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# Landau Damping of Intra-Bunch Oscillations

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# Octupoles for Landau Damping

Octupole magnets → tune spread → Landau damping

$$B_{\text{oct}} \propto x^3$$

$$\Delta Q_{\text{oct}} \propto a^2$$

**Dynamic  
Aperture**

A balance is necessary

**Landau  
Damping**

$$\Delta Q_{\text{coh}}^{\text{FCC}} \approx \Delta Q_{\text{coh}}^{\text{LHC}}$$

168 Octupoles are the essential part of the beam stability in LHC,  
FCC would need much more.

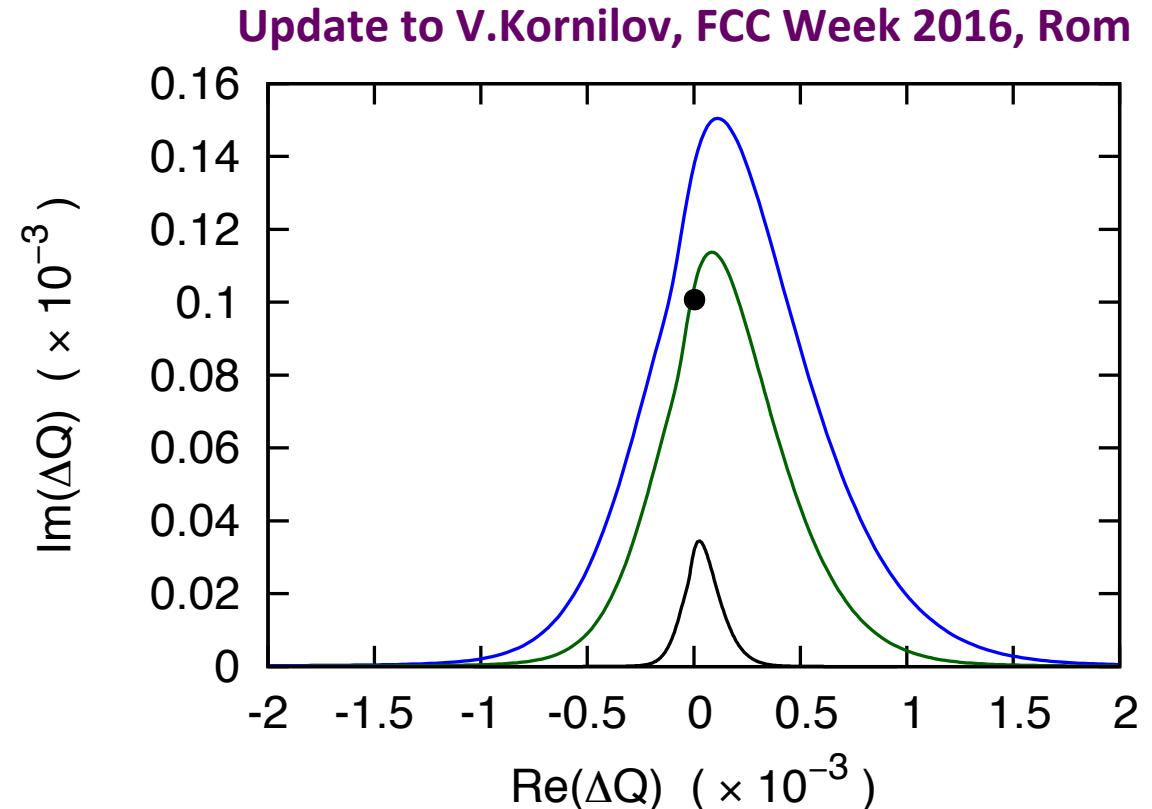
# Overview FCC Landau Octupoles

Blue:  $\Delta Q_{coh}$ -Damping as in LHC.  
**3554** Octupoles.

Green: enough damping for the  
(•) impedances.  
**2686** octupoles.

Black:  $N_{MO} = N_{MQ} = \mathbf{814}$

LHC: 168 octupoles.  
LHC octupole magnets are  
assumed here.



Stability Diagram:  
stable below the line,  
unstable above the line.

