FCC-ee Requirements on Beam Polarization and Energy Calibration

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Some references:

B. Montague, Phys.Rept. 113 (1984) 1-96;
Polarization at LEP CERN Yellow Report *88-02;*
AB. Beam Polarization in e+e- CERN-PPE-93-125 Adv.Ser.Direct.High Energy Phys. 14 (1995) 277-324;
Spin Dynamics in LEP http://dx.doi.org/10.1063/1.1384062
Precision EW Measts on the Z Phys.Rept.427:257-454,2006 arXiv:0509008v3
for FCC-ee: arXiv:1308.6176 ; arXiv:1506.00933 ; arXiv:1705.03003

Beam polarization is directly useable in lepton colliders

-- no polarized structure functions etc…

At Electroweak Scale there are two main uses

-1- transverse polarization for energy calibration by resonant depolarization

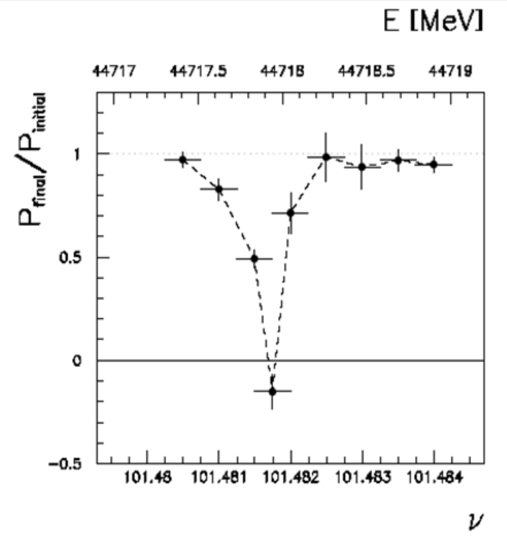
-2- e+e- longitudinal polarization combinations

-- as a way to control the spin of the e+e- system

Both can be used to improve precision measurements.

observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
M_Z MeV/c2	Input	91187.5 ±2.1	Z Line shape scan	0.005 MeV <±0.1 MeV	E_cal	QED corrections
Γ_Z MeV/c2	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ±2.3	Z Line shape scan	0.008 MeV <±0.1 MeV	E_cal	QED corrections
$R_l \equiv \frac{\Gamma_h}{\Gamma_l}$	α_s, δ_b	20.767 (25)	Z Peak	0.0001 (2-20)	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν's	2.984 ±0.008	Z Peak Z+γ(161 GeV)	0.00008 (40) 0.001	->lumi meast Statistics	QED corrections to Bhabha scat.
R_b	δ_b	0.21629 (66)	Z Peak	0.000003 (20-60)	Statistics, small IP	Hem. correlations
A_{LR}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_w^{eff}$ ±0.000006	4 bunch scheme	Design experiment
A_{FB}^{lept}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23099(53)		$\sin^2\theta_w^{eff}$ ±0.000006	E_cal & Statistics	
M_W MeV/c2	$\Delta\rho$, ε_3, ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2	Input	173200 ± 900	Threshold scan	~10 MeV	E_cal & Statistics	Theory limit at 50 MeV?

Beam Polarization can provide two main ingredients to Physics Measurements



1. **Transverse beam polarization provides beam energy calibration by resonant depolarization**

→ low level of polarization is required (~10% is sufficient)

→ at Z & W pair threshold comes naturally

→ at Z use of asymmetric wigglers at beginning of fills since polarization time is otherwise very long.

→ could be used also at ee $\rightarrow$ H(126) (depending on exact m_H !)

→ use 'single' non-colliding bunches and calibrate continuously during physics fills to avoid issues encountered at LEP

→ this is possible with e+ and e- Compton polarimeter (commercial laser)

→ should calibrate at energies corresponding to half-integer spin tune

→ must be complemented by analysis of «average E_beam» to E_CM relationship

Aim: Z mass & width to ~100 keV (stat: 10 keV) W mass & width to ~500 keV (stat : 300 keV)

For beam energies higher than ~90 GeV can use ee $\rightarrow$ Z γ or ee $\rightarrow$ WW events to calibrate E_{CM} at $\pm$5-10 MeV level: matches requirements for m_H and m_{top} measts

Beam Polarization can provide two main ingredients to Physics Measurements

2. Longitudinal beam polarization provides chiral e+e- system

-- High level of polarization is required (>40%)

-- Must compare with natural e+e- polarization due to chiral couplings of electrons (15%) or with final state polarization analysis for CC weak decays (100% polarized) (tau and top)

-- Physics case for Z peak is very well studied and motivated:

A_{LR} , A_{FB}^{Pol}(f) etc... (CERN Y.R. 88-06)

figure of merit is L.P² --> must not lose more than a factor ~10 in lumi.

self calibrating polarization measurement *→ spares

-- uses : enhance Higgs cross section (by 30%)

top quark couplings? final state analysis does as well (Janot **arXiv:1503.01325)**

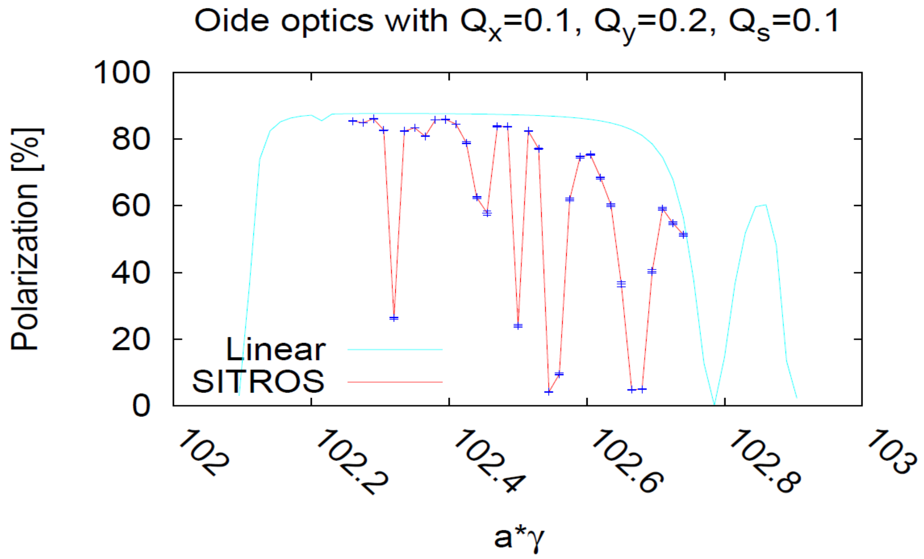
enhance signal, subtract/monitor backgrounds, for ee→WW , ee →H

-- requires High polarization level and often both e- and e+ polarization

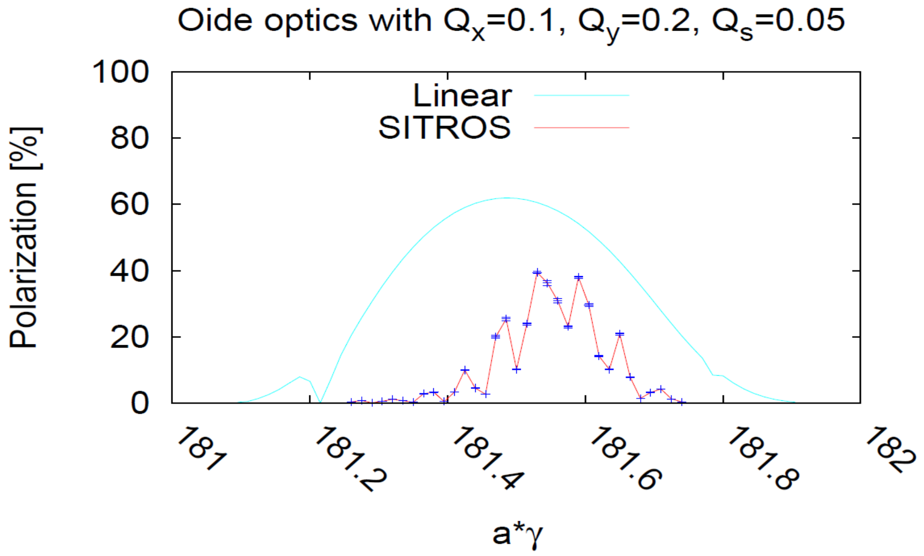
➔ **not interesting If loss of luminosity is too high**

