Pseudoscalar condensate from axial anomaly

Shu Lin 林树
Sun Yat-Sen University

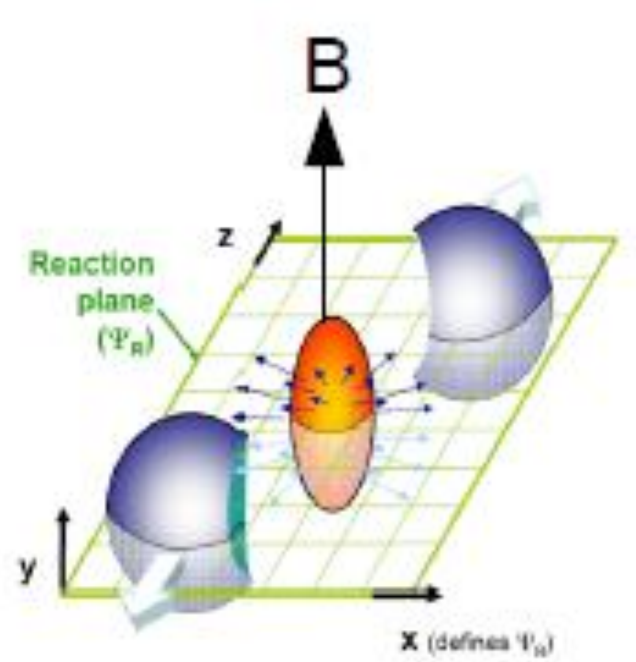
CCNU, Oct 27, 2016

Guo, SL. 1602.03952, PRD 2016
1610.05886

Outline

- Motivation
- Signature of chiral magnetic wave
- Quark mass correction to chiral separation effect
- Response functions to inhomgeneity
- Mode corresponds to pseudoscalar condensate spiral
- Summary

Local parity violation in heavy ion collisions



Chiral Magnetic Effect (CME)

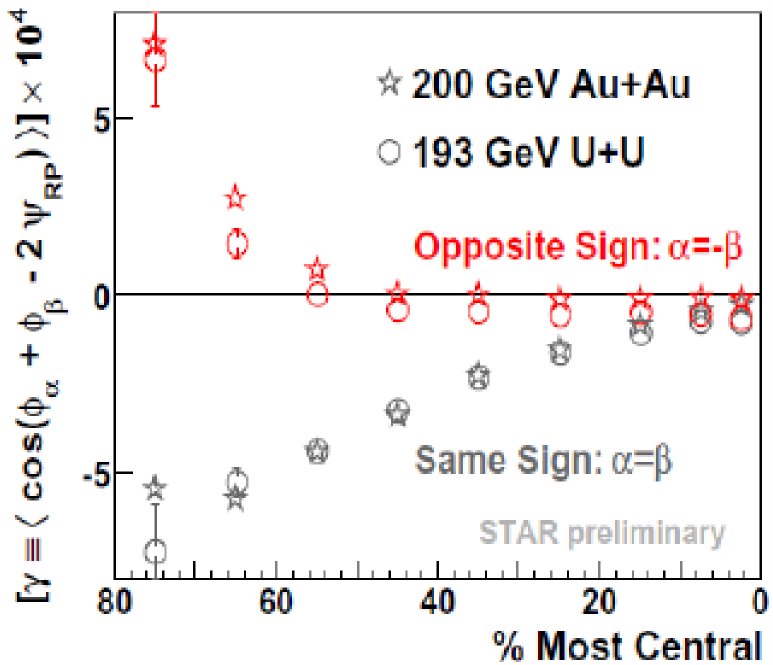
$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

QED anomaly

μ_A: chiral imbalance in QGP
$eB \sim m_\pi{}^2$: strong magnetic field in heavy ion collisions

Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008
Fukushima, Kharzeev, Warringa, PRD 2008

Experimental signature of CME



$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

Origin of chiral imbalance: QCD anomaly

$$\partial_\mu {j_A}^\mu = -\frac{g^2 N_f}{8\pi^2} tr(G\tilde{G})$$

Same charge correlation enhanced than opposite charge correlation due to CME

Measurement on an event-by-event basis

STAR collaboration, PRL (2014), 1404.1433

Analog of CME: Chiral separation effect

$$j_V = \frac{N_c \mu_A}{2\pi^2} eB \quad \xrightarrow{V \leftrightarrow A} \quad j_A = \frac{N_c \mu_V}{2\pi^2} eB$$

CME

CSE

Metlitski, Zhitnitsky, PRD (2005)

$$\langle N_V \rangle \neq 0$$

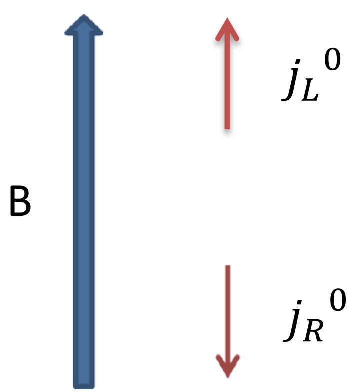
Yet experimentally not accessible

Chiral magnetic wave: an interplay of CME and CSE

$$j_V{}^1 = \frac{N_c\mu_A}{2\pi^2}eB - D_L\partial_1 j_V{}^0$$ Chiral magnetic effect + diffusion

$$j_A{}^1 = \frac{N_c\mu_V}{2\pi^2}eB - D_L\partial_1 j_A{}^0$$ Chiral separation effect + diffusion

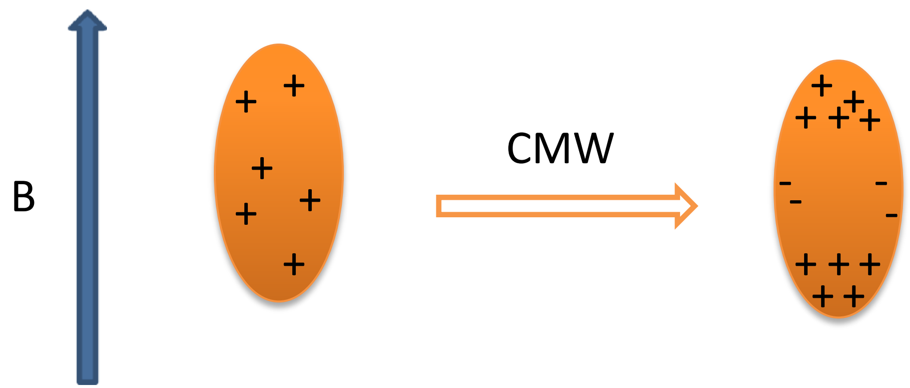
$$\partial_\mu j_V{}^\mu = 0, \quad \partial_\mu j_A{}^\mu = 0 \quad \Longrightarrow \quad \left(\partial_0 \mp \frac{N_c eB\alpha}{2\pi^2}\partial_1 - D_L\partial_1^2\right) j^0_{L,R} = 0$$



$$v_\chi = \frac{N_c eB\alpha}{2\pi^2} = \frac{N_c eB}{4\pi^2}\left(\frac{\partial\mu_L}{\partial j^0_L}\right) = \frac{N_c eB}{4\pi^2}\left(\frac{\partial\mu_R}{\partial j^0_R}\right)$$

Kharzeev, Yee, PRD (2011)

Charge dependent flow from CMW



Negative charges have larger pressure gradient than positive charges

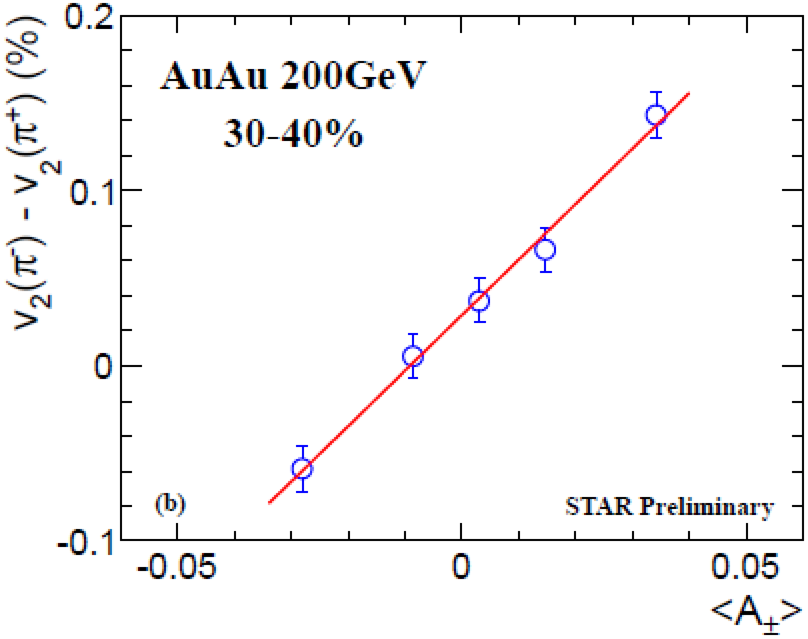
$$v_2(\pi^+) < v_2(\pi^-)$$

Charge dependent flow survives even after event averaging!

