Challenges in heavy-quark fragmentation

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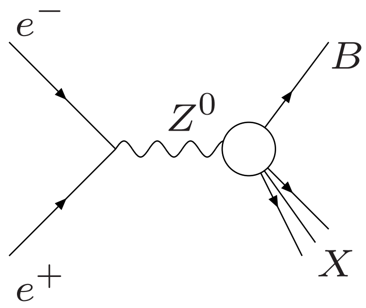
See also talks on b-fragmentation in TOP LHCWG

Heavy-quark (charm, bottom) production in different environments is interesting from the theoretical and experimental point of view

Tests of QCD, parton model and factorization, power corrections and hadronization

Systematics for top (m_t) and Higgs ($H \to b\bar{b}$) phenomenology

A case study: B-hadron production in e^+e^- annihilation



$m \gg \Lambda_{\rm QCD}$: perturbative QCD allows one to calculate the parton-level (b-quark) spectrum, but not the hadronization into a B

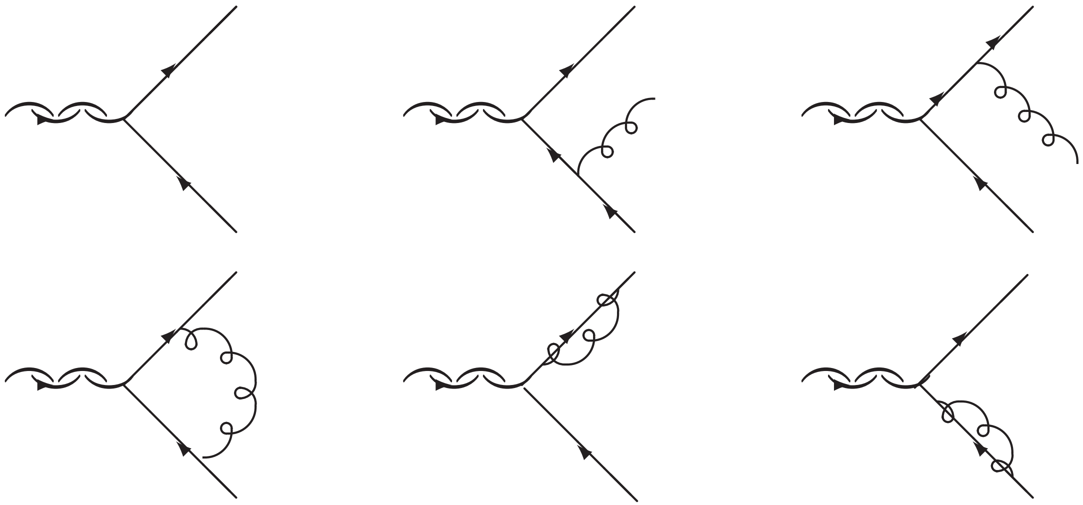
$$\sigma(e^+e^- \to B) = \sigma(e^+e^- \to b\bar{b}) \otimes D_{np}(b \to B)$$

$D_{np}(b \to B)$ contains parameters to be fitted to experimental data

Alternatively: non-perturbative corrections into an effective/frozen coupling constant

Heavy-quark production at NLO in e^+e^- annihilation

$$e^+e^- \to Z^0(q) \to b(p_b)\bar{b}(p_{\bar{b}})\ (g(p_g))$$



$$x_b = \frac{2p_b \cdot q}{q^2} = \frac{2E_b}{m_Z} \quad ; \quad \sigma_0 = \sigma(e^+e^- \to b\bar{b})$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi}\left\{\left[P_{qq}(x_b)\ln\frac{m_Z^2}{m_b^2} + A(x_b)\right] + \mathcal{O}\left[\left(\frac{m_b^2}{m_Z^2}\right)^p\right]\right\} + \mathcal{O}(\alpha_S^2)$$

$$P_{qq}(x_b) = C_F\left(\frac{1+x_b^2}{1-x_b}\right)_+ \ ; \ \alpha_S(\mu)\ln\frac{m_Z^2}{m_b^2} \simeq \mathcal{O}(1)\ ; \ \int_0^1 dx_b f(x_b)[g(x_b)]_+ = \int_0^1 dx_b[f(x_b)-f(1)]g(x_b)$$

$P_{qq}(x)$ and $A(x)$ contain terms $\sim 1/(1-x)_+$ and $\sim [\ln(1-x)/(1-x)]_+$

Several approaches to resum mass and threshold contributions, mostly differing in treatment of power corrections (Mele, Nason '91; Cacciari, Catani,'01; Kniehl, Kramer et al, '98-'08)

Perturbative fragmentation: factorizing hard-scattering massless coefficient function and massive perturbative fragmentation function (like PDFs)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b}(x_b, m_b \neq 0) = \frac{1}{\sigma_0}\sum_i \int_{x_b}^1 \frac{dz}{z}\frac{d\hat{\sigma}_i}{dz}(m_Z, m_i = 0, \mu_F) D_i\left(\frac{x_b}{z}, \mu_F, m_b\right) + \mathcal{O}\left[\left(\frac{m_b^2}{m_Z^2}\right)^p\right]$$

$D_i(x_b, \mu_F, m_b)$: perturbative fragmentation function (PFF) for $i \to b$

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dz} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \hat{A}(z)\right] \qquad (d = 4 - 2\epsilon)$$

$\overline{\mathrm{MS}}$ coefficient function $(i = b)$:

$$\left(\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dz}\right)^{\overline{\mathrm{MS}}} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\hat{A}(z)$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b}(m_b) = \left(\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dx_b}(m_b = 0)\right)^{\overline{\mathrm{MS}}} \otimes D_b^{\overline{\mathrm{MS}}}(m_b)$$