-- Obtaining high level of polarization in high luminosity collisions is delicate in top-up mode

45 GeV



80 GeV



At the Z obtain excellent polarization level but too slow for polarization in physics need wigglers for Energy calibration

At the W expectation similar to LEP at Z
→ enough for energy calibration

Simulations by Eliana Gianfelice

EUROPEAN ORGANIZATION FOR PARTICLE PHYSICS

CERN–EP/98–40
CERN–SL/98–12
March 11, 1998

Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties

The LEP Energy Working Group

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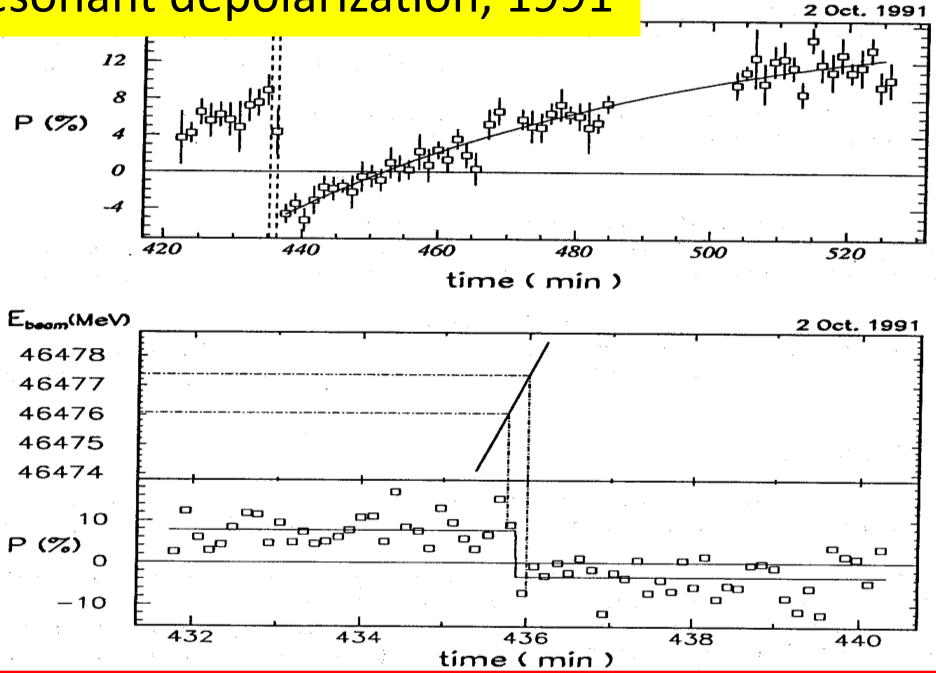
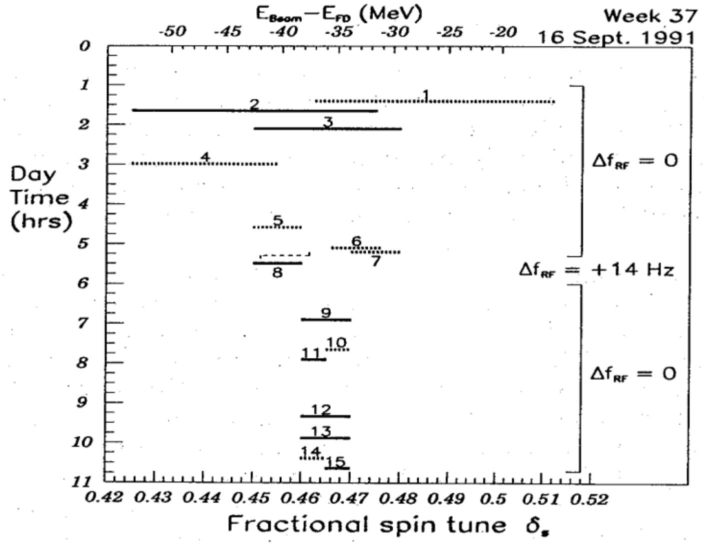
Abstract

The determination of the centre-of-mass energies from the LEP1 data for 1993, 1994 and 1995 is presented. Accurate knowledge of these energies is crucial in the measurement of the Z resonance parameters. The improved understanding of the LEP energy behaviour accumulated during the 1995 energy scan is detailed, while the 1993 and 1994 measurements are revised. For 1993 these supersede the previously published values. Additional instrumentation has allowed the detection of an unexpectedly large energy rise during physics fills. This new effect is accommodated in the modelling of the beam-energy in 1995 and propagated to the 1993 and 1994 energies. New results are reported on the magnet temperature behaviour which constitutes one of the major corrections to the average LEP energy.
The 1995 energy scan took place in conditions very different from the previous years. In particular the interaction-point specific corrections to the centre-of-mass energy in 1995 are more complicated than previously: these arise from the modified radiofrequency-system configuration and from opposite-sign vertical dispersion induced by the bunch-train mode of LEP operation.
Finally an improved evaluation of the LEP centre-of-mass energy spread is presented. This significantly improves the precision on the Z width.

LEP Resonant depolarization, 1991





$E_{beam} = 46{,}466.6 \pm 0.6$ MeV, e.g. precise to $\pm\ 1.5\ 10^{-5}$.

Figure 20: Polarization signal on 2 October 1991, showing the localization of the depolarizing frequency within the sweep.
Top: display of data points, with the frequency sweep indicated with vertical dashed lines. The full line represents the result of a fit with starting polarization $(-4.9 \pm 1.)\%$, polarization rise-time (60 ± 13) minutes, asymptotic polarization $(18.4 \pm 4.1)\%$.
Bottom: expanded view of the sweep period, with the individual data sets displayed (there are 10 sets per point); The frequency sweep lasted 7 data sets. The corresponding beam energy is shown in the upper box. Spin flip occurred between the two vertical dash-dotted lines.

