Gluon and Quark Fragmentation from LEP to FCC-ee

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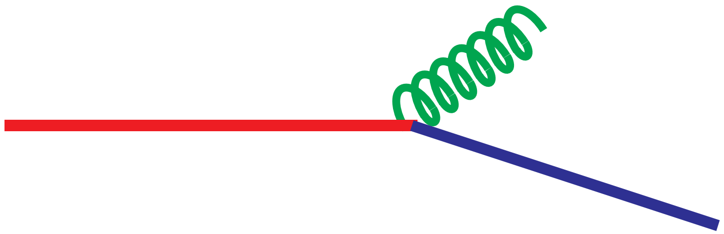
Results from

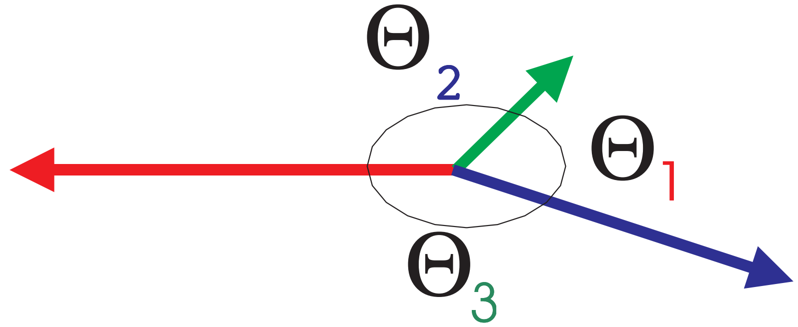
Aleph, Delphi, L3, Opal

Outline

- Basic Techniques at LEP
  Intrinsic Limitations for Gluon/Quark Comparisons
- Expectations and Basic Results
- Colour Coherence
- Fragmentation Functions from 3-Jet Events
- (Sub-)Jet-Rates, Splitting Kernels
- The Multiplicity in 3-Jet Events & 2 Gluon-Systems
- Identified Particles and Leading Systems
- What Can/Should be Done @ FCC-ee?

Comparing Gluons and Quarks in e^+e^- 3 Jet Events





- assign jets $\leftrightarrow$ partons at tree level
- identify g,q using E-ordering and/or displaced vertices (heavy q's)
- compare g to mix of q & g jets form light quark events
- unfold light-, b-quark, g-contribution by inverting purity matrix
- initially for symm. events (Y, Mercedes) generalised to all topologies $\rightarrow$ allows kinematical (topological) studies

- assignment of particles to jets requires algorithms
- determine parton kinematics from event topology
- dynamical studies require Lorentz invariant scales $\rightarrow$ transverse momentum type scales:

$$\kappa = Q = E_{jet} \cdot \sin\frac{\theta_{ij}}{2} \quad (\widehat{=} \frac{\sqrt{s}}{2})$$

Intrinsic Limits of g and q Comparison

- 3 Jet events not $q\bar{q}$ or gg systems
- Coherent emission from $q\bar{q}$g system
- Parton $\leftrightarrow$ jet identification (tree level);
  bias and smearing of parton properties due to hadronisation
- Finite energy; mass/valence particle effects stronger for gluons
- Overlap between jets $\rightarrow$ dependence on algorithm & on implicit cuts

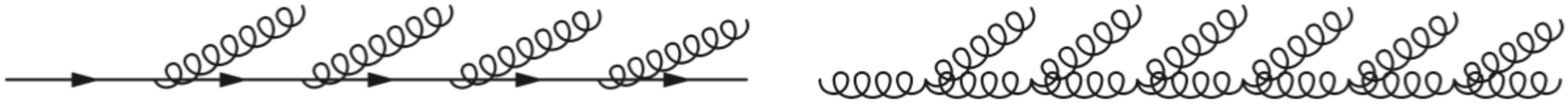
Minimise/measure difficulties by

- analysing only 3-jet event multiplicity
- use fully symmetric situation (may require boosting)
- analyse mainly fast hadrons (high x fragmentation)

- study energy-dependence

Expectations and Basic Results

Expect: hadrons (mainly) come from radiated soft gluons



Splittings: q-jets: $q \to qg$; g-jets: $g \to gg$

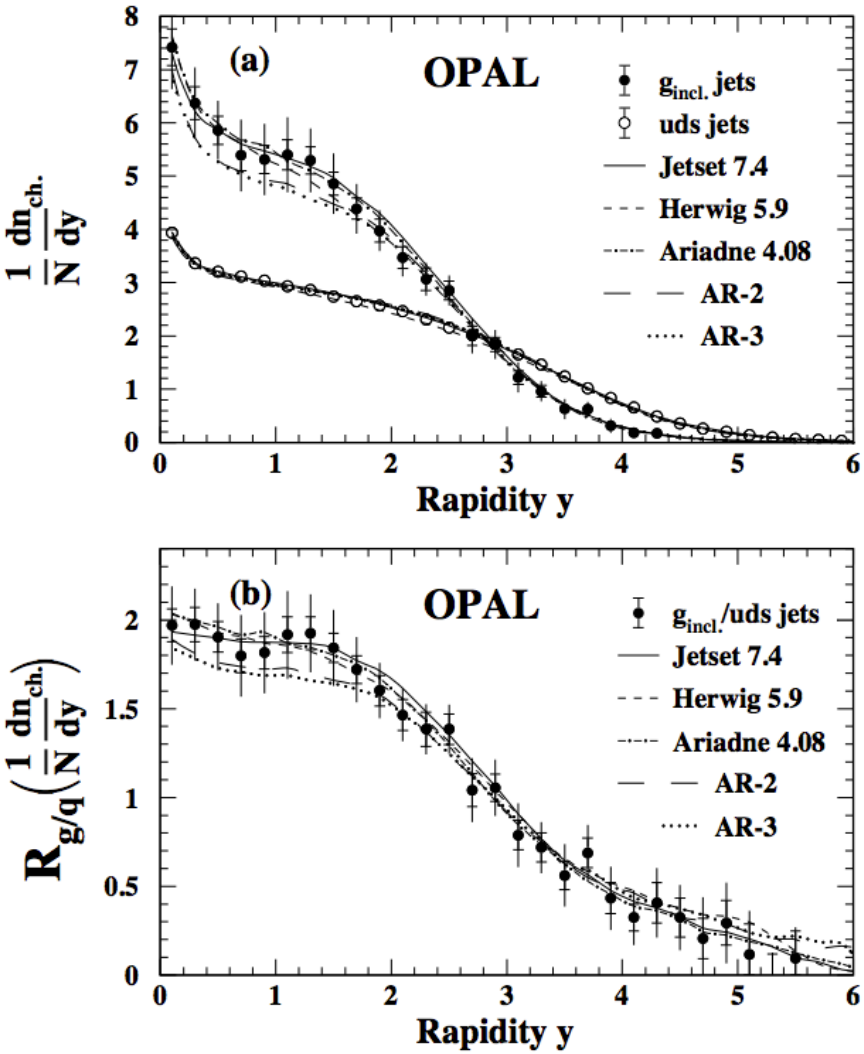
$\rightarrowtail$ Hadron multiplicity ratio $r_n = N_{gg}/N_{q\bar{q}} = C_A/C_F$=9/4=2.25

Similarly: C_A/C_F times bigger scaling violations; higher $p_\perp$ . . . .

