Resummation for event shapes and jet rates at FCC-ee

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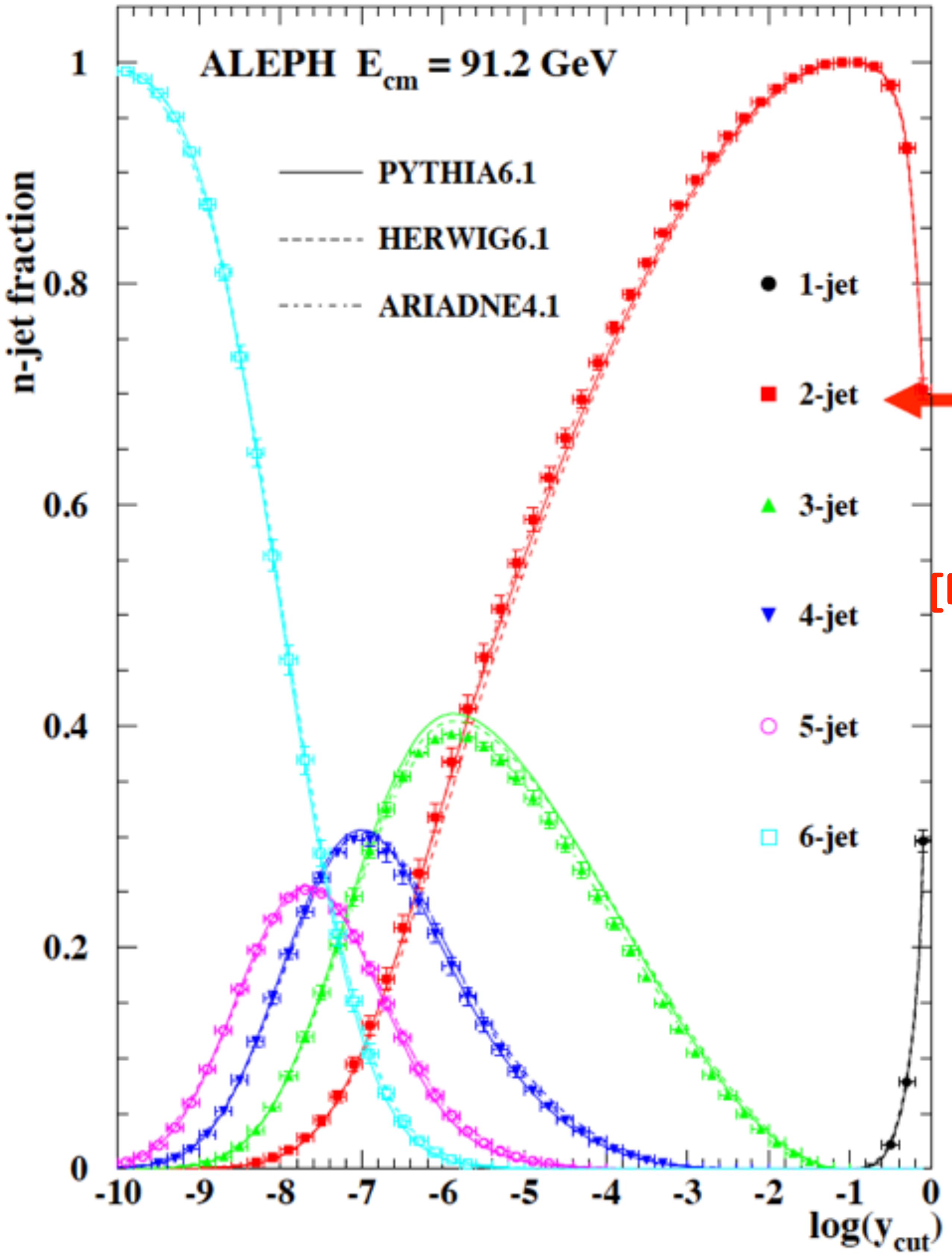
CERN

Parton Radiation and Fragmentation from LHC to FCC-ee - CERN, 22 November 2016

Fixed-order for $e^+e^- \to Z/\gamma^* \to \text{hadrons}$

E.g. exclusive jet rates (Durham kt algorithm)