variation of RF frequency to eliminate half integer ambiguity

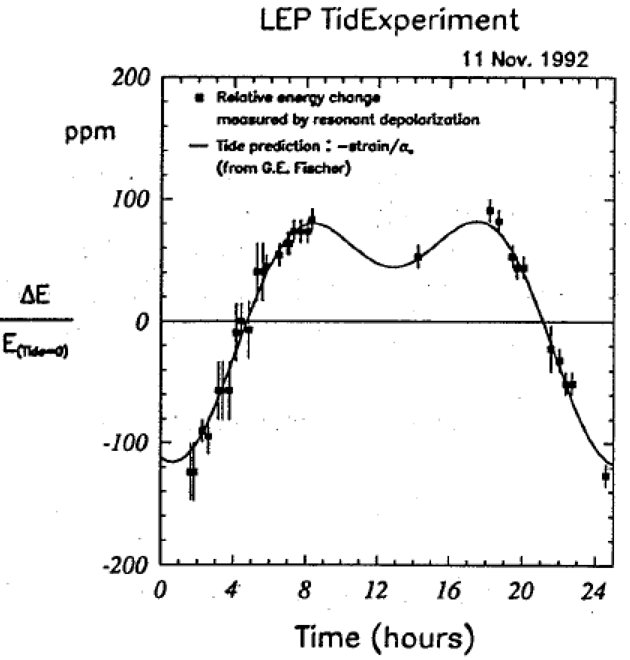
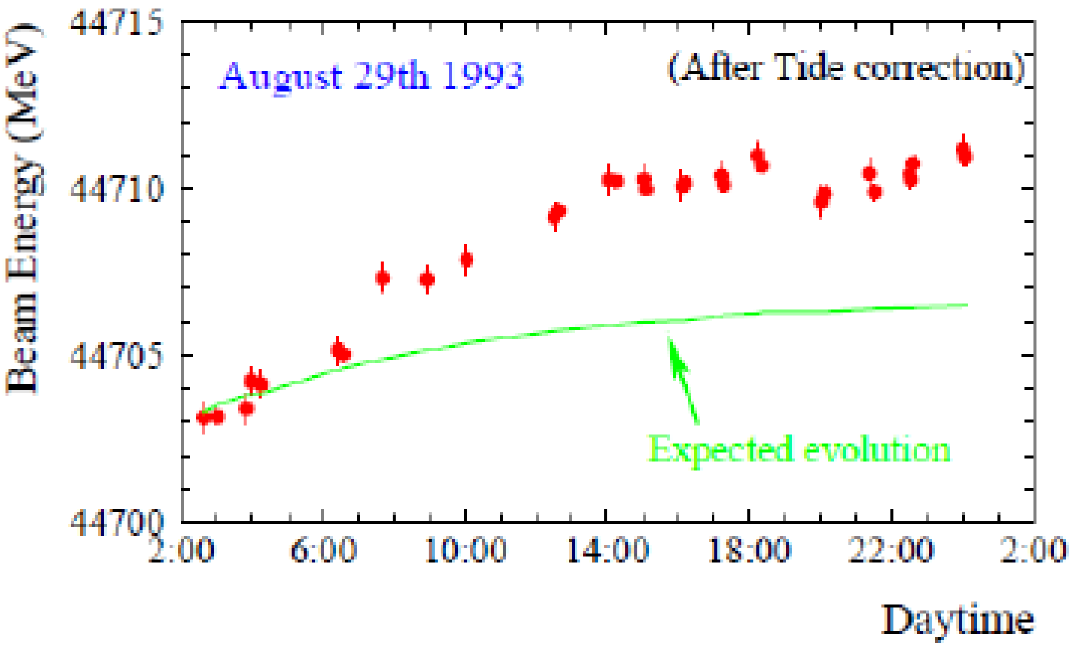


Figure 23: Beam energy variations measured over 24 hours compared to the expectation from the tidal LEP deformation.



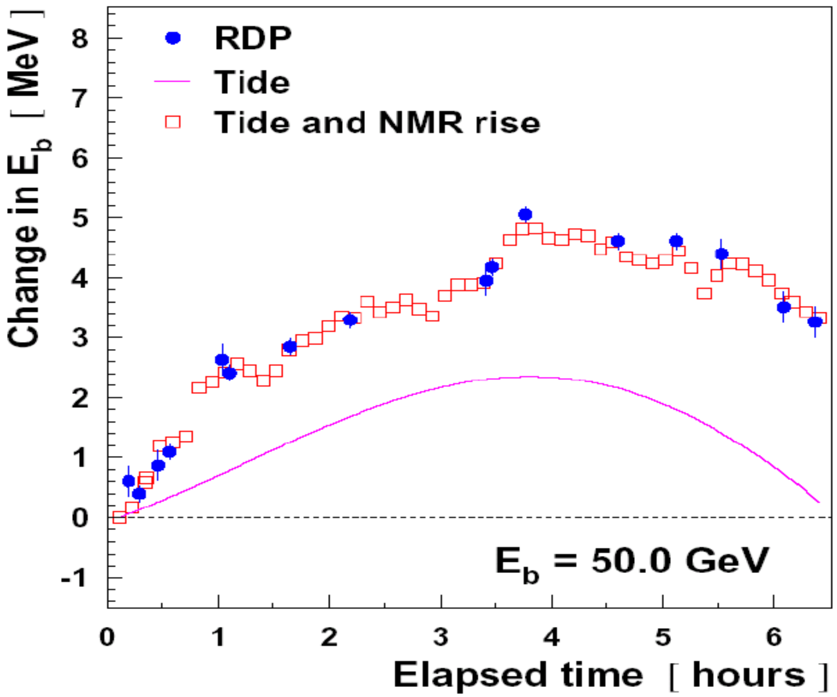
Many effects spoil the calibration if it is performed outside physics time

-- tides and other ground motion

-- RF cavity phases

-- histeresis effects and environmental effects (trains..etc)

Modelling of energy rise by (selected) NMR sampling of B-field is excellent !



(Experiment from 1999)

by 1999 we had an excellent model of the energy variations…
but we were not measuring the Z mass and width anymore
– we were hunting for the Higgs boson!

We have concluded that first priority is to achieve transverse polarization
in a way that allows continuous beam calibration by resonant depolarization
(energy measurement every ~10 minutes on ‘monitoring’ single bunches)

- This is a unique feature of circular e+e- storage ring colliders
- baseline runnig scheme defined with monitoring bunches
- the question of the residual systematic error requires further studies of the relationship between spin tune, beam energy at IRs, and center-of-mass energy

➔ target is 100keV at Z and W pair threshold energies

‘Do we want longitudinal polarization’?
we will discuss this in the following.

EXPERIMENTS ON BEAM-BEAM DEPOLARIZATION AT LEP

R. Assmann*, A. Blondel*, B. Dehning, A. Drees°, P. Grosse-Wiesmann, H. Grote, M. Placidi, R. Schmidt, F. Tecker†, J. Wenninger

PAC 1995

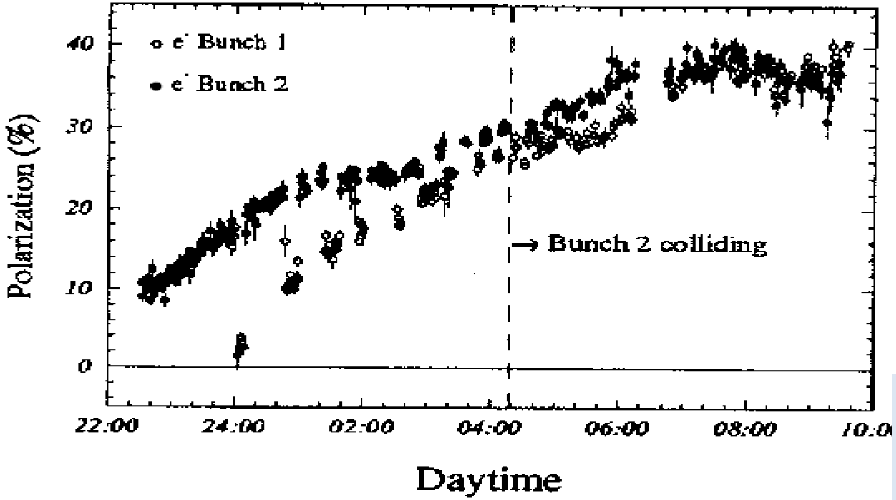


Figure. 3. Polarization level during third experiment

- With the beam colliding at one point, a polarization level of 40 % was achieved. The polarization level was about the same for one colliding and one non colliding bunch.
- It was observed that the polarization level depends critically on the synchrotron tune : when Q_s was changed by 0.005, the polarization strongly decreased.

experiment performed at an energy of 44.71 GeV the polarization level was 40 % with a linear beam-beam tune shift of about 0.04/IP. This indicates, that the beam-beam depolarization does not scale with the linear beam-beam tune shift at one crossing point. Other parameters as spin tune and synchrotron tune are also of importance

LEP:

This was only tried 3 times!