Cleo	$E_{\rm Jet} < 3.5{\rm GeV}$	$r_n = 1.04 \pm 0.02 \pm 0.05$
HRS	$E_{\rm Jet} = 9.7{\rm GeV}$	$r_n = 1.29 \pm 0.2({\rm stat.})^{+0.21}_{-0.20}({\rm syst.})$
Tasso	$E_{\rm Jet} = 11{\rm GeV}$	$r_n \simeq 1.$
Opal	$E_{\rm Jet} = 24.5{\rm GeV}$	$r_n = 1.02 \pm 0.04^{+0.06}_{-0.00}$
Opal	$E_{\rm Jet} = 24{\rm GeV}$	$r_n = 1.25 \pm 0.02 \pm 0.03$
Aleph	$E_{\rm Jet} = 24{\rm GeV}$	$r_n = 1.249 \pm 0.084 \pm 0.022$
Delphi	$\overline{E_{\rm Jet}} = 24{\rm GeV}$	$r_n = 1.241 \pm 0.015 \pm 0.025$
Opal	$E_{\rm Jet} = 40{\rm GeV}$	$r_n = 1.552 \pm 0.041 \pm 0.061$

Observe increase of r_n with energy.

What is going on?



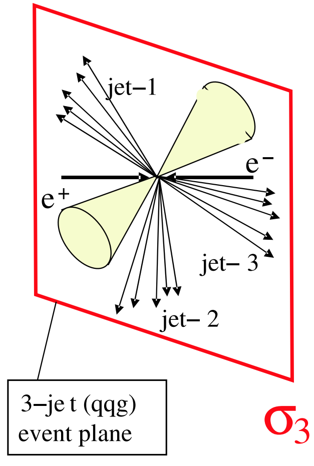
OPAL data:
g in one hemisphere recoils wrt 2 b-jets
($E_g = 40$ GeV, $\kappa \sim 37$ GeV)

compare to
q from "2-jet" event
($E_q = \kappa = 45.6$ GeV)

- small y
  hadrons produced first in time;
  $r = R \lesssim 2$; very close to expectation
  deviation due to
  - difference in scale (?),
  - coherent emission (?)
- $y > 3$; $R < 1$ more hadrons from q than g; diminishes overall ratio.
  - due to valence quarks/finite energy!

Coherent Particle Production at Large Angles

- Measure soft gluons/hadrons at large angles:
  Large wave-length $\rightarrow$ small resolution $\rightarrow$ coherent emission



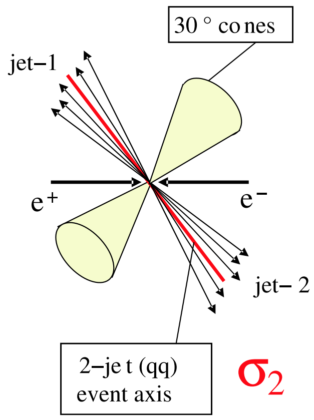
- Compare gluon radiation $\perp$ to $q\bar{q}g$ plane $\leftrightarrow$ $\perp$ to $q\bar{q}$ axis.
  Effective colour-charge depends on event topology

$$\frac{N_\perp^{q\bar{q}g}}{N_\perp^{q\bar{q}}} = \frac{C_A}{C_F} \cdot r_t = \underbrace{\frac{C_A}{C_F}}_{\text{colour factor}} \cdot \frac{1}{4}\left[\widehat{q\,g} + \widehat{\bar{q}\,g} - \underbrace{\frac{1}{N_C^2}\widehat{q\,\bar{q}}}_{\text{destr. interf.}}\right]$$

Khoze, Ochs, Lupia

$$\widehat{i\,j} = 2\sin^2\frac{\Theta_{ij}}{2}$$

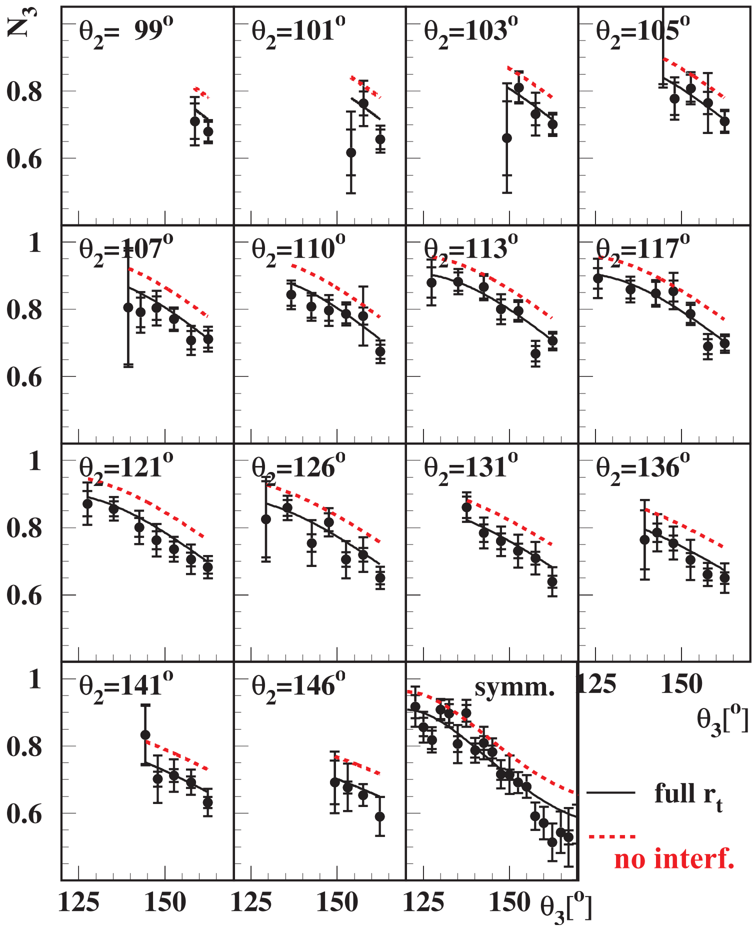
Emitter is $q\bar{q}$-like for $r_t \sim 0.5$,
gg-like for $r_t \rightarrow 1$.
Ratio $\propto$ to C_A/C_F in LO! NO parameters, absolute prediction!

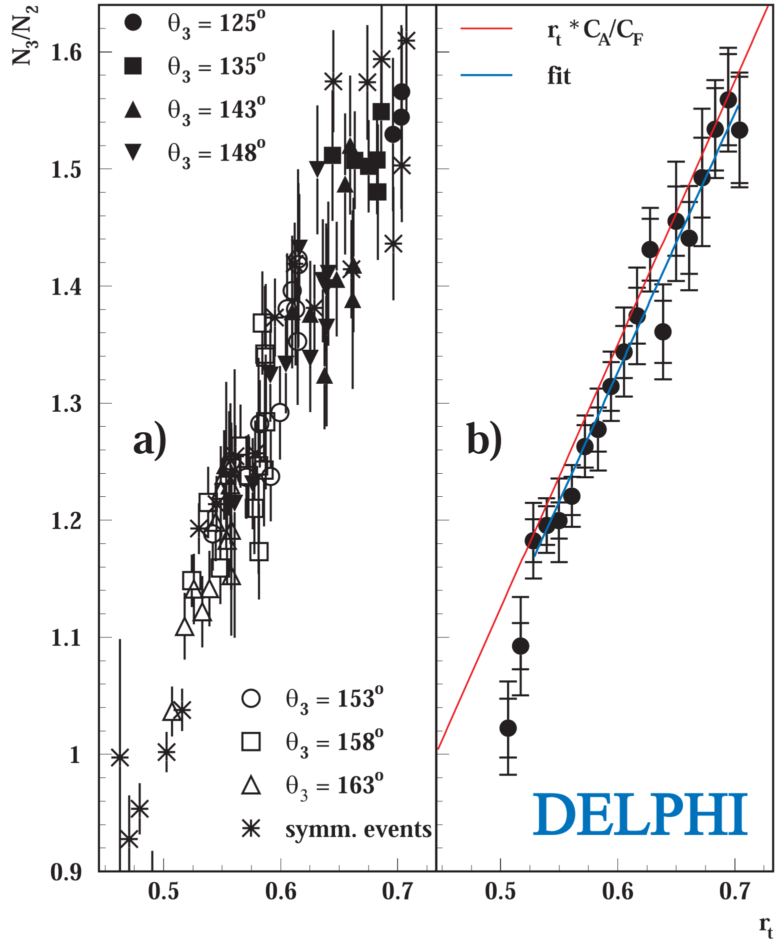


- Experimentally identify partons with k_t-jets (at fixed y_{cut}):
  defines 2 and 3-jet events, excludes $\geq$ 4-jet events $\leftrightarrow$ LO

