Landau Damping of Intra-Bunch Oscillations

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Octupoles for Landau Damping

Octupole magnets $\rightarrow$ tune spread $\rightarrow$ Landau damping

$$B_{\text{oct}} \propto x^3$$
$$\Delta Q_{\text{oct}} \propto a^2$$

Dynamic Aperture $\Rightarrow$ A balance is necessary $\Rightarrow$ Landau Damping

$$\Delta Q_{\text{coh}}^{\text{FCC}} \approx \Delta Q_{\text{coh}}^{\text{LHC}}$$

168 Octupoles are the essential part of the beam stability in LHC, FCC would need much more.

V.Kornilov, FCC Week 2016, Rom
Overview FCC Landau Octupoles

Blue: $\Delta Q_{\text{coh}}$ – Damping as in LHC. **3554** Octupoles.

Green: enough damping for the (●) impedances. **2686** octupoles.

Black: $N_{\text{MO}} = N_{\text{MQ}} = 814$

LHC: 168 octupoles. LHC octupole magnets are assumed here.

Stability Diagram:
stable below the line, unstable above the line.

Update to V. Kornilov, FCC Week 2016, Rom
Dispersion Relation

\[ \Delta Q_\text{coh} \int \frac{1}{\Delta Q_\text{oct} - \Omega/\omega_0} \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y = 1 \]

complex coherent tune shift for the beam without damping

The solution: collective mode frequency \( \Omega \) for the given impedance and beam

This dispersion relation has been used for the LHC planning, and confirmed in specific measurements.

L. Laslett, V. Neil, A. Sessler, 1965
D. Möhl, H. Schönauer, 1974
J. Berg, F. Ruggiero, CERN SL-96-71 AP 1996
Stability Diagram

\[ I_{\text{oct}}^F = +500 \, \text{A} \]
\[ I_{\text{oct}}^D = +500 \, \text{A} \]

\[ \Delta Q_y \left( \times 10^{-3} \right) \]
\[ \Delta Q_x \left( \times 10^{-3} \right) \]

\[ \mathrm{Im}(\Delta Q) \left( \times 10^{-3} \right) \]
\[ \mathrm{Re}(\Delta Q) \left( \times 10^{-3} \right) \]

footprint

\[ J_x + J_y < 6\varepsilon_x \]

6\varepsilon_x in action (3.5\sigma_x in ampl)
gives \approx the damping extent

Tune spread provides Landau damping
Intra-Bunch Oscillations

But this is a 2D dispersion relation.
What about Gaussian bunches and intra-bunch oscillations?
Not known so far.

The practical relevance:
The higher-order modes would probably need less than 3600 octupoles.
The damping of the $k=0$ mode can be provided by a feedback system.
The PIC code PATRIC

- 2.5D sliced bunches
- Self-consistent space-charge, frozen space-charge
- Impedances, Wakes
- Snapshot domain (space), fixed-location domain (time)
- Tune shifts, spectra, instabilities verified with analytical theories:
- Verified vs. HEADTAIL (CERN)
- Landau damping simulations, head-tail modes with space-charge:
  V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 114201 (2010)
Particle Tracking Simulations

- Start with a small eigenmode perturbation
- Apply an impedance (resistive-wall here)
- Apply octupoles

Stable due to octupoles

Unstable

exponential growth $\exp(N/N_{\text{ins}})$
Particle Tracking Simulations

Accurately determining the stability thresholds

Stable due to octupoles

Above the threshold
1. Case: the mode k=0

Surprising agreement
• 2D dispersion relation vs. 3D Gaussian bunches
• Stability due to phase-mixing and not purely Landau damping (involved discussion)
Particle Tracking Simulations

Similar simulation scans for the k=1 and k=2 modes

- Intra-Bunch oscillation produce a small global offset
- The growth rates are smaller than for the k=0 mode.
  Here: factor 4 for k=1, factor 6 for k=2
Summary of Simulation Results

- The octupoles provide a similar stability to the high-order modes.
- The instability growth rate and the tune spread are related (DR!).
- Basically, a 2D mode ↔ particles interaction all along the bunch.
RF Quadrupole

A. Grudiev, PRSTAB 17, 011001 (2014)

For LHC:

L = 0.15 m, 6 cavities
E = 46 MV/m
\( \omega = 800 \text{ MHz}, \lambda = 0.375 \text{ m} \)

The incoherent tune shift:

\[
\Delta Q_{RFQ}(z) = \pm \frac{\beta k_2}{4\pi} \cos(\omega z / c)
\]

The related tune spread should provide Landau damping
RF Quadrupole

\[ \Delta Q_{RFQ}(z) = \Delta Q_{0RF} \cos(\omega z/c) \]

- Tune spread (rms $\delta Q_{rfq} = 0.4 \Delta Q_{0RF}$)
- Global tune shift (average $\Delta Q_{rfq} = 0.6 \Delta Q_{0RF}$)
- Modification of the chromaticity effect
- → Affects the instability drive
- Tune spread is longitudinal: in every slice zero spread
The needed RFQ tune spread is much bigger (factor ≈5–10)

RFQ can provide stability (like ξ). Does it provide Landau damping?
Conclusions

• Nearly 3600 LHC-octupoles are needed at FCC to ensure the transverse stability

• Stability of intra-bunch oscillations (k≥1 modes) due to octupoles corresponds to the 2D Landau damping DR. → the true Landau damping and higher tolerable impedances, or less octupoles

• RF Quadrupole provides stability only by factors ≈5–10 larger tune spreads → existence of Landau damping is not clear, it can be the instability drive modification (like ξ)