Best result: P = 40% , ξ^*_y= 0.04 , one IP

FCC-ee

Assuming 2 IP and ξ^*_y= 0.01 ➔

reduce luminosity, 10^{10} Z @ P~30%

Reduction of polarization due to continuous injection

The colliding bunches will lose intensity continuously due to collisions.
In FCC-ee with 4 IPs, L= 28 10^{34}/cm2/s beam lifetime is 213 minutes
In FCC-ee with 2 IPs, L= 1.4 10^{36}/cm2/s beam life time is 55minutes

Luminosity scales inversely to beam life time.
The injected e+ and e- are not polarized → asymptotic polarization is reduced.
Assume here that machine has been well corrected and beams
(no collisions, no injection) can be polarized to nearly maximum.
(*Eliana Gianfelice in Rome talk*)

- 45 GeV
  - limit ΔE=50 MeV (extrapolating from LEP)
  - 4 wigglers with B^+= 0.7 T
  - 10% polarization in 2.9 h for energy calibration

(polarization time is 26h)

We have simulated the simultaneous effect of

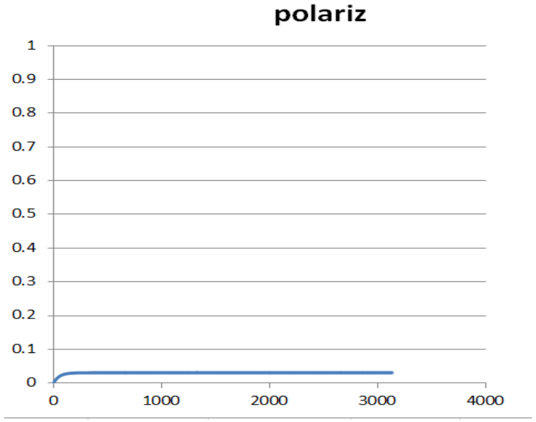
-- natural polarization

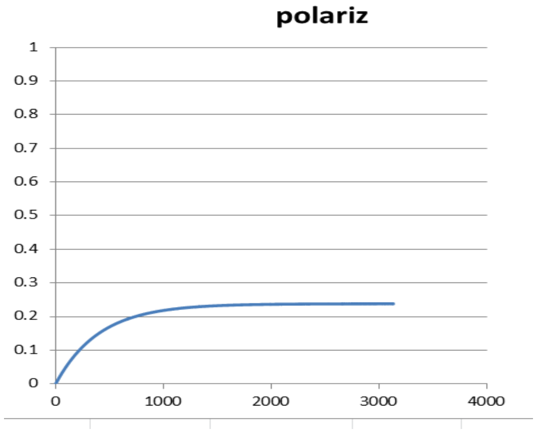
-- beam consumption by e+e- interactions

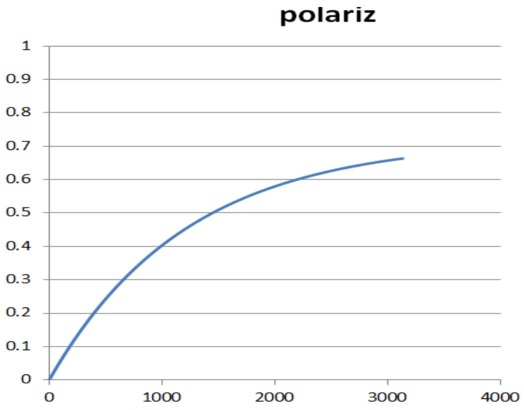
-- replenishment with unpolarized beams

assuming ***optimistically*** a maximal 90% asymptotic polarization

Running at full luminosity	Running at 10% Lumi	Running at 1% Lumi
P_max=0.03! P_eff=0.03	P_max=0.24, P_eff=0.21	P_max=0.66, P_eff=0.5







ΔA_{LR} scales as $1/\sqrt{(P^2 L)}$

Lumi loss factor	L.10^34	**Figure of merit: sum(P²L)**	Peff	Pmax
1	220	0.195	0.03	0.03
2	110	0.367	0.059	0.06
4	55	0.627	0.1078	0.11
6	37	0.805	0.149	0.16
8	27	0.924	0.184	0.2
10	22	1.003	0.214	0.24
12	18	1.053	0.24	0.27
15	15	1.09	0.27	0.32
18	12	**1.101**	0.3	0.35
22	10	1.088	0.33	0.4
26	8	1.059	0.354	0.43
30	7	1.023	0.37	0.46
40	5	0.92	0.41	0.52

Optimum around a reduction of luminosity by a factor 18.

This is still a luminosity of ~10^{35} per IP… and the effective polarization is 30%.
This is equivalent to a 100% polarization expt with luminosity reduced by 180.

observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
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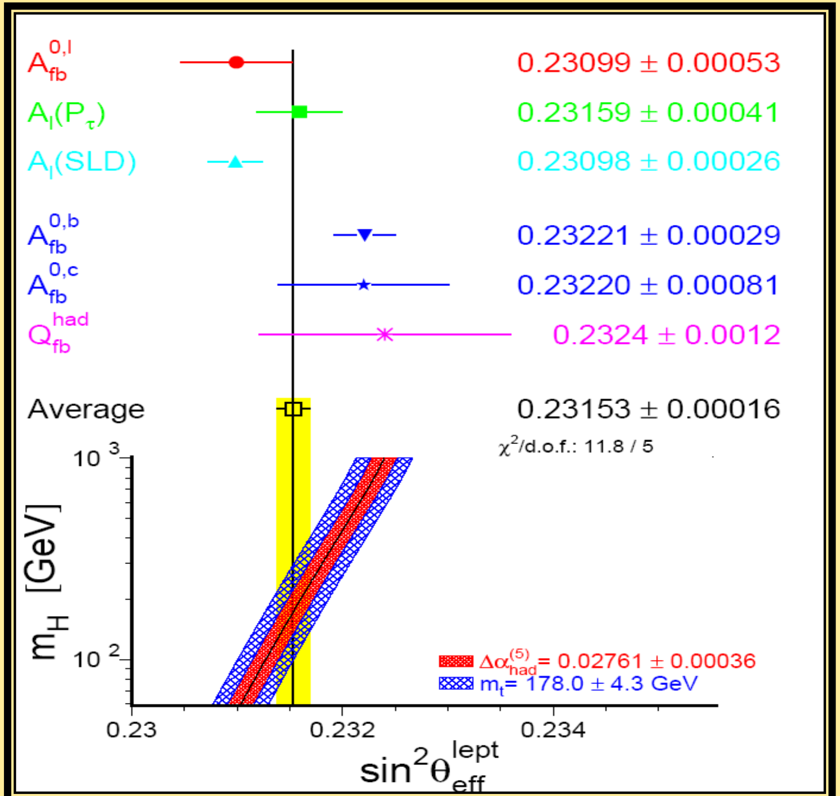
A_{LR}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_w^{eff}$ ±0.000006	4 bunch scheme	Design experiment
A_{FB}^{lept}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23099(53)		$\sin^2\theta_w^{eff}$ ±0.000006	E_cal & Statistics	
M_W MeV/c2	$\Delta\rho$, ε_3, ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2	Input	173200 ± 900	Threshold scan	~10 MeV	E_cal & Statistics	Theory limit at 50 MeV?