Resummation of mass logarithms through evolution equations:

DGLAP equations for PFFs (singlet $j = g$ and non-singlet $j = b$ contributions):

$$\frac{d}{d\ln\mu_F^2} D_i(x,\mu_F,m_b) = \sum_j \int_x^1 \frac{dz}{z} P_{ji}\left(\frac{x}{z},\alpha_S(\mu_F)\right) D_j(z,\mu_F,m_b)$$

Initial conditions are process-independent

$$D_b^{\rm ini} = \delta(1-x) + \frac{\alpha_S C_F}{2\pi}\left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_{0F}^2}{m_b^2} - 2\ln(1-x) - 1\right)\right]_+ \quad D_g^{\rm ini} = \frac{\alpha_S T_R}{2\pi}\left[x^2 + (1-x)^2\right]\ln\frac{\mu_{0,F}^2}{m_b^2}$$

Solution in Mellin moment space ($\mu_{0F} \to \mu_F$ evolution):

$$D_N(\mu_F,m_b) = D_N(\mu_{0F},m_b)\exp\left\{\frac{P_N^{(0)}}{2\pi b_0}\ln\frac{\alpha_S(\mu_{0F})}{\alpha_S(\mu_F)} + \frac{\alpha_S(\mu_{0F})-\alpha_S(\mu_F)}{4\pi^2 b_0}\left[P_N^{(1)} - \frac{2\pi b_1}{b_0}P_N^{(0)}\right]\right\}$$

$\mu_{0F} \simeq m_b$, $\mu_F \simeq m_Z$: resumming LLs $\alpha_S^n \ln^n(m_Z^2/m_b^2)$, NLLs $\alpha_S^n \ln^{n-1}(m_Z^2/m_b^2), \ldots$

Large-x resummation: $\frac{1}{(1-x)_+} \to \ln N$, $\left[\frac{\ln(1-x)}{1-x}\right]_+ \to \ln^2 N$

LLs $\alpha_S^n \ln^{n+1} N$; NLLs $\alpha_S^n \ln^n N$, $\ldots$ can be resummed via standard techniques

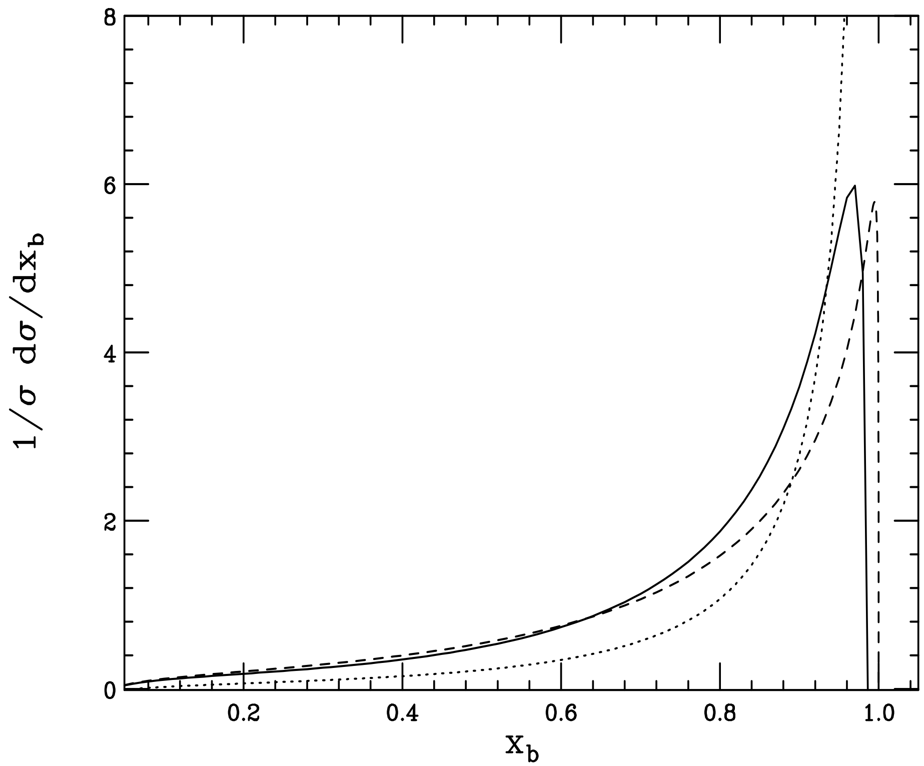
Phenomenology carried out at NLO+NLL (e^+e^-, top decays, $H \to b\bar{b}$, DIS, $pp \to b\bar{b}$)

All ingredients to promote investigation to NNLO+NNLL are available

(Rijken, Van Neerven '97, Mitov, Moch'06, Melnikov, Mitov '04; Mitov '05; Mitov, Moch, Vogt '06; Mitov, Moch '06)

b-quark energy spectrum in e^+e^- annihilation at the Z^0 pole

m_Z=91.19 GeV, m_b=5 GeV, $\mu_F = \mu_R = m_Z$, $\mu_{0R} = \mu_{0F} = m_b$, $\Lambda_{\overline{\rm MS}}$=200 GeV



Solid: soft and collinear resummation Dashes: only collinear resummation

Dots: massive NLO without resummation

Negative spectrum at large x: lack of non-perturbative corrections (how about NNLO?)

