Bose-Einstein Correlations in e^+e^- annihilation

(a review)

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Workshop on Parton Radiation and Fragmentation from LHC to FCC-ee
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Introduction - BEC

$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Longrightarrow \frac{\rho_2(\vec{Q})}{\rho_0(\vec{Q})}$ or $\frac{\rho_2(Q)}{\rho_0(Q)}$ ρ_0 = 2-particle density of 'reference sample'

$\vec{Q} = \vec{p}_1 - \vec{p}_2 \qquad Q = \sqrt{-(p_1 - p_2)^2}$

Assuming particles produced incoherently
with spatial source density of emission points $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\ \mathrm{e}^{iQx}\, S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda = 0$ if production completely coherent

Assuming $S(x)$ is a spherically symmetric Gaussian distribution
with radius r, $\Longrightarrow$

$$R_2(Q) = 1 + \lambda\ \mathrm{e}^{-(Qr)^2}$$

Problems with this approach

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Gaussian distribution of particle emitters
  seems unlikely in $\mathrm{e^+e^-}$ annihilation
  — jets
- static source, *i.e.*, no t-dependence
  certainly wrong

Final-State Interactions

1. Coulomb
   - form not certain
     (usually use Gamow factor)
     overcorrects!
   - for R_2, a few % in lowest Q bin
   - double if $+,-$ ref. sample
   - often neglected for R_2
   - but not negligible for R_3
2. Strong interaction - s-wave $\pi\pi$ phase shifts can be incorporated together with Coulomb into the formula for R_2

   Osada, Sano, Biyajima, Z.Phys. C72(1996)285)

   tends to increase λ, decrease r -
   Not used by LEP experiments

Reference Sample

Common choices:

1. $+,-$ pairs
   But different resonances than $+,+$
2. Mixed events – pair particles from different events
   But destroys all correlations, not just BEC

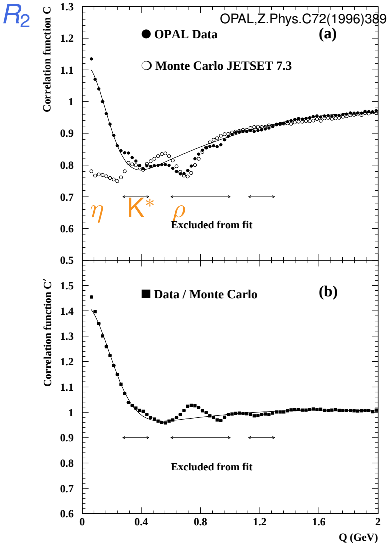
correct by MC (no BEC):

$$\rho_0 \Longrightarrow \rho_0 \frac{\rho_2^{\mathrm{MC}}}{\rho_0^{\mathrm{MC}}}$$

$$R_2 = \frac{\rho_2}{\rho_0} \Longrightarrow \frac{\rho_2}{\rho_0} \Big/ \frac{\rho_2^{\mathrm{MC}}}{\rho_0^{\mathrm{MC}}}$$ ‘double ratio’

– But is the MC correct?

ref. sample, ρ_0, from $+,-$ pairs



Long-range correlations inadequately treated in ref. sample:

$R_2(Q) \propto (1+\lambda\, e^{-Q^2 r^2})\cdot(1+\delta Q)$ or even $\cdot(1+\delta Q+\epsilon Q^2)$

What have we learned from LEP?

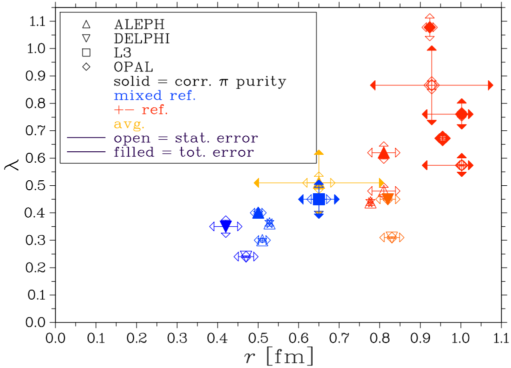
The simple picture is inadequate

1. R_2 is not Gaussian
2. $R_2 \neq 1 + \lambda|\widetilde{S}(Q)|$
   i.e. $\neq 1+$ positive-definite term
   $\exists$ also a region where $R_2 < 1$,
   i.e., anticorrelation
3. $R_2 \neq R_2(Q)$, but $R_2(\vec{Q})$
4. R_2 depends on jet structure