# Dispersion Relation

L.Laslett, V.Neil, A.Sessler, 1965

D.Möhl, H.Schönauer, 1974

J.Berg, F.Ruggiero, CERN SL-96-71 AP 1996

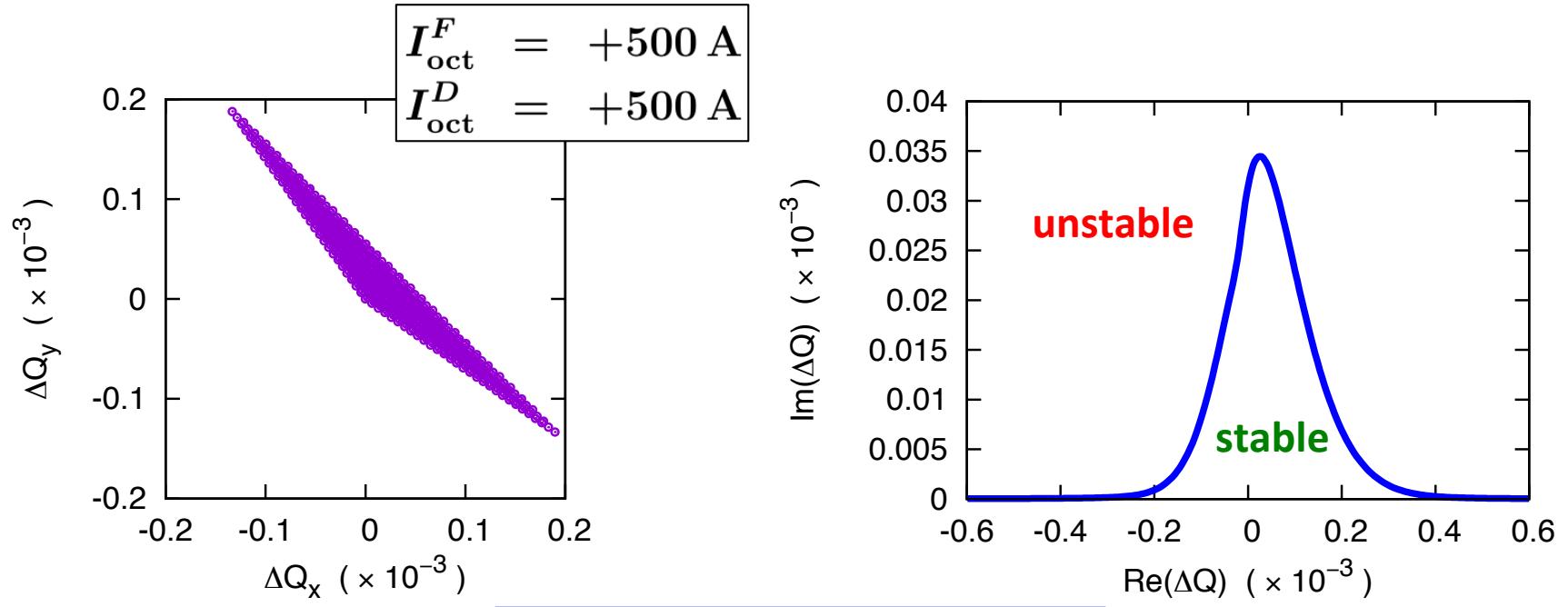
$$\Delta Q_{\text{coh}} \int \frac{1}{\Delta Q_{\text{oct}} - \Omega/\omega_0} J_x \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y = 1$$

complex coherent tune shift for  
the beam without damping

The solution: collective mode frequency  $\Omega$   
for the given impedance and beam

This dispersion relation has been used for the LHC planning,  
and confirmed in specific measurements.

# Stability Diagram



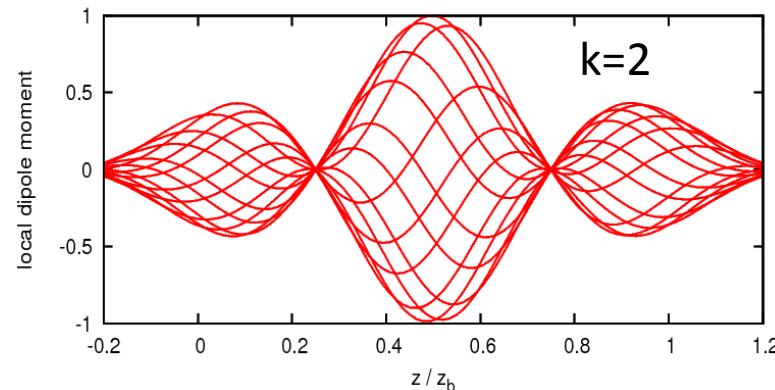
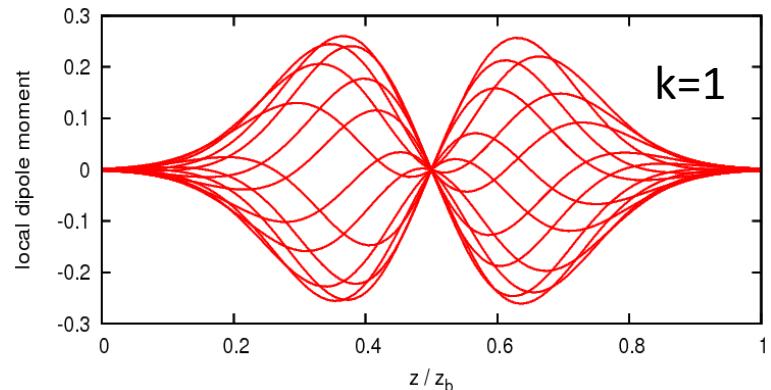
footprint  
 $J_x + J_y < 6\varepsilon_x$   
 $6\varepsilon_x$  in action ( $3.5\sigma_x$  in ampl)  
gives  $\approx$  the damping extent

Tune spread provides Landau damping

# Intra-Bunch Oscillations

But this is a 2D dispersion relation.

What about Gaussian bunches and intra-bunch oscillations?  
Not known so far.



The practical relevance:

The higher-order modes would probably need less than 3600 octupoles.  
The damping of the  $k=0$  mode can be provided by a feedback system.