Hadron-level results: fits of hadronization models to $e^+e^- \to B$ data (G.C.,Drollinger'05)

B-hadrons from SLD, OPAL (mesons and baryons, mainly Λ_b) an ALEPH (only mesons), mostly in chains $B \to D^* \ell\nu$, $D^* \to D\pi$, $D \to K(n\pi)$

NLO+NLL calculation - power law: $D_{\rm np}(x_B,\gamma) = (1+\gamma)(2+\gamma)x_B(1-x_B)^\gamma$

Best fit (no correlations, $0.18 \le x \le 0.94$): $\gamma = 17.178 \pm 0.303$, $\chi^2/\mathrm{dof} = 46.2/53$

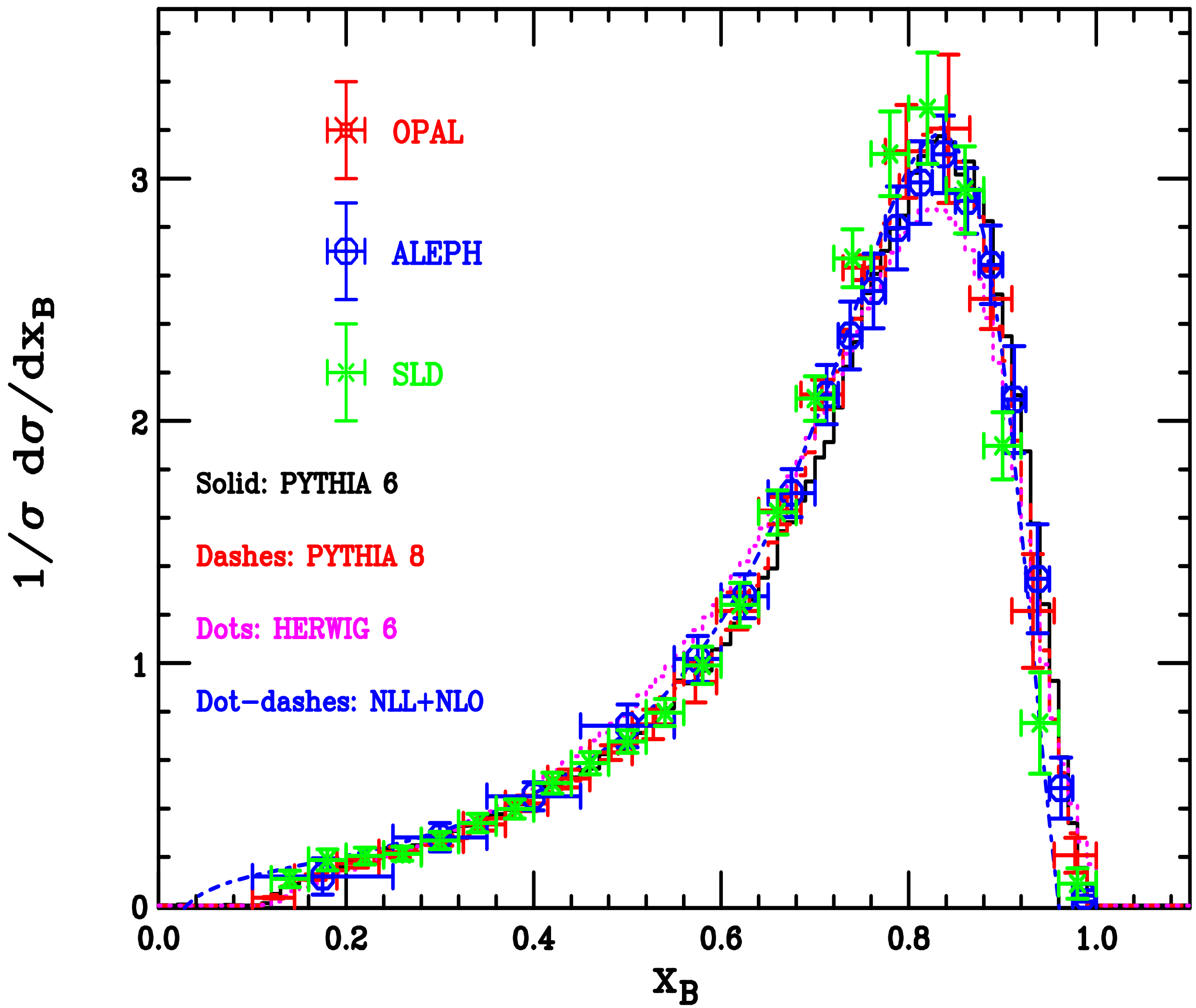
Monte Carlo generators: string (PYTHIA) and cluster (HERWIG) hadronization model

Discussions within the top-quark LHC working group (see also G.C. and F.Mescia,'10)

$$\text{Lund/Bowler fragmentation function}: f_B(z) \propto \frac{1}{z^{1+brm_b^2}}(1-z)^a \exp(-bm_T^2/z)$$

HERWIG 6	PYTHIA 6 (8)
CLSMR(2) $=$ 0.3	PARJ(41) (StringZ:aLund) $=$ 0.55 $[a]$
DECWT $=$ 0.7	PARJ(42) (StringZ:bLund) $=$ 1.08 $[b]$
CLPOW $=$ 2.1	PARJ(46) (StringZ:rFactB) $=$ 0.85 $[r]$
PSPLT(2) $=$ 0.33	
χ^2/dof $=$ 222.4/61	χ^2/dof $=$ 109.5/61 (45.9/61)

Comparison with e^+e^- data



In progress:

POWHEG (private version with $e^+e^- \to q\bar{q}$), MC@NLO

NLO corrections should have small impact on the fits

Results in moment space

$$\Gamma_N = \int_0^1 dz \; z^{N-1} \frac{1}{\Gamma} \frac{d\Gamma}{dz}(z)$$

e^+e^- annihilation $\sigma_N^B = \sigma_N^b D_N^{np}$

σ_N^B measured ; σ_N^b calculated ; D_N^{np} fitted

top decay: $\Gamma_N^B = \Gamma_N^b D_N^{np} = \Gamma_N^b \sigma_N^B / \sigma_N^b$

