Simulations for FCC-ee beam self-polarization

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Introduction

- *Resonant de-polarization* has been proposed for accurate beam energy calibration ($\ll$ 100 keV) at 45 and 80 GeV beam energy.
  It relies on the relationship $\boldsymbol{\nu_{spin}}=\boldsymbol{a}\boldsymbol{\gamma}$ [a].

- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  The effect is in practice restricted to a limited range of values of machine size and beam energy because
  - of the build-up rate
  - it is jeopardized by machine imperfections (spin/orbital motion resonances) which affects the reachable level of polarization in particular at high energy.

[a] $\boldsymbol{a}$ = gyromagnetic anomaly

Sokolov-Ternov polarization

Beam get vertically polarized in the ring guiding field

$$P_\infty^{\mathrm{ST}} = 92.3\% \qquad\qquad \tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint\frac{ds}{|\rho|^3}$$

For FCC-e^+e^- with $\rho \simeq 10424$ m, fixed by the maximum attainable dipole field for the hh case, it is

E (GeV)	τ_{pol} (h)	$\tau_{10\%}$ (*) h
45	256	29
80	14	1.6

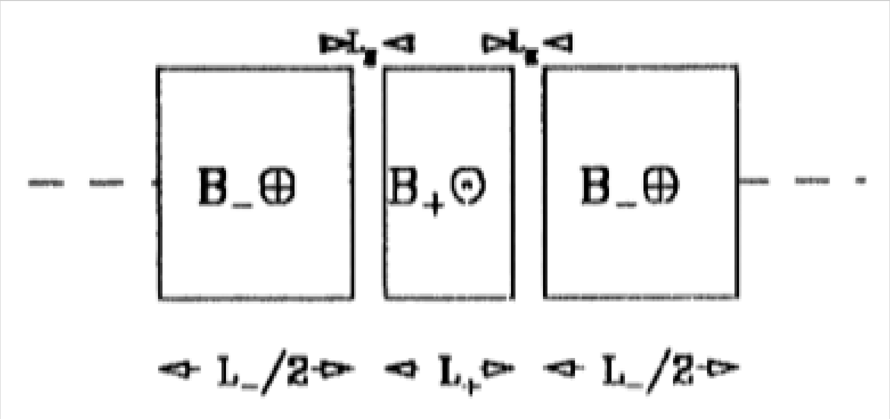
(*) Time needed to reach P=10% for energy calibration

$$\tau_{10\%} = -\tau_p \times \ln(1 - 0.1/P_\infty)$$

Polarization wigglers

τ_p may be reduced by introducing wigglers, a sequence of vertical dipole fields with alternating direction, as proposed for LEP.

The LEP polarization wigglers:



For 4 LEP-like wigglers with $B_+/B_-(=L_-/L_+)\simeq$ 6 and B^+=0.7 T
it is $\tau_{10\%}\simeq$2.9 h at 45 GeV.

Horizontal emittance

$$\epsilon_x = C_q \gamma^2 \frac{\mathcal{I}_5}{J_x \mathcal{I}_2} \qquad \mathcal{I}_2 \equiv \oint ds \frac{1}{\rho^2}$$

$$\mathcal{I}_5 \equiv \oint ds \frac{\beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2}{|\rho|^3}$$

Also if nominally D_x=0 the wiggler may increase the horizontal emittance

$$\Delta \mathcal{I}_5 \simeq \frac{1}{15\pi^3} \frac{< \beta_x >_w \ell_w}{\rho_w^5} \lambda_w$$

For the 1 mm β^* optics ($90^0/90^0$ deg FODO) the horizontal emittance at 45 GeV increases from 90 pm to 500 pm.

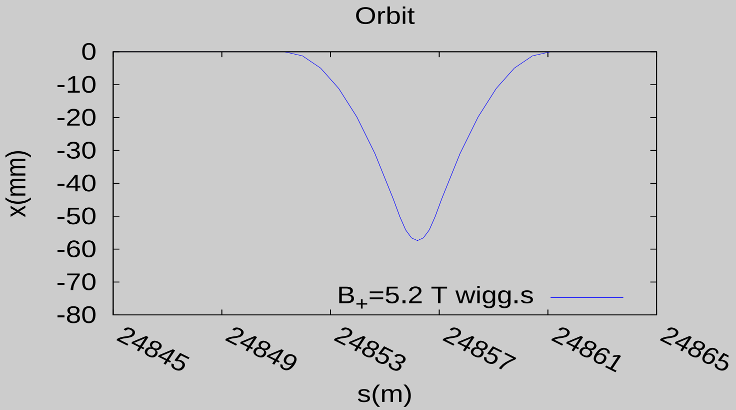
The emittance increase can be mitigated by choosing a shorter wiggler period, λ_w.

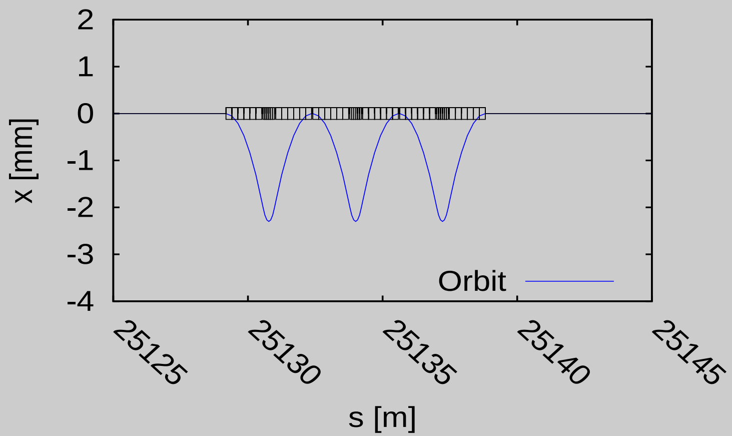
LEP-like

(orbit shown for B^+=5.2 T)

3 periods

(for B^+=0.7 T)





- Emittance is 100 pm with proposed wigglers turned on.
- Energy spread and τ_p as with previous design.

Polarization in *real* storage rings

Sokolov-Ternov effect
in the guiding dipole field

Perturbations
(v-bends, vertical orbit in quads etc.)

$\downarrow$ Polarisation

$\downarrow$ Depolarisation

$\searrow$ $\swarrow$

Equilibrium polarisation $\quad (< \boldsymbol{P}_{\infty}^{\mathbf{ST}})$

Tools

Accurate simulations are necessary for evaluating the *actual* polarization level to be expected.

- MAD-X used for simulating quadrupole misalignments and orbit correction
- SITROS (by J. Kewish) used for computing the resulting polarization. Tracking code with 2th order orbit description and non-linear spin motion. Used for HERA-e in the version improved by M. Böge and M. Berglund.

SLIM by A. Chao (and SITROS/SITF) may be used for linear calculations.