$$v_2^{\pm} = v_2 \mp \left(\frac{q_e}{\bar{\rho}_e}\right) A_{\pm} \qquad A_{\pm} \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

Burnier, Kharzeev, Liao, Yee PRL 2011

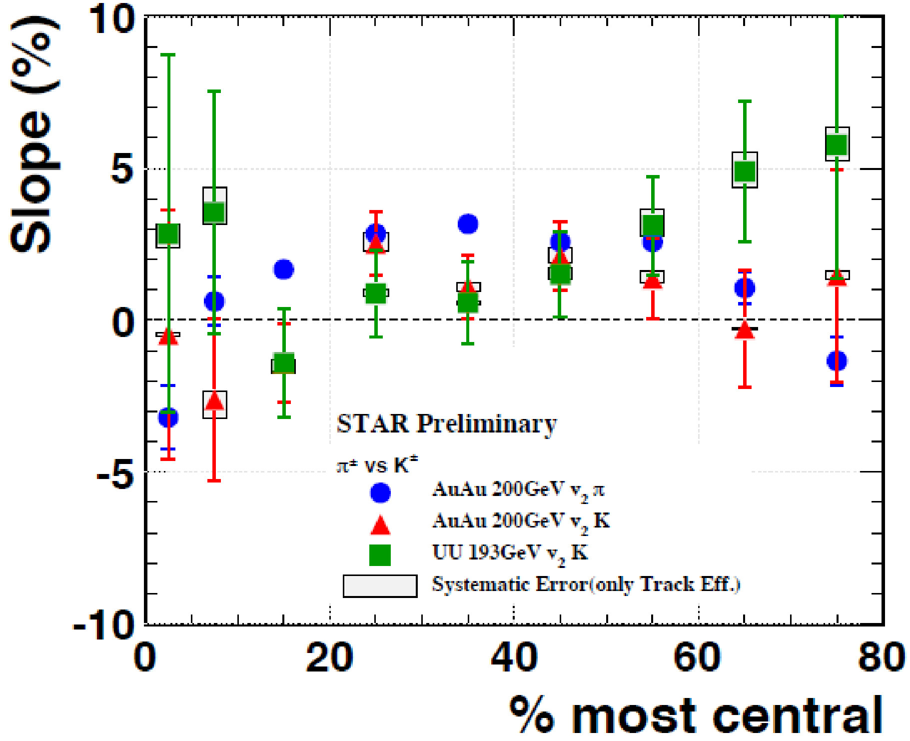
Experimental signature of CMW



Hongwei Ke, J.Phys.Conf.Ser (2012)

Further confirmation from Kaon flow?

Quark mass dependence of CMW



Qiye Shou, J.Phys.Conf.Ser (2014)

Quark mass term in axial anomaly

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma},$$

Pseudoscalar QED anomaly QCD anomaly

when $m \ll T$, neglect mass term above

HIC at RHIC, $T \lesssim 350MeV$
Strange quark mass $m \sim 100MeV$

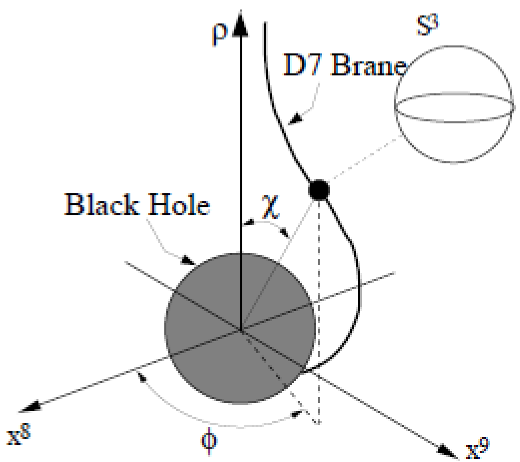
Quark mass effects:

- modify fluctuation/dissipation of axial charge.
- modify CSE, but not modify CME

The D3/D7 model

N_c D3 branes + N_f D7 branes

gluons quarks

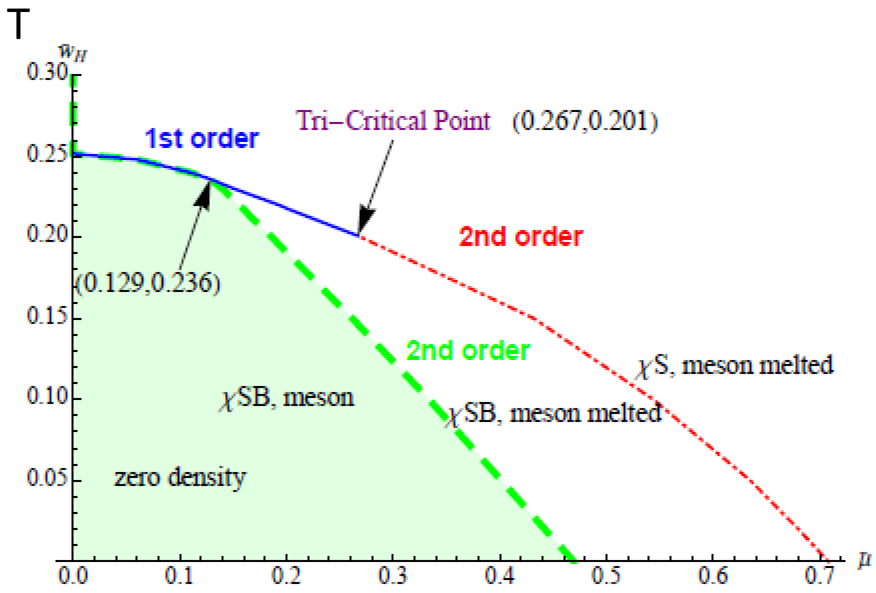


	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D7	×	×	×	×	×	×	×	×		

$N_c \gg N_f$

quarks in gluon plasma $T \neq 0$

The finite μ and B background

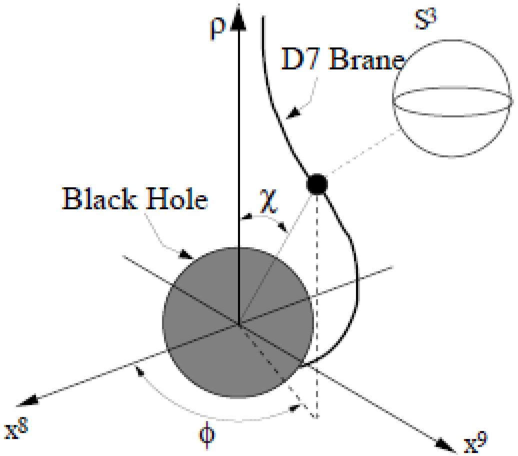


Similar structures as one varies m and B

No anomaly effect

Evans et al, JHEP 2010

Axial anomaly in D3/D7 model



axial-symmetry realized as rotation
in x8-x9 plane

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}};$$

$$S = \mathcal{N}\int d^5x\left(-\frac{1}{2}\sqrt{-G}G^{MN}\partial_M\phi\partial_N\phi - \frac{1}{4}\sqrt{-H}F^2\right) - \mathcal{N}\kappa\int d^5x\Omega\epsilon^{MNPQR}F_{MN}F_{PQ}\partial_R\phi$$

$$\partial_\mu\left(\frac{\delta S}{\delta\partial_\mu\phi}\right) + \partial_\rho\left(\frac{\delta S}{\delta\partial_\rho\phi}\right) = 0$$

$$J_R^\mu = \int d\rho\frac{\delta S}{\delta\partial_\mu\phi} \qquad \partial_\mu J_R^\mu + \frac{\delta S}{\delta\partial_\rho\phi}\Big|_{\rho=\rho_h}^{\infty} = 0$$

ϕ dual to $mi\bar{\psi}\gamma^5\psi + \cdots + \mathcal{N}E\cdot B$

Hoyos et al, JHEP (2011)

