FCC-ee Requirements on Beam Polarization and Energy Calibration

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**Some references:**
B. Montague, Phys.Rept. 113 (1984) 1-96;
Polarization at LEP CERN Yellow Report 88-02;
Spin Dynamics in LEP [http://dx.doi.org/10.1063/1.1384062](http://dx.doi.org/10.1063/1.1384062)
Beam polarization is directly useable in lepton colliders
   -- no polarized structure functions etc...

At Electroweak Scale there are two main uses

   -1- transverse polarization for energy calibration by resonant depolarization

   -2- e+e- longitudinal polarization combinations
      -- as a way to control the spin of the e+e- system

Both can be used to improve precision measurements.
<table>
<thead>
<tr>
<th>Observable</th>
<th>Physics</th>
<th>Present Precision</th>
<th>FCC-ee stat Syst Precision</th>
<th>FCC-ee key</th>
<th>Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ MeV/c²</td>
<td>Input</td>
<td>$91187.5 \pm 2.1$</td>
<td>$0.005 \text{ MeV} &lt;\pm 0.1 \text{ MeV}$</td>
<td>E_cal</td>
<td>QED corrections</td>
</tr>
<tr>
<td>$\Gamma_Z$ MeV/c²</td>
<td>$\Delta \rho (T)$ (no $\Delta \alpha$)</td>
<td>$2495.2 \pm 2.3$</td>
<td>$0.008 \text{ MeV} &lt;\pm 0.1 \text{ MeV}$</td>
<td>E_cal</td>
<td>QED corrections</td>
</tr>
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<td>$R_I \equiv \frac{\Gamma_{h}}{\Gamma_{l}}$</td>
<td>$\alpha_s, \delta_b$</td>
<td>$20.767 (25)$</td>
<td>$0.0001 (2-20)$</td>
<td>Statistics</td>
<td>QED corrections</td>
</tr>
<tr>
<td>$N_v$</td>
<td>Unitarity of PMNS, sterile $\nu$'s</td>
<td>$2.984 \pm 0.008$</td>
<td>$0.00008 (40)$</td>
<td>$0.001$</td>
<td>Measurements to Bhabha scat.</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$\delta_b$</td>
<td>$0.21629 (66)$</td>
<td>$0.000003 (20-60)$</td>
<td>Statistics, small IP</td>
<td>Hem. correlations</td>
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<tr>
<td>$A_{LR}$</td>
<td>$\Delta \rho, \varepsilon_3, \Delta \alpha$ (T, S)</td>
<td>$\sin^2 \theta_{w_{\text{eff}}} = 0.23098(26)$</td>
<td>$\sin^2 \theta_{w_{\text{eff}}} \pm 0.000006$</td>
<td>$4$ bunch scheme</td>
<td>Design experiment</td>
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<tr>
<td>$A_{FB}^{\text{lept}}$</td>
<td>$\Delta \rho, \varepsilon_3, \Delta \alpha$ (T, S)</td>
<td>$\sin^2 \theta_{w_{\text{eff}}} = 0.23099(53)$</td>
<td>$\sin^2 \theta_{w_{\text{eff}}} \pm 0.000006$</td>
<td>E_cal &amp; Statistics</td>
<td></td>
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<tr>
<td>$M_W$ MeV/c²</td>
<td>$\Delta \rho, \varepsilon_3, \varepsilon_2, \Delta \alpha$ (T, S, U)</td>
<td>$80385 \pm 15$</td>
<td>$0.3 \text{ MeV} &lt;\pm 0.5 \text{ MeV}$</td>
<td>E_cal &amp; Statistics</td>
<td>QED corrections</td>
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<tr>
<td>$m_{\text{top}}$ MeV/c²</td>
<td>Input</td>
<td>$173200 \pm 900$</td>
<td>$\sim 10 \text{ MeV}$</td>
<td>E_cal &amp; Statistics</td>
<td>Theory limit at 50 MeV?</td>
</tr>
</tbody>
</table>
Beam Polarization can provide two main ingredients to Physics Measurements