Other aspects of BEC:

1. $\sqrt{s}$ dependence
2. comparison of 2-π and 3-π BEC
   suggests complete incoherence,
   but large errors
3. $r_{\pi^0\pi^0} \approx$ or $< r_{\pi^\pm\pi^\pm}$?
   LEP inconclusive
4. cross talk?, i.e. is there BEC if π's from different jets?
   suggests no,
   but very large uncertainties

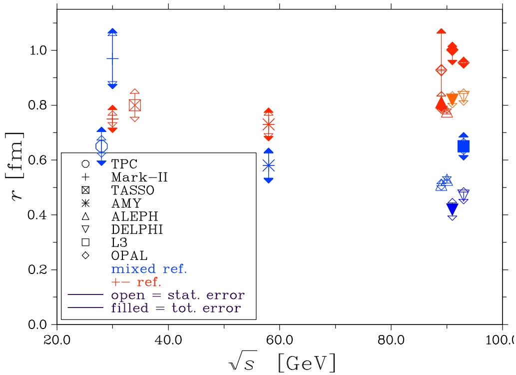
Results from R_2, $\sqrt{s} = M_Z$ (Gaussian parametrization)



– correction for π purity increases λ

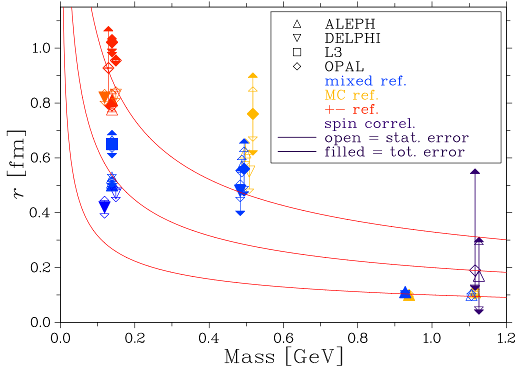
– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little

$\sqrt{s}$ dependence of r



No evidence for $\sqrt{s}$ dependence
But uncertainties large

Mass dependence of *r* — BEC and FDC



No evidence for $r \sim 1/\sqrt{m}$

$r_{\pi\text{-}\pi} \approx r_{\text{K-K}}$

$r(\text{mesons}) > r(\text{baryons})$

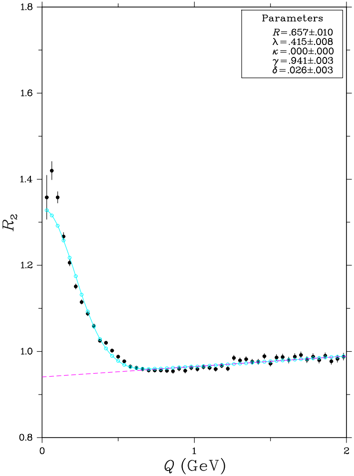
$r(\text{baryons})$ is very small — $r_{\text{p}} = 0.1$ fm while size of p is 1 fm ???

Disclaimer

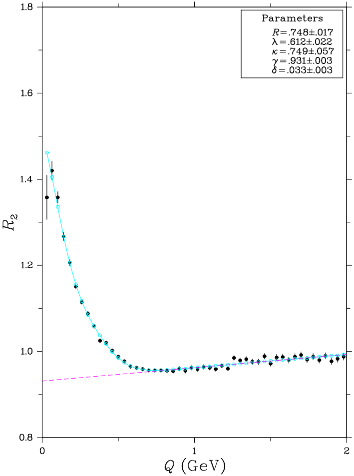
- There are many BEC measurements with pions.
- There are also BEC measurements with kaons, and FDC measurements with protons, lambdas, but fewer.
- From here on I will concentrate on pion results.