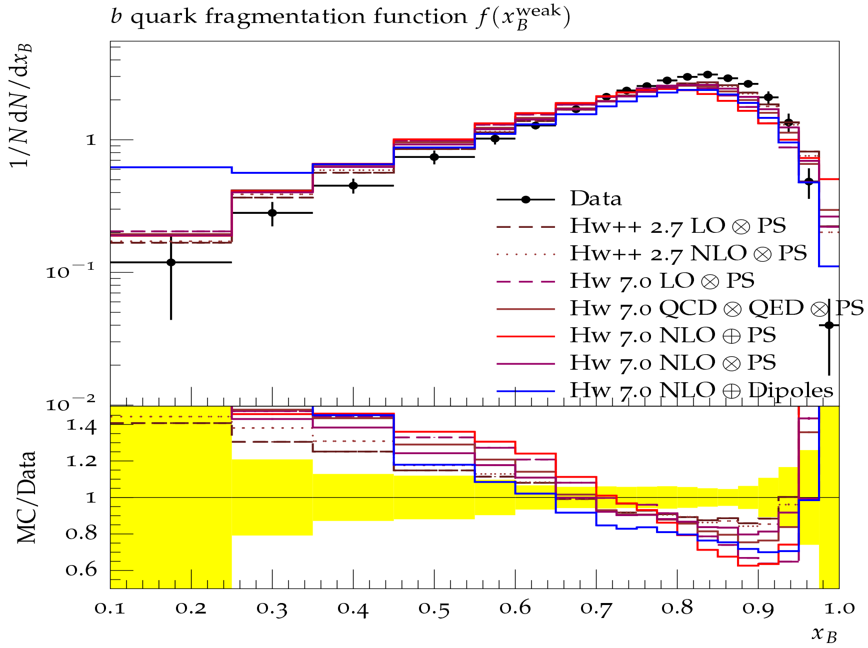
Fits to DELPHI data in moment space

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	0.7153±0.0052	0.5401±0.0064	0.4236±0.0065	0.3406±0.0064
e^+e^- NLL σ_N^b	0.7801	0.6436	0.5479	0.4755
$D_N^{\rm np}$	0.9169	0.8392	0.7731	0.7163
e^+e^- HW σ_N^B	0.7113	0.5354	0.4181	0.3353
e^+e^- PY σ_N^B	0.7162	0.5412	0.4237	0.3400
t-dec. NLL Γ_N^b	0.7883	0.6615	0.5735	0.5071
t-dec. NLL $\Gamma_N^B = \Gamma_N^b D_N^{\rm np}$	0.7228	0.5551	0.4434	0.3632
t-dec. HW Γ_N^B	0.7325	0.5703	0.4606	0.3814
t-dec. PY Γ_N^B	0.7225	0.5588	0.4486	0.3688

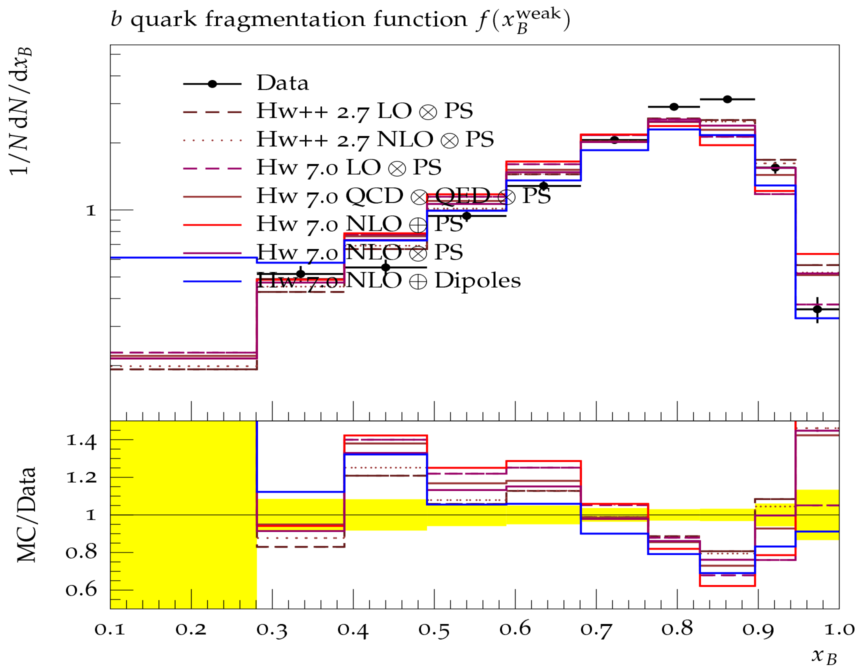
Extension to NNLO/NNLL is feasible

Results with HERWIG 7 (from https://herwig.hepforge.org/plots/herwig7.0/)