1. Transverse beam polarization provides beam energy calibration by resonant depolarization
   - low level of polarization is required (~10% is sufficient)
   - at Z & W pair threshold comes naturally
   - at Z use of asymmetric wigglers at beginning of fills since polarization time is otherwise very long.
   - could be used also at ee → H(126) (depending on exact $m_H$ !)
   - use ‘single’ non-colliding bunches and calibrate continuously during physics fills to avoid issues encountered at LEP
   - this is possible with e+ and e- Compton polarimeter (commercial laser)
   - should calibrate at energies corresponding to half-integer spin tune
   - must be complemented by analysis of «average E_beam» to $E_{CM}$ relationship

Aim: Z mass & width to ~100 keV (stat: 10 keV) W mass & width to ~500 keV (stat : 300 keV)

For beam energies higher than ~90 GeV can use ee → Zγ or ee → WW events to calibrate $E_{CM}$ at ±5-10 MeV level: matches requirements for $m_H$ and $m_{top}$ measts.
Beam Polarization can provide two main ingredients to Physics Measurements

2. Longitudinal beam polarization provides chiral e+e- system
   -- High level of polarization is required (>40%)
   -- Must compare with natural e+e- polarization due to chiral couplings of electrons (15%)
     or with final state polarization analysis for CC weak decays (100% polarized) (tau and top)
   -- Physics case for Z peak is very well studied and motivated:
     \( A_{LR}, \ A_{FB}^{Pol}(f) \) etc... (CERN Y.R. 88-06)
     figure of merit is \( L.P^2 \) --> must not lose more than a factor \( \sim 10 \) in lumi.
     self calibrating polarization measurement \( * \rightarrow \) spares

   -- uses: enhance Higgs cross section (by 30%)
     top quark couplings? final state analysis does as well (Janot arXiv:1503.01325)
     enhance signal, subtract/monitor backgrounds, for ee \( \rightarrow \) WW, ee \( \rightarrow \) H
   -- requires High polarization level and often both e- and e+ polarization
     \( \rightarrow \) not interesting If loss of luminosity is too high

   -- Obtaining high level of polarization in high luminosity collisions is delicate in top-up mode
At the Z obtain excellent polarization level but too slow for polarization in physics. Need wigglers for energy calibration.

At the W expectation similar to LEP at Z → enough for energy calibration.

Simulations by Eliana Gianfelice
Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties

The LEP Energy Working Group

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Abstract

The determination of the centre of mass energies from the LEP1 data for 1993, 1994 and 1995 is presented. Accurate knowledge of these energies is crucial in the measurement of the Z resonance parameters. Improved understanding of the LEP energy behaviour accumulated during the 1995 energy scan is detailed, while the 1993 and 1994 measurements are revised. For 1993 these supersede the previously published values. Additional instrumentation has allowed the detection of an unexpectedly large energy rise during physics fills. This new effect is accommodated in the modelling of the beam energy in 1995 and propagated to the 1993 and 1994 energies. New results are reported on the magnet temperature behaviour which constitutes one of the major corrections to the average LEP energy. The 1995 energy scan took place in conditions very different from the previous years. In particular the interaction point specific corrections to the centre of mass energy in 1995 are more complicated than previously: these arise from the modified radiofrequency-system configuration and from opposite-sign vertical dispersion induced by the bunch-train mode of LEP operation. Finally an improved evaluation of the LEP centre of mass energy spread is presented. This significantly improves the precision on the Z width.
Figure 20: Polarization signal on 2 October 1991, showing the localization of the depolarizing frequency within the sweep.
Top: display of data points, with the frequency sweep indicated with vertical dashed lines. The full line represents the result of a fit with starting polarization ($-4.9 \pm 1.1\%$), polarization rise-time ($60 \pm 13$) minutes, asymptotic polarization ($18.4 \pm 4.1\%$).
Bottom: expanded view of the sweep period, with the individual data sets displayed (there are 10 sets per point); The frequency sweep lasted 7 data sets. The corresponding beam energy is shown in the upper box. Spin flip occurred between the two vertical dash-dotted lines.