Measuring $\sin^2\theta_W^{eff}$ (m_Z)

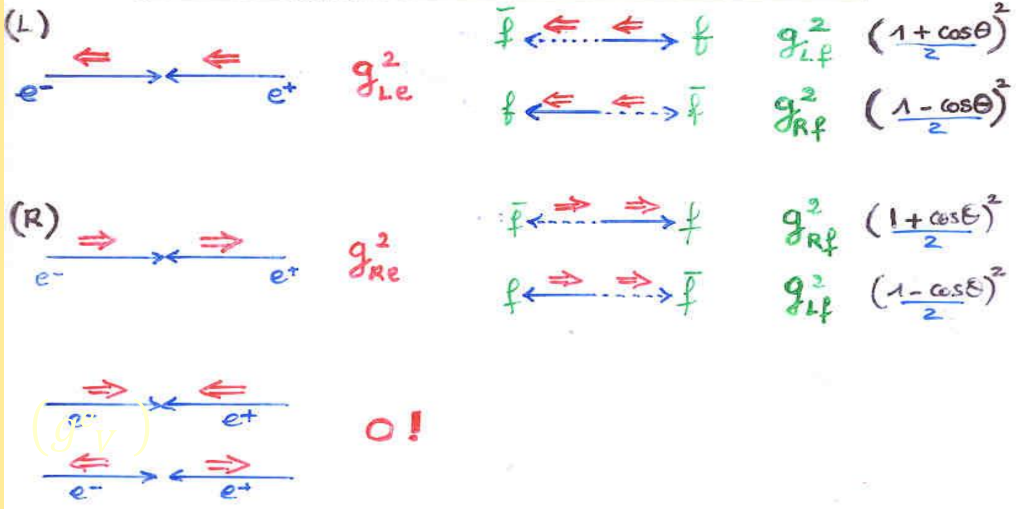
$$\sin^2\theta_W^{eff} \equiv \tfrac{1}{4}\,(1 - g_V/g_A)$$

$$g_V = g_L + g_R$$

arXiv:0509008



Helicity effects in $e^+e^- \to f\bar{f}$



Pol^d BEAM $\Rightarrow$

$$A_{LR} = \frac{\sigma_L^{tot} - \sigma_R^{tot}}{\sigma_L^{tot} + \sigma_R^{tot}} = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} \equiv \mathcal{A}_e = \frac{2\,g_{Ve}\,g_{Ae}}{g_{Ve}^2 + g_{Ae}^2}$$

$$A_{FB}^{Pol\,f} = \frac{\sigma_L^{Ff} - \sigma_L^{Bf} - (\sigma_R^{Ff} - \sigma_R^{Bf})}{\sigma_L^{Ff} + \sigma_L^{Bf} + \sigma_R^{Ff} + \sigma_R^{Bf}} = \frac{3}{4}\mathcal{A}_f$$

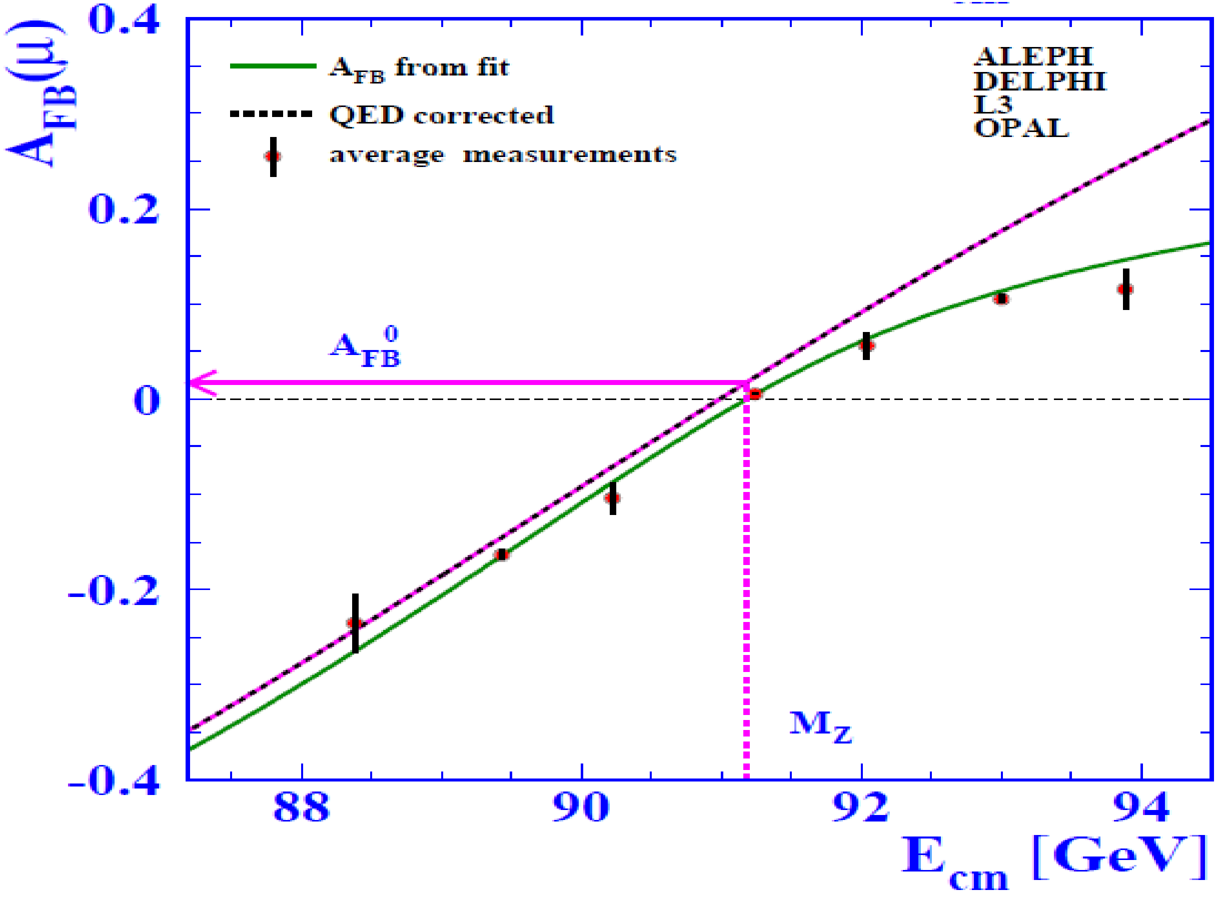
no Pol available:

$$\boxed{A_{FB} = \frac{\sigma_U^{Ff} - \sigma_U^{Bf}}{\sigma_U^{Ff} + \sigma_U^{Bf}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f}$$

Pol^n analyses τ

$$\langle P_f \rangle = \frac{\sigma_U^R - \sigma_U^L}{\sigma_U^R + \sigma_U^L} = -\mathcal{A}_f$$

$$A_{FB}^{Pol} = \frac{\sigma_U^{RF} - \sigma_U^{LF} - (\sigma_U^{RB} - \sigma_U^{LB})}{\sigma_U^{RF} + \sigma_U^{LF} + \sigma_U^{RB} + \sigma_U^{LB}} = -\frac{3}{4}\mathcal{A}_e$$



$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^{0}
M_Z
88
90
92
94
E_{cm} [GeV]

	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^{9}	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$	$4.5\ 10^{-5}$
$\Delta\ E_{cm}$ (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$	
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$	$6\ 10^{-6}$ +?

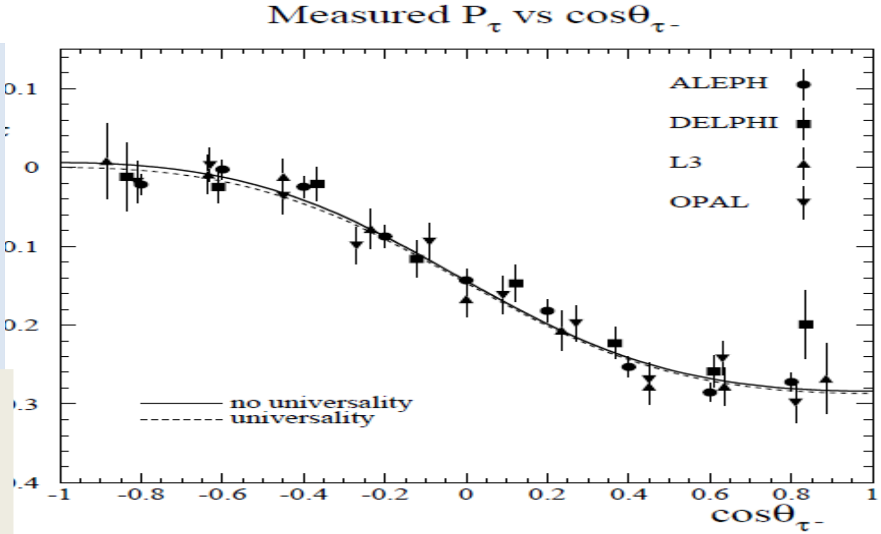
All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ ($3 10^{-5}$) or the comparison with m_W (500keV)

But this precision on $\Delta\sin^2\theta^{lept}_W$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.
At FCC-ee

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

Already syst. level of 6 10^{-5} on $\sin^2\theta^{eff}_W$
much improvement possible
by using dedicated selection
e.g. tau$\rightarrow$ π ν to avoid had. model



4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP ex-
nts. Only the statistical errors are shown. The values are not corrected for radiation,
ence or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.