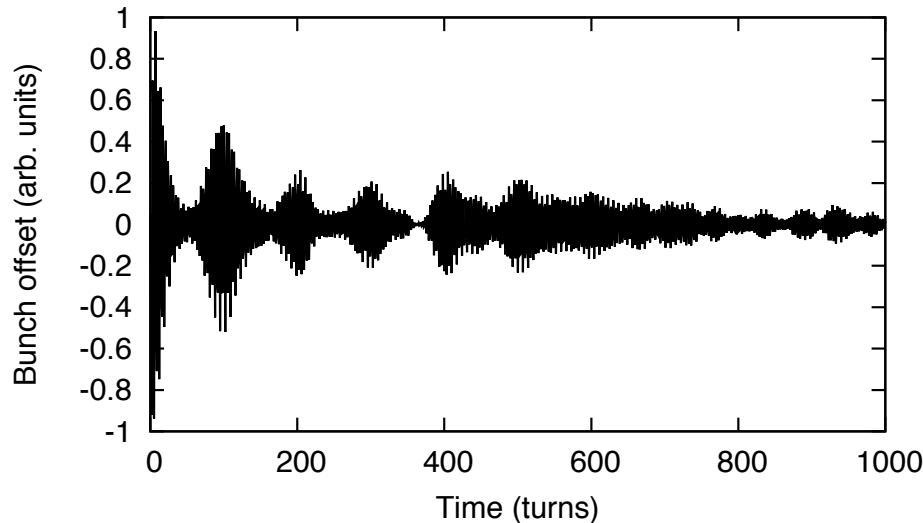
# Particle Tracking Simulations

## The PIC code PATRIC

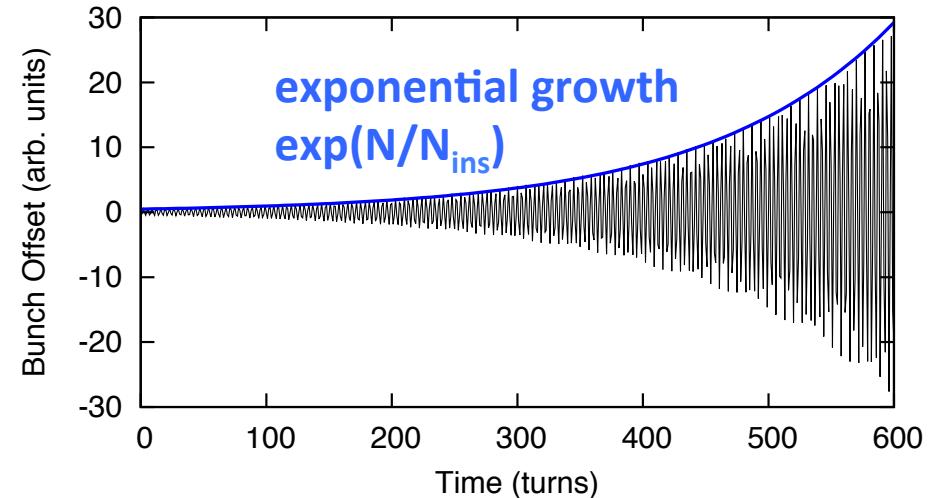
- 2.5D sliced bunches
- Self-consistent space-charge, frozen space-charge
- Impedances, Wakes
- Snapshot domain (space), fixed-location domain (time)
- Tune shifts, spectra, instabilities verified with analytical theories:
  - V. Kornilov and O. Boine-Frankenheim, Proc. of ICAP2009, San Francisco (2009)
  - O.Boine-Frankenheim, V.Kornilov, Proc. of ICAP2006 (2006)
- Verified vs. HEADTAIL (CERN)
- Landau damping simulations, head-tail modes with space-charge:
  - V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)

# Particle Tracking Simulations

- Start with a small eigenmode perturbation
- Apply an impedance (resistive-wall here)
- Apply octupoles