An example: pseudoscalar term induces correction to CSE

$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

Non-renormalization

Fukushima, Kharzeev, Warringa, PRD 2008

$$j_A = \frac{N_c \mu_V}{2\pi^2} eB + O(m^2)$$

Should lead to modified CMW

structure of correction to CSE

$$\nabla \cdot \mathbf{j}_5 = C\tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} + 2M_q i\bar{\psi}\gamma^5\psi \quad \equiv \sigma_5$$

massless case: $\nabla \cdot \mathbf{j}_5 = -\nabla \cdot \left(C\mu_q \tilde{\mathbf{B}}\right) \Rightarrow \mathbf{j}_5 = -C\mu_q \tilde{\mathbf{B}}.$

massive case: σ_5 P odd, T odd, while B P even, T odd, μ_q P even, T even

$$\sigma_5 = g(M_q^2, T, \mu, \tilde{B})\tilde{\mathbf{B}} \cdot \nabla\mu_q \Longrightarrow \mathbf{j}_5 = -C\mu_q\tilde{\mathbf{B}} + 2g(M_q^2, T, \mu_q, \tilde{B})\mu_q\tilde{\mathbf{B}}.$$

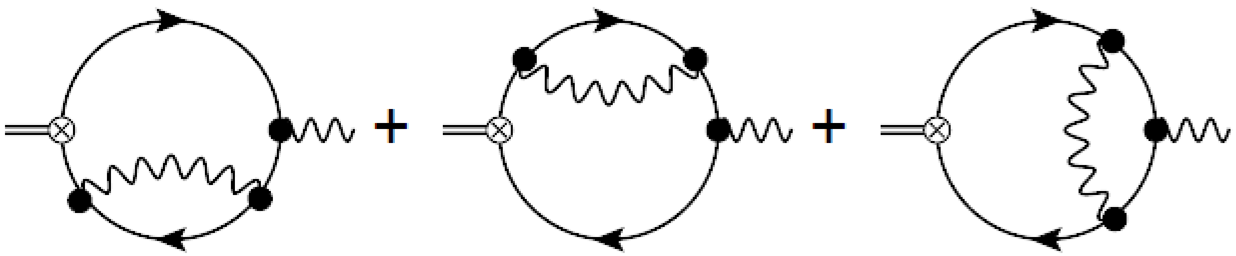
$g = \#\frac{M_q^2}{T^2} + o(M_q^2)$ when $\mu_q \ll T, B \ll T^2$

Guo, SL. 1610.05886

Compare to QED calculation at T=0

free theory $\mathbf{j}_5 = e\mathbf{B}\sqrt{\mu^2 - m^2}/(2\pi^2)$

perturbative correction



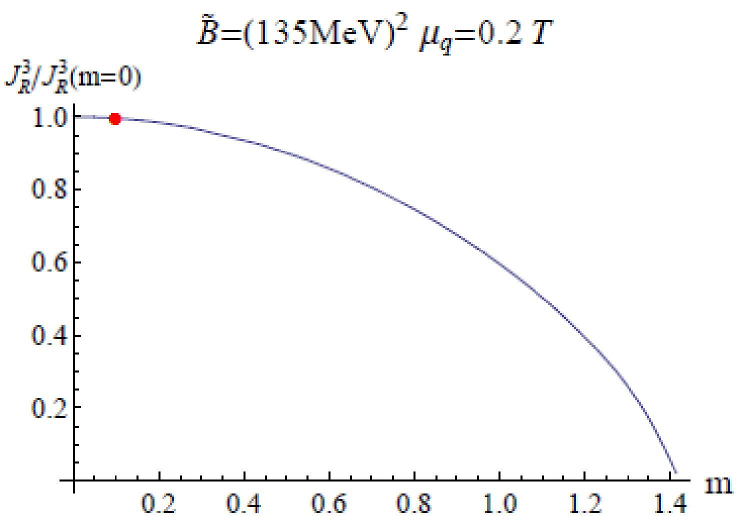
$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3 \mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

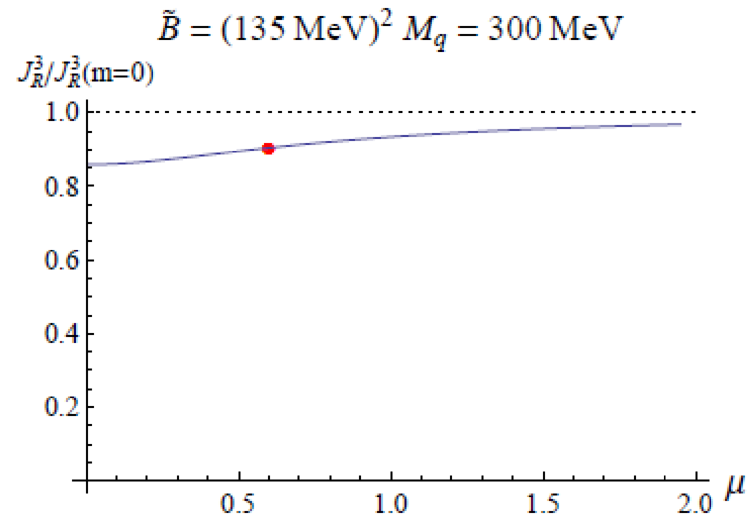
m_γ IR cutoff

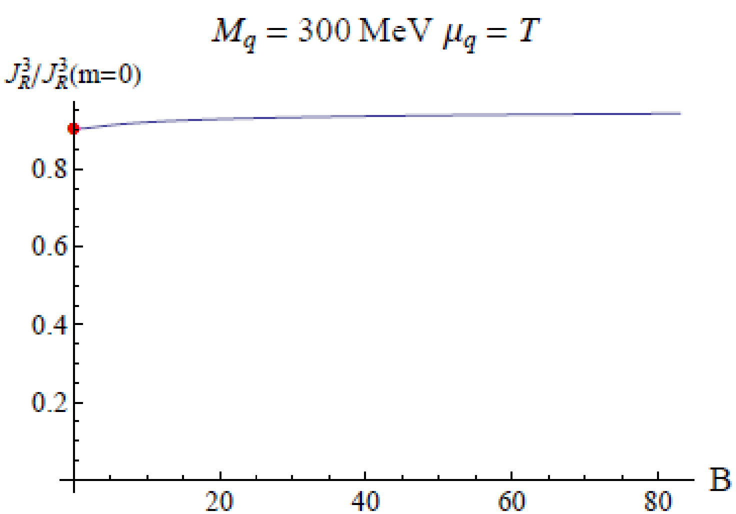
Radiative correction may induce non-analytic term in m

Gorbar et al, PRD 2013

CSE correction from D3/D7 model







$$\Delta j_5 = -\# \frac{{M_q}^2}{T^2} \mu B + o({M_q}^2)$$

Correction numerically small

M_q suppresses CSE

μ_q and B enhance CSE (suppress magnitude of correction)

small k-scaling of static correlators

$$\sigma \equiv \bar{\psi}\psi \qquad \sigma_5 \equiv im\bar{\psi}\gamma^5\psi \qquad n \equiv \bar{\psi}\gamma^0\psi.$$

$$G_{\mu\nu}(k) = \int d^4(x-y)e^{i\vec{k}\vec{x}}\langle O_\mu(x), O_\nu(y)\rangle$$

$$\mu, \nu = \sigma,\ n,\ \sigma_5$$

$k \parallel B$

$$k \to 0, \quad \begin{array}{c} G_{\sigma 5n}, G_{\sigma 5\sigma} \sim O(kB) \\ G_{\sigma 5\sigma 5} \sim O(k^2) \end{array}$$