Concluding remarks

1. There are very strong arguments for precision energy calibration with transverse polarization at the Z peak and W threshold.

2. Given the likely loss in luminosity, and the intrinsic uncertainties in the extraction of the weak couplings, the case for longitudinal polarization is limited

➔ **We have concluded that first priority is to achieve transverse polarization**
in a way that allows continuous beam calibration by resoenant depolarization

- this is all possible with a very high precision, both at the Z and the W.
  calibration at higher energies can be made from the data themselves at sufficient level.

- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy
  with the aim of achieving a precision of O(100 keV) on E_CM

spares

Longitudinal polarization at FCC-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of **$\sin^2\theta^{\ell ept}_W$** $= e^2/g^2\ (m_z)$
(-- not to be confused with -- $\sin^2\theta_W = 1 - m_w^2/m_z^2$

Useful references from the past:
«polarization at LEP» CERN Yellow Report 88-02
Precision Electroweak Measurements on the Z Resonance
Phys.Rept.427:257-454,2006 http://arxiv.org/abs/hep-ex/0509008v3
GigaZ @ ILC by K. Moenig

$\Delta\rho$: $\equiv \varepsilon_1$ $$\Gamma_\ell = (1+\Delta\rho)\,\frac{G_F\, M_Z^3}{24\pi\sqrt{2}}\left(1+\left(\frac{g_{V\ell}}{g_{A\ell}}\right)^2\right)\left(1+\frac{3}{4}\frac{\alpha}{\pi}\right)$$

ε_3 $$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,M_Z^2}\,\frac{1}{1+\Delta\rho}\,\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

δ_{vb} $$\Gamma_b = (1+\delta_{vb})\,\Gamma_d\left(1-\underset{\propto\, m_b^2/M_Z^2}{\text{mass corrections}}\right)$$

ε_2 $$M_W^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\sin^2\theta_W^{eff}}\cdot\frac{1}{(1-\varepsilon_3+\varepsilon_2)}$$

$\sin^2\theta_W^{eff}$ is defined from

$$\boxed{\sin^2\theta_W^{eff} = \frac{1}{4}\left(1-\underbrace{\frac{g_{V\ell}}{g_{A\ell}}}\right) = \sin^2\theta_{W\,lept}^{eff}}$$

obtained from asymmetries at the Z.

also Δr $$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\,G_F}\cdot\frac{1}{\left(1-\frac{M_W^2}{M_Z^2}\right)}\,\frac{1}{(1-\Delta r)}$$

$$\Delta r = \Delta\alpha - \frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\,\frac{\cos^2\theta_W}{\sin^2\theta_W}\,\varepsilon_3 + \frac{c^2-s^2}{s^2}\,\varepsilon_2$$

EWRCs

relations to the well measured

G_F m_Z α_{QED}

at first order:

$\Delta\rho = \alpha/\pi\ (m_{top}/m_Z)^2$

$-\ \alpha/4\pi\ \log\,(m_h/m_Z)^2$

$\varepsilon_3 = \cos^2\theta_w\ \alpha/9\pi\ \log\,(m_h/m_Z)^2$

$\delta_{vb} = 20/13\ \alpha/\pi\ (m_{top}/m_Z)^2$

complete formulae at 2d order
including strong corrections
are available in fitting codes

e.g. ZFITTER , GFITTER

Extracting physics from $\sin^2\theta^{lept}_W$

1. Direct comparison with m_Z

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\;\frac{1}{1+\Delta\rho}\;\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

Uncertainties in m_{top}, $\Delta\alpha(m_z)$, m_H , etc….

$\Delta\sin^2\theta^{lept}_W$ ~ $\Delta\alpha(m_z)$ /3 = 10^{-5} if we can reduce $\Delta\alpha(m_z)$ (see P. Janot)

2. Comparison with m_W/m_Z

Compare above formula with similar one:

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\cdot\frac{1}{1-\left(-\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho+2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3+\frac{c^2-s^2}{s^2}\varepsilon_2\right)}$$

Where it can be seen that **$\Delta\alpha(m_z)$** cancels in the relation.
The limiting error is the error on m_W.
For **Δm_W= 0.5 MeV this corresponds to $\Delta\sin^2\theta^{lept}_W = 10^{-5}$**

Assume for now ONE experiment at ECM=91.2

Luminosity «baseline» with beta*=1mm : 2.1 10^{36}/cm^2/s = 2 pb^{-1}/s,
Sigma_had = 31 10^{-33}cm^2 ➔ 6.5 10^{11} qq events/10^7 year/exp.

Consider 3 years of 10^7 s
2 10^{12} Z→ $q\bar{q}$ events (typical exp at LEP was 4.10^6)
4 10^{11} Z→ $b\bar{b}$
10^{11} Z→ $\mu\mu$, $\tau\tau$ each

Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare

ILC measurement of A_{LR} with 10^9 Z and P_{e-} =80%, P_{e+} =30%

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{Pol}(\tau)$ with 2.10^{12} Z

Comparing A_{LR} (P) and A_{FB} ($\mu\mu$)

Both measure the weak mixing angle as **defined** by the relation $A_\ell = \frac{(g^e{}_L)^2-(g^e{}_R)^2}{(g^e{}_L)^2+(g^e{}_R)^2}$

with $(g^e{}_L) = \frac{1}{2} - \sin^2\theta^{lept}_W$ and $(g^e{}_R) = -\sin^2\theta^{lept}_W$ $\quad A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$

$A_{LR} = A_e$

$A_{FB}{}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_\ell^2$

-- $\mathbf{A_{FB}{}^{\mu\mu}}$ is measured using muon pairs (5% of visible Z decays) and unpolarized beams

-- $\mathbf{A_{LR}}$ is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization

both with very small experimental systematics

-- **parametric sensitivity** $\quad \frac{dA_{FB}{}^{\mu\mu}}{d\sin^2\theta^{lept}_W} = 1.73$ **vs** $\frac{dA_{LR}}{d\sin^2\theta^{lept}_W} = 7.9$

Measurement of A_{LR}

electron bunches	$1\Leftarrow$	2	3	$4\Leftarrow$
positron bunches	1	$2\Rightarrow$	3	$4\Rightarrow$
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

$$\sigma_1 = \sigma_u (1 - P^-_e A_{LR})$$

$$\sigma_2 = \sigma_u (1 + P^+_e A_{LR})$$

$$\sigma_3 = \sigma_u$$

$$\sigma_4 = \sigma_u [1 - P^+_e P^-_e + (P^+_e - P^-_e) A_{LR}]$$

Verifies polarimeter with experimentally measured cross-section ratios

statistics

ΔA_{LR} = 0.0025 with about 10^6 Z^0 events,

ΔA_{LR} = 0.000045 with 5.10^{10} Z and 30% polarization in collisions.