  Compare multiplicity $\perp$ to 3-jet plane to the one $\perp$ 2-jet axis
  Systematics: variation of y_{cut}, θ_{cone} and cluster algorithms

Compare to Data





Homogenous straight line fit to symmetric & general topologies:

Colour coherence & destruct. interference observed!

$$C_A/C_F = 2.211 \pm 0.014_{(\mathrm{stat.})} \pm 0.045_{(\mathrm{sys.})}$$

Quark Fragm. Funct. in 3 Jet Events

Extract scale dependent fragmentation functions of quarks (and gluons) from 3 jet events.

Compare flavour-inclusive

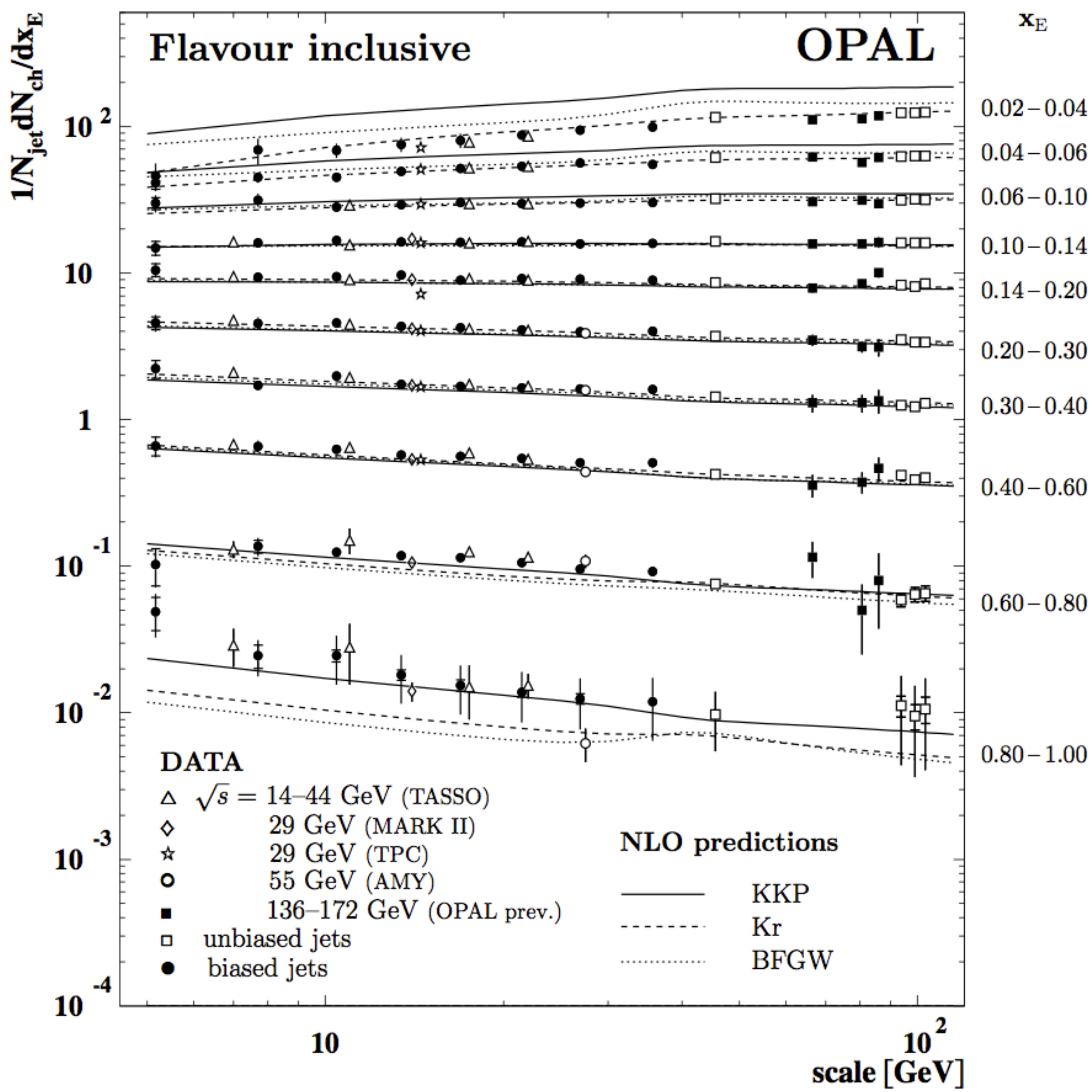
- FF from e^+e^- at different $\sqrt{s}/2$
- FF from quark jets in 3 jet events

$$\kappa = E_{jet} \cdot \sin\frac{\theta_{ij}}{2} \qquad \text{quark}$$

$$p_\perp = \frac{1}{2}\sqrt{\frac{s_{qg}s_{\bar{q}g}}{s}} \qquad \text{gluon}$$

to theoretical parameterisations (KKP –, Kr - -, BFGW . . .)

Reasonable agreement for quarks!



Gluon Fragm. Funct. in 3 Jet Events

Compare

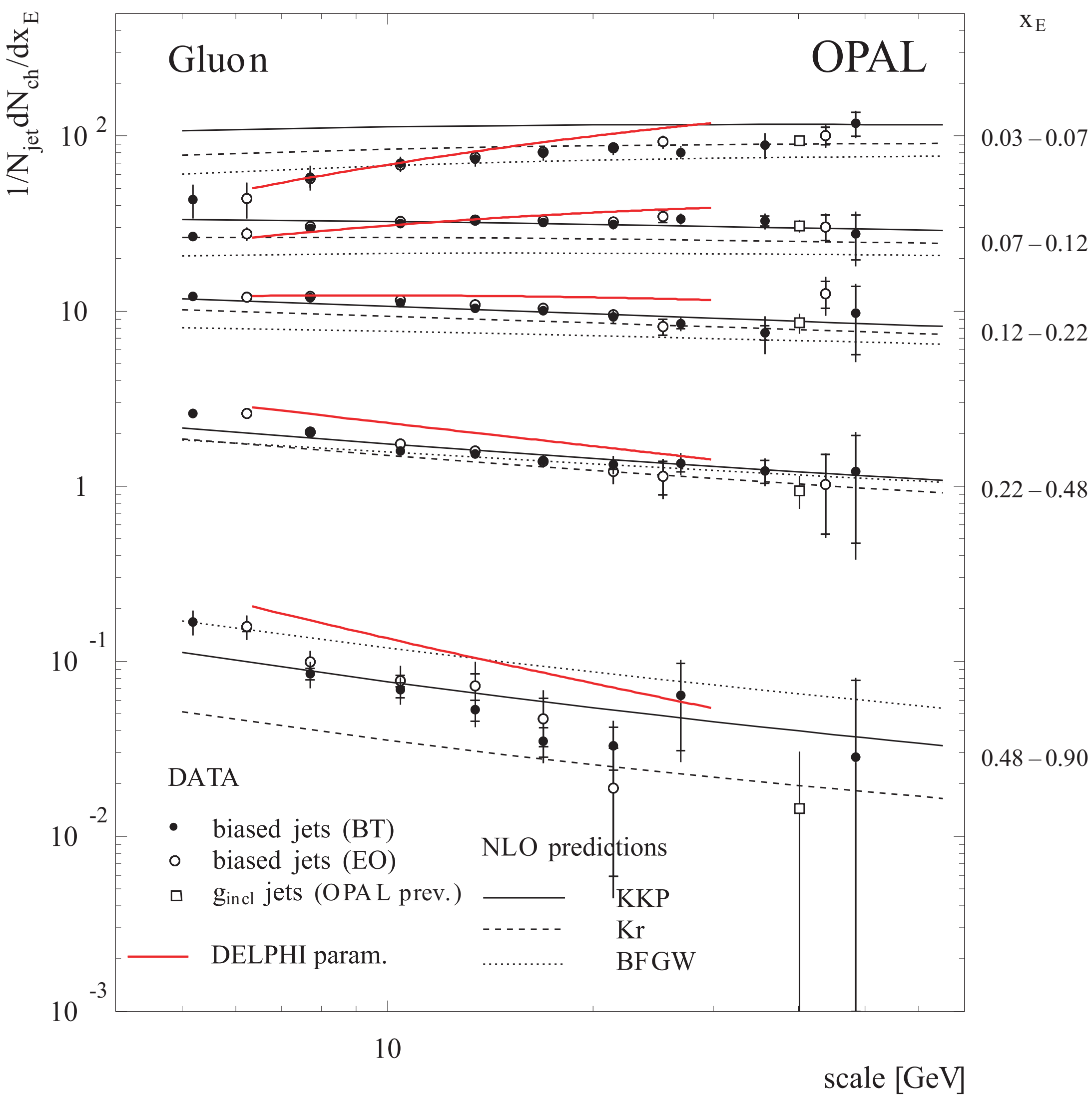
- OPAL's "inclusive" and "boosted" analyses
- older DELPHI 3 jet analysis (parameterised –, different x bins)