$$\mathbf{j}_5 = -C\mu\mathbf{B} + g(M_q^2, T, \mu_q, B)\mu\mathbf{B}.$$

$$g = \frac{2G_{\sigma 5n}}{ikB} \sim {M_q}^2$$

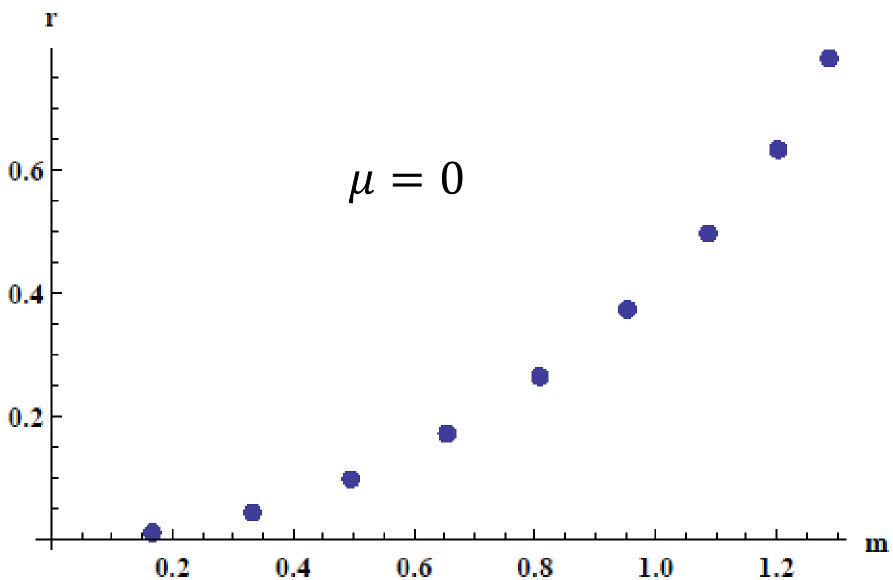
$$g \sim {M_q}^2$$

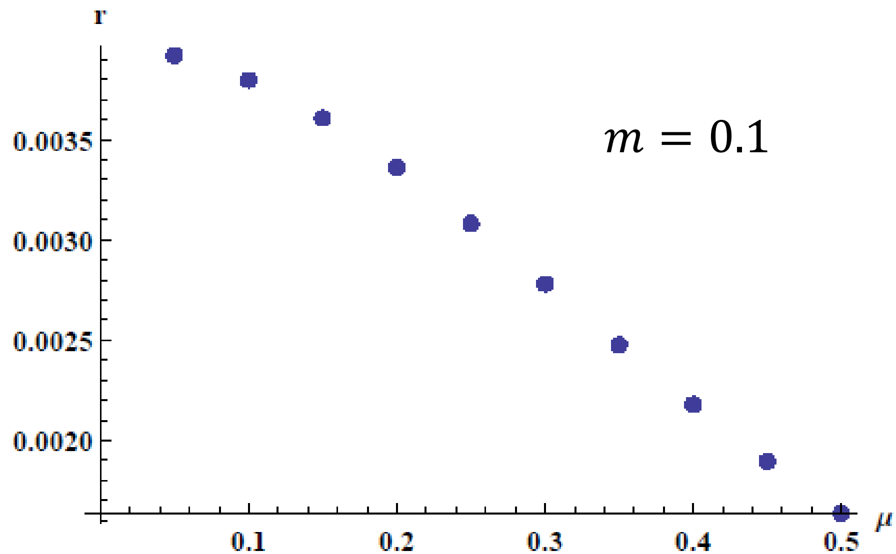
CSE correction proportional to correlator $G_{\sigma 5n}$
accessible on the lattice!

Wigner function approach by Qun Wang et al

Guo, SL. 1610.05886

$G_{\sigma 5n}$ versus μ and m





$$r = -\frac{\sigma_5}{\tilde{E}\tilde{B}}$$

$$r \sim g \sim \# \frac{{M_q}^2}{T^2}$$

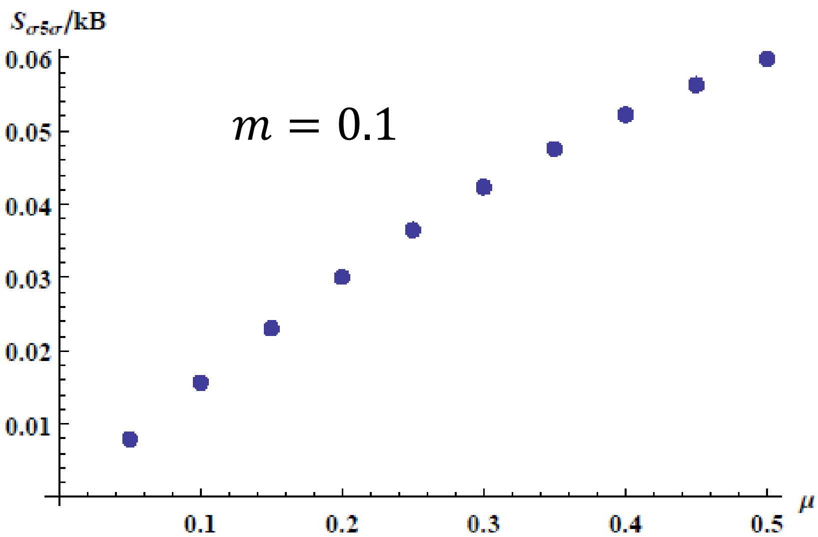
Response of σ_5 to parallel $\tilde{E}$ and $\tilde{B}$

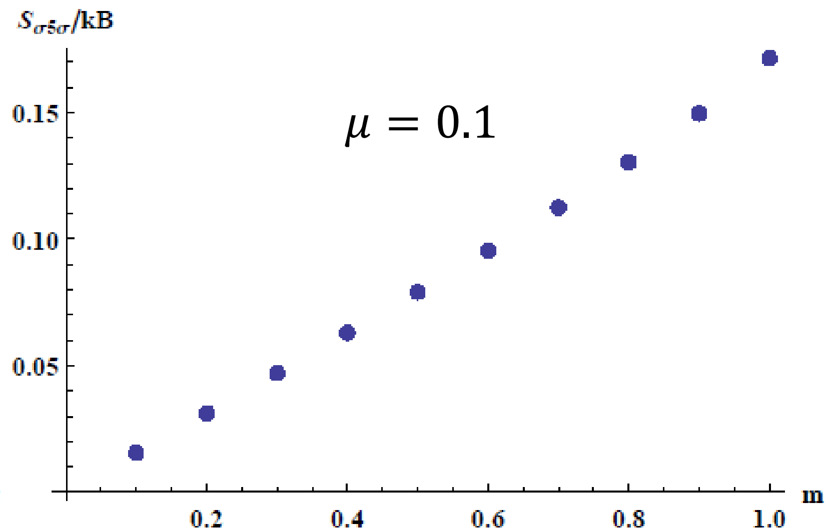
$$\sigma_5 = g(M_q^2, T, \mu, \tilde{B})\tilde{\mathbf{B}} \cdot \nabla \mu_q$$