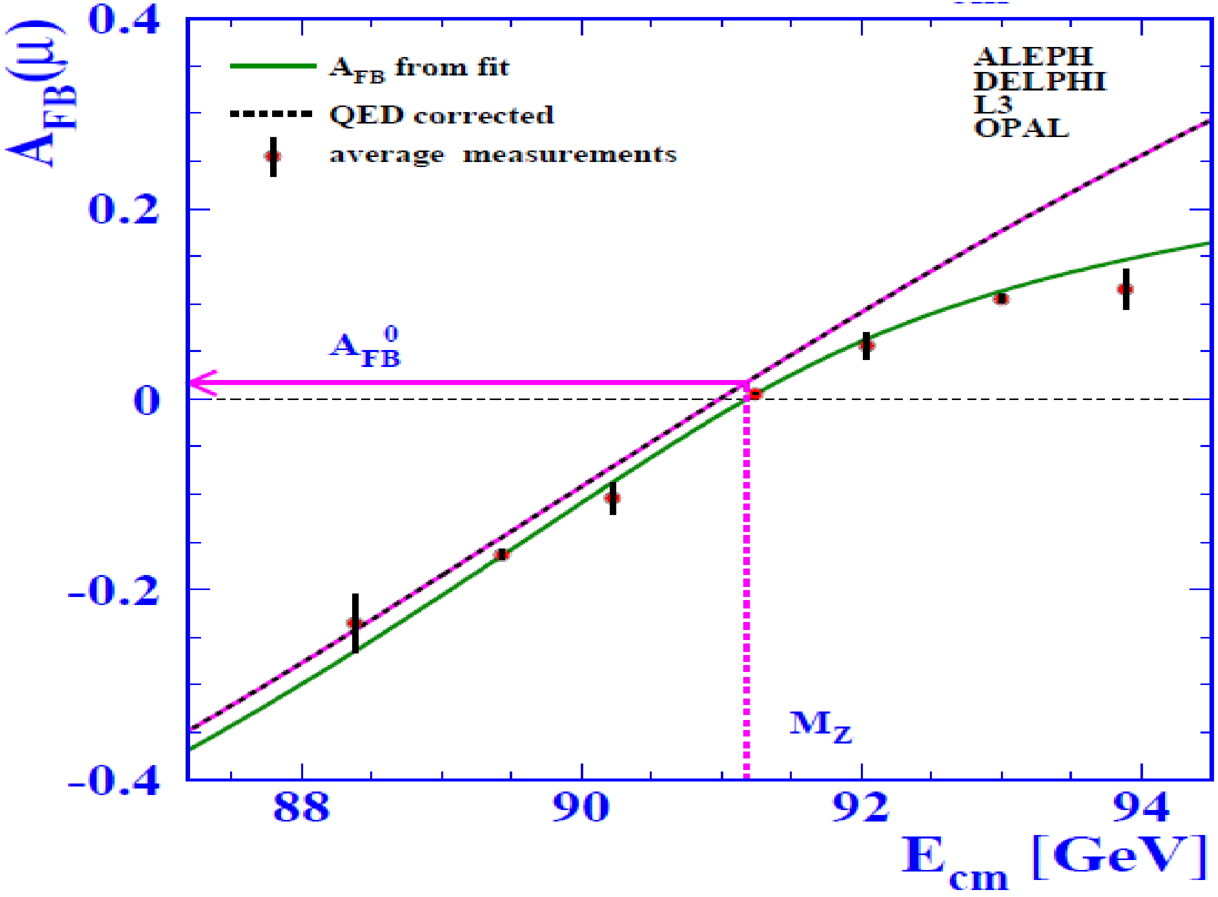
$\Delta\sin^2\theta_w^{eff}$ (stat) = $O(2.10^{-6})$

Will consider two sources of errors

-- statistics
-- uncertainty on center-of-mass energy (relative to the Z mass)

main inputs taken from
arXiv:hep-ex/0509008v3 precision measurements on the Z resonance
Phys. Rep. 427:257-454,2006

there are other uncertainties but they are very small for A_{FB}
This is a lower limit estimate for A_{LR} ; the systematics related to knowledge of the beam polarization (80% for e-, 30% for e+) should also be taken into account



$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^{0}
M_Z
88
90
92
94
E_{cm} [GeV]

	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^{9}	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$	$4.5\ 10^{-5}$
$\Delta\ E_{cm}$ (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$	
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$	$6\ 10^{-6}$ +?

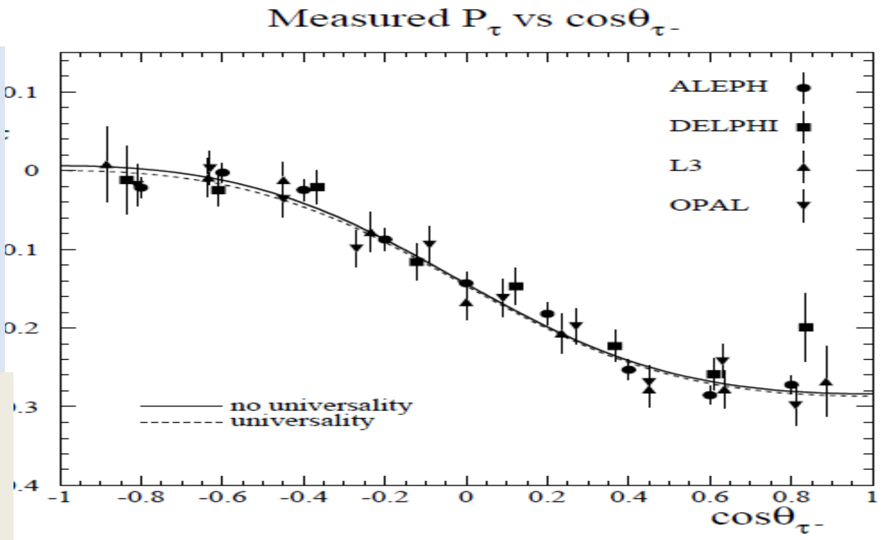
All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ (310^{-5}) or the comparison with m_W (500keV)

But this precision on $\Delta \sin^2\theta^{\ell ept}_W$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.
At FCC-ee

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

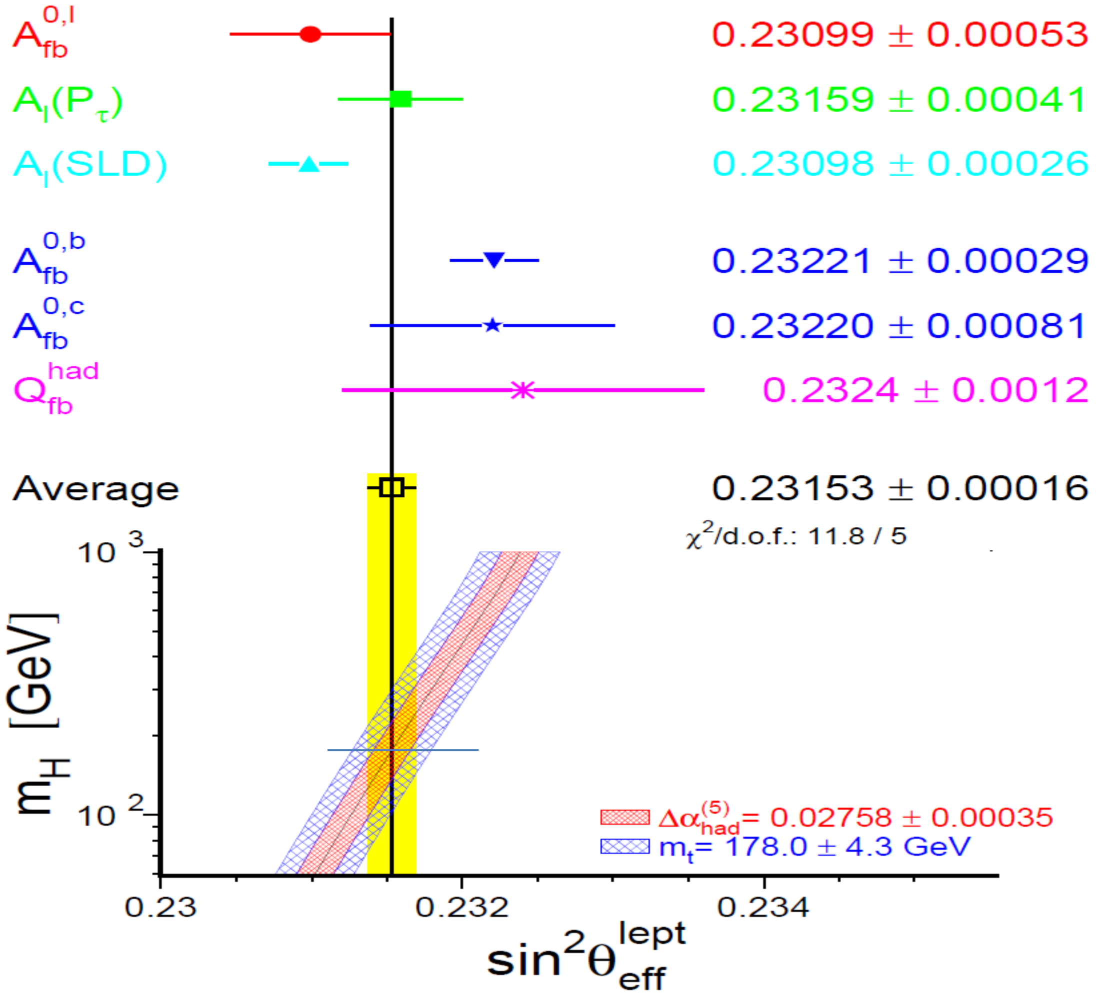
Already syst. level of 6 10^{-5} on $\sin^2\theta^{eff}_W$
much improvement possible
by using dedicated selection
e.g. tau→ π ν to avoid had. model