Simulations for a *toy* ring

Much simplified optics (only FODO cells and dispersion-free regions for accommodating the wigglers)

- $60^o/60^o$ FODO and $90^o/90^o$ FODO studied
- 4 wigglers at 45 GeV
- one BPM+CV close to each v-focusing quad
- δy^Q_{rms}= 200 μm
- orbit correction by SVD

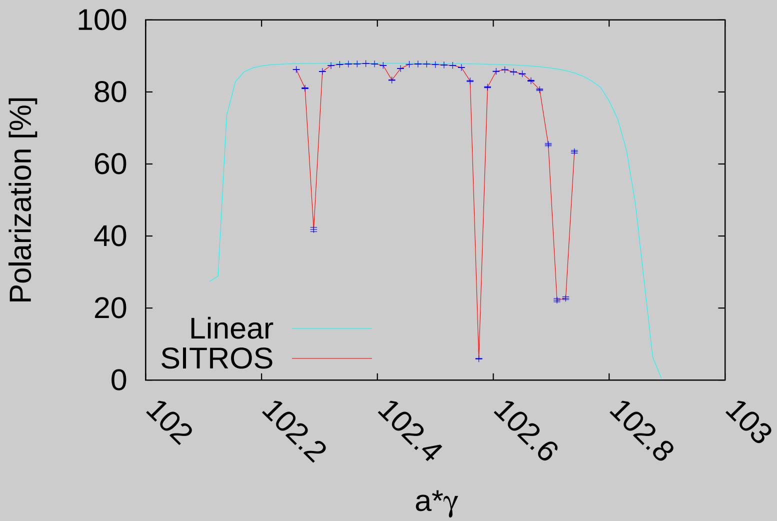
45 GeV beam energy – $90^o/90^o$ FODO

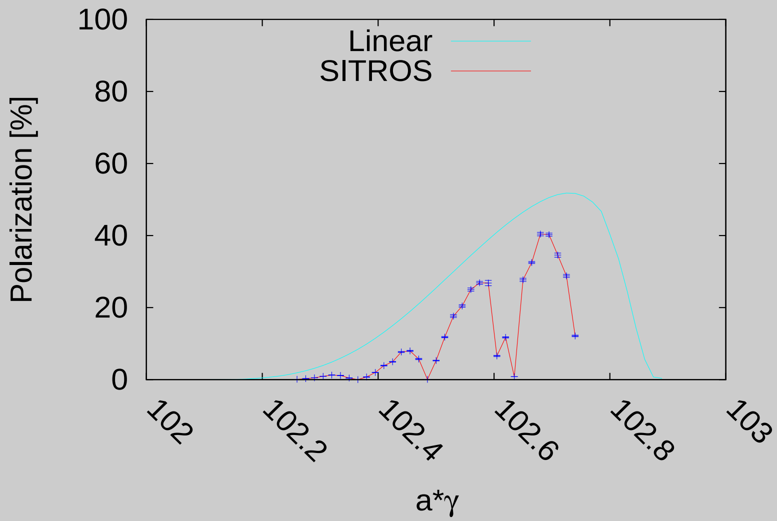
no BPMs errors

- y_{rms}=0.08 mm
- $|\delta\hat{n}_0|_{rms}$=0.3 mrad

with 10% BPMs calibration errors

- y_{rms}=1.4 mm
- $|\delta\hat{n}_0|_{rms}$ =10 mrad





45 GeV beam energy

with BPMs errors

+ harmonic bumps 45 GeV

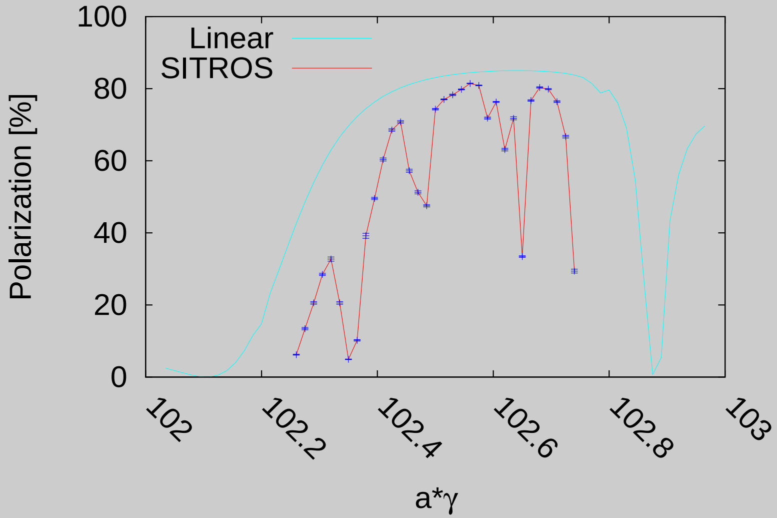
- $|\delta\hat{n}_0|_{rms}$=6.2 mrad

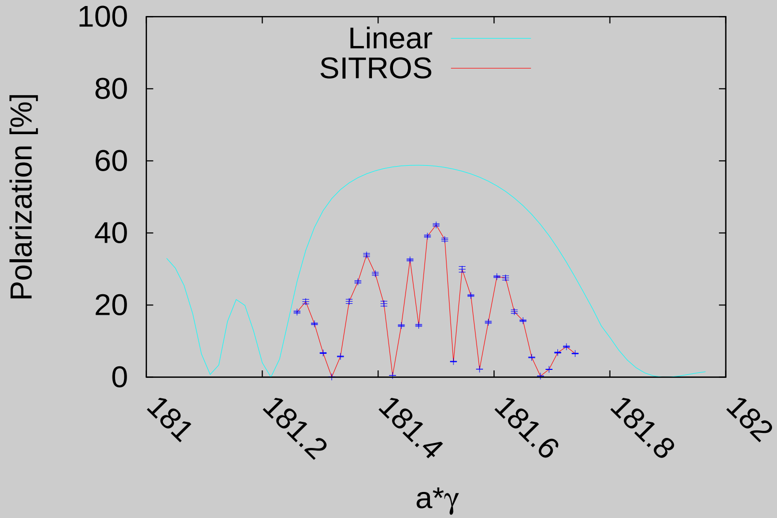
80 GeV beam energy

with BPMs errors

+ harmonic bumps 80 GeV

- $|\delta\hat{n}_0|_{rms}$ =14 mrad (it was 35 mrad after SVD)





$\leadsto$ Polarization feasible for calibration purposes at 45 and 80 GeV in presence of relatively large errors (quads and BPMs) for a much simplified ring.