not Gaussian — try Edgeworth expansion

$$R_2(Q) = (1 + \lambda \, e^{-Q^2 r^2} \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]) \cdot (1 + \delta Q)$$



Gaussian ($\kappa = 0$) CL$= 10^{-14}$



Edgeworth expansion CL$= 18\%$

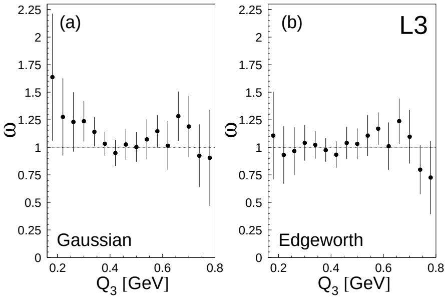
But note large δ – see later

3π BEC

with $R_3^{\text{genuine}}(Q_3) = R_3(Q_3) -$ contribution from 2-π $\Longrightarrow$ $\omega = \dfrac{R_3^{\text{genuine}}(Q_3)-1}{2\sqrt{R_2(Q_3)-1}}$

Using R_3^{genuine} from data, R_2 from fit

L3, PLB540 (2002) 185



Conclusion: Data consistent with $\omega = 1$,

i.e., with completely incoherent pion production

Possibly a problem for string models!

But large uncertainties

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

- Naively expect same BEC for $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$
- Hadronization with local charge conservation, *e*.g., string,

  $$\Longrightarrow r_{00} < r_{\pm\pm}$$

  But most π's from resonances — dilutes this effect.

- Many measurements of BEC with charged π's
- but few with π^0's

  in e^+e^-: L3, P.L. B524 (2002) 55
  OPAL, P.L. B559 (2003) 131

Selection:

OPAL	L3
$p_{\pi^0} > 1.0$ GeV	$E(\pi^0) < 6.0$ GeV
2-jet, $T > 0.9$	all events

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

BEC from Z decays
Gaussian parametrization

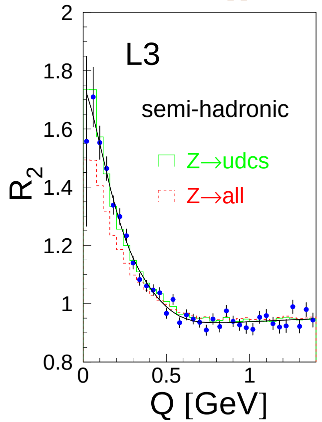
	Expt.	ρ_0	r (fm)	λ
00	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.31 ± 0.10	0.16 ± 0.09
	OPAL $E_\pi > 1$, 2-jet	mix	0.59 ± 0.09	0.55 ± 0.14
$\pm\pm$	L3	mix	0.65 ± 0.04	0.45 ± 0.07
	L3 3-π	mix	0.65 ± 0.07	0.47 ± 0.08
	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.46 ± 0.01	0.29 ± 0.03
	OPAL	$+-$	$1.00^{+0.03}_{-0.10}$	0.76 ± 0.06

- L3: $r_{00} < r_{\pm\pm}$ and $\lambda_{00} < \lambda_{\pm\pm}$, both 1.5σ
- ALEPH, DELPHI find $r_{\pm\pm}(\mathrm{mix})/r_{\pm\pm}(+-) \approx 0.68, 0.51$
  Applying this to OPAL $r_{\pm\pm} \approx 0.6 \pm 0.1$ – So, $r_{00} \approx r_{\pm\pm}$
- L3 and OPAL $\pi^0\pi^0$ results disagree by 2σ
- Is the L3-OPAL $\pi^0\pi^0$ difference due to E_π and/or 2-jet selection ???