to theoretical parameterisations (KKP –, Kr - -, BFGW . . .)

Agreement OK

Ratio g/q-energy slopes consistent with $C_A/C_F = 9/4$

But:
g-slope at high x stronger than expected from theory



Influence of Hadronisation Smearing

Jets measured from hadrons, parton properties influenced by hadronisation!

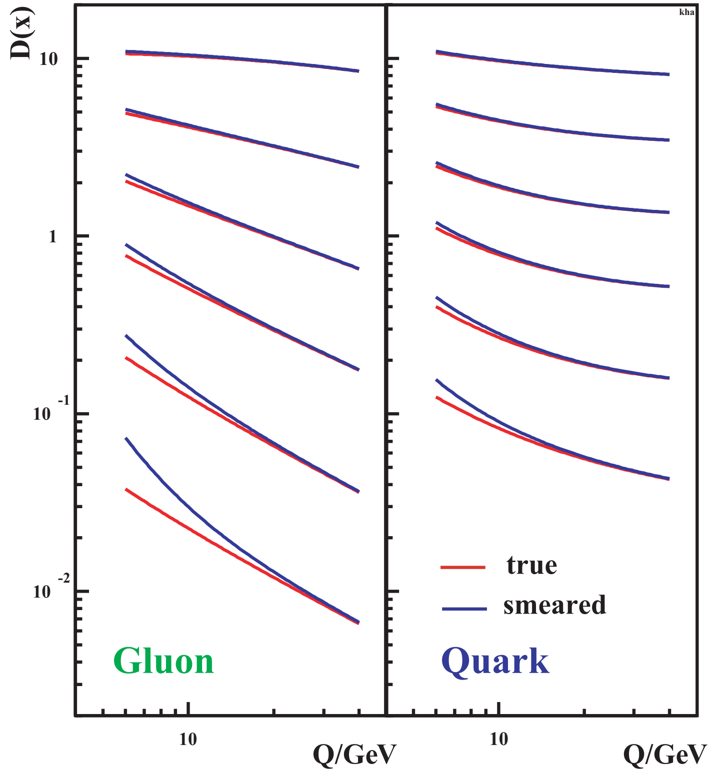
$z = E_{hadron}/E_{parton}$ replace parton by jet energy $\rightarrow z = E_{hadron}/E_{jet}$

- Hadronisation smearing
  $\rightarrow$ jet-parton $\sphericalangle$-resolution $\sim 3^\circ$ at Z
  $\rightarrow$ parton energy smearing (tube model)
  $\propto\sim 1/E_{parton}$
- Effect on FF's is $\propto$ slope of FF
  $\Rightarrow$ strong overestimate of
  gluon FF at small E and high x

This principle problem is there for
any jet analysis!

- Less of a problem for multiplicities
  (small "slow" variations)

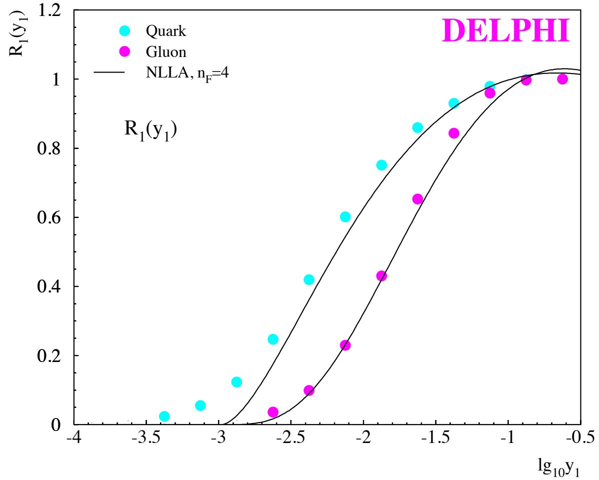
Cure (?) hadronisation unfolding $\rightarrow$ systematics!



Crude analytical estimate!

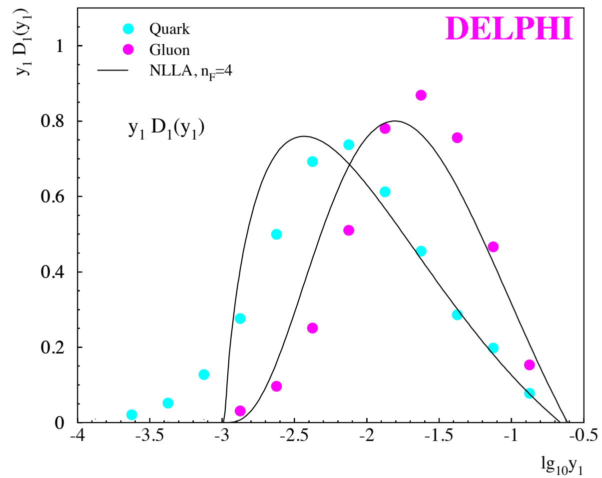
Sudakov Δ, Jet Rate $R_1(y)$ & Splitting Kernels Γ

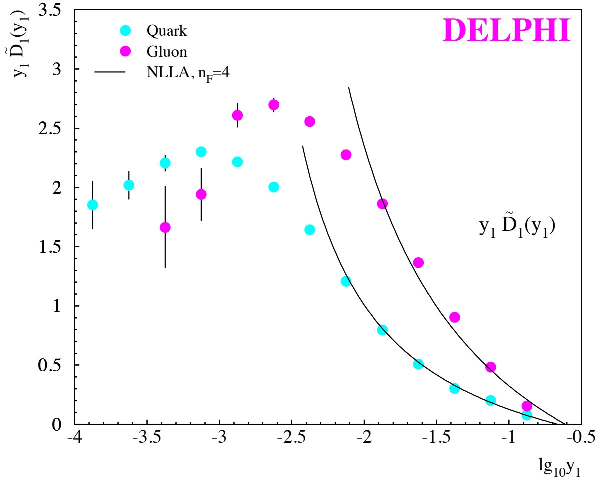
$$\Delta(y) = R_1(y) = \frac{N_1(y)}{N_{tot}} = \int_0^y D_1(y')dy' \qquad \Gamma_{p\to p',p"} = \tilde{D}_1(y) = \frac{1}{N_1(y)} \cdot \frac{\Delta N_1(y)}{\Delta y}$$