Orbit sensitivity to misalignments

$$< z_{rms} >= F\, \delta z^Q_{rms} \qquad z = x, y$$

with

$$F \equiv \frac{1}{2\sqrt{2}|\sin \pi Q_z|}\sqrt{< \beta_z >}\sqrt{\Sigma_{i=1}^{NQ}\beta_i (k\ell)_i^2}$$

$$< z'_{rms} > \simeq \sqrt{\frac{< \gamma_z >}{< \beta_z >}} < z_{rms} >$$

For the β^*_y=1 mm optics it is $\hat{\beta}_y$=9.8 km at QC1R.
With $90^o/90^o$ FODO and q_y=0.2 and δy^Q_{rms}=200 μm

	F	$< y_{rms} >$ (mm)	$< y'_{rms} >$(mrad)
all quads	613	123	13
w/o IPs doublets	141	28	3
"toy" ring	65	13	0.1

"Tricks" needed for introducing misalignments errors in the simulation (!):

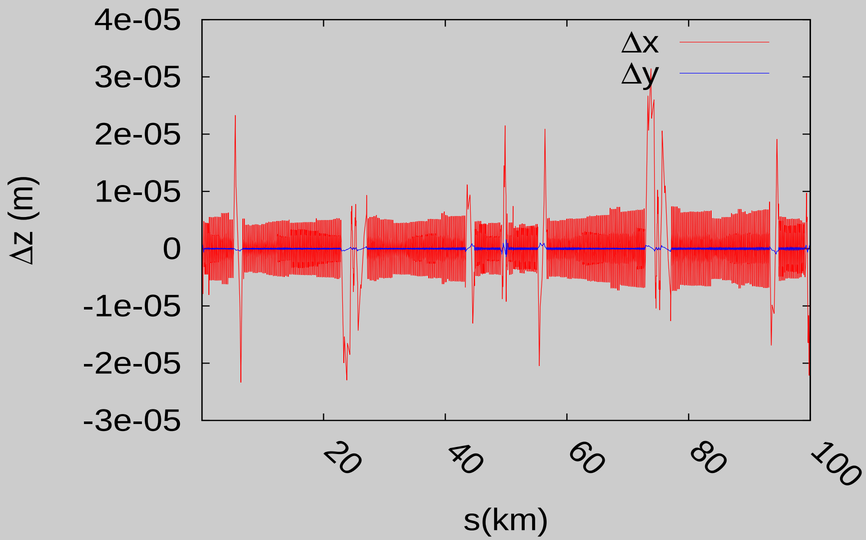
- Move tunes away from integer ("*set up*" tunes)
  - q_x: 0.1 $\rightarrow$ 0.2
  - q_y: 0.2 $\rightarrow$ 0.3
- Switch sextupoles off
- Add errors to "arc" quads in steps of 10 μm (!) and correct by each step with large number (some hundreds) correctors
- Add errors to each doublet quadrupole in steps of 5 μm (!!) and correct with close by correctors

In the process for each quadrupole the misalignment increment $\Delta\delta z^{Q_i}$ is kept *constant* .

A lengthy procedure not feasible in a *real* machine. In practice: use *"relaxed"* optics and one-turn steering through correction dipoles for establishing a closed orbit.

But for many seeds machine became *unstable* when sextupoles were turned on at the very end!

An example.
Sextupoles *off/on* but at 45% for getting a stable machine:
orbit is almost unchanged by the sextupoles.



Explanation of the "mystery": The phase advance between the sextupoles around the IPs being 180^o and their strengths having opposite signs, they produce a *coupling wave* when the beam offset at those sextupoles are *anti-symmetric* wrt IP. Indeed moving the betatron tunes closer, the sextupole strengths must be further reduced to get a stable machine.

$\rightsquigarrow$ The vertical beam position at those sextupoles must be extremely well controled!

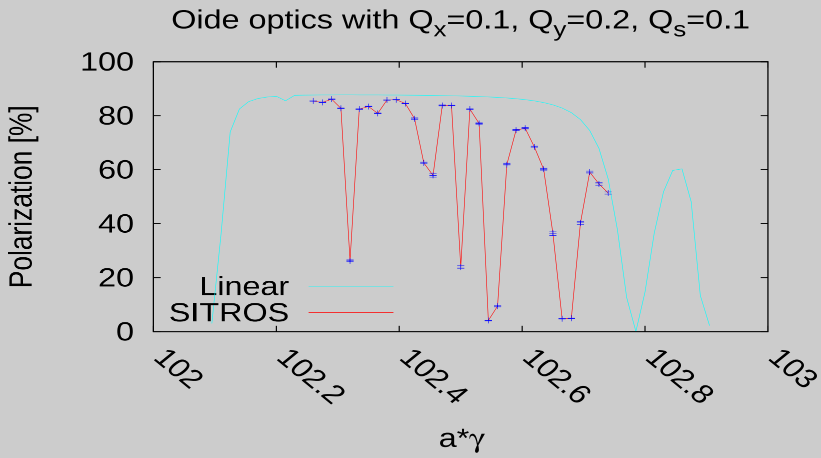
And now polarization!

45 GeV beam energy

45 GeV case with 4 wigglers (LEP-like) in dispersion free regions.

δy^{Q}_{rms}=200μm, but *no* BPMs errors: y_{rms}=0.049 mm

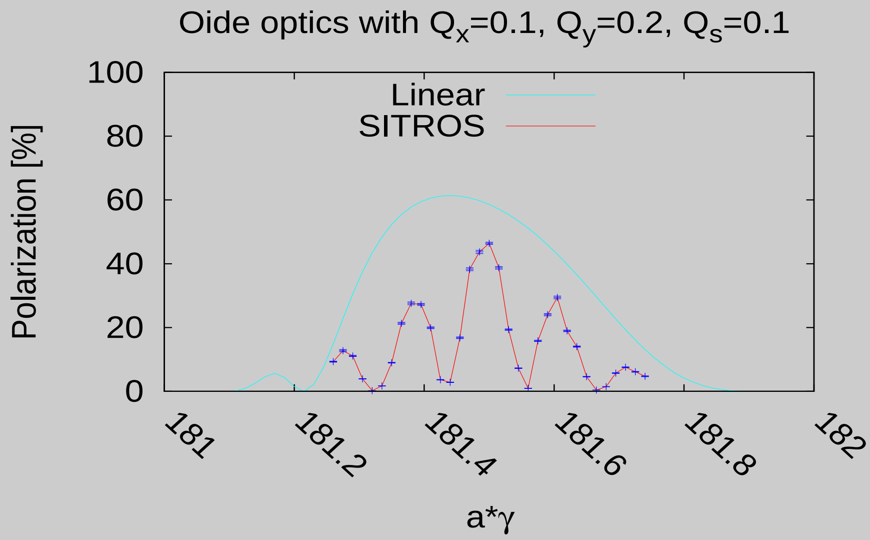
$|\delta\hat{n}|_{0,rms}$=0.4 mrad, no harmonic bumps



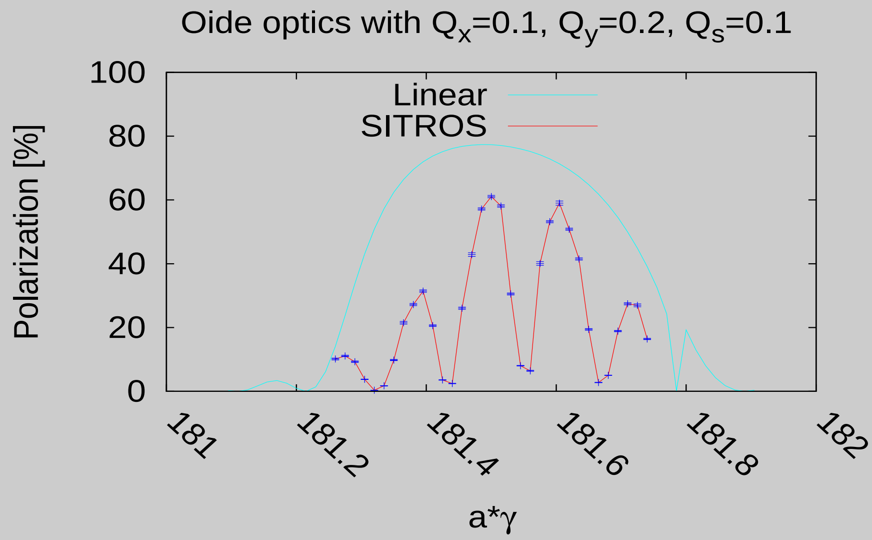
80 GeV beam energy

Same error realization as at 45 GeV: $|\boldsymbol{\delta\hat{n}}|_{\boldsymbol{0,rms}}$=2 mrad