response suppressed in μ

$\Longrightarrow$ enhanced CSE

$G_{\sigma5\sigma}$ versus μ and m



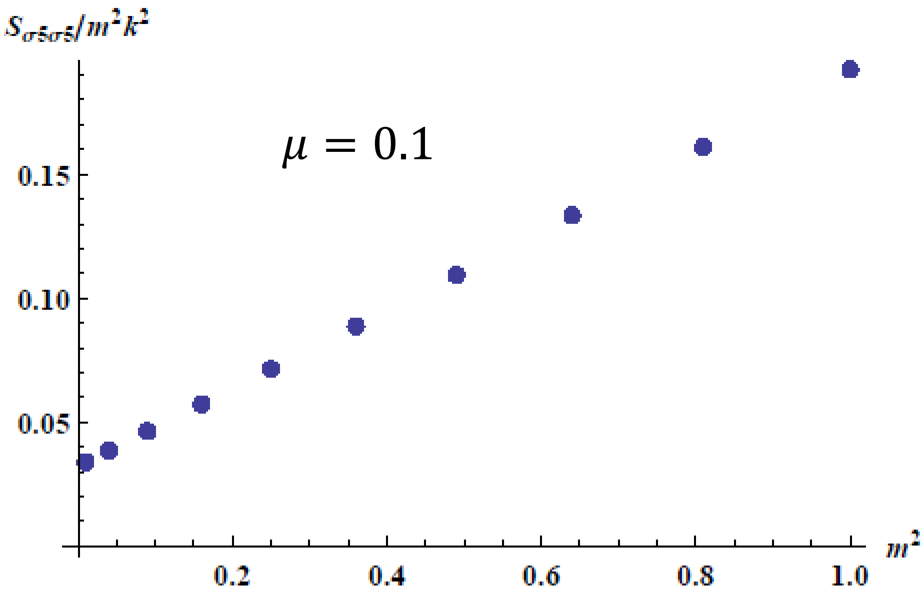


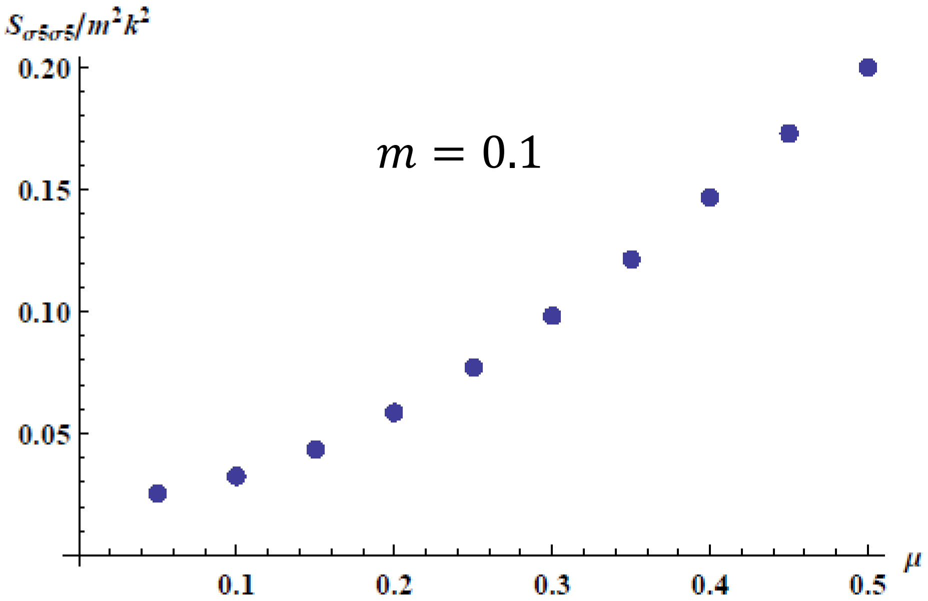
Response of σ_5 to inhomogeneous mass

$$\sigma_5 = 2\#\frac{M_q\mu_q}{T^2}\tilde{\mathrm{B}} \cdot \nabla M_q + o(M_q^2)$$

$$G_{\sigma5\sigma} = 2i\#M_q k\tilde{B}\mu_q + o(M_q)$$

$G_{\sigma 5\sigma 5}$ versus μ and m





Response of σ_5 to chiral shift $\nabla\phi$

$\nabla\phi$ defined as chiral shift
by Gorbar et al, PRD 2010

$$\mathcal{L} \to \mathcal{L} + j^5 \cdot \nabla\phi$$

$$\nabla \cdot \mathbf{j}_5 = 2s_5 = h(m^2, \mu, B)\nabla^2\phi$$

$$\mathbf{j}_5 = h(m^2, \mu, B)\nabla\phi.$$

chiral shift induces axial current

Response to chiral shift $\nabla\phi$

$$G_{ab} = G^*_{ab} = G_{ba}$$

$G_{\sigma 5 n}$ response of σ_5 to $\nabla\mu$ $\Longleftrightarrow$ $G_{n\sigma 5}$ response of n to $\nabla\phi$

$G_{\sigma 5\sigma}$ response of σ_5 to ∇m $\Longleftrightarrow$ $G_{\sigma\sigma 5}$ response of σ to $\nabla\phi$

Guo, SL. 1610.05886

Understanding small k-scaling

Lagrangian $m\bar{\psi}e^{i\phi\gamma^5}\psi.$

$\text{chiral rotation, } \psi \to e^{-i\gamma^5\phi/2}\psi$

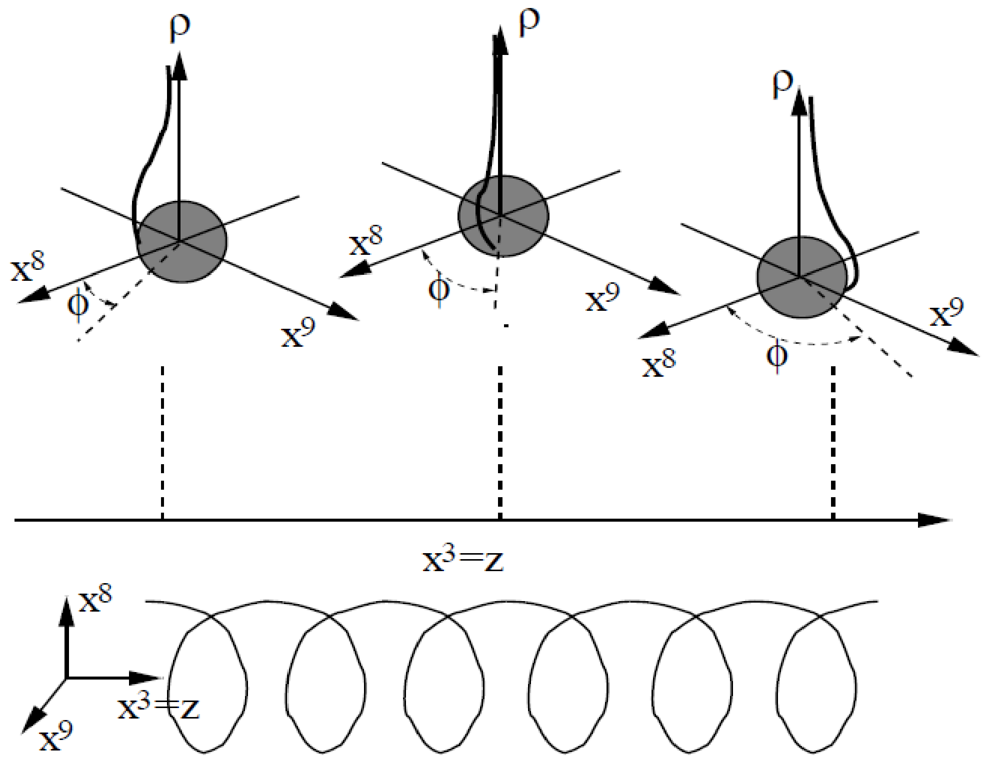
$$m\bar{\psi}e^{i\phi\gamma^5}\psi \to m\bar{\psi}\psi - \frac{\partial_\mu\phi}{2}\bar{\psi}\gamma^\mu\gamma^5\psi$$

Hoyos, Nishioka, O'Bannon

Physical source $\nabla\phi$

$$G_{\sigma 5n}, G_{\sigma 5\sigma} \sim O(kB)$$
$$G_{\sigma 5\sigma 5} \sim O(k^2)$$