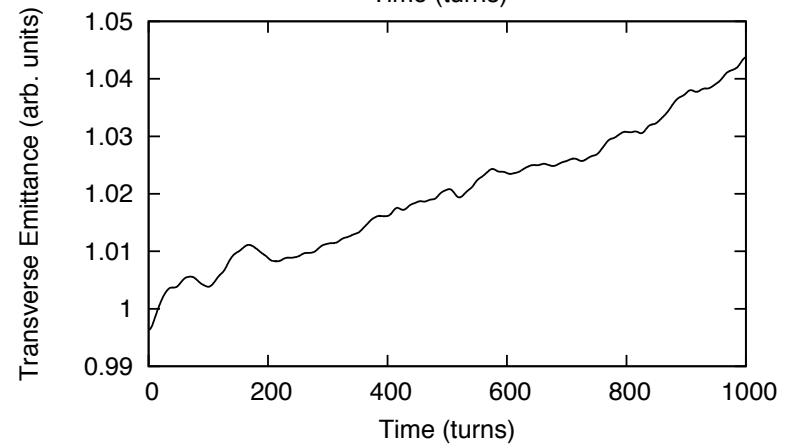
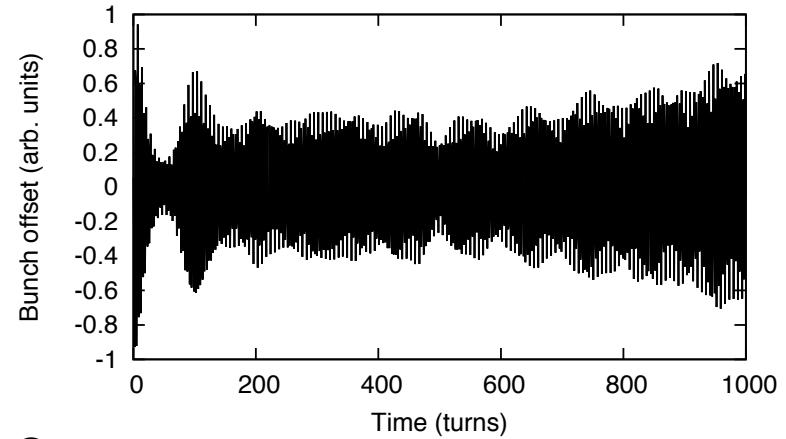
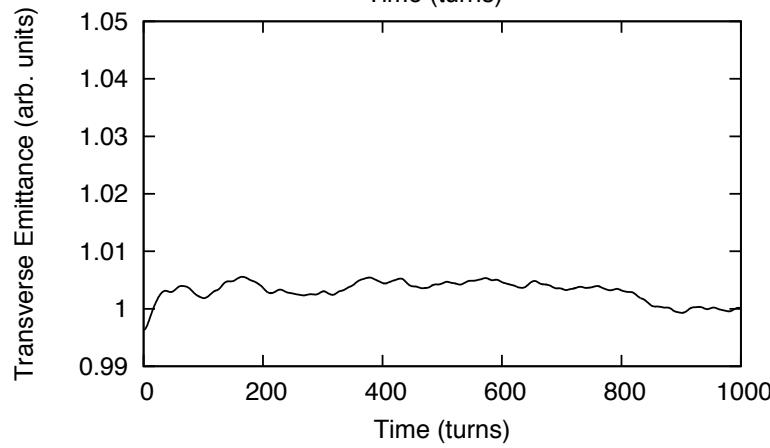
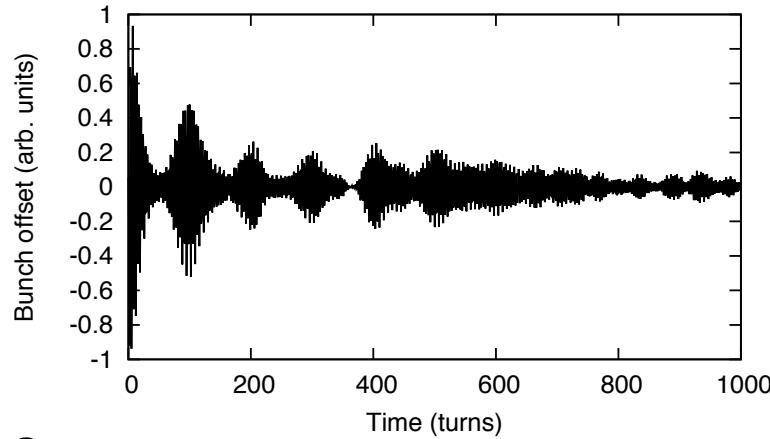
Stable due to octupoles



Unstable

# Particle Tracking Simulations

Accurately determining the stability thresholds

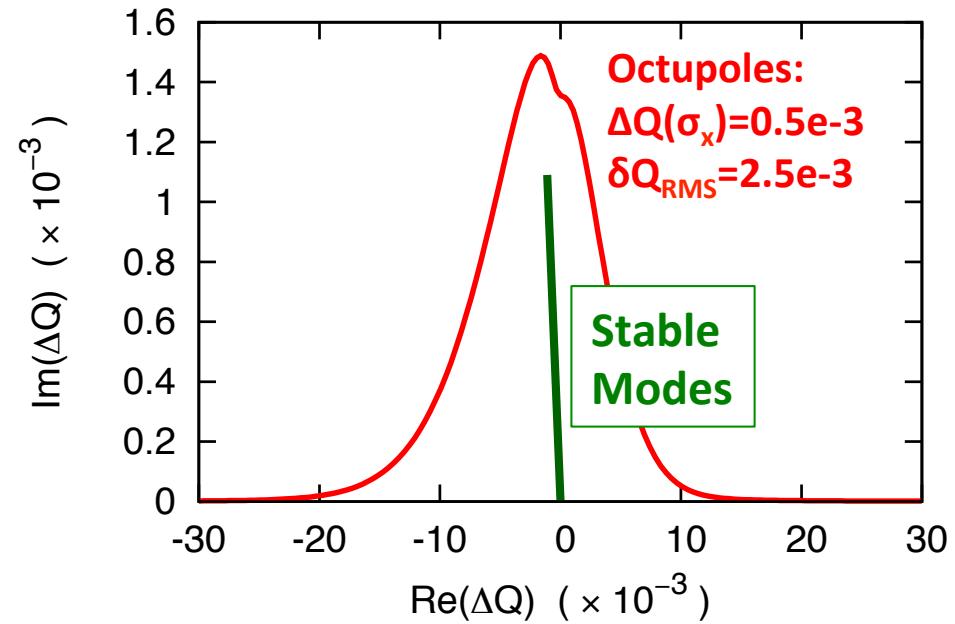
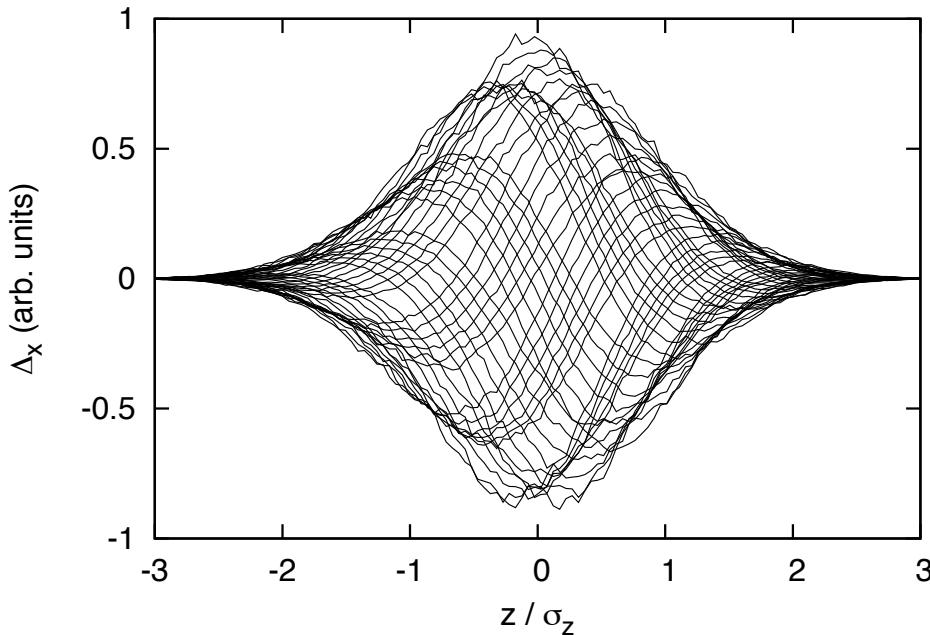