w/o harmonic bumps

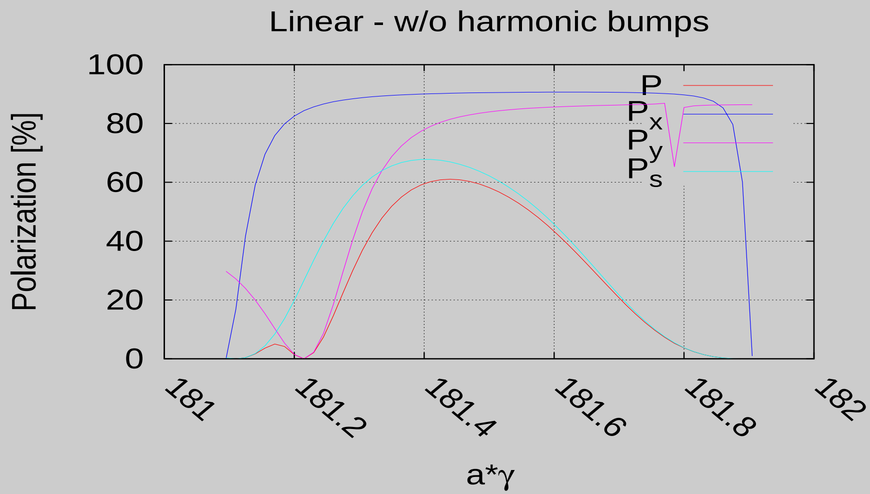


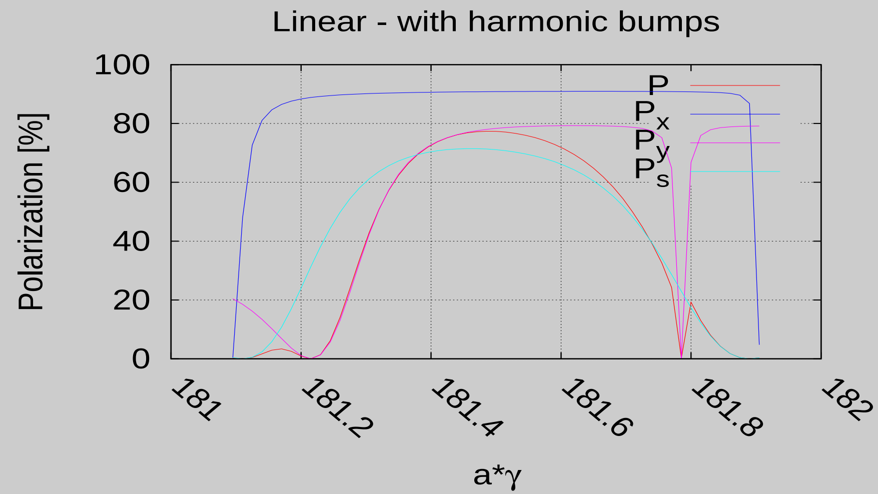
with



There is may be space for improvements:

the harmonic bumps increase ϵ_y from 12.8 pm to 19.4 pm and P_y get reduced





Vertical emittance preservation

FCC-$e^{\pm}$ design relies on ultra-flat beams (from http://tlep.web.cern.ch/)

	Z		WW	H	$t\bar{t}$
Beam energy [GeV]	45.6		80	120	175
ϵ_x [nm]	0.2	0.09	0.26	0.61	1.3
ϵ_y [pm]	1	1	1	1.2	2.5
β_x^* [m]	0.5	1	1	1	1
β_y^* [mm]	1	2	2	2	2
σ_x^* [μm]	10	9.5	16	25	36
σ_y^* [nm]	32	45	45	49	70

Vertical emittance is generated by

- non vanishing vertical closed orbit at quadrupoles which introduces radial magnetic fields and thus vertical dispersion: the orbit correction should take care of it;
- roll angle of the quadrupoles around $\hat{s}$ which introduces coupling between horizontal and vertical motion
  - generation of vertical dispersion if $D_x \neq 0$ at the tilted quad, out of reach for usual orbit correction;
  - transfer of the (large) horizontal emittance into the vertical; it can be described by *coupling functions*, $w^{\pm}$, which for a single source at s_{skq}, write

$$w_{\pm}(\theta) = -\frac{C_{\pm}^{skq}}{4\sin\pi Q_{\pm}} \mathrm{e}^{-iQ_{\pm}[s-s_{skq}-\pi\,\mathrm{sign}(s-s_{skq})]/R}$$

with $Q_{\pm} \equiv Q_x \pm Q_y$ and

$$C_{\pm}^{skq} \equiv \frac{\ell}{2}\sqrt{\beta_x\beta_y}\,\underbrace{\frac{e}{p}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right)}_{K_{sqk}}\mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

- Vertical dispersion may be measured by BPMs.
- Coupling functions may be measured by BPMs with Turn-by-Turn capability.

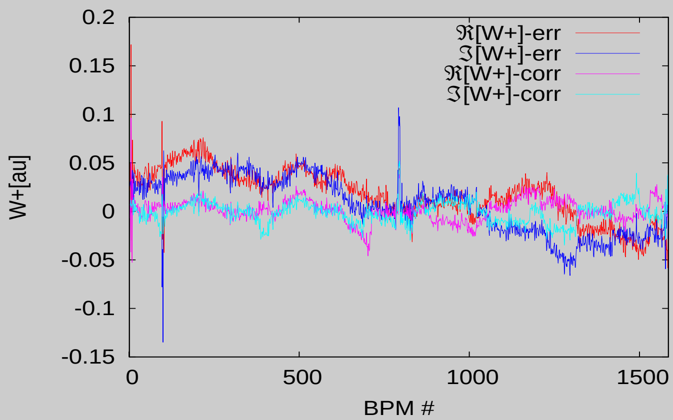
It is possible to correct vertical spurious dispersion and betatron coupling through skew quadrupoles.

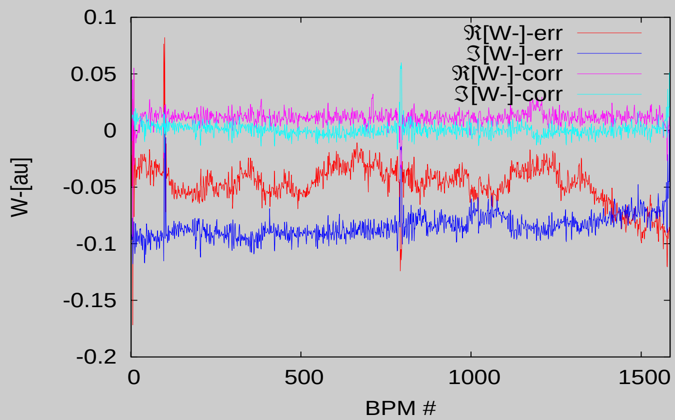
They introduce extra-radial fields which may affect polarization!