$E_{beam} = 46,466.6 \pm 0.6$ MeV, e.g. precise to $\pm 1.5 \times 10^{-5}$.
Many effects spoil the calibration if it is performed outside physics time
-- tides and other ground motion
-- RF cavity phases
-- histeresis effects and environmental effects (trains...etc)
by 1999 we had an excellent model of the energy variations… but we were not measuring the Z mass and width anymore – we were hunting for the Higgs boson!
We have concluded that first priority is to achieve transverse polarization in a way that allows continuous beam calibration by resonant depolarization (energy measurement every ~10 minutes on ‘monitoring’ single bunches)

- This is a unique feature of circular e+e- storage ring colliders
- baseline running scheme defined with monitoring bunches
- the question of the residual systematic error requires further studies of the relationship between spin tune, beam energy at IRs, and center-of-mass energy

⇒ target is 100keV at Z and W pair threshold energies

‘Do we want longitudinal polarization’?
we will discuss this in the following.
EXPERIMENTS ON BEAM-BEAM DEPOLARIZATION AT LEP


PAC 1995

- With the beam colliding at one point, a polarization level of 40% was achieved. The polarization level was about the same for one colliding and one non colliding bunch.
- It was observed that the polarization level depends critically on the synchrotron tune: when $Q_x$ was changed by 0.005, the polarization strongly decreased.

Experiment performed at an energy of 44.71 GeV the polarization level was 40% with a linear beam-beam tune shift of about 0.04/IP. This indicates that the beam-beam depolarization does not scale with the linear beam-beam tune shift at one crossing point. Other parameters as spin tune and synchrotron tune are also of importance.

LEP:
This was only tried 3 times!
Best result: $P = 40\%$, $\xi y^* = 0.04$, one IP

FCC-ee
Assuming 2 IP and $\xi y^* = 0.01 \Rightarrow$
reduce luminosity, $10^{10} Z @ P \sim 30\%$
Reduction of polarization due to continuous injection

The colliding bunches will lose intensity continuously due to collisions.
In FCC-ee with 4 IPs, $L = 28 \times 10^{34}/\text{cm}^2/\text{s}$ beam lifetime is 213 minutes.
In FCC-ee with 2 IPs, $L = 1.4 \times 10^{36}/\text{cm}^2/\text{s}$ beam life time is 55 minutes.

Luminosity scales inversely to beam life time.
The injected $e^+$ and $e^-$ are not polarized $\rightarrow$ asymptotic polarization is reduced.
Assume here that machine has been well corrected and beams (no collisions, no injection) can be polarized to nearly maximum.
(Eliana Gianfelice in Rome talk)

- 45 GeV
  - limit $\Delta E = 50$ MeV (extrapolating from LEP)
  - 4 wigglers with $B^+ = 0.7$ T
  - 10% polarization in 2.9 h for energy calibration

(polarization time is 26h)
We have simulated the simultaneous effect of
-- natural polarization
-- beam consumption by e+e- interactions
-- replenishment with unpolarized beams
assuming *optimistically* a maximal 90% asymptotic polarization

<table>
<thead>
<tr>
<th>Running at full luminosity</th>
<th>Running at 10% Lumi</th>
<th>Running at 1% Lumi</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_max=0.03! P_eff=0.03</td>
<td>P_max=0.24, P_eff=0.21</td>
<td>P_max=0.66, P_eff=0.5</td>
</tr>
</tbody>
</table>
\[ \Delta A_{LR} \text{ scales as } 1/\sqrt{P^2 L} \]

<table>
<thead>
<tr>
<th>Lumi loss factor</th>
<th>L.10^{34}</th>
<th>Figure of merit: \text{sum}(P^2 L)</th>
<th>Peff</th>
<th>Pmax</th>
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<tbody>
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<td>0.03</td>
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<td>2</td>
<td>110</td>
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<td>0.059</td>
<td>0.06</td>
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<td>4</td>
<td>55</td>
<td>0.627</td>
<td>0.1078</td>
<td>0.11</td>
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<td>6</td>
<td>37</td>
<td>0.805</td>
<td>0.149</td>
<td>0.16</td>
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<td>8</td>
<td>27</td>
<td>0.924</td>
<td>0.184</td>
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<td>10</td>
<td>22</td>
<td>1.003</td>
<td>0.214</td>
<td>0.24</td>
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<tr>
<td>12</td>
<td>18</td>
<td>1.053</td>
<td>0.24</td>
<td>0.27</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>1.09</td>
<td>0.27</td>
<td>0.32</td>
</tr>
<tr>
<td><strong>18</strong></td>
<td><strong>12</strong></td>
<td><strong>1.101</strong></td>
<td><strong>0.3</strong></td>
<td><strong>0.35</strong></td>
</tr>
<tr>
<td>22</td>
<td>10</td>
<td>1.088</td>
<td>0.33</td>
<td>0.4</td>
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<td>26</td>
<td>8</td>
<td>1.059</td>
<td>0.354</td>
<td>0.43</td>
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<tr>
<td>30</td>
<td>7</td>
<td>1.023</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>0.92</td>
<td>0.41</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Optimum around a reduction of luminosity by a factor 18.