Statistics and Systematics make any conclusions tenuous

Another source of $q\bar{q}$: W

$e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$



BE(W) = BE(Z →light quarks)

$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$ $\sqrt{s} \approx$ 190–200 GeV

If independent decay of W^+W^-,
i.e., no BEC between pions from different W's

$$\begin{aligned}
\rho_{4q}(p_1,p_2) = \ & \rho^+(p_1,p_2) && 1,2 \text{ from } W^+ \\
+ \ & \rho^-(p_1,p_2) && 1,2 \text{ from } W^- \\
+ \ & \rho^+(p_1)\rho^-(p_2) && 1 \text{ from } W^+,\ 2 \text{ from } W^- \\
+ \ & \rho^+(p_2)\rho^-(p_1) && 1 \text{ from } W^-,\ 2 \text{ from } W^+
\end{aligned}$$

Assuming $\rho^+ = \rho^- = \rho_{2q}$, W separation $\sim$ 0.1 fm

$$\rho_{4q}(p_1,p_2) = 2\rho_{2q}(p_1,p_2) + 2\rho_{2q}(p_1)\rho_{2q}(p_2)$$

Inter-W BEC $\Longrightarrow$ W decays *not* independent
$\Longrightarrow$ this relation does *not* hold.

Measure

- $\rho_{4q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$
- $\rho_{2q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$
- $\rho_{2q}(p_1)\rho_{2q}(p_2)$ from $\rho_{\text{mix}}(p_1,p_2)$ obtained by mixing $\ell^+\nu q\bar{q}$ and $q\bar{q}\ell^-\nu$ events

$W^+W^- \to q\bar{q}q\bar{q}$

Measure violation of

$$\rho_{4q}(Q) = 2\rho_{2q}(Q) + 2\rho_{mix}(Q)$$

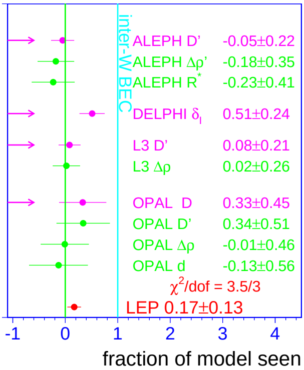
by

$$\Delta\rho(Q) = \rho_{4q}(Q) - [2\rho_{2q}(p_1, p_2) + 2\rho_{mix}(p_1, p_2)]$$

$$D(Q) = \frac{\rho_{4q}(Q)}{2\rho_{2q}(Q) + 2\rho_{mix}(Q)}$$

$$\delta_I(Q) = \frac{\Delta\rho(Q)}{2\rho_{mix}(Q)}$$

Compare to expectation of BE_{32} model in PYTHIA



$\delta_I(Q)$ measures genuine inter-W BEC

DELPHI: $0.51 \pm 0.24 \quad \sim 2\sigma$

average: $0.17 \pm 0.13 \quad \sim 1\sigma$

Conclusion: BEC (mostly) between π's from same string

But errors are large

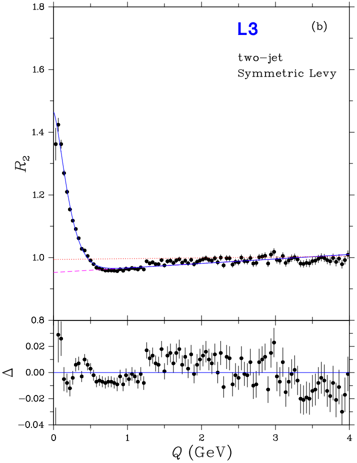
and event selection (4 well separated jets)

suppresses small Q for π pairs from different strings

Results – 'Classic' Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$
  Fit: $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^\alpha\right)$
  $0 < \alpha \leq 2$
  Gaussian if $\alpha = 2$
  Fit: $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$ Anti-Correlation!

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point highly correlated with momentum of produced particle:

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$

  where for 2-jet events, $a = 1/m_t$

  $\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2}$ is the "longitudinal" proper time

  and $m_t = \sqrt{E^2 - p_z^2}$ is the "transverse" mass

  and dist. of prod. points about their mean is very narrow (δ-function)

- Then, with $H(\tau)$ the distribution of proper time

$$R_2(p_1, p_2) = 1 + \lambda \,\mathrm{Re} \widetilde{H}\left(\frac{a_1 Q^2}{2}\right) \widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau) \exp(i\omega\tau)$$

- Assume a one-sided Lévy distribution for $H(\tau)$

  3 parameters:
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.