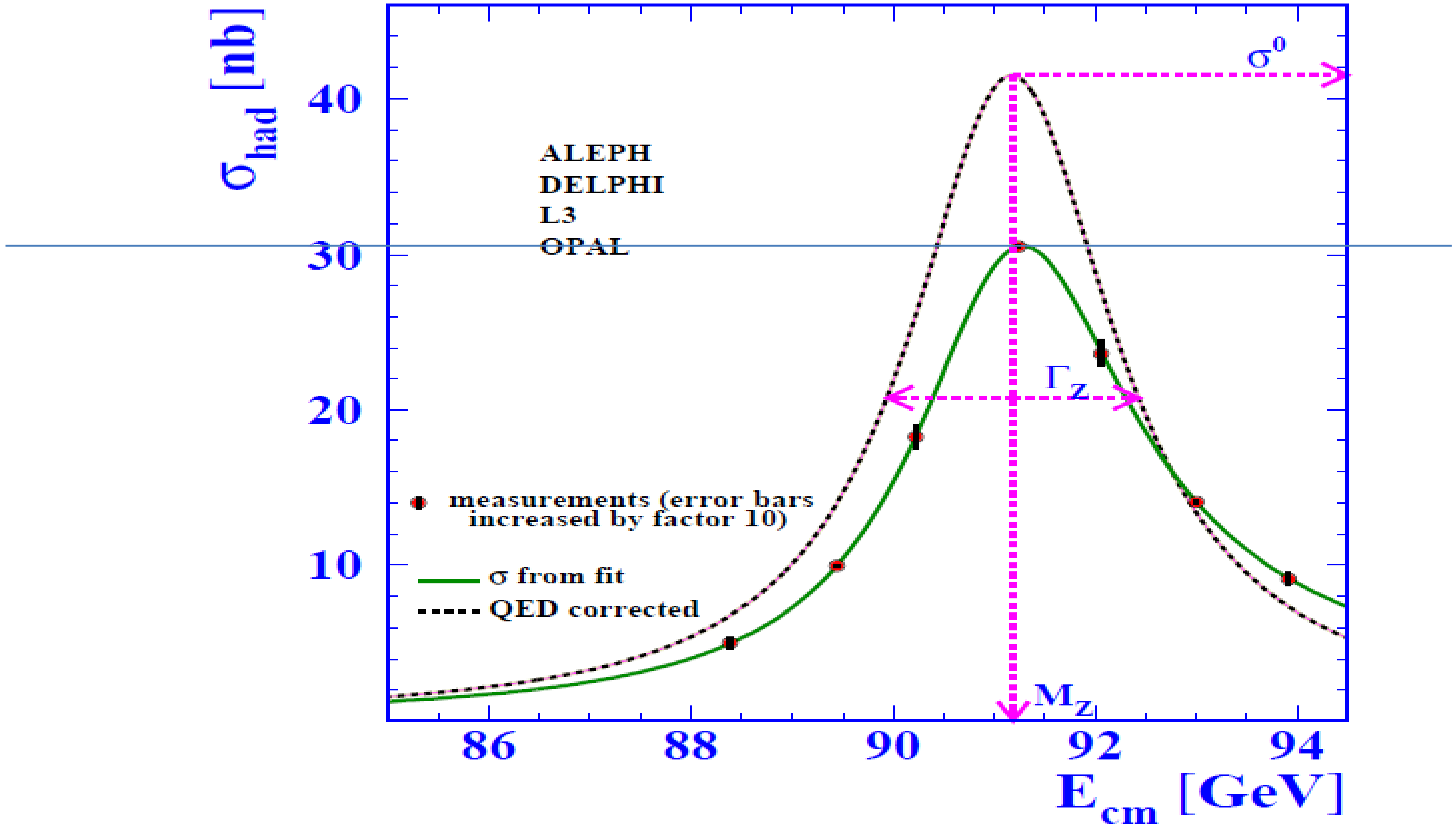
4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau-}$ as measured by each of the LEP ex-...nts. Only the statistical errors are shown. The values are not corrected for radiation, ...ence or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.



$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
$A_l(SLD)$ 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23221 ± 0.00029
$A_{fb}^{0,c}$ 0.23220 ± 0.00081
Q_{fb}^{had} 0.2324 ± 0.0012
Average 0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$



σ_{had} [nb]
40
30
20
10
86
88
90
92
94
E_{cm} [GeV]
ALEPH
DELPHI
L3
OPAL
σ^0
Γ_Z
M_Z
measurements (error bars increased by factor 10)
σ from fit
QED corrected

Going through the observables

the weak mixing angle as **defined** by the relation

$$A_\ell = \frac{2 g^e{}_V \, g^e{}_A}{(g^e{}_V)^2 + (g^e{}_A)^2} = \frac{(g^e{}_L)^2 - (g^e{}_R)^2}{(g^e{}_L)^2 + (g^e{}_R)^2}$$

with $(g^e{}_L) = ½ - \sin^2\theta^{lept}_W$ and $(g^e{}_R) = -\sin^2\theta^{lept}_W$

$A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$ very sensitive to $\sin^2\theta^{lept}_W$!

Or

$A_{LR} = A_e$ measured from $(\sigma_{vis,L} - \sigma_{vis,R}) / (\sigma_{vis,L} - \sigma_{vis,R})$

(total visible cross-section had + $\mu\mu$ + $\tau\tau$ (35 nb) for 100% Left Polarization

$$g_{Vf} = \sqrt{\mathcal{R}_f}\,(T_3^f - 2Q_f\mathcal{K}_f \sin^2\theta_W)$$

$$g_{Af} = \sqrt{\mathcal{R}_f}\,T_3^f \,.$$

$A_{FB}{}^{\mu\mu} = ¾ A_e A_\mu = ¾ A_\ell^2$

$$\begin{aligned} A_{FB}^{0,f} &= \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \\ A_{LR}^0 &= \mathcal{A}_e \\ A_{LRFB}^0 &= \frac{3}{4}\mathcal{A}_f \\ \langle\mathcal{P}_\tau^0\rangle &= -\mathcal{A}_\tau \\ A_{FB}^{pol,0} &= -\frac{3}{4}\mathcal{A}_e \,. \end{aligned}$$

$$\begin{aligned} A_{FB} &= \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ A_{LR} &= \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} \\ A_{LRFB} &= \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} \,. \end{aligned}$$