Stable due to octupoles

Above the threshold

# Particle Tracking Simulations

## 1. Case: the mode k=0

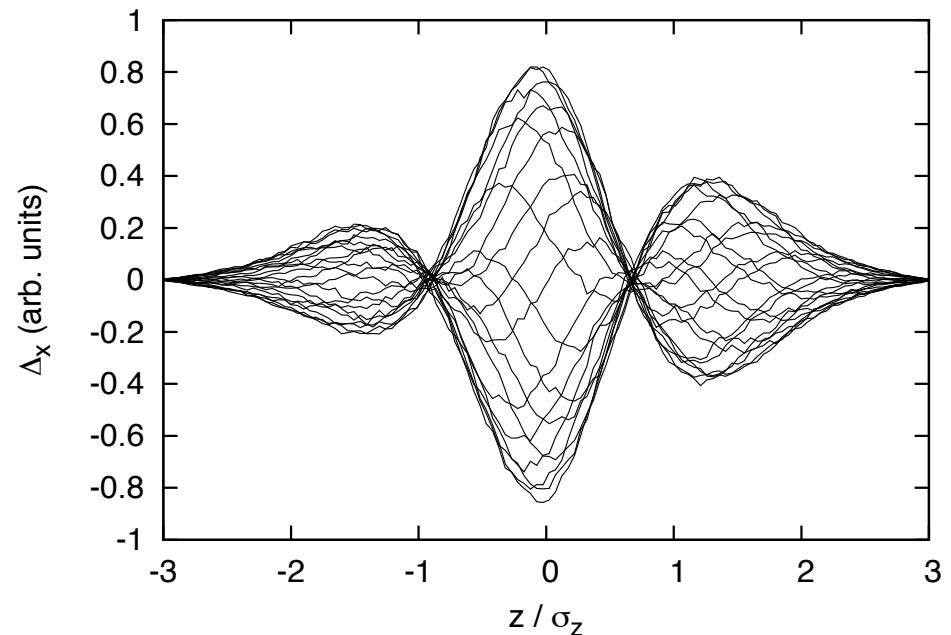
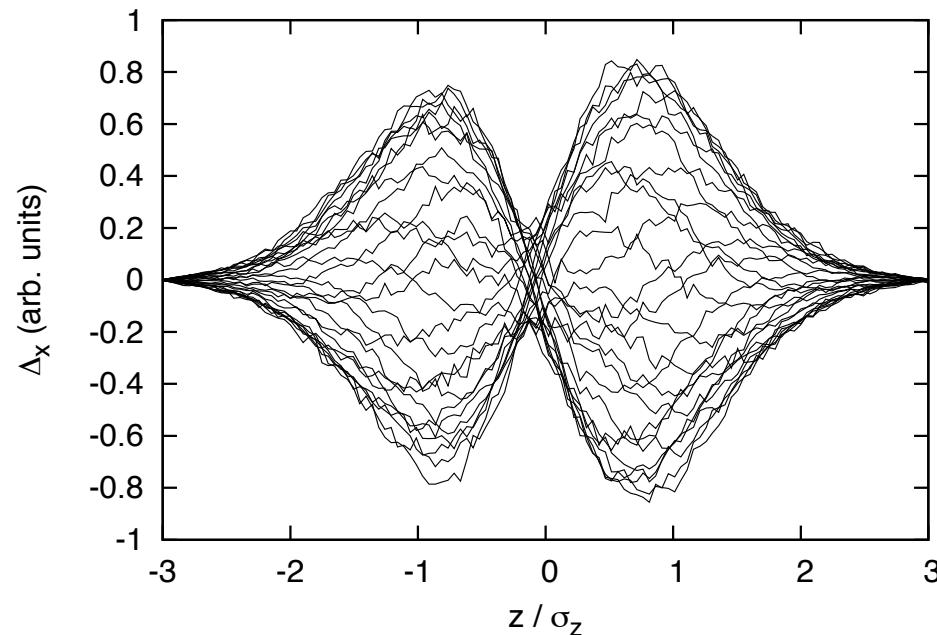