- Then, R_2 depends on Q, a_1, a_2

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \cdot \exp\left[- \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(- (RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

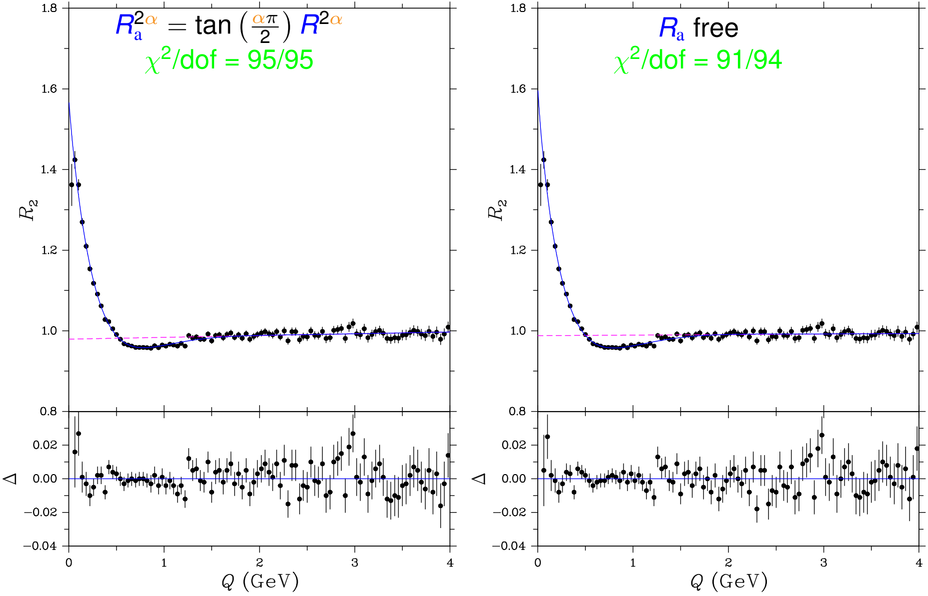
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay



Elongation?

- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $r_{\mathrm{side}}/r_{\mathrm{L}} \approx 0.8$ for all events
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function? or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation

Elongation in the Simplified τ-model?

LCMS: $$Q^2 = Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2 - (\Delta E)^2$$
$$= Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2\left(1-\beta^2\right) , \qquad \beta = \frac{p_{1\mathrm{out}}+p_{2\mathrm{out}}}{E_1+E_2}$$

Replace $$R^2 Q^2 \Longrightarrow A^2 = R_\mathrm{L}^2 Q_\mathrm{L}^2 + R_\mathrm{side}^2 Q_\mathrm{side}^2 + \rho_\mathrm{out}^2 Q_\mathrm{out}^2$$

Then in τ-model,
$$R_2(Q_\mathrm{L}, Q_\mathrm{side}, Q_\mathrm{out}) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) A^{2\alpha}\right) \exp\left(-A^{2\alpha}\right)\right]$$
$$\cdot \left(1 + \epsilon_\mathrm{L} Q_\mathrm{L} + \epsilon_\mathrm{side} Q_\mathrm{side} + \epsilon_\mathrm{out} Q_\mathrm{out}\right)$$

for 2-jet events:

		χ^2/dof	CL
τ-model	$R_\mathrm{side}/R_\mathrm{L} = 0.61 \pm 0.02$	14847/14921	66%
Edgeworth	$r_\mathrm{side}/r_\mathrm{L} = 0.64 \pm 0.02$	14891/14919	56%
	consistent		

Elongation is real

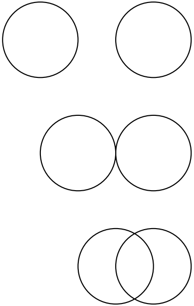
But τ-model must be modified: Q-dependence $\Longrightarrow$ $\vec{Q}$-dependence

Another way to get Anti-correlation

Białas and Zalewski, Phys. Lett. B727(2013)182

Białas, Florkowski and Zalewski, Phys. Lett. B748(2015)9

Pions are not point particles.