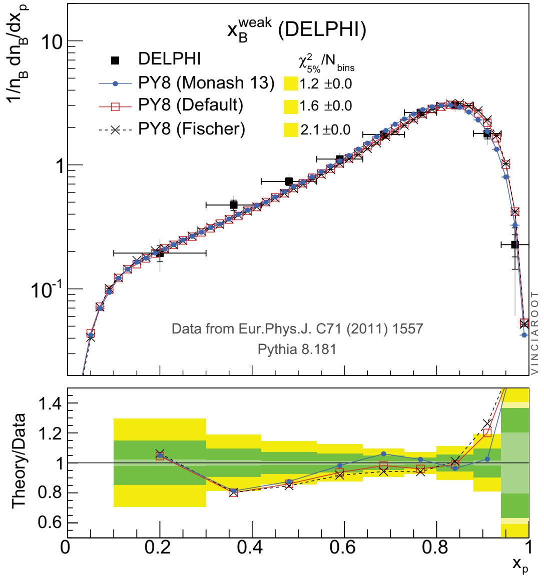
ALEPH data

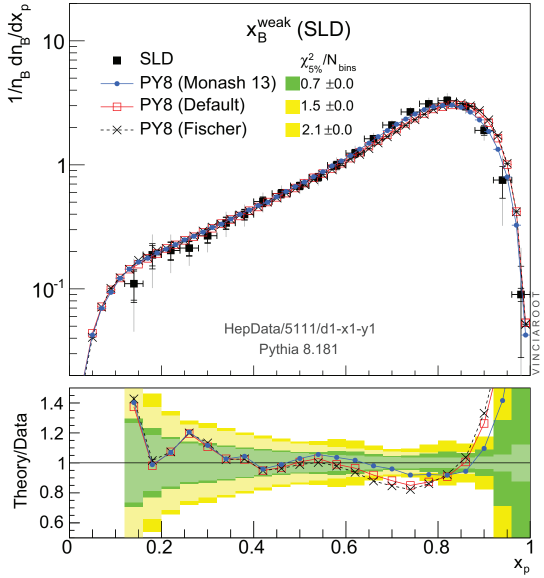


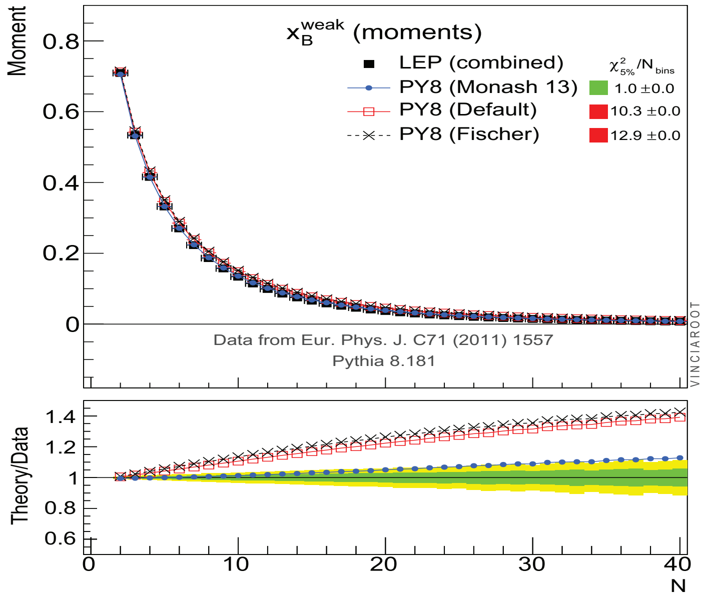
DELPHI data



PYTHIA 8.1 with Monash tuning for b-fragmentation (Skands, Carrazza, Rojo, '14)





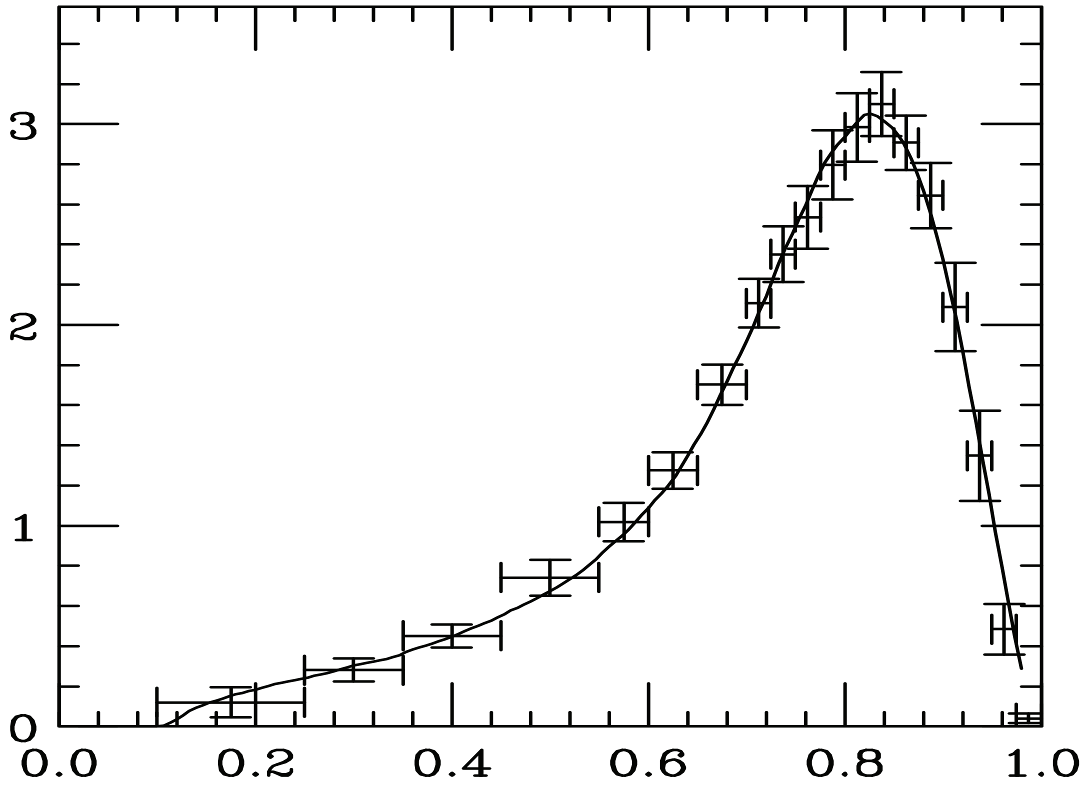


Preliminary results with POWHEG+PYTHIA 8 (E.Bagnaschi)