### Surprising agreement

- 2D dispersion relation vs. 3D Gaussian bunches
- Stability due to phase-mixing and not purely Landau damping  
(involved discussion)

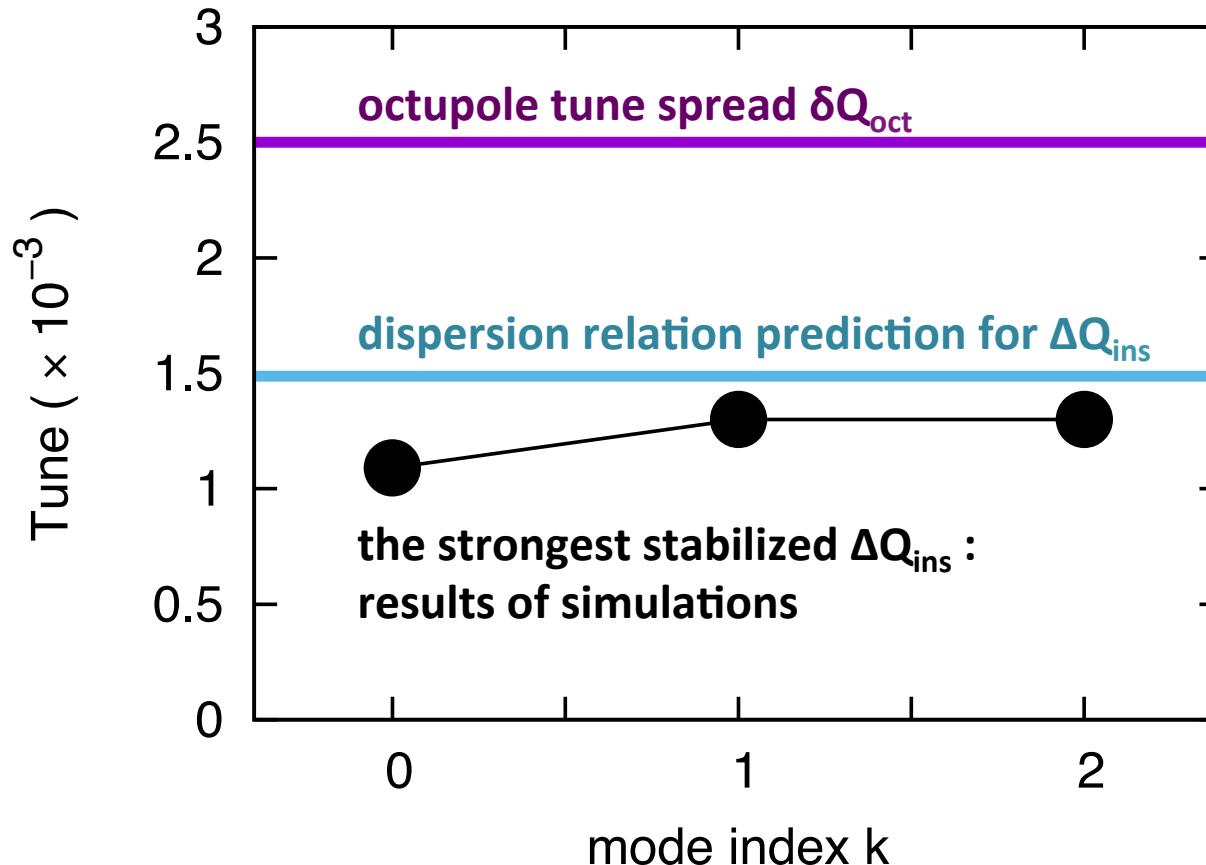
# Particle Tracking Simulations

Similar simulation scans for the k=1 and k=2 modes



- Intra-Bunch oscillation produce a small global offset
- The growth rates are smaller than for the k=0 mode.  
Here: factor 4 for k=1, factor 6 for k=2

# Summary of Simulation Results



- The octupoles provide a similar stability to the high-order modes
- The instability growth rate and the tune spread are related (DR!)
- Basically, a 2D mode↔particles interaction all along the bunch

# RF Quadrupole

A.Grudiev, PRSTAB 17, 011001 (2014)

A.Grudiev, et.al., HB2014, East Lansing, USA, (2014)

M.Schenk, et.al., HB2016, Malmö, Sweden (2016)