Some results of coupling/dispersion correction

- δy^{Q}_{rms}=200 μm (including doublets)
- 250 μrad quadrupole roll angle (including doublets)
- 1086 BPMs w/o errors
- orbit corrected with 1086 CVs down to y_{rms}=0.05 mm
- coupling/dispersion correction with 289 skew quadrupoles

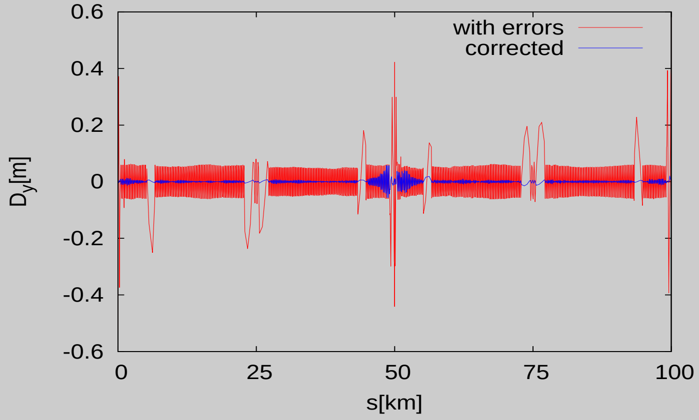
Coupling functions at BPMs





Global coupling decreased from 0.019 to 0.002.

Vertical dispersion



Effect on emittance at 45 GeV (MAD-X)

	ϵ_x (pm)	ϵ_y (pm)	ratio
design goal	90	1	0.011
before orbit correction	-	-	-
after orbit correction	88.1	8.4	0.095
+ coupling/dispersion correction	88.6	0.9	0.010

80 GeV

	iteration#	ϵ_x (pm)	ϵ_y (pm)	ratio
design goal	-	260	1	0.004
unperturbed	-	279	0	0
after orbit correction	-	270.6	31.7	0.117
+coupling/dispersion correction	1	279.5	2.5	0.009
	2	280.5	1.3	0.005
	3	280.2	0.8	0.003

Right choice of weights for the 5 quantities to be minimized is very important!

Effect on polarization

β_y^*=1 mm optics with:

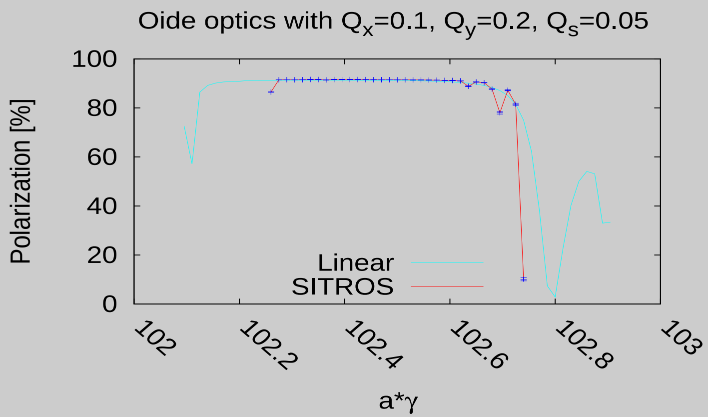
- δy_{rms}^Q=200 μm,
- 250 μrad quadrupole roll angle,
- in addition: maximum horizontal mis-alignments (0.5 μm) to get a stable machine w/o correction (no horizontal correction yet implemented!),
- y_{rms}=0.05 mm after orbit correction by SVD (no BPMs errors).

$$< |K| \times \ell >_{skews} \simeq 0.00022\,\mathrm{m}^{-1}$$

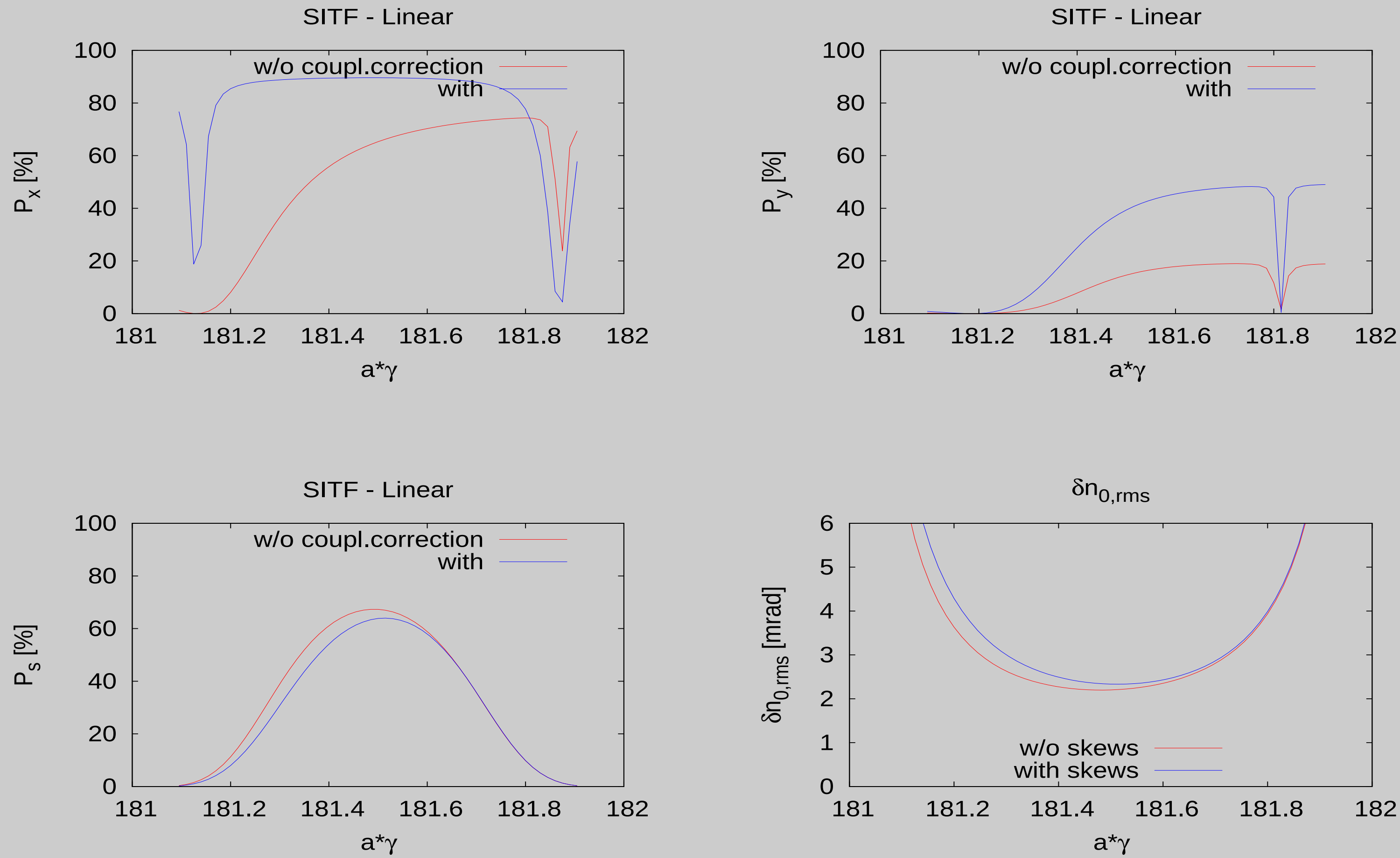
$$x_{co} \simeq 0.000037\,\mathrm{m} \quad \text{(radiation + coupling)}$$