POWHEG has a private version with $e^+e^- \to q\bar{q}$ at NLO

It needs fitting to data, because of NLO hard scattering

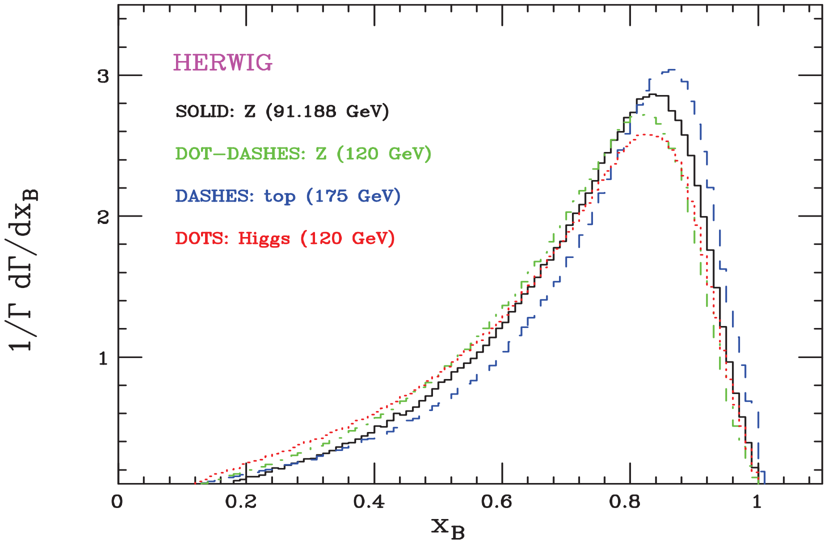
Tuning to ALEPH data: $a = 0.8 \pm 0.19$; $b = 0.85 \pm 0.17$; $r = 0.85 \pm 0.02$; $\chi^2 \sim 10^{-3}$

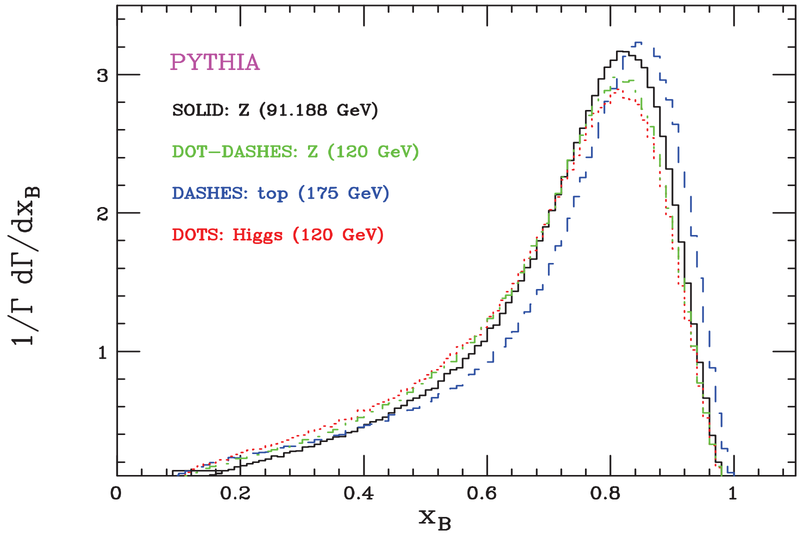


Crucial to model b-fragmentation in top decays

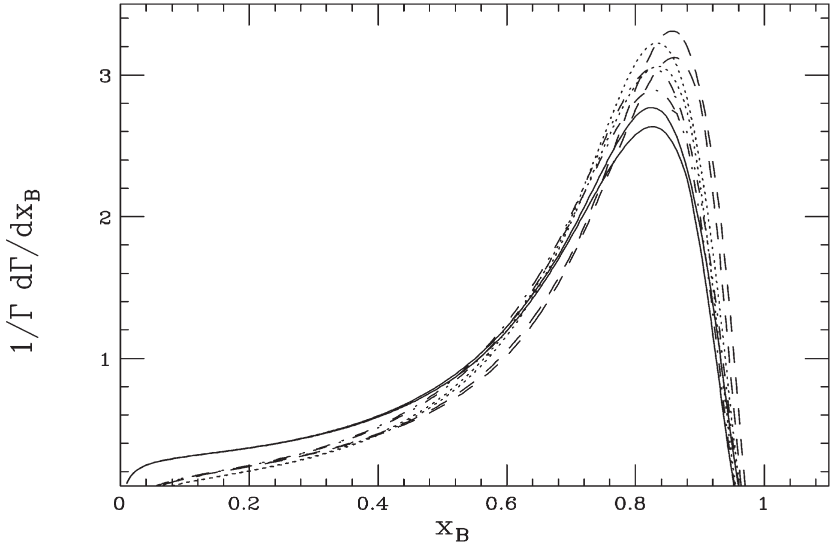
Comparing b-fragmentation in Z, Higgs and top decays - $x_B = 2p_B \cdot q/q^2$, $q = p_{Z,H,t}$

Monte Carlo generators:





NLO+NLL resummations Solid: H; Dashes: t; Dots: Z at 91 GeV; Dot-dashes Z at 120 GeV



