_c and asymptotically free limit;
- The heat capacity is mainly contributed from Goldstone modes for quarksonic matter, but deconfinement gives extra contribution from gluons and quarks
- Inverse magnetic catalysis is found in pion superfluidity.

Thank you very much!