$$\rightarrow \theta_y^{orb} \simeq 8.1\,\mathrm{nrad}$$

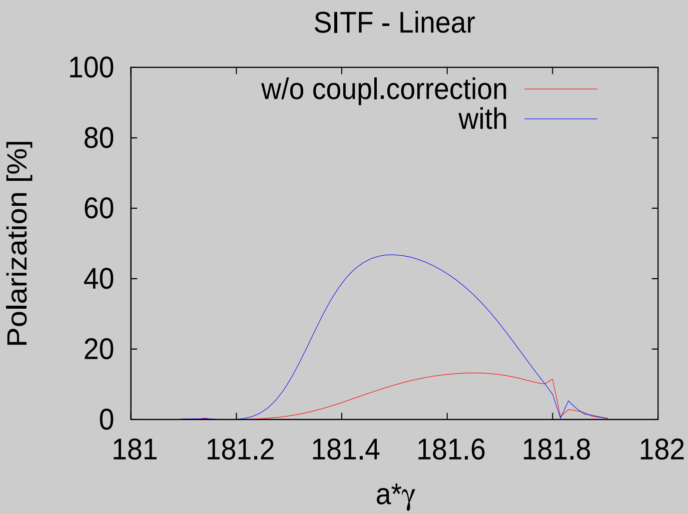
Polarization after coupling/dispersion correction

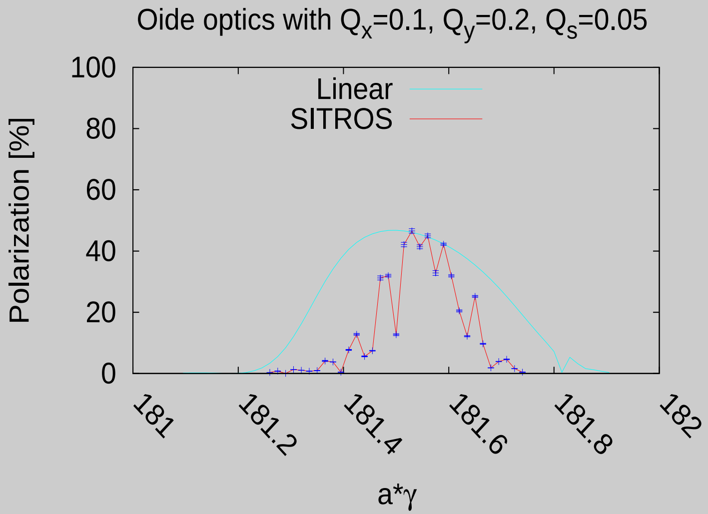


At 80 GeV the spin kick is $\mathbf{181} \times \mathbf{8.1}$=1.4 $\boldsymbol{\mu}$rad.



The effect on $\boldsymbol{\delta\hat{n}_0}$ is still small and the overall effect of the coupling/dispersion correction is beneficial.





Further studies

Improve closed orbit simulations

- Add
  - horizontal quadrupoles misalignement
  - horizontal correctors
  - BPMs close to h-focusing quads
- Include BPMs errors
  - couple quad/BPM misalignments: is there a difference?
- Reduce misalignments to smaller yet realistic values

New parameters and conditions

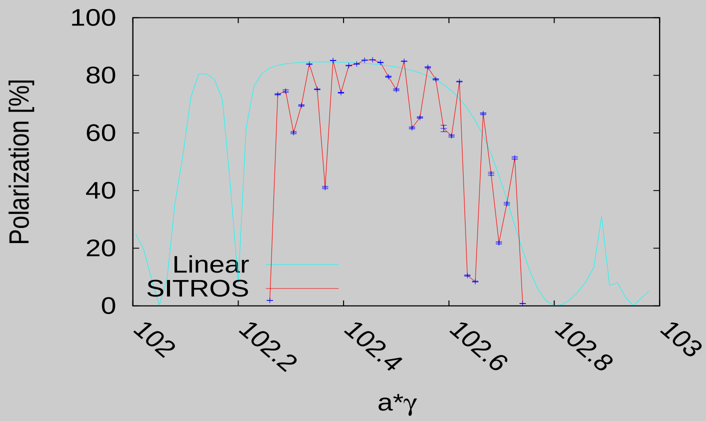
- 8 doublet quads
  - δx^q_{rms}=δy^q_{rms}=50 μm
  - 50 μrad roll angle
- all other quads
  - δx^q_{rms}=δy^q_{rms}=100 μm
  - 100 μrad roll angle
- 3148 BPMs *coupled* to near-by-quadrupole.

Orbit corrected with 1574 CHs +1586 CVs (one particular seed) down to

- x_{rms}= 75 μm w/o BPMs errors $\rightarrow$ 92 μm with BPMs offset&tilt
- y_{rms}= 41 μm w/o BPMs errors $\rightarrow$ 57 μm with BPMs offset&tilt

For the seed presented:

a) machine stable with errors in the quads only, $|C^{-}| \simeq 0.015$ and $D_y^{rms} \simeq 7$ mm



b) stable but large coupling ($|C^{-}|$=0.074) and emittance ($\epsilon_x \simeq 140$ pm, $\epsilon_y \simeq 17$ pm at 45 GeV) with BPMs offset and tilt;

c) Twiss failure with calibration errors as small as 0.1% (?).

Remedies, at least for case b)

- correction of dispersion rather than orbit
- measurement of BPM tilt ϕ from "*true*" and measured orbit values