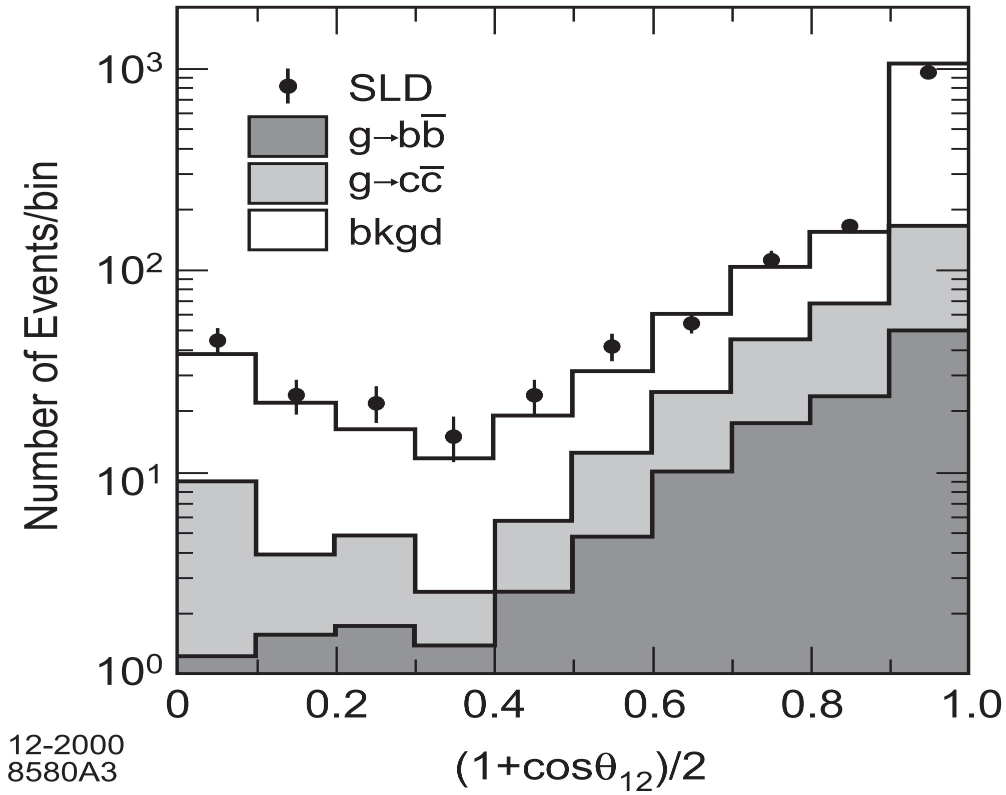
Challenging $g \to b\bar{b}$ $(c\bar{c})$ splitting and singlet fragmentation functions

Typically b/c-jet pairs at small angles: $Z \to q\bar{q}g$, $g \to c\bar{c}(b\bar{b})$

Already measured at LEP and SLD:

OPAL: $g_{c\bar{c}} = (3.20 \pm 0.21 \pm 0.38) \times 10^{-2}$;
ALEPH: $g_{c\bar{c}} = (3.23 \pm 0.48 \pm 0.53) \times 10^{-2}$

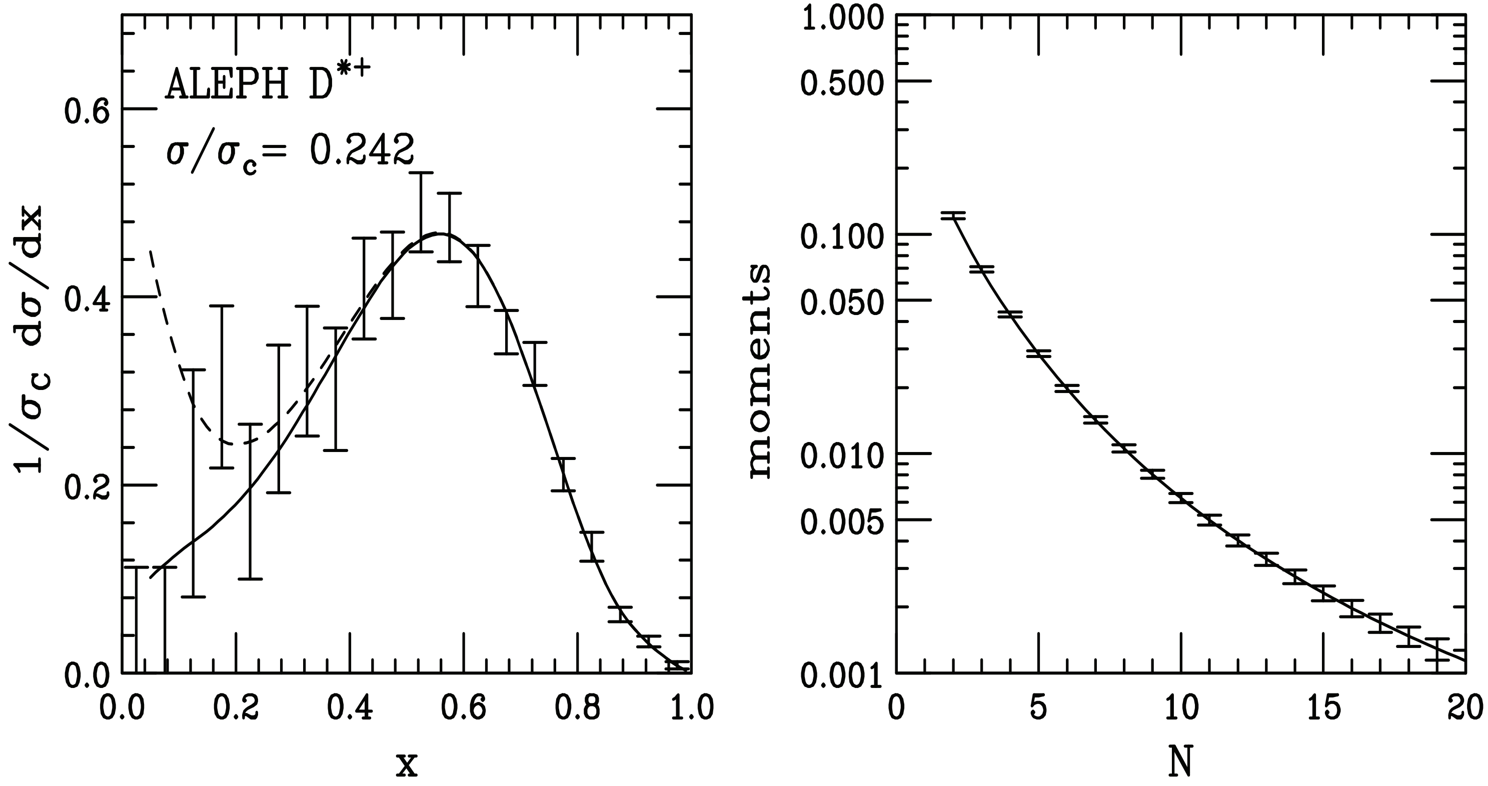
SLD: $g_{b\bar{b}} = (2.44 \pm 0.59 \pm 0.34) \times 10^{-3}$
DELPHI: $g_{b\bar{b}} = [2.1 \pm 1.1 \pm 0.9) \times 10^{-3}$
ALEPH: $g_{b\bar{b}} = (2.77 \pm 0.42 \pm 0.57) \times 10^{-3}$