NNLO, i.e. $1 + \alpha_s + \alpha_s^2 + \alpha_s^3$

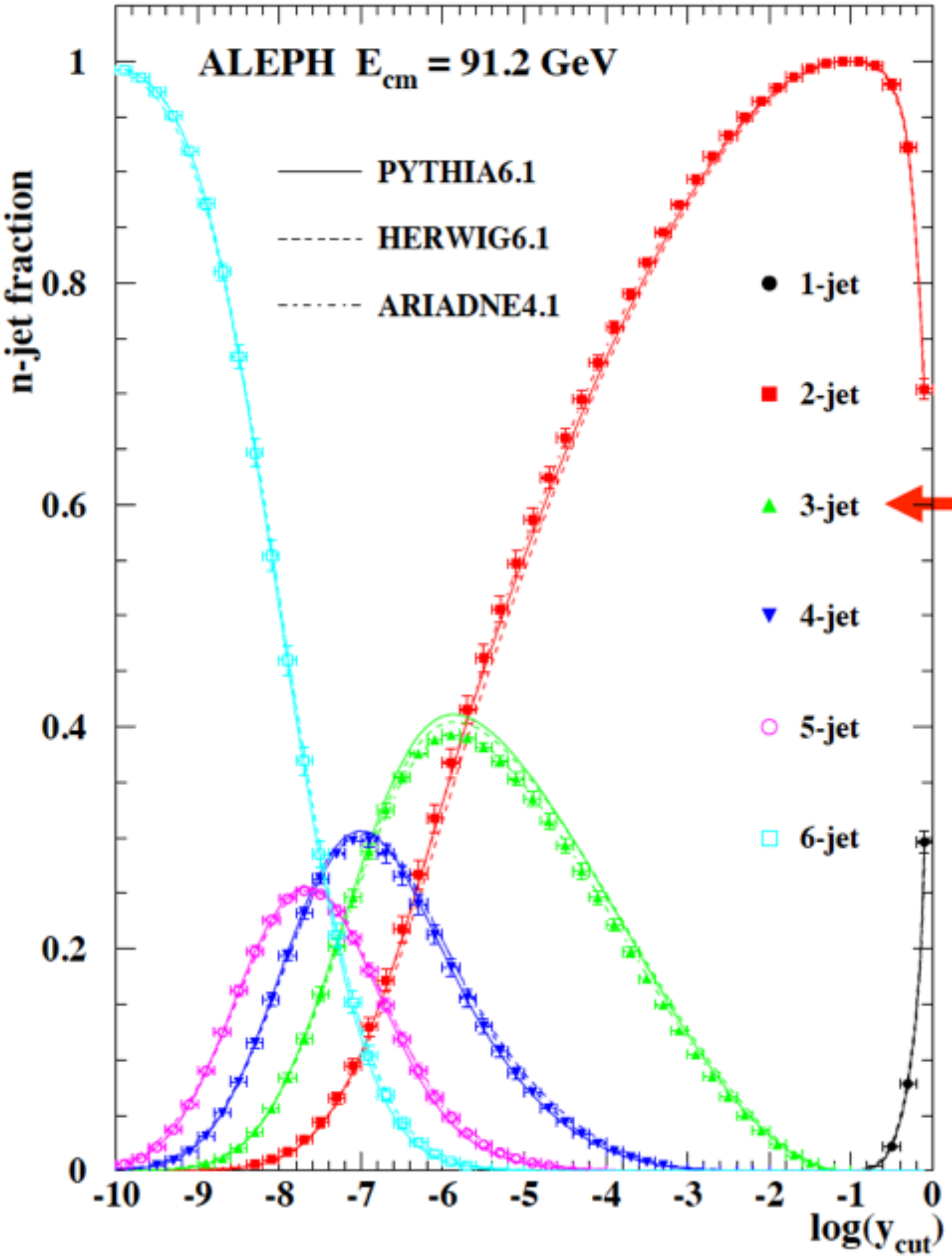
[Gehrmann Gehrmann Glover Heinrich ‘08]

[Weinzierl ‘09]