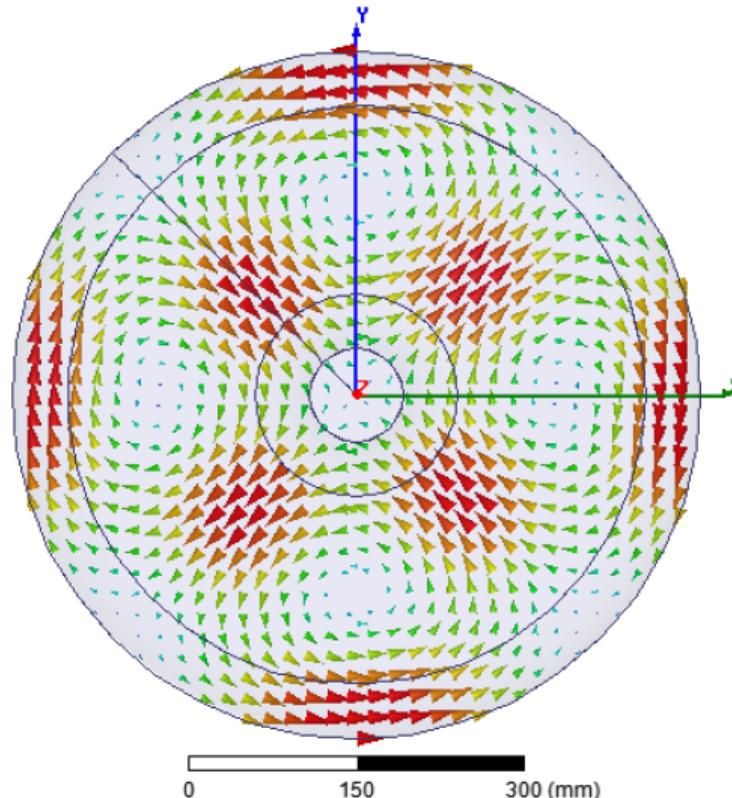


Figure 1: Magnetic field distribution in the transverse plane of the TM quadrupolar mode cavity of the RFQ.

For LHC:

$L = 0.15 \text{ m}$ , 6 cavities

$E = 46 \text{ MV/m}$

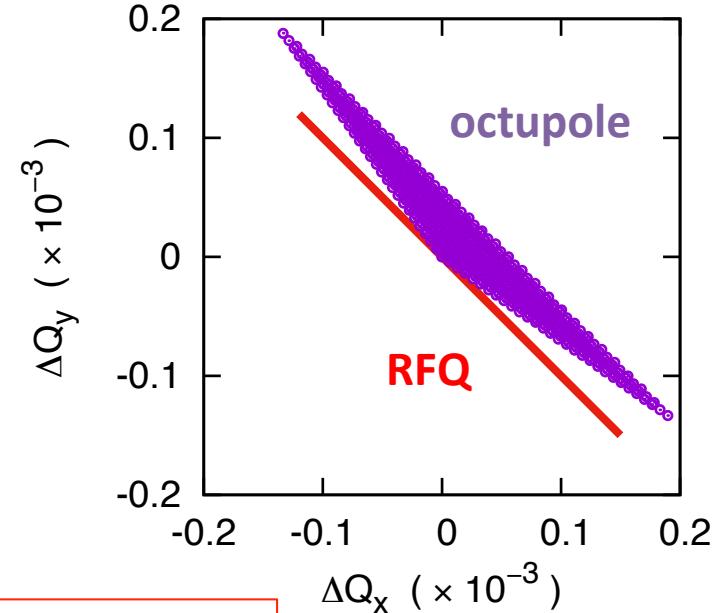
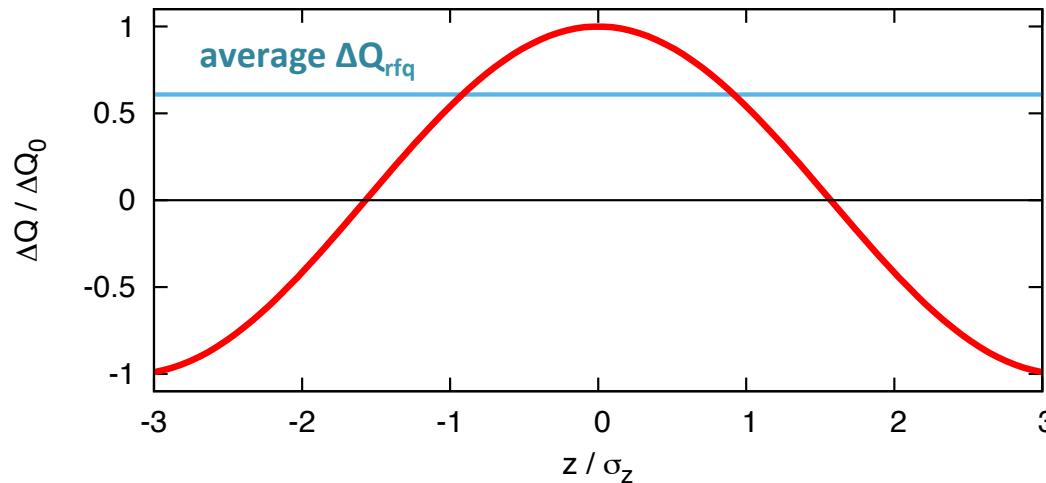
$\omega = 800 \text{ MHz}$ ,  $\lambda = 0.375 \text{ m}$