SLD data vs. signal and background for $g \to b\bar{b}$ $(c\bar{c})$ θ_{12}: angle between b/c-jets

Heavy-meson fragmentation and $g \to c\bar{c}$ splitting (Cacciari, Nason, Oleari'06)

Inclusion of singlet, but not necessary to fit data on charm and bottom fragmentation

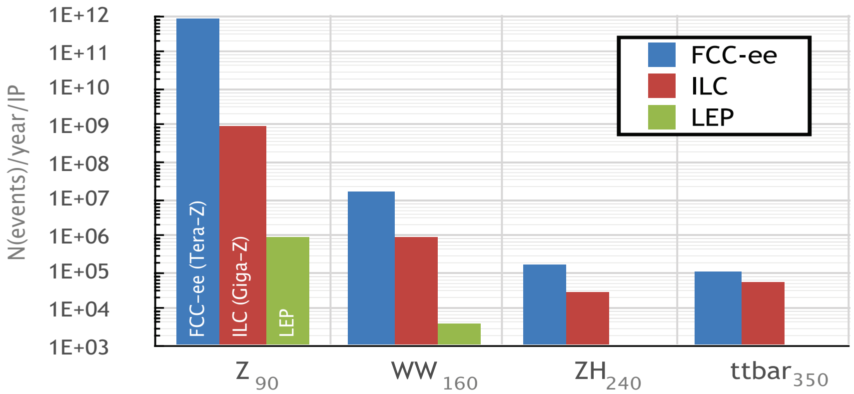


FCC-ee: higher statistics and granularity of calorimeter and vertex detector allow one to be sensitive to $g \to b\bar{b}$ splitting, through a 'double tags'

Unique environment to test singlet vs non singlet contributions to heavy-quark fragmentation

Perspectives at FCC-ee - $\mathcal{L}_{\rm int} = 1\ \rm ab^{-1}$ (see P.Skands and D.d'Enterria, 1610.06254)

$\sqrt{s}$ (GeV):	90 (Z)	125 (eeH)	160 (WW)	240 (HZ)	350 ($t\bar{t}$)	350 (WW→H)
σ	43 nb	290 ab	4 pb	200 fb	0.5 pb	25 fb
$\mathcal{L}$/IP (cm^{-2} s^{-1})	$4.3 \cdot 10^{36}$	$2.2 \cdot 10^{36}$	$7.6 \cdot 10^{35}$	$1.8 \cdot 10^{35}$	$5 \cdot 10^{34}$	$5 \cdot 10^{34}$
$\mathcal{L}_{\rm int}$ (ab^{-1}/yr, 2 IPs)	86	45	15	3.5	1.0	1.0
Events/year (2 IPs)	$3.7 \cdot 10^{12}$	$1.3 \cdot 10^{4}$	$6.1 \cdot 10^{7}$	$7.0 \cdot 10^{5}$	$5 \cdot 10^{5}$	$2.5 \cdot 10^{4}$
Years needed (2 IPs)	2.5	1.5	1	3	0.5	3



Statistics 10^5 higher than LEP $\Rightarrow$ statistical uncertainties reduced by 30

At LEP/SLD R_b tagging one b-jet and efficiency by means of 'double tag' method

Current value: $R_b = 0.21629 \pm 0.00066$ systematics and statistics equally shared

SLD better than LEP thanks to more granular vertex detector and smaller beam spot

FCC-ee will have smaller beam spot size than SLD and next-generation vertex detector

A precision of 2-5$\times 10^{-5}$ can be reached on R_b (TLEP WG, JHEP'04)

Exclusive analyses and better fragmentation (meson/baryon) extraction (e.g. $B \to J/\psi X$)

Feasibility for accurate measurements of $gg \to c\bar{c}$ $(b\bar{b})$

Conclusions

Heavy-flavour fragmentation is challenging at FCC-ee

Resummations: state of the art is NLO+NLL - extension to NNLO+NNLL feasible

Monte Carlo codes for b-fragmentation: HERWIG and PYTHIA tuned to data

NLO+shower generators (POWHEG, MC@NLO) need to be fitted to heavy-flavour fragmentation data

$g \to c\bar{c}$ $(b\bar{b})$ measured at LEP and challenging at FCC-ee thanks to high statistics and detector granularity

Exclusive measurements will allow one to distinguish b- and c-flavoured hadrons and separate fragmentation spectra (charged vs neutral, spin 1 vs spin 0, baryons vs mesons)

Precise b-fragmentation measurements crucial for top and Higgs phenomenology

Higher-order calculations and tuned Monte Carlo generators should be able to meet the precision requirements of FCC-ee