This is still a luminosity of \( \sim 10^{35} \) per IP... and the effective polarization is 30%.
This is equivalent to a 100% polarization expt with luminosity reduced by 180.
<table>
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<tr>
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<th>Present precision</th>
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<th>Challenge</th>
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<td>$M_Z$ MeV/c²</td>
<td>Input</td>
<td>91187.5 ±2.1</td>
<td>Z Line shape scan</td>
<td>0.005 MeV &lt;±0.1 MeV</td>
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<td>$\Gamma_Z$ MeV/c²</td>
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<td>2495.2 ±2.3</td>
<td>Z Line shape scan</td>
<td>0.008 MeV &lt;±0.1 MeV</td>
<td>E_cal</td>
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<td>$R_l \equiv \frac{\Gamma_h}{\Gamma_l}$</td>
<td>$\alpha_s, \delta_b$</td>
<td>20.767 (25)</td>
<td>Z Peak</td>
<td>0.0001 (2-20)</td>
<td>Statistics</td>
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<td>$N_v$</td>
<td>Unitarity of PMNS, sterile $\nu$'s</td>
<td>2.984 ±0.008</td>
<td>Z Peak $Z+\gamma (161 \text{ GeV})$</td>
<td>0.000008 (40) 0.001</td>
<td>-&gt;lumi meast Statistics</td>
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<td>$A_{LR}$</td>
<td>$\Delta \rho, \varepsilon_3, \Delta \alpha (T, S )$</td>
<td>$\sin^2 \theta_w^{\text{eff}} 0.23098(26)$</td>
<td>Z peak, Long. polarized</td>
<td>$\sin^2 \theta_w^{\text{eff}} \pm 0.000006$</td>
<td>4 bunch scheme</td>
</tr>
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<td>$A_{FB}^{\text{lept}}$</td>
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<td></td>
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<tr>
<td>$M_W$ MeV/c²</td>
<td>$\Delta \rho, \varepsilon_3, \varepsilon_2, \Delta \alpha (T, S, U)$</td>
<td>80385 ± 15</td>
<td>Threshold (161 GeV)</td>
<td>0.3 MeV &lt;0.5 MeV</td>
<td>E_cal &amp; Statistics</td>
</tr>
<tr>
<td>$m_{\text{top}}$ MeV/c²</td>
<td>Input</td>
<td>173200 ± 900</td>
<td>Threshold scan</td>
<td>~10 MeV</td>
<td>E_cal &amp; Statistics</td>
</tr>
</tbody>
</table>
Measuring $\sin^2 \theta_W^{\text{eff}} (m_Z)$

$\sin^2 \theta_W^{\text{eff}} \equiv \frac{1}{4} (1 - g_V/g_A)$

$g_V = g_L + g_R$

arXiv:0509008
<table>
<thead>
<tr>
<th></th>
<th>$A_{FB}^{\mu\mu}$ @ FCC-ee</th>
<th>$A_{LR}$ @ ILC</th>
<th>$A_{LR}$ @ FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>visible Z decays</td>
<td>$10^{12}$</td>
<td>$10^9$</td>
<td>$5.10^{10}$</td>
</tr>
<tr>
<td>muon pairs</td>
<td>$10^{11}$</td>
<td>beam polarization</td>
<td>90%</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$ (stat)</td>
<td>$3 \times 10^{-6}$</td>
<td>$\Delta A_{LR}$ (stat)</td>
<td>$4.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta E_{cm}$ (MeV)</td>
<td>0.1</td>
<td>2.2</td>
<td>?</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$ (E$_{CM}$)</td>
<td>$9.2 \times 10^{-6}$</td>
<td>$\Delta A_{LR}$ (E$_{CM}$)</td>
<td>$4.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$\Delta A_{LR}$</td>
<td>$5.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta \sin^2 \theta_{lept}$</td>
<td>$5.9 \times 10^{-6}$</td>
<td>$7.5 \times 10^{-6}$</td>
<td>$6 \times 10^{-6}$ +?</td>
</tr>
</tbody>
</table>