The incoherent tune shift:

$$\Delta Q_{\text{RFQ}}(z) = \pm \frac{\beta k_2}{4\pi} \cos(\omega z/c)$$

The related tune spread should provide Landau damping

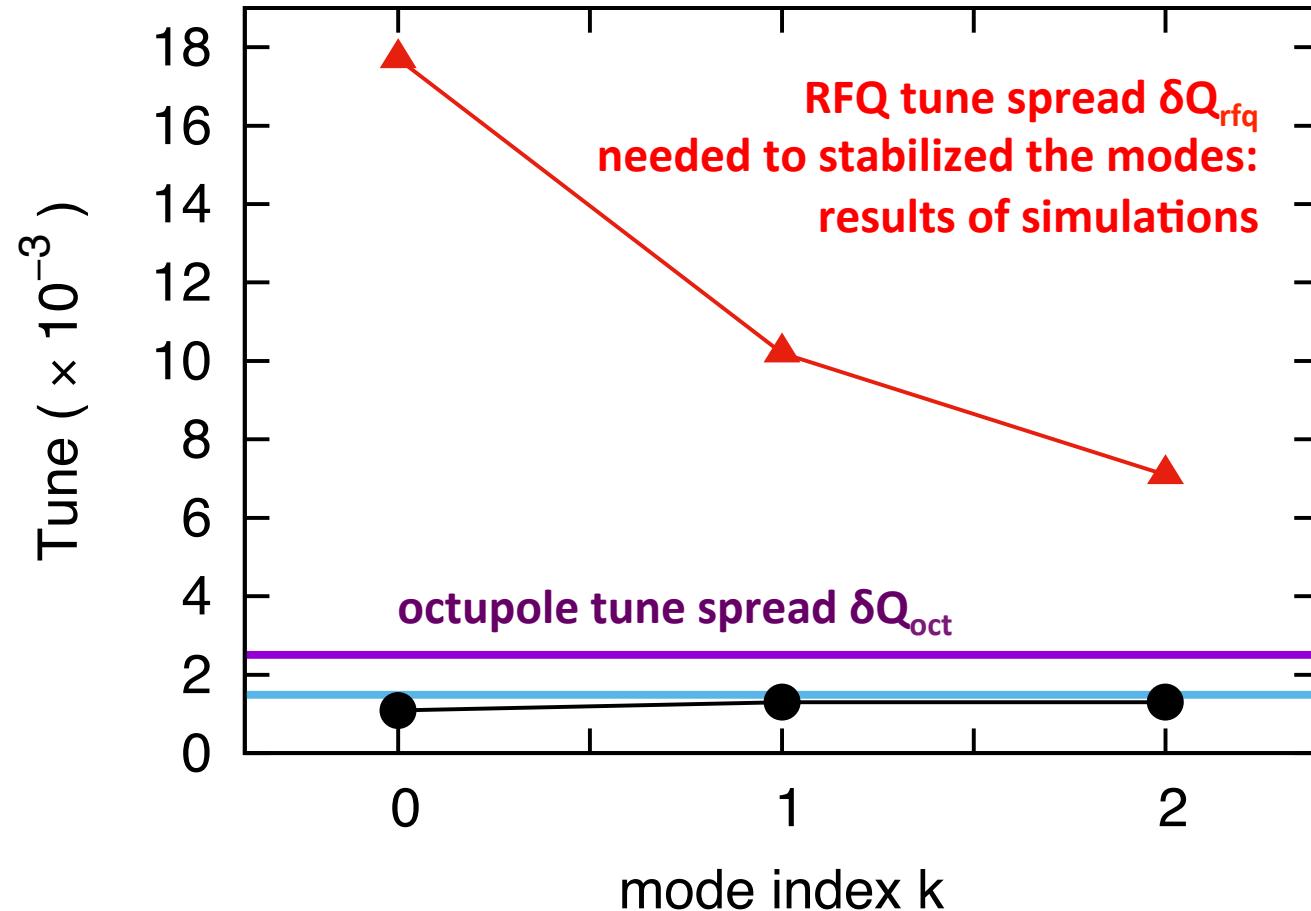
# RF Quadrupole



$$\Delta Q_{\text{RFQ}}(z) = \Delta Q_{0\text{RF}} \cos(\omega z/c)$$

- Tune spread (rms  $\delta Q_{\text{rfq}} = 0.4 \Delta Q_{0\text{RF}}$ )
- Global tune shift (average  $\Delta Q_{\text{rfq}} = 0.6 \Delta Q_{0\text{RF}}$ )
- Modification of the chromaticity effect
- → Affects the instability drive
- Tune spread is longitudinal: in every slice zero spread

# Summary of Simulation Results



The needed RFQ tune spread is much bigger (factor  $\approx 5-10$ )

RFQ can provide stability (like  $\xi$ ). Does it provide Landau damping?

# Conclusions

- Nearly 3600 LHC-octupoles are needed at FCC to ensure the transverse stability
- Stability of intra-bunch oscillations ( $k \geq 1$  modes) due to octupoles corresponds to the 2D Landau damping DR.  
→ the true Landau damping and higher tolerable impedances, or less octupoles
- RF Quadrupole provides stability only by factors  $\approx 5\text{--}10$  larger tune spreads  
→ existence of Landau damping is not clear, it can be the instability drive modification (like  $\xi$ )