← 1-jet rate

diff. 1-jet rate →

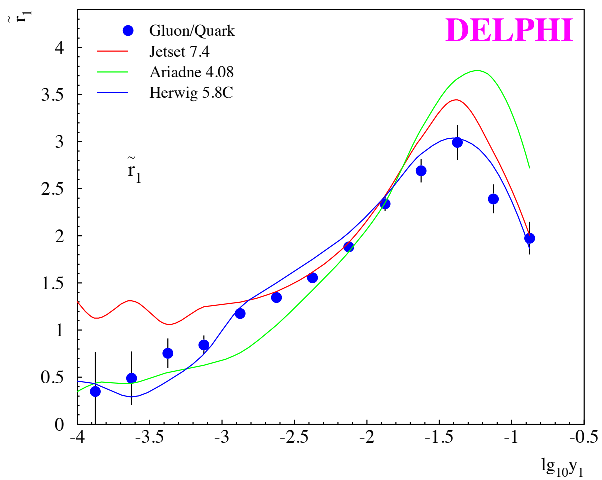




← splitting kernel

g/q kernel ratio →

Expect $\sim C_A/C_F$



Generalisation to Higher Rank Splittings/Subjets

Reminds to Zeno's paradox

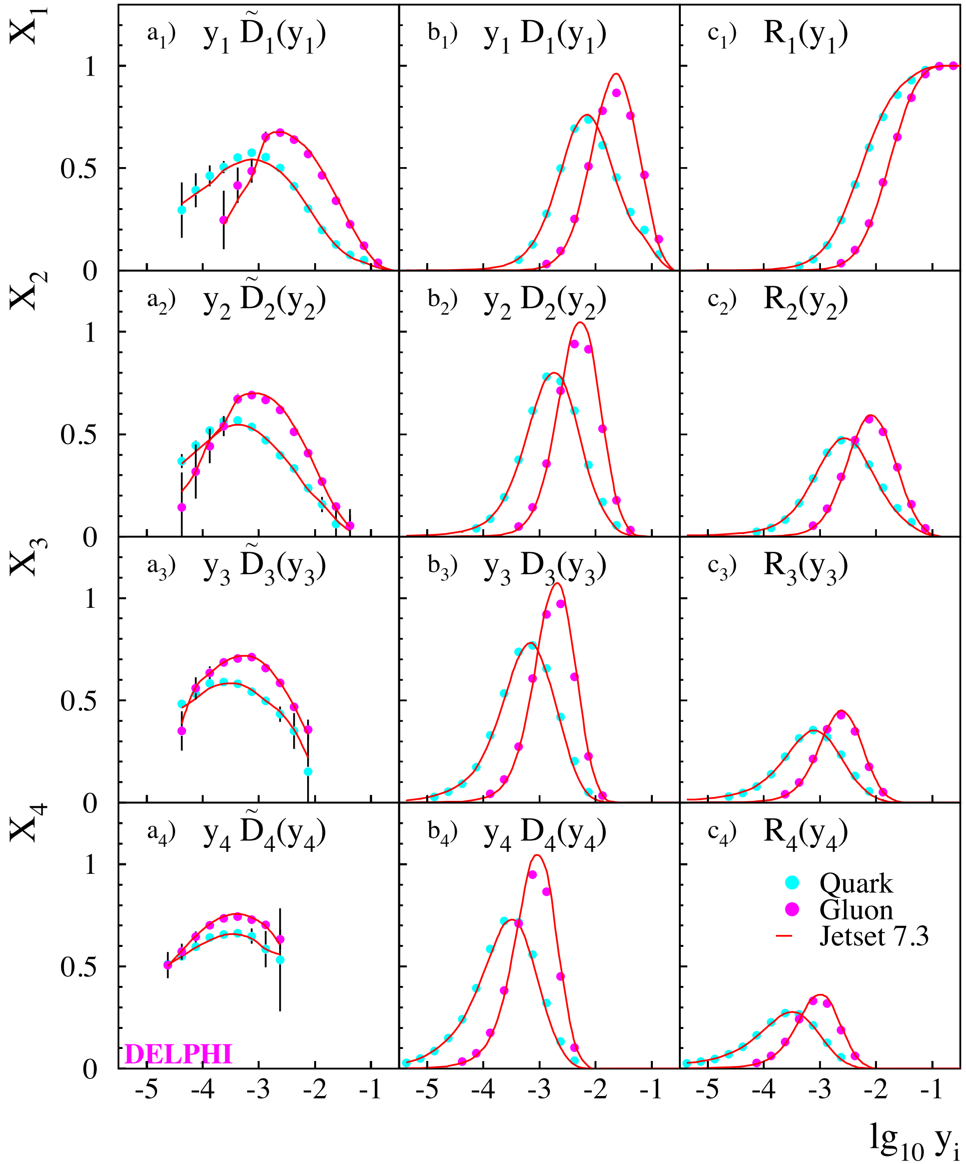
Gluon splitting starts at higher y cmp. to quark.

Quark then keeps up even for high ranks (middle).

Explains "smallish" multiplicity in gluon jets.

For high rank:
Splitting probability about equal for g and q.

All jets are gluon dominated!

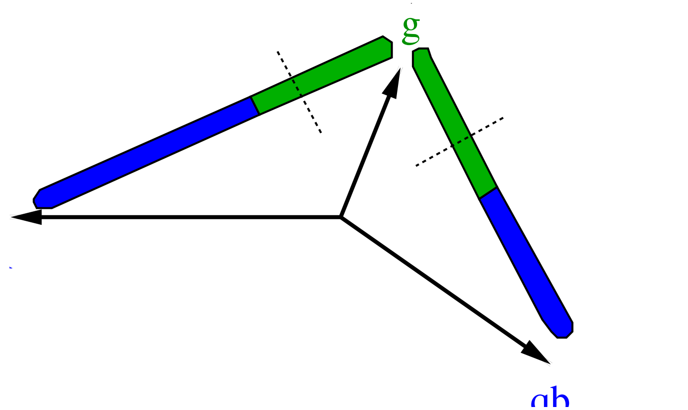


Topology Dependence of 3 Jet Event Multiplicity

Measure overall multiplicity of 3 jet events

$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Le}) = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Le}) \quad \text{(A)}$$

$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Lu}) = N_{q\bar{q}}(L, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Lu}) \quad \text{(B)}$$



with

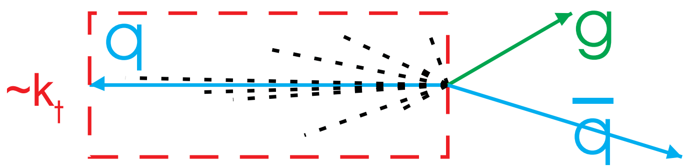
Eden, Gustafson, Khoze

$$L = \ln\left(\frac{s}{\Lambda^2}\right), \quad L_{q\bar{q}} = \ln\left(\frac{s_{q\bar{q}}}{\Lambda^2}\right), \kappa_{Lu} = \ln\left(\frac{s_{qg} s_{\bar{q}g}}{s\Lambda^2}\right), \quad \kappa_{Le} = \ln\left(\frac{s_{qg} s_{\bar{q}g}}{s_{q\bar{q}}\Lambda^2}\right)$$

Prediction accounts for coherence effects by choice of scales

Division into $q\bar{q}$ and gluon part is arbitrary

$\rightarrow$ differing definitions of gluon multiplicity $\leftrightarrow$ differing scales.