All exceeds the theoretical precision from $\Delta \alpha (m_Z)$ ($3 \times 10^{-5}$) or the comparison with $m_W$ (500keV)

But this precision on $\Delta \sin^2 \theta_{lept}$ can only be exploited at FCC-ee!
The forward backward tau polarization asymmetry is very clean. Dependence on $E_{CM}$ same as $A_{LR}$ negl. At FCC-ee

ALEPH data 160 pb$^{-1}$ (80 s @ FCC-ee)

Already syst. level of $6 \times 10^{-5}$ on $\sin^2 \theta_w$ much improvement possible by using dedicated selection e.g. $\tau \rightarrow \pi \nu$ to avoid had. model

<table>
<thead>
<tr>
<th></th>
<th>ALEPH</th>
<th></th>
<th>DELPHI</th>
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<th>L3</th>
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<th>OPAL</th>
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<tr>
<td></td>
<td>$\delta A_T$</td>
<td>$\delta A_o$</td>
<td>$\delta A_T$</td>
<td>$\delta A_o$</td>
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<td>$\delta A_o$</td>
<td>$\delta A_T$</td>
<td>$\delta A_o$</td>
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<tr>
<td>ZFITTER</td>
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<td>had. decay model</td>
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<td>0.0001</td>
<td>0.0025</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 4.2: The magnitude of the major common systematic errors on $A_T$ and $A_o$ by category for each of the LEP experiments.
Concluding remarks

1. There are very strong arguments for precision energy calibration with transverse polarization at the Z peak and W threshold.

2. Given the likely loss in luminosity, and the intrinsic uncertainties in the extraction of the weak couplings, the case for longitudinal polarization is limited.

We have concluded that first priority is to achieve transverse polarization in a way that allows continuous beam calibration by resonant depolarization.

- this is all possible with a very high precision, both at the Z and the W. Calibration at higher energies can be made from the data themselves at sufficient level.

- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy with the aim of achieving a precision of $O(100 \text{ keV})$ on $E_{CM}$.
spares
Longitudinal polarization at FCC-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of \( \sin^2 \theta_{\text{lept}}^W = e^2/g^2 \) (\( m_z \))

(-- not to be confused with -- \( \sin^2 \theta_W = 1 - m_w^2/m_z^2 \))

Useful references from the past:
«polarization at LEP» CERN Yellow Report 88-02
Precision Electroweak Measurements on the Z Resonance
GigaZ @ ILC by K. Moenig
relations to the well measured
\[ G_F, m_Z, \alpha, \text{QED} \]

at first order:
\[ \Delta \rho = \frac{\alpha}{\pi} \left( \frac{m_{\text{top}}}{m_Z} \right)^2 \]
\[ - \frac{\alpha}{4\pi} \log \left( \frac{m_h}{m_Z} \right)^2 \]

\[ \varepsilon_3 = \cos^2 \theta_w \frac{\alpha}{9\pi} \log \left( \frac{m_h}{m_Z} \right)^2 \]

\[ \delta_{vb} = \frac{20}{13} \frac{\alpha}{\pi} \left( \frac{m_{\text{top}}}{m_Z} \right)^2 \]

complete formulae at 2d order including strong corrections are available in fitting codes

e.g. ZFITTER, GFITTER
Extracting physics from $\sin^2\theta^{\text{lept}}_W$

1. Direct comparison with $m_Z$

\[
\sin^2\theta^{\text{lept}}_W \Delta\alpha(m_Z) = \frac{\pi \alpha (m_Z^2)}{\sqrt{2}} \sqrt{1 + \Delta \rho} \frac{1}{1 - \frac{E_3}{\cos^2 \theta_W}}
\]

Uncertainties in $m_{\text{top}}$, $\Delta \alpha(m_z)$, $m_H$, etc.

$\Delta \sin^2\theta^{\text{lept}}_W \sim \Delta \alpha(m_z) / 3 = 10^{-5}$ if we can reduce $\Delta \alpha(m_z)$ (see P. Janot)

2. Comparison with $m_W/m_Z$

Compare above formula with similar one:

\[
\sin^2\theta_W \cos^2\theta_W = \frac{\pi \alpha (m_Z^2)}{\sqrt{2}} \sqrt{1 - (1 - \frac{E_3}{\cos^2 \theta_W})}
\]

Where it can be seen that $\Delta \alpha(m_z)$ cancels in the relation.