Arbitrary k: toward spiral phase

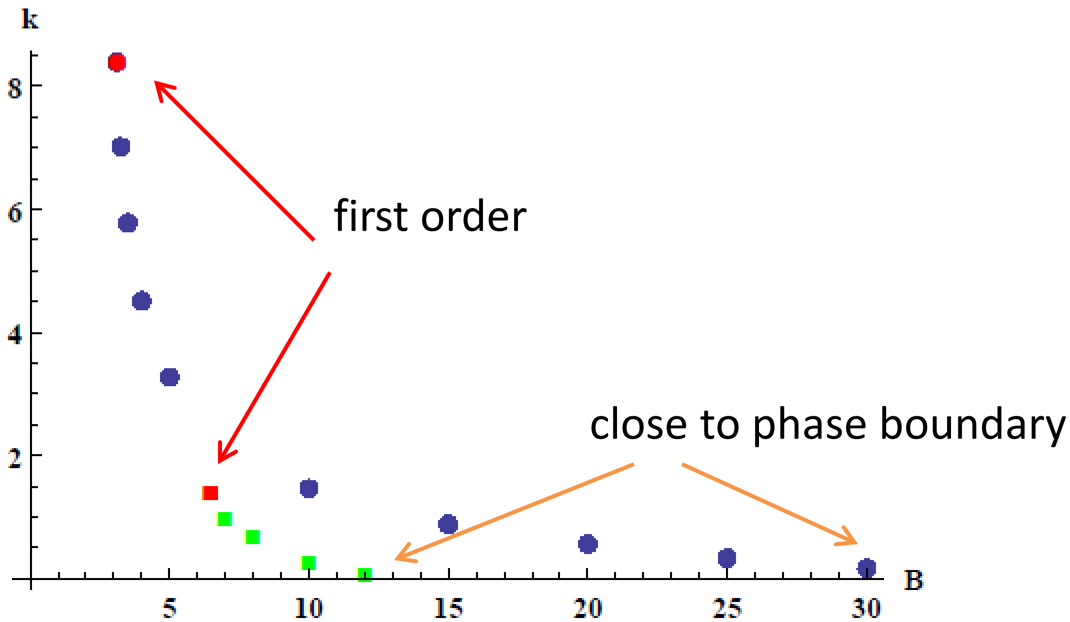


D7 brane being a point in x8-x9 plane, spiral phase in x3 direction

Kharzeev and Yee, PRD 2011

$\sigma_5 \neq 0$

Inhomogeneous normalizable mode



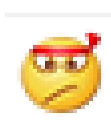
dot $\mu = 3$, square $\mu = 1$

inhomogeneous normalizable mode exists beyond critical B
Critical B drops with μ

Analog in confined phase, solution
by Brauner, Yamomoto 2016

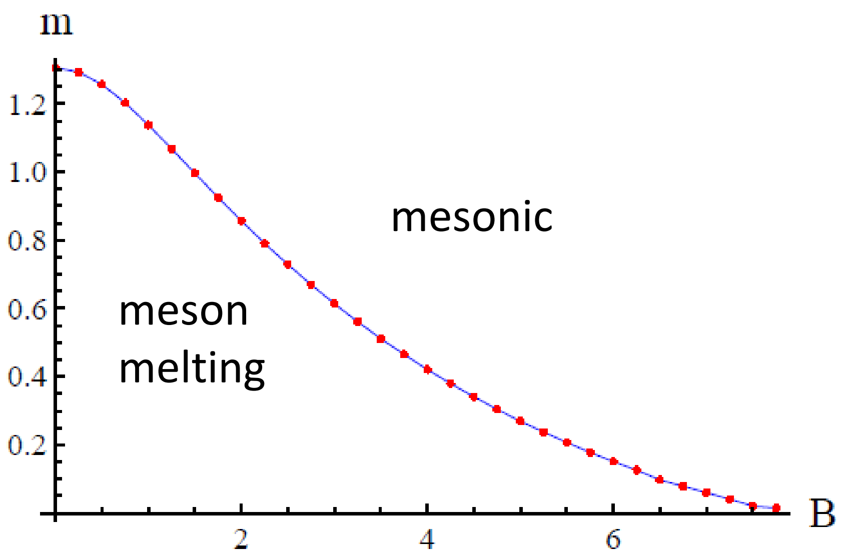
Summary

- Quark mass correction to CSE.
- Response of σ_5, σ, n to inhomogeneous ϕ, m, μ follows scaling law in small k regime.
- Formation of spiral phase beyond critical B suggested by existence of normalizable mode at finite k regime.
- Nonlinear solution featured by spiral in σ_5, σ, n?



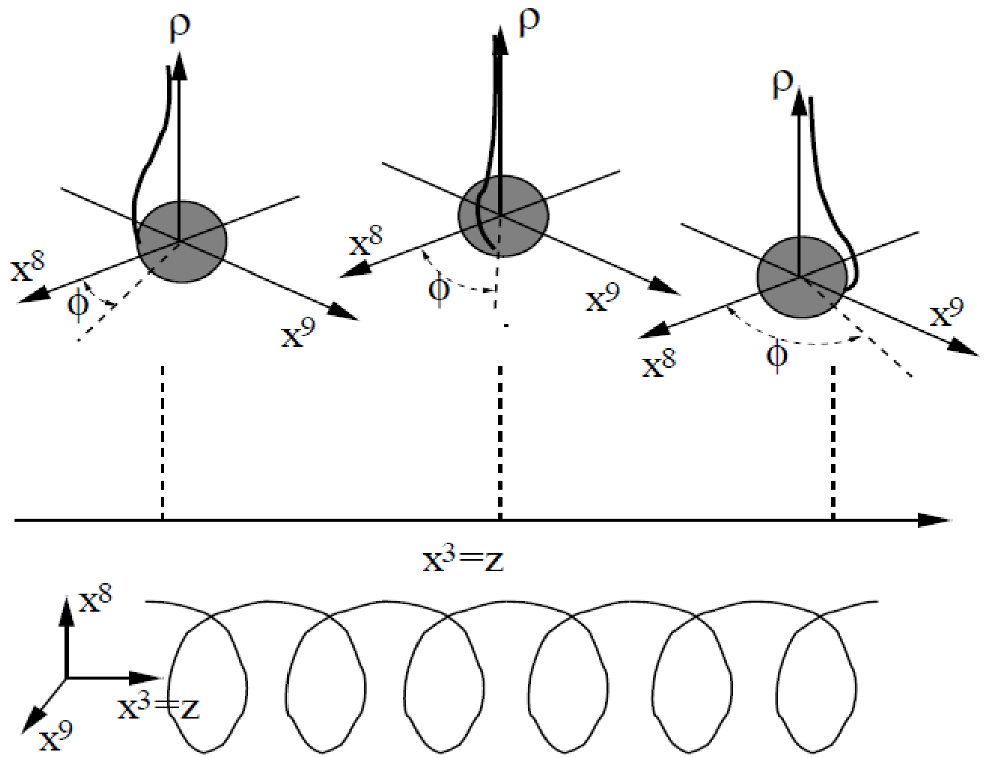
Thank you!

$\mu = 0$



Mateos et al, PRL 2006
Filev et al, JHEP 2007
Erdmenger et al, JHEP 2007

Spiral phase and correction to CSE



D7 brane being a point in x8-x9 plane, spiral phase in x3 direction

Kharzeev and Yee, PRD 2011

In spiral phase $i\bar{\psi}\gamma^5\psi \neq 0$ induces correction to CSE in massive case

In progress

Frameworks for axial charge dynamics

chiral kinetic theory (Berry curvature)

Son, Yamamoto, PRL (2012)
Stephanov, Yin, PRL (2012)
Pu, Gao, Wang et al, PRL (2012), (2013), PRD (2014)

Q. Wang's talk

hydrodynamics (axial charge)

Son, Surowka, PRL (2009)
Neiman, Oz, JHEP (2011)

Relativistic hydrodynamics for HIC

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j^V{}_\lambda$$
$$\partial_\mu j_V{}^\mu = 0$$
$$\partial_\mu j_A{}^\mu = C E^\mu B_\mu$$

Son, Surowka, PRL (2009)

with QED anomaly,
without QCD anomaly

$$j_V{}^\mu = n_V u^\mu + \nu_V{}^\mu$$
$$j_A{}^\mu = n_A u^\mu + \nu_A{}^\mu$$

talks by Jiang, Huang and Liao

Anomalous part:

CVE   CME

$$\nu_V{}^\mu = C\mu_V\mu_A\omega^\mu + C\mu_A B^\mu$$
$$\nu_5{}^\mu = C/2(\mu_V{}^2 + \mu_A{}^2 + \cdots)\omega^\mu + C\mu_V B^\mu$$

CSE

However, in HIC we need QCD anomaly to generate axial charge!

$\langle N_A \rangle = 0, \langle N_A{}^2 \rangle \neq 0$

Axial charge stochastic,
hydrodynamic noise necessary!

How hydro noise is included

Conserved charge as an example

$$\partial_\mu J^\mu = 0$$

w/o noise

$J^0 = n$ charge density $J_k = -D\partial_k n$ diffusive current

with noise

$J^0 = n$

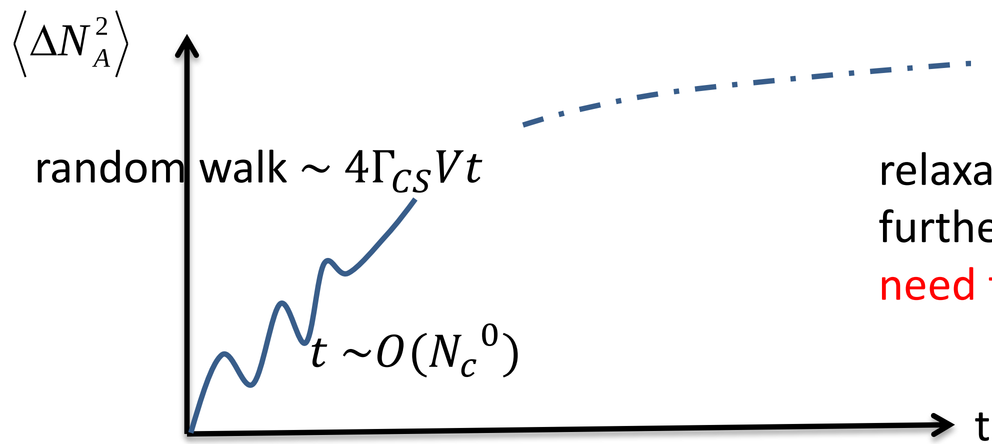
$$J_k = -\underbrace{D\partial_k n}_{\text{dissipation}} + \underbrace{r_k}_{\text{fluctuation}}$$

$$\langle r_i(\mathbf{x},t)\, r_k(\mathbf{x}',t')\rangle = C\delta_{ik}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Yan's talk

Kovtun, 1205.5040

Axial charge from topological fluctuation



$t \sim \tau = \frac{\chi T}{2\Gamma_{CS}} \sim O(N_c)$

relaxation: presence of n_A affects further topological charge fluctuation.
need fermion dynamics