pions far apart – BEC

pions close together

pions overlapping – no longer pions – So, no BEC

This excluded volume leads to anti-correlation dip.
at approx. the right place – different for Long, side, out

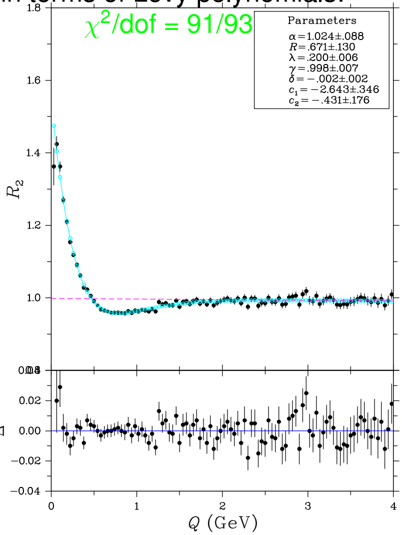
Another parametrization: Levy Polynomial Expansion

De Kock, Eggers, Csögő, ArXiv:1206.1680

Expansion of the Symmetric Lévy distribution in terms of Lévy polynomials:

$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

$$G = \exp\left(-|rQ|^{\alpha}\right) \left[1 + \sum_{n=1}^{\infty} c_n L_n(rQ \mid \alpha) \right]$$



Fits L3 data as well as simplified τ-model

Advantage of Levy exp:
model independent

Advantage of simplified τ-model:
anticorrelation region is
simply related to one parameter, R_a

Multiplicity/Jet/rapidity dependence in τ-model

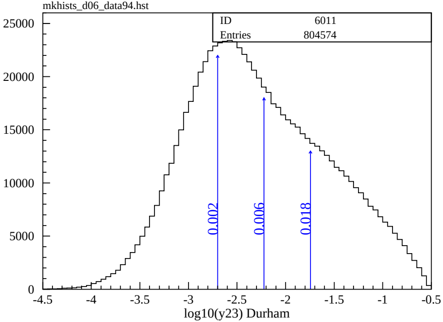
Use simplified τ-model, $\tau_0 = 0$
to investigate multiplicity and jet dependence

To stabilize fits against large correlation of parameters α and *R*, fix $\alpha = 0.44$

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all 'distances' between jets are $> y_{\text{cut}}$
  - instead, stop combining when there are 3 jets left
  - y_{23} is the smallest 'distance' between any 2 of the 3 jets
- y_{23} is value of y_{cut} where number of jets changes from 3 to 2



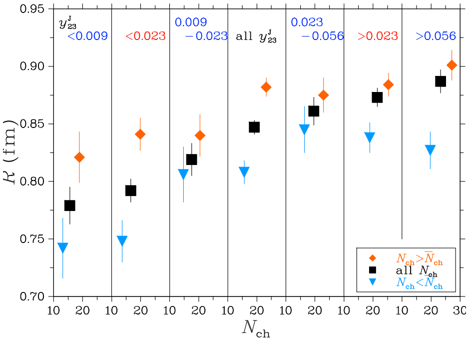
define regions of y_{23}^{D} (Durham):

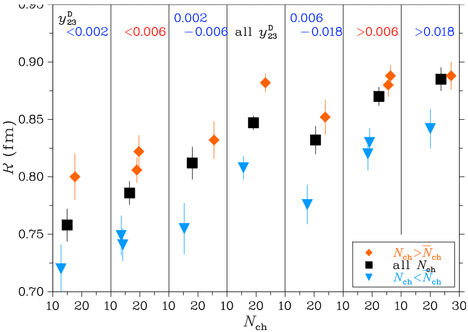
$y_{23}^{\text{D}} < 0.002$	narrow two-jet	or		
$0.002 < y_{23}^{\text{D}} < 0.006$	less narrow two-jet		$y_{23}^{\text{D}} < 0.006$	two-jet
$0.006 < y_{23}^{\text{D}} < 0.018$	narrow three-jet		$0.006 < y_{23}^{\text{D}}$	three-jet
$0.018 < y_{23}^{\text{D}}$	wide three-jet			