$$x_m = \ \ x_t \cos\phi + y_t \sin\phi$$
$$y_m = -x_t \sin\phi + y_t \cos\phi$$

$$\phi = \tan^{-1}[(x_m y_t - x_t y_m)/(x_m x_t + y_m y_t)]$$

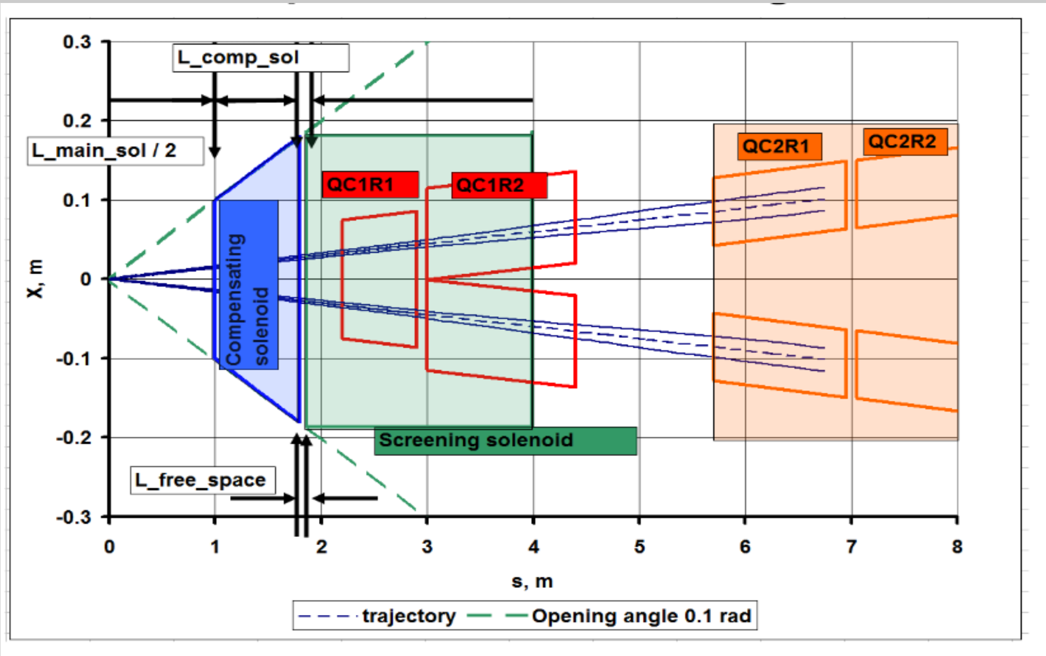
By exciting a bump in one plane around a given BPM and measuring the orbit change in both planes, ϕ can be computed *without need of a model*, because either x_t or y_t are zero.

Effect of experiment solenoid

Experiment solenoids

- tilt the polarization axis $\hat{n}_0$
- shift the spin tune breaking the $a\gamma$ relationship.

B_s=2 T, ℓ_s^m=2 m, θ_{cross}=30 mrad



(S. Sinyatkin, FCCee IR Workshop 2017)

45 GeV:

x_{rms}=14.9 μm

y_{rms}=0.9 μm

$|\delta\hat{n}_0|_{rms}$=0.010 mrad

P_{lin}=88%

80 GeV:

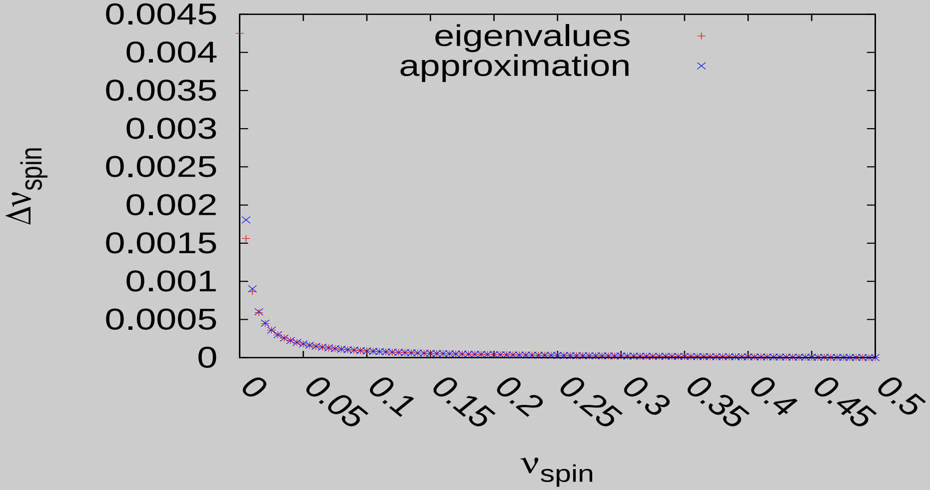
x_{rms}=4.7 μm

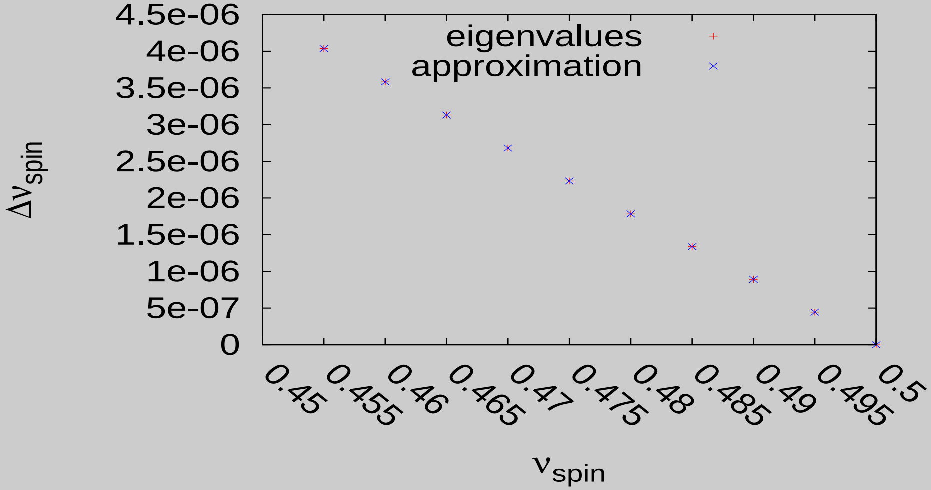
y_{rms}=0.5 μm

$|\delta\hat{n}_0|_{rms}$=0.001 mrad

P_{lin}=88%

Solenoid spin tune shift with ℓ_{sol}=2 m and B_{sol}=2 T at 45 GeV, solenoid axis aligned with the beam





At ν_{spin}=0.475 it is $\Delta\nu_{spin} \simeq 2.2\times10^{-6}$ [a] which gives a energy error of 1 KeV for one *uncompensated* solenoid.

For the actual configuration, with 2 compensated solenoids at 45.156 GeV it is (SLIM) $\Delta\nu_{spin} \simeq 1.6\times10^{-6}$ ie $\Delta E \simeq$0.71 KeV.

The effect of solenoids can be also measured.

[a] in full agreement with SLIM for one solenoid aligned with the beam

Some considerations on energy calibration through resonant depolarization

Many phenomena affecting the beam energy at collision will be described by A. Bogomyagkov.

The relationships $\nu_{spin} = a\gamma$ holds for a purely *planar* ring.