Chern-Simon diffusion rate $\quad \Gamma_{CS} = \int d^4x \langle q(x) q(0) \rangle$

$q \sim trG\tilde{G}$ topological charge density

weak coupling extrapolation: $\Gamma_{CS} \sim 30 \alpha_s{}^4 T^4$ — Moore, Tassler, JHEP 2011

strong coupling: $\Gamma_{CS} = \alpha_s{}^2 N_c{}^2 T^4 / 16\pi$ — Son, Starinets, JHEP 2002

strong coupling w/B: $\Gamma_{CS} \sim \alpha_s{}^2 N_c{}^2 B T^2$ — Basar, Kharzeev, PRD 2012

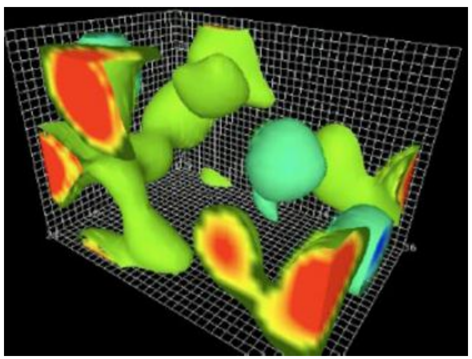
Topological fluctuation as hydro noise

Size of QGP >> fluid cell >> size of topological fluctuation

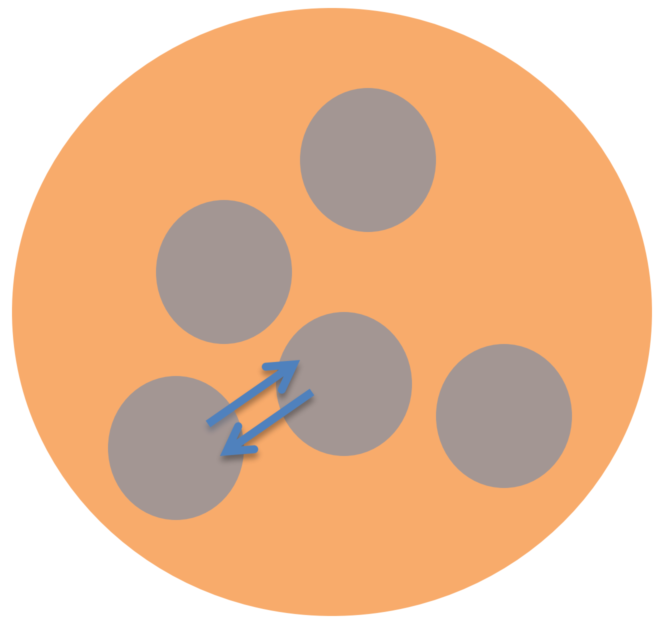
Axial charge fluctuation localized in fluid cell

Topological transition additional source of noise!

within one fluid cell



between fluid cells



The Sakai-Sugimoto model (D4/D8)

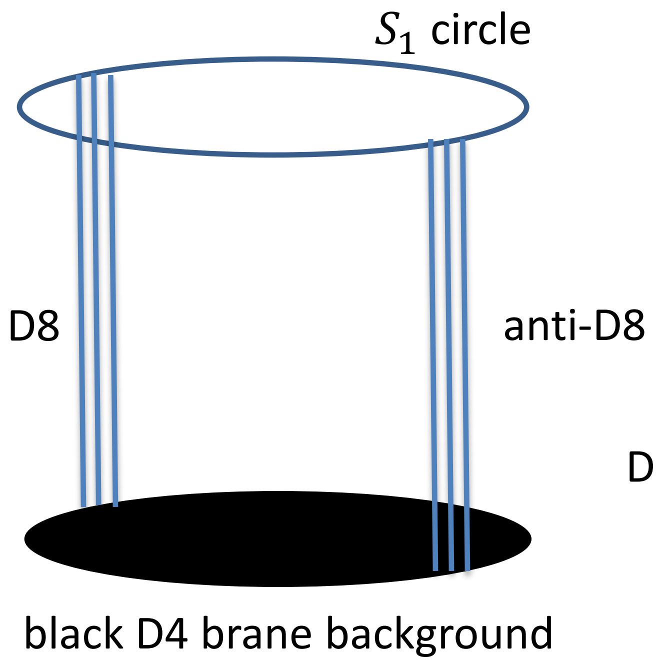
N_c D4 branes wrapped on S^1 circle + N_f D8/anti D8 branes being a point on S^1.

$\Uparrow$ gluons

$\Uparrow$ left/right handed quarks

mass gap $M_{KK} = \frac{1}{R_4}$

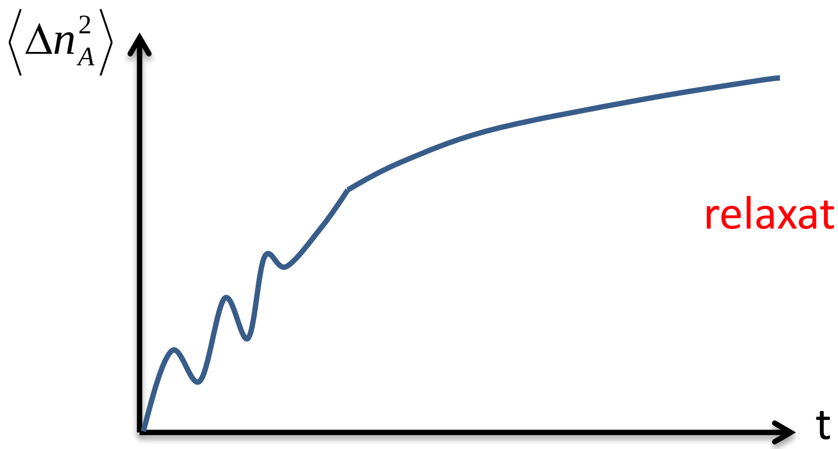
Sakai, Sugimoto, Prog.Theor.Phys, 2005



Deconfined, chiral symmetry restored

Aharony et al, Annal. Phys. 2006

Axial charge relaxation



relaxation, explicit in our model with fermions

Response of q to n_A

$$q = \frac{\Gamma_{CS}}{\chi T} n_A \Longrightarrow \frac{dn_A}{dt} = -2q = -\frac{2\Gamma_{CS}}{\chi T} n_A = -\frac{n_A}{\tau_{sph}}$$

χ: static susceptibility $\quad \tau_{sph} = \frac{\chi T}{2\Gamma_{CS}}$: relaxation time

consistent with early statistical argument

Also work by Akamatsu, Rothkopf, Yamamoto, JHEP 2016

Iatrakis, SL, Yin, JHEP 2015

Stochastic hydrodynamics for axial charge

Dynamical equation

$$\partial_t n_A(t,x) + \nabla \cdot j_A(t,x) = -2q(t,x)$$

Constitutive equations

$$j_A(t,x) = -D\nabla n_A(t,x) + \xi(t,x)$$

$$q(t,x) = \frac{n_A(t,x)}{2\tau_{\rm sph}} + \xi_q(t,x)$$

Non-topological fluctuation $\langle \xi_i(t,x)\xi_j(t,x')\rangle = 2\sigma T \delta_{ij}\delta(t-t')\delta^3(x-x')$

topological fluctuation $\langle \xi_q(t,x)\xi_q(t,x')\rangle = \Gamma_{\rm CS}\delta(t-t')\delta^3(x-x')$

Iatrakis, SL, Yin, JHEP 2015

Time evolution of axial charge from stochastic hydrodynamics

$$C_{nn}(t,x) \equiv \langle [n_A(t,x) - n_A(0,x)][n_A(t,0) - n_A(0,0)] \rangle$$

$$C_{nn}(t,\boldsymbol{x}) = (\chi T)\left[\delta^3(\boldsymbol{x}) - \frac{1}{(8\pi D t)^{3/2}} e^{-\frac{2t}{\tau_{\rm sph}}} e^{-\frac{|\boldsymbol{x}|^2}{8Dt}}\right]$$

within cell

across cells

Early time $t \ll \tau_{\rm sph}$

$$C_{nn}(t,\boldsymbol{x}) \approx 4\Gamma_{\rm CS}\, t\, \delta^3(\boldsymbol{x})$$

Late time $t \gg \tau_{\rm sph}$

$$C_{nn}(t\to\infty,\boldsymbol{x}) \to (\chi T)\,\delta^3(\boldsymbol{x})$$ thermodynamic limit