and similarly for y_{23}^{J} (JADE): 0.009, 0.023, 0.056

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY





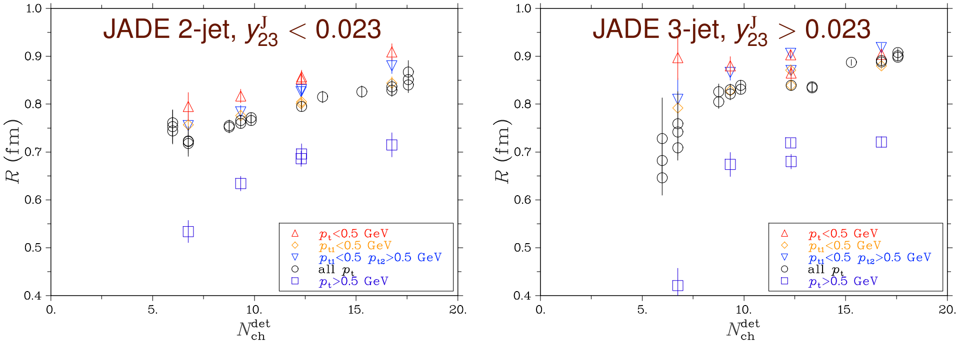
- R increases with y_{23}, *i.e.*, going from narrow 2-jet to wide 3-jet
- R increases with multiplicity at all y_{23}

m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)



- for both 2-jet and 3-jet events,
  R decreases with m_t for all N_{ch}
  smallest when both particles at high m_t

Jets and Rapidity

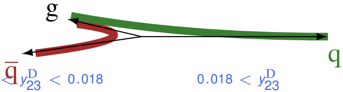
order jets by energy: $E_1 > E_2 > E_3$

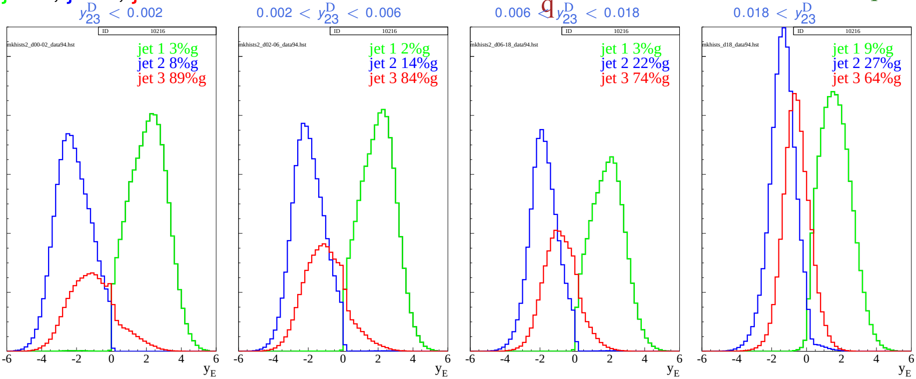
Note: thrust only defines axis $|\vec{n}_\mathrm{T}|$, not its direction.

Choose positive thrust direction such that jet 1 is in positive thrust hemisphere

rapidity, y_E, of particles from

jet 1, jet 2, jet 3:



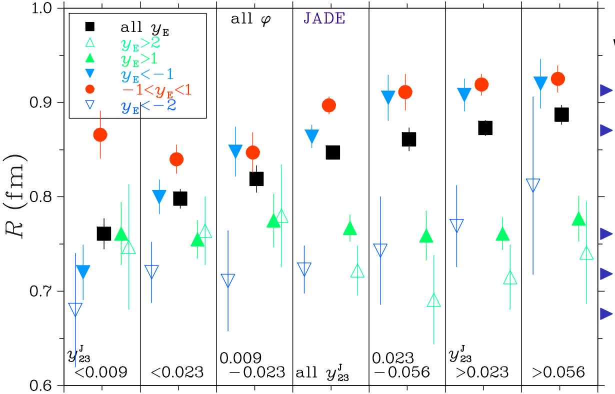


- $y_\mathrm{E} > 1$ almost all jet 1 — almost all quark
- $y_\mathrm{E} < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_\mathrm{E} < 1$ jet-3 enriched — largely gluon