The phase space of the $q\bar{q}$-pair is restricted by the gluon jet $\rightarrow$ requires correction

Topology Dependence of 3 Jet Event Multiplicity

- In the Dipole Model energy slopes of gg and $q\bar{q}$ systems are related by:

$$\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{C_A}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)$$

- $N_{q\bar{q}}(E_{\mathrm{cm}})$ measured by various e^+e^--experiments
- Solution leaves constant of integration N_0 free

  $\rightarrow$ To be determined from a single measurement of N_{gg} for the prediction.

  $\rightarrow$ Take CLEO-data from $\chi_b'(J=2) \rightarrow gg$ decay at $E_{\mathrm{cm}} = 9.9132\mathrm{GeV}$

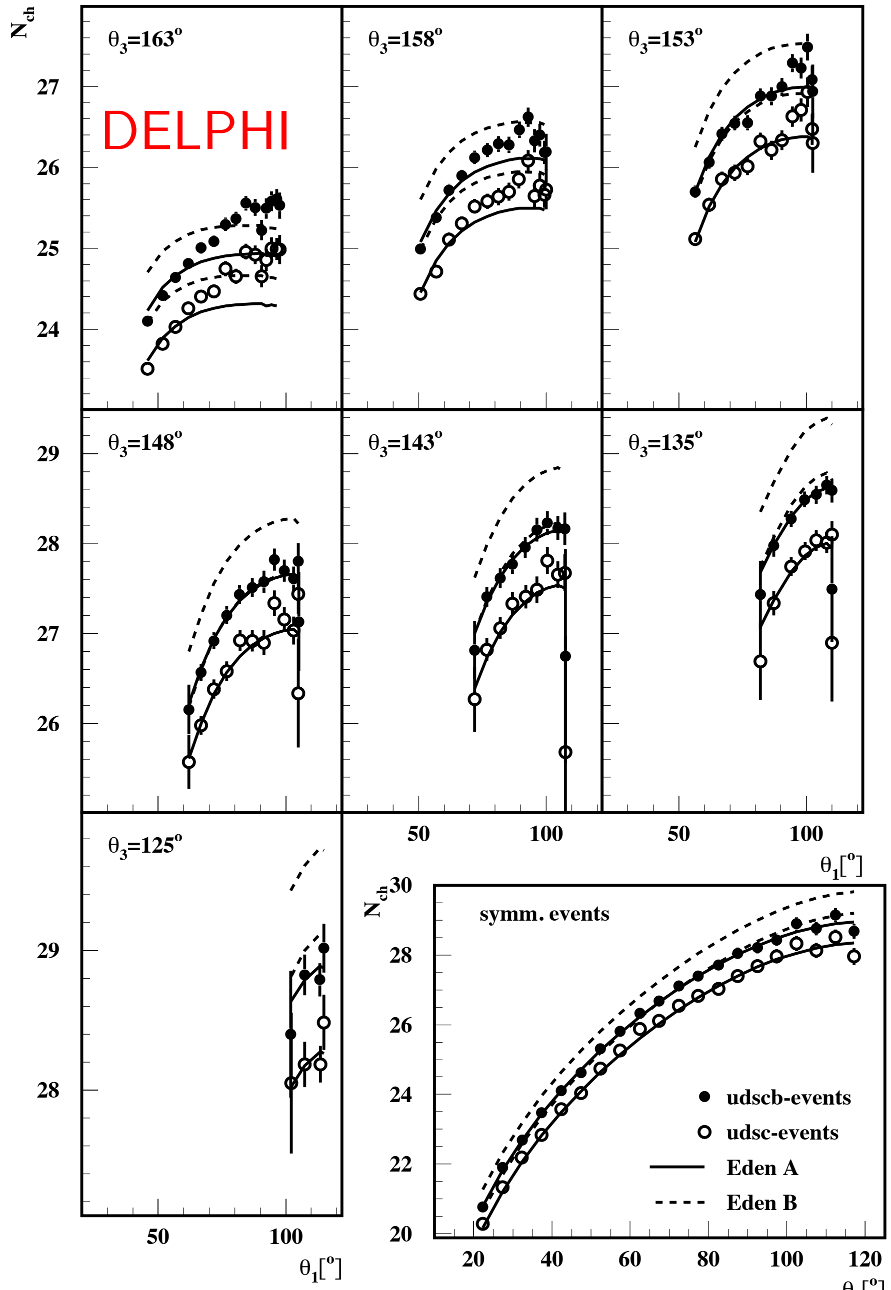
Analysis:

- Select 3 jet events without cut on y_{cut} (AoD, Cambridge, Durham, PHYJET)
- Apply (tiny) hadronisation correction
- Compare general and symmetric topologies
- Compare $udscb$ and $udsc$ events $\rightarrow$ constant offset $N_0 \sim 0.6$ due to b-events
- Compare solutions Eden (A) and (B)
- Leave N_0 free $\rightarrow$ only slope determines measurement of C_A/C_F !!!

The 3 Jet Multiplicity

- Compare $udsc$ ($\circ$) and $udscb$ ($\bullet$) data
- Eden A ——
  – Very good agreement for symmetric and general topologies
- Eden B - - - (dismissed by DELPHI)
  – χ^2 inacceptable in global fit
- OPAL used Eden B (sym. events only)
- DELPHI result based only on Eden A

$$\frac{C_A}{C_F} = 2.261 \pm 0.014_{\text{stat.}} \pm 0.036_{\text{exp.}}$$
$$\pm 0.052_{\text{theo.}} \pm 0.041_{\text{clus.}}$$



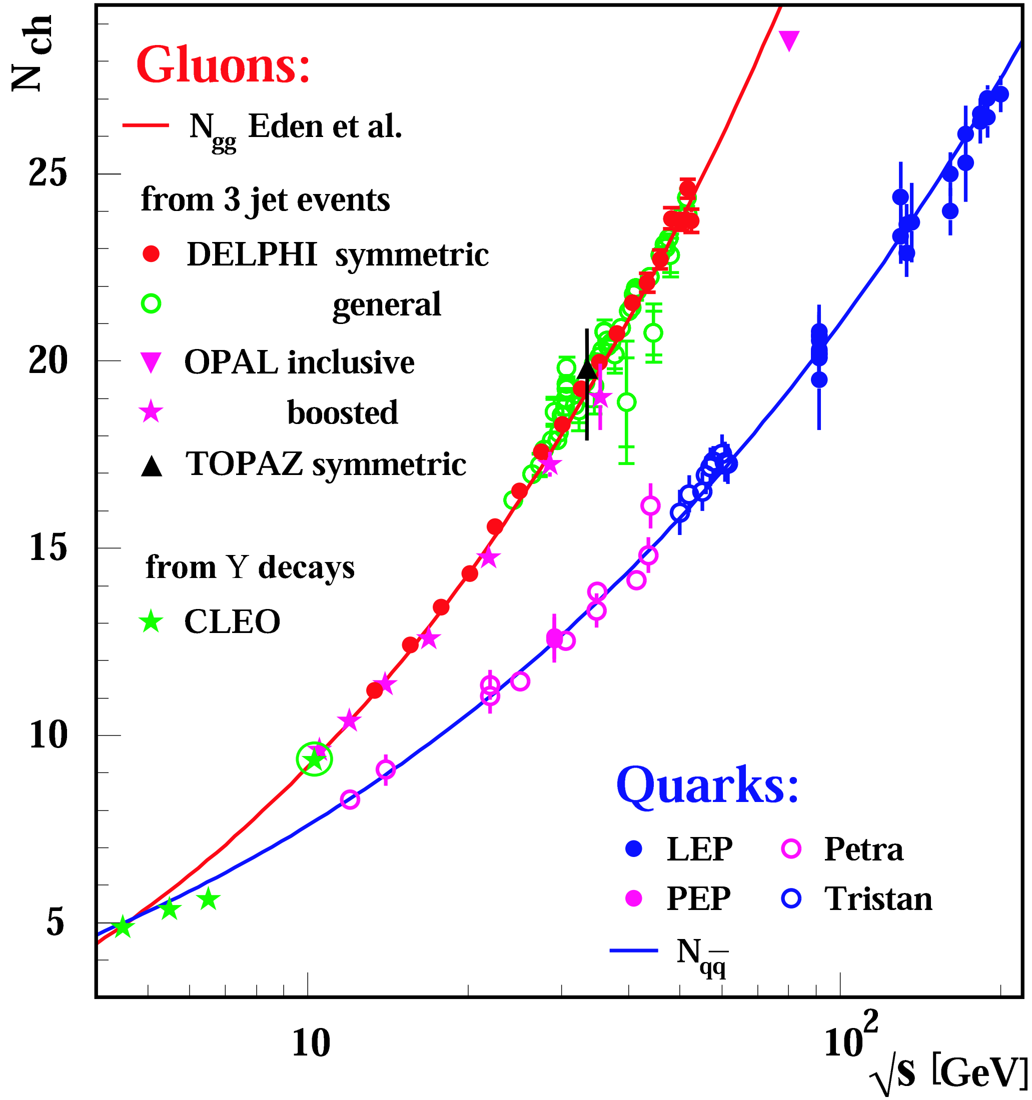
Energy dependence of the Gluon Multiplicity N_{gg}