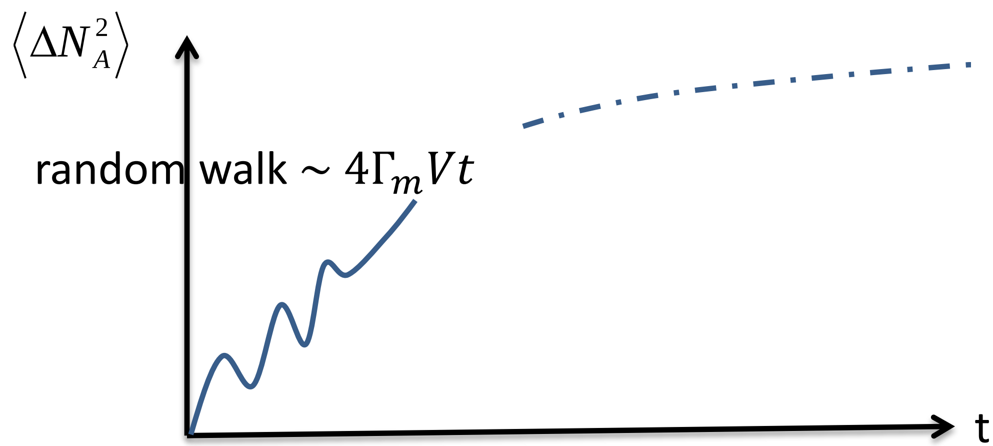
Mass diffusion rate

$$O_\eta = mi\bar{\psi}\gamma^5\psi + \cdots$$

$$G_{\eta\eta}(\omega) = \int dt \langle [O_\eta(t), O_\eta(0)] \rangle \Theta(t) e^{i\omega t} \sim \frac{-i\omega\Gamma_m}{2T} \qquad \text{as } \omega \to 0$$

Γ_m analogous to Γ_{CS}

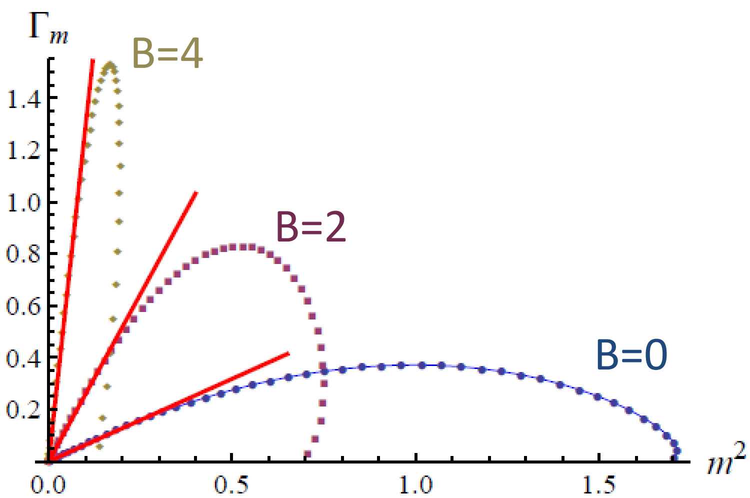
mass diffusion rate CS diffusion rate (absent in D3/D7)



enhance axial charge fluctuation

Guo, SL, PRD (2016)

Mass diffusion rate



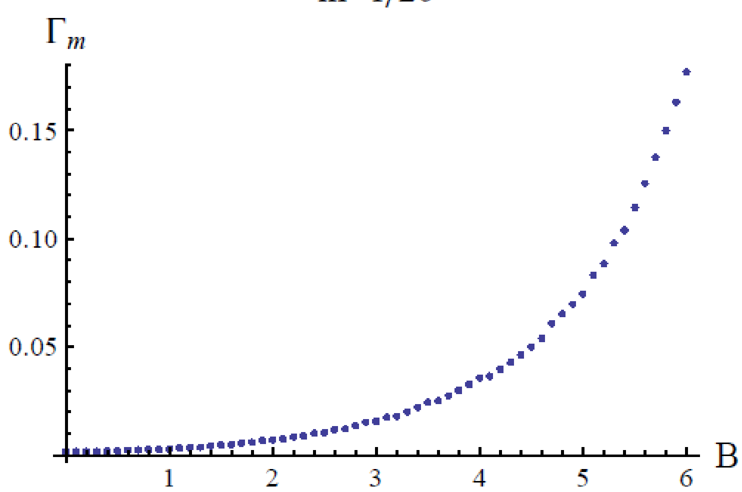
$$\Gamma_m \sim m^2 F(B)$$

Measure of helicity flipping rate

Magnetic field enhances helicity flipping rate

Guo, SL, PRD (2016)

m=1/20



$$B = m_\pi{}^2, T = 300MeV, M = M_s, N_f = 1$$

$$\Gamma_m \sim 6\Gamma_{cs}$$

Mass diffusion significant compared to Chern-Simon diffusion

Dynamical susceptibility from CME

Define dynamical axial chemical potential using CME $J(\omega) = C\mu_A(\omega)B(\omega)$

susceptibility $\chi(\omega) = \dfrac{n_A(\omega)}{\mu_A(\omega)}$

$\chi \sim O(\omega^{-1})$ as $\omega \to 0$ divergent suseptibility Guo, SL, PRD (2016)

- Spontaneous generation of axial charge costs no energy (diffusion)
- Leakage of axial charge from quarks to adjoint reservoir (specific to D3/D7 model)

while m=0 has a finite χ as $\omega \to 0$

Phenomenology? finite m versus ω

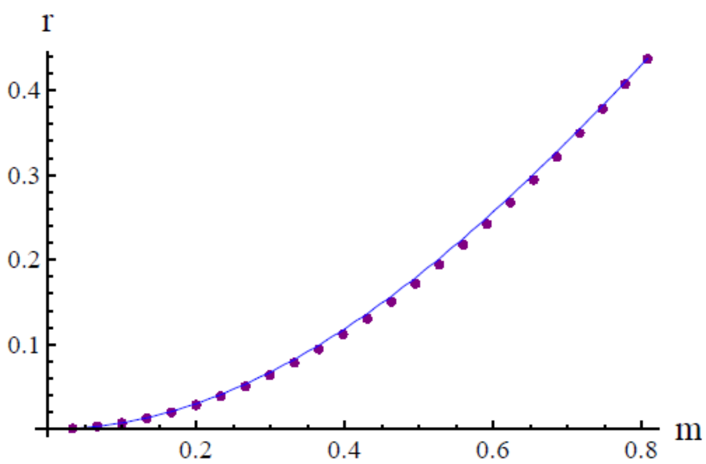
Mass dissipation rate

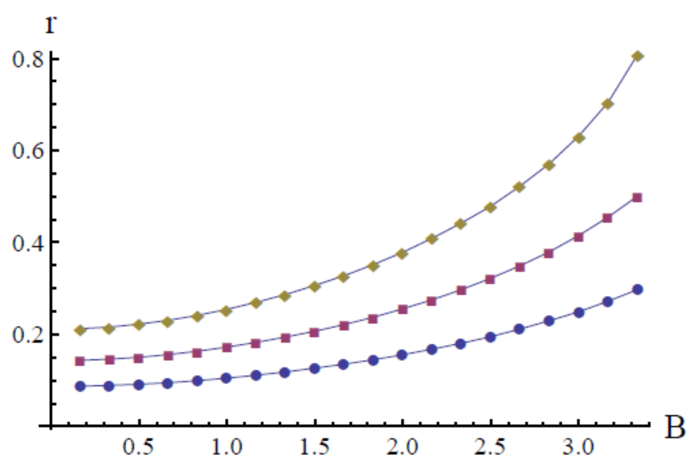
$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}},$$

O_η $\quad$ $E \cdot B$

$$r = \frac{O_\eta}{\mathcal{N}E \cdot B}.$$

Produce axial charge by setting up parallel E and B fields for $\omega \to 0$





Mass term more effective in dissipating axial charge at large m and B

r<1: axial charge survives the hydro limit for massive quarks?

Guo, SL, PRD (2016)

Mass dissipation rate

Adjoint reservoir

$\rho = \rho_h$

1-r

One unit of
axial charge
$E \cdot B$ fields

r

$$\partial_\mu J^\mu_R + \frac{\delta S}{\delta \partial_\rho \phi}\Big|^{\infty}_{\rho=\rho_h} = 0$$

Dissipates through
mass term

$\rho = \infty$

Effectively only r unit of axial charge is produced, all dissipates through the mass term in hydro limit

Guo, SL, PRD (2016)

Consistent with relaxation time approximation

Landsteiner et al, JHEP 2015

τ_{rel} increases with B, decreases with m

Phenomenology? finite τ_{hydro} versus τ_{rel}