Jets and Rapidity – simplified τ-model – L3 preliminary

To stabilize fits against large correlation of α, R, fix $\alpha = 0.44$

Select particle pairs by rapidity of pair



With y^J_{23},

- all y: R increases
- 'pure' q jet, $y_E > 1$, or $y_E < -1$ & y^J_{23} small, or $y_E < -2$: R const.
- $R_{-1<y_E<1} > R_{\text{'pure' q}}$
- $R_{y_E<-1}$ increases
- at large y^J_{23} $R_{-1<y_E<1} = R_{y_E<-1}$

Conclusion (Durham agrees):

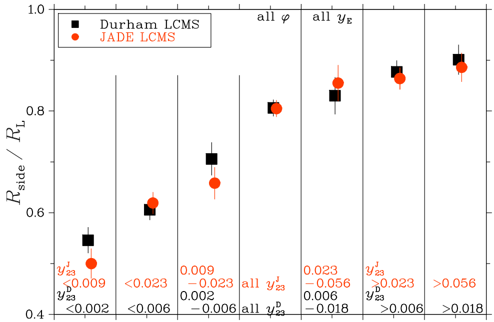
Increase in R with y^J_{23} is due to appearance of gluon jet

τ-model elongation – L3 preliminary

ad hoc extension of τ-model: in LCMS

$$R_2(Q) \Longrightarrow R_2(\vec{Q}), \qquad \vec{Q} = \{Q_\mathrm{L}, Q_\mathrm{side}, Q_\mathrm{out}\}$$

$$R^2 Q^2 \Longrightarrow = R_\mathrm{L}^2 Q_\mathrm{L}^2 + R_\mathrm{side}^2 Q_\mathrm{side}^2 + \rho_\mathrm{out}^2 Q_\mathrm{out}^2$$



- Durham, JADE agree
- Elongation decreases with y_{23}, $R_\mathrm{side} \approx 0.5$–$0.9\, R_\mathrm{long}$
- agrees with Gaussian/Edgeworth fits (all events)

  Gaussian: $r_\mathrm{side}/r_\mathrm{L} = 0.80 \pm 0.02 \pm^{0.03}_{0.18}$

  Edgeworth: $r_\mathrm{side}/r_\mathrm{L} = 0.81 \pm 0.02 \pm^{0.03}_{0.19}$

Conclusions

1. LEP has made a good start in investigating fragmentation with BEC
   But, statistics limited to
   - 1-D parametrizations
   - or very global 3-D parametrizations
2. Anticorrelation region
   - On what does it depend?
   - Is strong *x*-*p* correlation (as in τ-model) the correct explanation?
   - pion size?
   - something else?
   - 3-D fits needed for different regions, *e*.g., y_{23}, y, m_t
3. Parametrization
   - model independent, *e*.g., Lévy polynomial expansion
   - τ-model
     - Known to be inadequate: elongation
     - Particularly suspect: assumption of strong *x*-*p* corelation in transverse plane
     - relaxing this correlation requires addtional parameters, dimensions
   - other model?
4. Does *r* depend on mass, charge? π-K-p, π^0-$\pi^\pm$

Desiderata

1. π/K/p identification
2. good track efficiency (enters as the square for pairs)
3. good two-track resolution
4. good π^0 measurement
5. good K^0, Λ measurement
6. good b-tag efficiency
7. much higher statistics than LEP
   - enable narrower bins to better determine BEC parametrization
   - enable more differential look at event structure, *e*.g., $R_{\text{in plane}} = R_{\text{out of plane}}$?
   - L3 analyses I showed used 10^6 events
   - an example: is R the same for quark, gluon?
     need pure gluon jets: double b-tag qqg event with large E_g
     $\Longrightarrow$ about 1/1000 of the events
     So 10^9 events needed to do for gluon what we now do for quarks
   - 1-D to 3-D requires N^3_{bins} as many events
     For 100 bins 10^6 events $\Longrightarrow$ 10^{12} events
   - expected 10^{12} Z events per year per expt looks pretty good