- Determine Gluon contribution:

  $N_{gg}(\kappa_{Le}) = 2 \cdot$
  $(N_{q\bar{q}g}(\theta_1) - N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) - N_0)$

- Agreement between:
  - general & symmetric topologies
  - experiments (except similar OPAL anal.)
  - data and Eden prediction

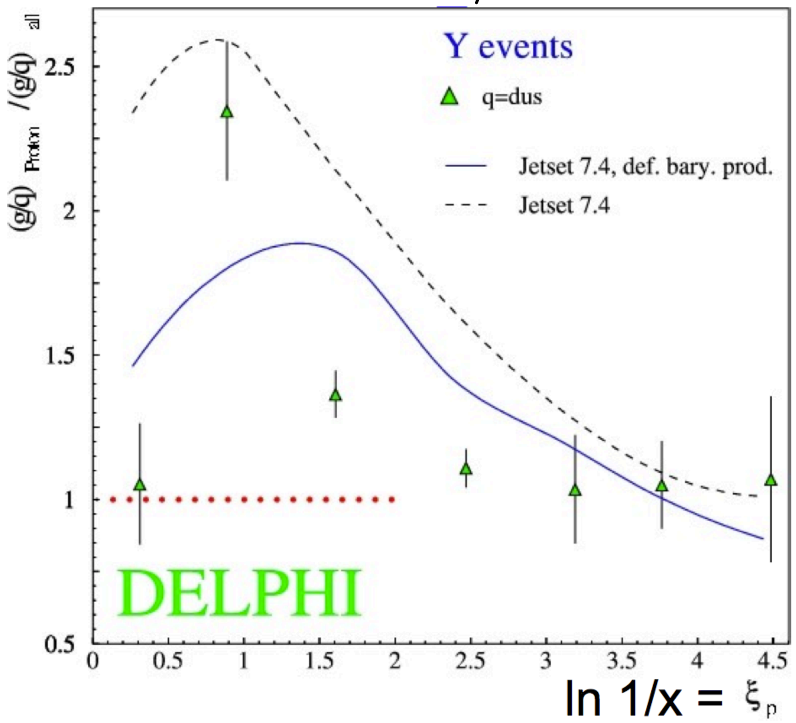
- N_{gg}: E-slope $\sim$ twice that of $N_{q\bar{q}}$ $\rightarrow$ illustrates colour factor ratio



Identified Particles

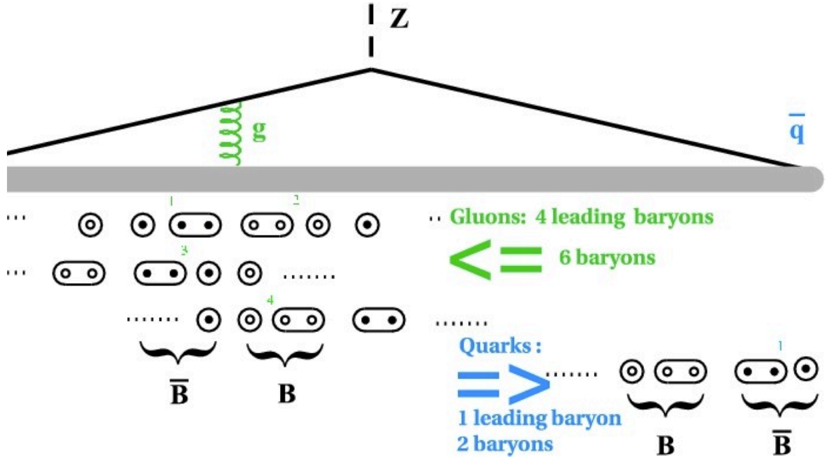
Observe increased baryon production in gluon-fragmentation (typ. $+20\%$)

plot double ratio: $g/q\big|_p \Big/ g/q\big|_{all}$



additional leading baryons
reasonably reproduced in string picture

use Fermis Golden Rule



Cluster model:
would require $g \to (qq)\overline{(qq)}$ splitting

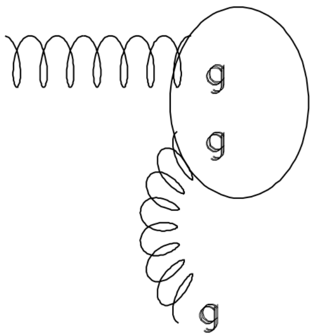
CLEO additional increase for
$\Upsilon \to gg\gamma; ggg$ (beyond model):

- $\Lambda \quad * 2$
- $p, \bar{p} \quad + 20\%$

not confirmed at LEP !

Octet Neutralisation ?

Expected (known → PGF) for gluons

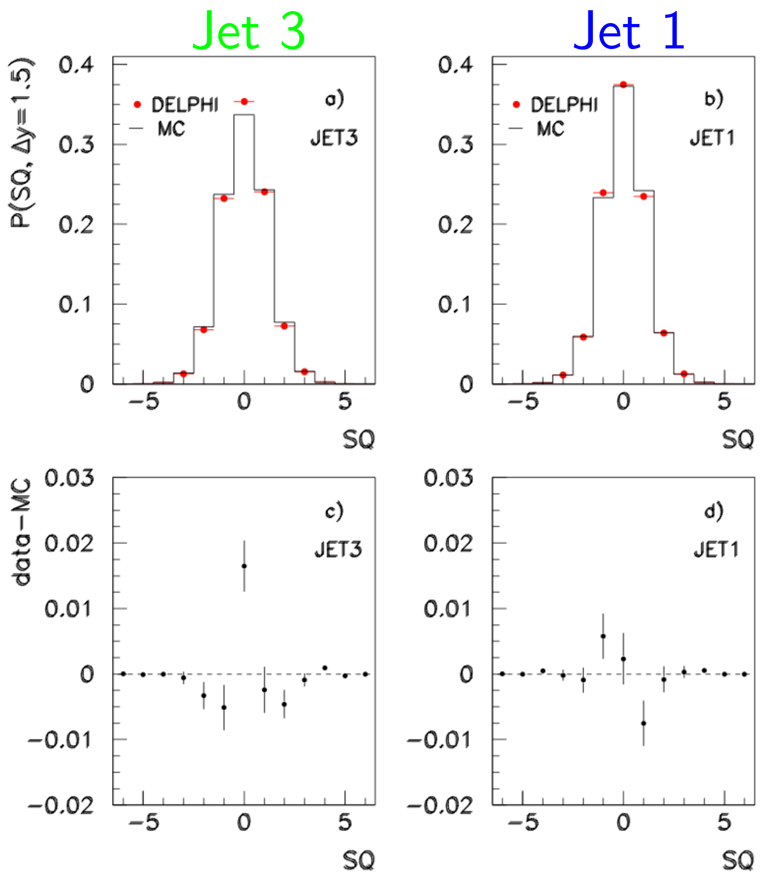


enhances:

- production of leading neutral systems;
- glueballs (?); isoscalars

NOT confirmed in std. fragmentation!