The limiting error is the error on $m_W$.

For $\Delta m_W = 0.5 \text{ MeV}$ this corresponds to $\Delta \sin^2\theta^{\text{lept}}_W = 10^{-5}$
Assume for now ONE experiment at ECM=91.2

Luminosity «baseline» with beta*=1mm : $2.1 \times 10^{36}/\text{cm}^2/\text{s} = 2 \text{ pb}^{-1}/\text{s}$,
Sigma_had = $31 \times 10^{-33}\text{cm}^2 \Rightarrow 6.5 \times 10^{11} \text{qq events}/10^7 \text{ year/\exp}$.

Consider 3 years of $10^7 \text{s}$
2 $10^{12} Z \rightarrow \text{qq events}$ (typical exp at LEP was $4.10^6$)
4 $10^{11} Z \rightarrow \text{bb}$
10$11 Z \rightarrow \mu\mu, \tau\tau$ each
Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare
ILC measurement of $A_{LR}$ with $10^9 Z$ and $P_{e-} = 80\%, \, P_{e+} = 30\%$

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{\text{pol}}(\tau)$ with $2.10^{12} Z$
Comparing $A_{LR}$ (P) and $A_{FB}$ ($\mu\mu$)

Both measure the weak mixing angle as defined by the relation

$$A_{\ell} = \frac{(g^e_L)^2 - (g^e_R)^2}{(g^e_L)^2 + (g^e_R)^2}$$

with $(g^e_L) = \frac{1}{2} \sin^2 \theta_{lept} W$ and $(g^e_R) = -\sin^2 \theta_{lept} W$  \( A_{\ell} \approx 8(1/4 - \sin^2 \theta_{lept} W) \)

$A_{LR} = A_e$

$A_{FB}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_{\ell}^2$

--- $A_{FB}^{\mu\mu}$ is measured using muon pairs (5% of visible Z decays) and unpolarized beams

--- $A_{LR}$ is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization

    both with very small experimental systematics

--- parametric sensitivity \( \frac{dA_{FB}^{\mu\mu}}{dsin^2 \theta_{lept} W} = 1.73 \) vs \( \frac{dA_{LR}}{dsin^2 \theta_{lept} W} = 7.9 \)
**Measurement of $A_{LR}$**

<table>
<thead>
<tr>
<th>electron bunches</th>
<th>1 $\leftrightarrow$ 2 3 4 $\leftrightarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>positron bunches</td>
<td>1 2 $\Rightarrow$ 3 4 $\Rightarrow$</td>
</tr>
<tr>
<td>cross sections</td>
<td>$\sigma_1$ $\sigma_2$ $\sigma_3$ $\sigma_4$</td>
</tr>
<tr>
<td>event numbers</td>
<td>$N_1$ $N_2$ $N_3$ $N_4$</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sigma_1 &= \sigma_u (1 - P^-_e \Lambda_{LR}) \\
\sigma_2 &= \sigma_u (1 + P^+_e \Lambda_{LR}) \\
\sigma_3 &= \sigma_u \\
\sigma_4 &= \sigma_u [1 - P^+_e P^-_e + (P^+_e - P^-_e) \Lambda_{LR}] \\
\end{align*}
\]

Verifies polarimeter with experimentally measured cross-section ratios

Statistics

\[
\Delta A_{LR} = 0.0025 \text{ with about } 10^6 Z^0 \text{ events},
\Delta A_{LR} = 0.000045 \text{ with } 5.10^{10} Z \text{ and 30% polarization in collisions.}
\]

$\Delta \sin^2 \theta_{\text{eff}} \text{ (stat)} = O(2.10^{-6})$
Will consider two sources of errors