K. Yokoya (1988) and Barber et al. (1994) spin tune shift (*first order*)

$$\Delta\nu_s^{(1)} = \frac{1}{2\pi} R(a\gamma + 1) \int_0^{2\pi} d\theta (\hat{n}_0 \cdot \hat{y}) x''_{co}$$

that is $\Delta\nu_s^{(1)}$=0 always for a planar designed ring. The *second order* term is

$$\Delta\nu_s^{(2)} = \frac{1}{4\pi} R^2 (a\gamma + 1)^2 \Im\Big[\frac{1}{e^{-i2\pi\nu_s^0} - 1} \int_0^{2\pi} d\theta\, h^*(\theta) y''_{co} \int_\theta^{\theta+2\pi} d\theta' h(\theta') y''_{co}\Big]$$

with

$$h(\theta) = (\hat{m}_0 + i\hat{l}_0) \cdot \hat{x}$$

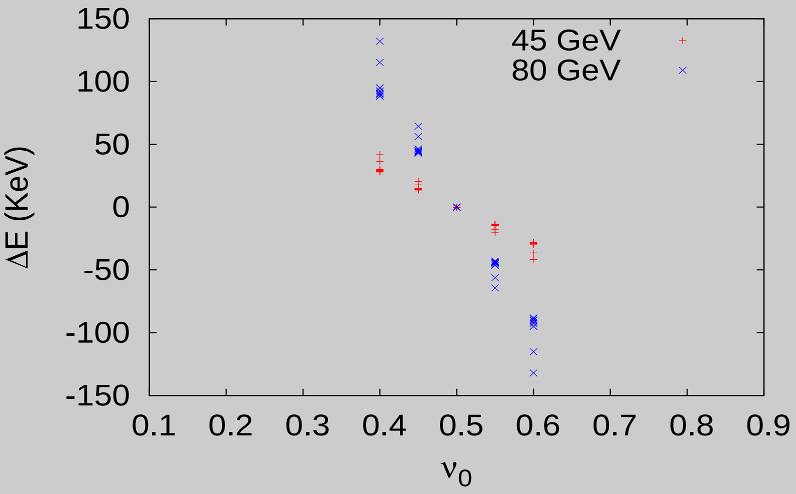
$$y'' = -K(y - \delta_y^Q) + \Big(\frac{\Delta B}{B\rho}\Big)_{cor}$$

The effect of closed orbit distortion has been evaluated for LEP by using a simplified model by R. Assmann and J. P. Koutchouk (1994) who found that for half-integer ν_s^0 it is $\Delta\nu_s$=0 in first and second order in the extra-spin rotations. For $\nu_s^0 \neq 0.5$ it is

$$< \Delta\nu_s >= \frac{\cot \pi\nu_s^0}{8\pi}(a\gamma)^2 \Big[< \Sigma_q (K\ell)_q^2 y_q^2 > + < \Sigma_k \theta_k^2 > \Big]$$

with $y_q \equiv K(y - \delta_y^Q)$.

Evaluating this expression over 10 seeds (β_y^*=1 mm optics, w/o BPMs errors)



Effect of RF electric field (term $\vec{\beta} \times \vec{\mathcal{E}}_{RF}$ in BMT-equation)[a]

	ΔE (KeV)
45 GeV	2 $\times y'_{rms}$
80 GeV	16 $\times y'_{rms}$

y'_{rms} = rms slope in mrad. With

$$< y'_{rms} > \simeq \sqrt{\frac{< \gamma_y >}{< \beta_y >}} < y_{rms} > \simeq 0.1 < y_{rms} >$$

the contribution from the RF electric field should be small.

[a] From Yu. I. Eidelman et al. formulas

Summary and outlook.

Results for the 45 GeV and 80 GeV case have been presented for the β^*_y=1 mm, $90^0/90^0$ deg FODO.

- The large sensitivity of the orbit to vertical misalignment of quads makes the orbit correction difficult. However a machine set-up would take place always with a "relaxed" optics.
- The orbit at the SYL and SYR sextupoles must be extremely well controled.
- The exercise on the "toy" ring has shown the importance of the BPMs errors on the orbit correction quality
  - BPMs errors <u>must</u> be included

- At 80 GeV, $\boldsymbol{\delta\hat{n}_0}$ due to the *same* misalignments increases and although the energy spread is the same as at 45 GeV with wigglers, the polarization is lower!
  - Large harmonic bumps cause a vertical emittance increase.
    - ∗ With the toy ring it was proven that using dispersion-free 5-coils harmonic bumps, $\boldsymbol{\delta\hat{n}_0}$ can be corrected w/o spoiling the vertical emittance.

End of the Episode

Thanks!

Polarization wigglers

τ_p may be reduced by introducing wigglers, a sequence of vertical dipole fields with alternating direction

$$\tau_p^{-1} = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3} \Big] \qquad F \equiv \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0 C}$$

Polarization

$$P_\infty = \frac{8}{5\sqrt{3}} \frac{\oint ds \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds \frac{1}{|\rho|^3}} \propto \tau_p \Big[\int_{dip} ds \, \frac{\hat{B}_d \cdot \hat{n}_0}{|\rho_d|^3} + \int_{wig} ds \, \frac{\hat{B}_w \cdot \hat{n}_0}{|\rho_w|^3} \Big]$$

$\hat{n}_0 \equiv \hat{y}$ in a perfectly planar ring.

Constraints:

- $x' = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, B_w = 0$ (vanishing field integral)
- $x = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, sB_w = 0$ (true for symmetric field)
- P large $\Rightarrow \int_{wig} ds \, B_w^3$ must be large

Derbenev-Kondratenko expression for equilibrium polarization

$$P_{\rm DK} = \frac{8}{5\sqrt{3}} \frac{\oint ds < \frac{1}{|\rho|^3}\hat{b}\cdot(\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{s})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \delta})^2\right] >}$$

with $\hat{b} \equiv \vec{v} \times \dot{\vec{v}}/|\vec{v} \times \dot{\vec{v}}|$

$\partial\hat{n}/\partial\delta$ ($\delta \equiv \delta E/E$) quantifies the depolarizing effects resulting from the trajectory perturbations consequent to photon emission. Corresponding polarization rate

$$\tau_{\rm DK}^{-1} = P_{\rm ST}\frac{r_e\gamma^5\hbar}{m_0 C}\oint < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left(\frac{\partial \hat{n}}{\partial \delta}\right)^2\right] >$$

Perfectly planar machine: $\partial\hat{n}/\partial\delta$=0.

In presence of radial fields: $\partial\hat{n}/\partial\delta \neq$0 and large when

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer} \qquad \nu_{spin} \simeq a\gamma$$

Usually the dominant higher order resonances are the *synchrotron sidebands* of the first order resonances.

Some considerations on energy calibration through resonant depolarization

It is based on the relationship $\nu_{spin} = a\gamma$.

Many phenomena affecting the CM energy at collision will be described by A. Bogomyagkov.

Effects of various nature

- "pitfalls" in determining the mean energy from depolarization
- beam energy dependence upon
  - azimuth (*sawtooth* effect)
  - orbit length $\longrightarrow$ "continuous" monitoring
- short luminosity lifetime (1-3 hours) calls for top-up injection $\longrightarrow$ use of non-colliding bunches for polarization
  - non-colliding bunches may have a different energy
- a basic problem: is it always $\nu_{spin} = a\gamma$?

The solenoid spin tune shift must be taken into account when the energy is measured by resonant depolarization. Effect of one solenoid:

$$\Delta\nu_{spin} \simeq -\frac{1}{2\sin(2\pi\nu_{spin})}\left[\cos\phi\cos(2\pi\nu_{spin}) - \cos(2\pi\nu_{spin}) + \cos\phi - 1\right]$$

with

$$\phi = (1+a)\frac{e}{p}B_{sol}\ell_{sol}$$

The spin tune may be also computed from the eigenvalues of the perturbed one-turn spin transport matrix

$$M = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} \cos(2\pi\nu^0_{spin}) & -\sin(2\pi\nu^0_{spin}) & 0 \\ \sin(2\pi\nu^0_{spin}) & \cos(2\pi\nu^0_{spin}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$