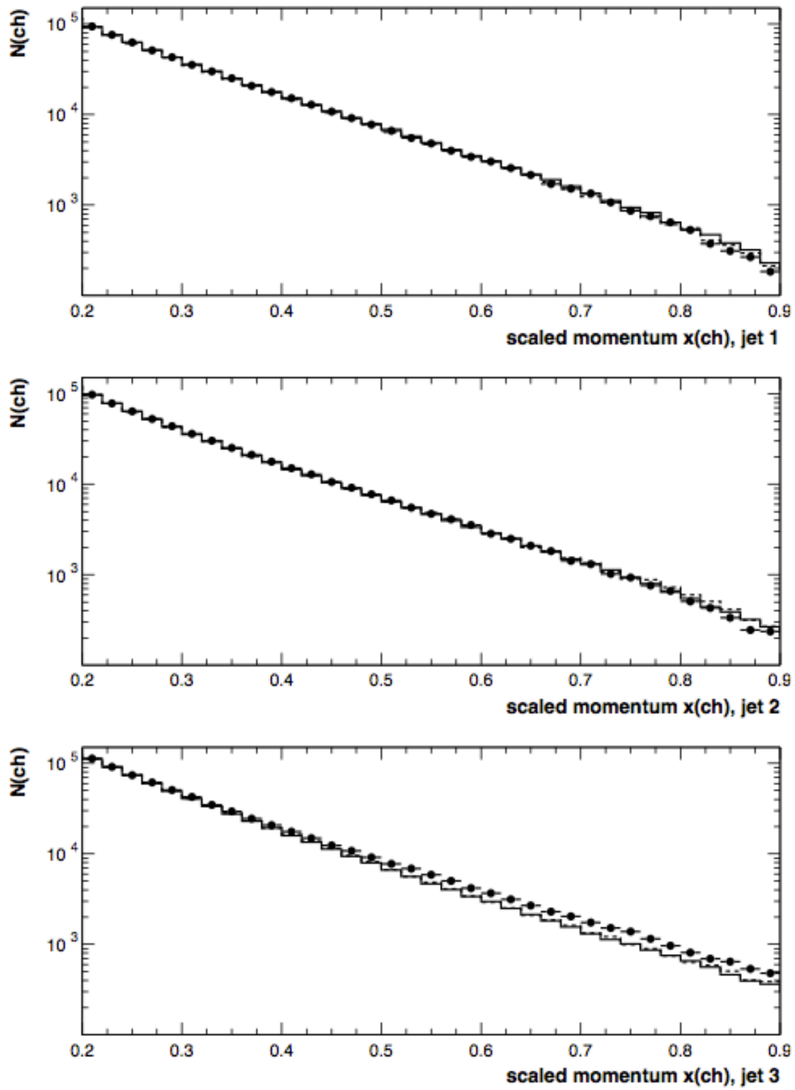
Signature: leading neutral system with rapidity gap $\Delta y \sim 1.5$



Tiny ($\lesssim 2\%$) excess of "fast" neutral systems at small inv. mass ($\lesssim 2\,\text{GeV}$)

- seen by ADO
- unidentfied (no excess of $\eta, \Phi \ldots$)

Refection in Gluon Fragmentation Function ?

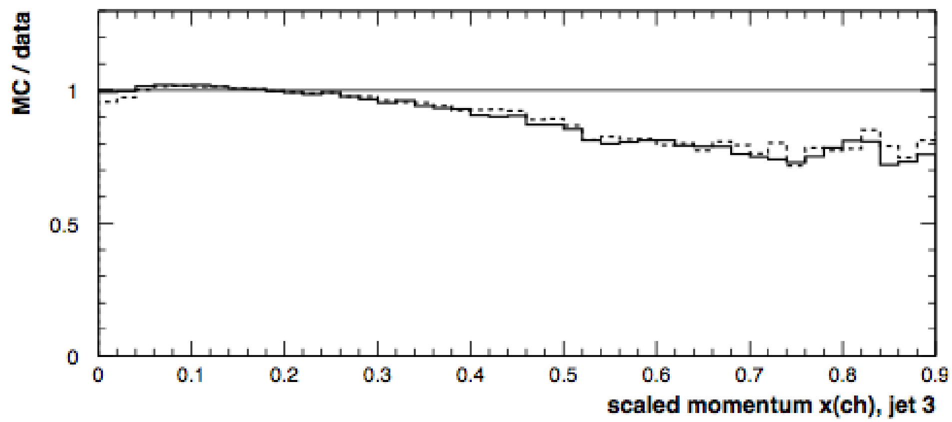


ALEPH

(unpublished, shown by G. Rudolph, Trento & ÖFKT Krems)

Scaled momentum spectra jor jets in 3 jet events (jet 1,2,3 E-ordered)

"Gluon"-jet (3) spectrum "harder" than predicted by JETSET/Ariadne, full simulation !



Overall size ($\mathcal{O}(2\%)$ of multiplicity) similar to excess of neutal systems

What should be done ?

	Z	125 GeV	160 GeV	240 GeV	350 GeV
$\mathcal{L}_{int}/Exp.y./\mathrm{ab}$	22	11	3.8	.87	.21
$\sim \#q\bar{q}/$Exp.	10^{12}	10^{9}	10^{8}	10^{7}	10^{6}
$\sim \#3 - jets_{tagged}$	10^{10}	10^{7}	10^{6}	10^{5}	10^{4}

$+$ radiative Events: statistics $>$ LEP from $\sim$40 GeV to 240 GeV!

Consequences for gluon to quark comparisons

- study of "any" dyn. dependence with negligible stat. unvertainty frag. functions, splitting kernels . . .
- mitigate systematics/resolution by unfolding, control using E-dependence
- fundamental problems remain ($q\bar{q}g$, not gg vs. $q\bar{q}$, tree level association, "parton" resolution)
- $q\bar{q}g$-multiplicity: topology dependence can be cross-checked vs. explicit E-dependence, slope derivatives, check of dead cone effect, . . .

What should be done ? (cont.)

- Enormous statistic allows for measurement of rare or difficult to measure processes.

- Compare esp. leading particles in *gluon* and quark jets:
  - search for octet-fragmentation
  - isoscalars $(\Phi, \omega, f(1710), \ldots)$ / glueballs
  - measurements mass-plots of resonances incl. (!) neutrals
  - check baryons + resonances $(\Delta^{0,++},\ \Lambda,\ \Lambda(1520),\ \ldots\ )$

Analysis: when does resonance belong to a jet? How to combine a resonance?

- More ideas? (depend on low E results (JLAB), lattice calculations)

Experimentally: need some particle id., high resolution e.m. calorimetry