-- statistics
-- uncertainty on center-of-mass energy (relative to the Z mass)

main inputs taken from
arXiv:hep-ex/0509008v3 precision measurements on the Z resonance

there are other uncertainties but they are very small for $A_{FB}$
This is a lower limit estimate for $A_{LR}$; the systematics related to knowledge of
the beam polarization (80% for e-, 30% for e+) should also be taken into account
<table>
<thead>
<tr>
<th></th>
<th>$A_{FB}^{\mu\mu}$ @ FCC-ee</th>
<th>$A_{LR}$ @ ILC</th>
<th>$A_{LR}$ @ FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>visible Z decays</td>
<td>$10^{12}$</td>
<td>visible Z decays $10^9$</td>
<td>$5.10^{10}$</td>
</tr>
<tr>
<td>muon pairs</td>
<td>$10^{11}$</td>
<td>beam polarization 90%</td>
<td>30%</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$ (stat)</td>
<td>$3 \times 10^{-6}$</td>
<td>$\Delta A_{LR}$ (stat) $4.2 \times 10^{-5}$</td>
<td>$4.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta E_{cm}$ (MeV)</td>
<td>0.1</td>
<td>2.2</td>
<td>?</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$ ($E_{CM}$)</td>
<td>$9.2 \times 10^{-6}$</td>
<td>$\Delta A_{LR}$ ($E_{CM}$) $4.1 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$\Delta A_{LR}$ $5.9 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \sin^{2}\theta_{\text{lept}}^{w}$</td>
<td>$5.9 \times 10^{-6}$</td>
<td>$7.5 \times 10^{-6}$</td>
<td>$6 \times 10^{-6}$ +?</td>
</tr>
</tbody>
</table>

All exceeds the theoretical precision from $\Delta \alpha (m_Z)$ ($310^{-5}$) or the comparison with $m_w$ (500keV)

But this precision on $\Delta \sin^{2}\theta_{\text{lept}}^{w}$ can only be exploited at FCC-ee!
The forward-backward tau polarization asymmetry is very clean. Dependence on $E_{CM}$ same as $A_{LR}$ negl. At FCC-ee

ALEPH data 160 pb$^{-1}$ (80 s @ FCC-ee)

Already syst. level of 6 $10^{-5}$ on $\sin^2\theta_\text{eff}^W$ much improvement possible by using dedicated selection e.g. tau$\rightarrow$ $\pi$ $\nu$ to avoid had. model
$A_{f_{V}}^{0,l}$

$A_{V}(P_T)$

$A_{V}(SLD)$

$A_{f_{V}}^{0,b}$

$A_{f_{V}}^{0,c}$

$Q_{f_{V}}^{had}$

Average

$0.23099 \pm 0.00053$

$0.23159 \pm 0.00041$

$0.23098 \pm 0.00026$

$0.23221 \pm 0.00029$

$0.23220 \pm 0.00081$

$0.2324 \pm 0.0012$

$\chi^2/\text{d.o.f.}: 11.8/5$

$m_H$ [GeV]

$\Delta\alpha_{\text{had}}^{(5)} = 0.02758 \pm 0.00035$

$m = 178.0 \pm 4.3 \text{ GeV}$
Going through the observables

the weak mixing angle as **defined** by the relation

\[ A_{\ell} = \frac{2g^e_V g^e_A}{(g^e_V)^2+(g^e_A)^2} = \frac{(g^e_L)^2-(g^e_R)^2}{(g^e_L)^2+(g^e_R)^2} \]

with \((g^e_L) = \frac{1}{2} \sin^2 \theta_{\text{lept}} W\) and \((g^e_R) = -\sin^2 \theta_{\text{lept}} W\)

\[ A_{\ell} \approx 8(1/4 - \sin^2 \theta_{\text{lept}} W) \] very sensitive to \(\sin^2 \theta_{\text{lept}} W\)!

Or

\[ A_{LR} = A_e \text{ measured from } \left( \sigma_{\text{vis},L} - \sigma_{\text{vis},R} \right) / \left( \sigma_{\text{vis},L} - \sigma_{\text{vis},R} \right) \]

( total visible cross-section had +µµ +ττ (35 nb) for 100% Left Polarization

\[ A_{FB}^{\mu\mu} = \frac{3}{4} A_e A_{\mu} = \frac{3}{4} A_{\ell}^2 \]

\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \]

\[ A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{\langle |P_e| \rangle} \]

\[ A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \frac{1}{\langle |P_e| \rangle} \]

\[ A_{FB}^0 = \frac{3}{4} A_e A_f \]

\[ A_{LR}^0 = A_e \]

\[ A_{LRFB}^0 = \frac{3}{4} A_f \]

\[ \langle P_T^0 \rangle = - A_t \]

\[ A_{FB}^{\text{pol,0}} = -\frac{3}{4} A_e \]