[Del Duca Duhr Kardos Somogyi Szor Trocsanyi Tullipant ’16]

Slide from A. Banfi’s talk at this workshop in 2015

Fixed-order for $e^+e^- \to Z/\gamma^* \to$ hadrons



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[Gehrmann Gehrmann Glover Heinrich ‘08]
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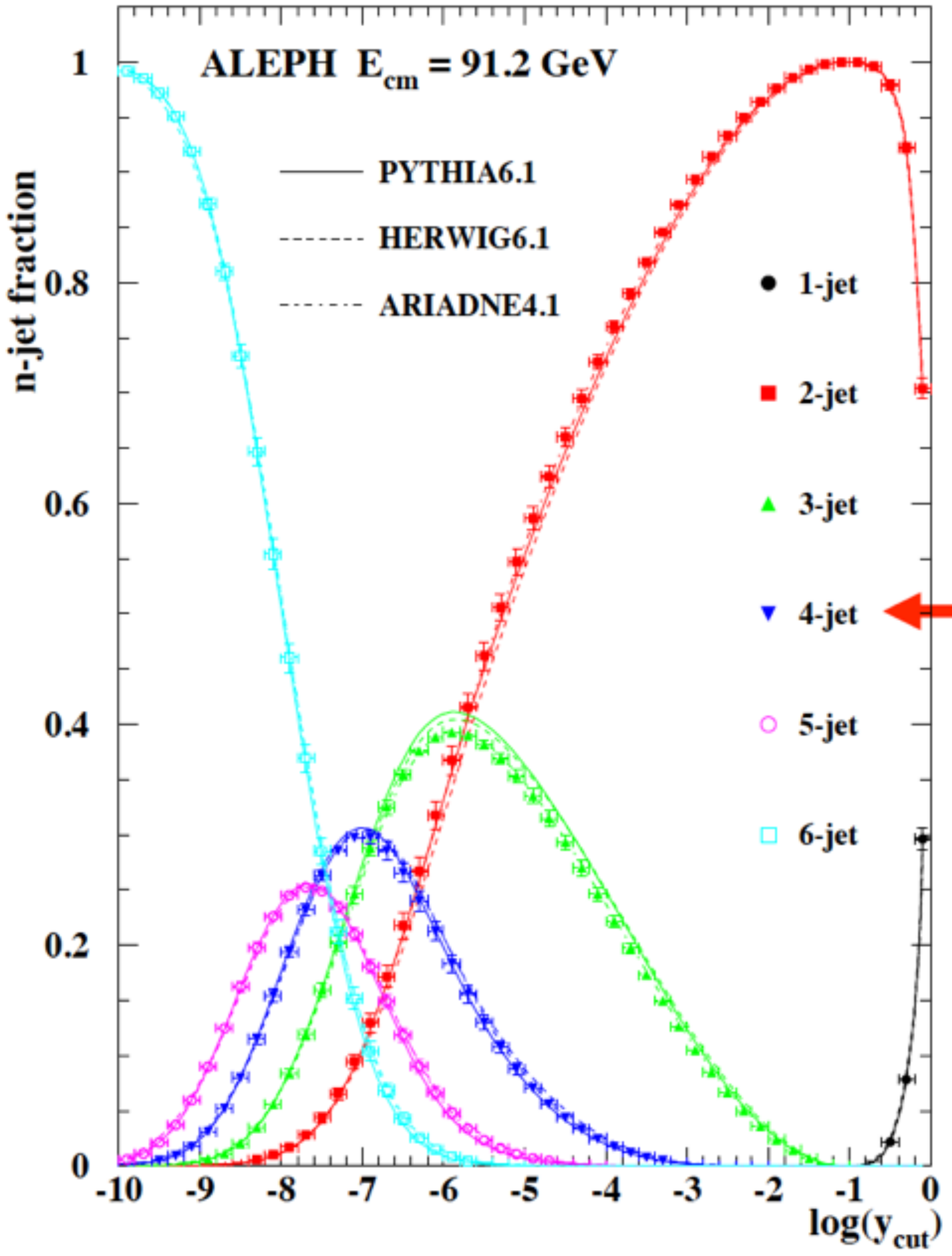
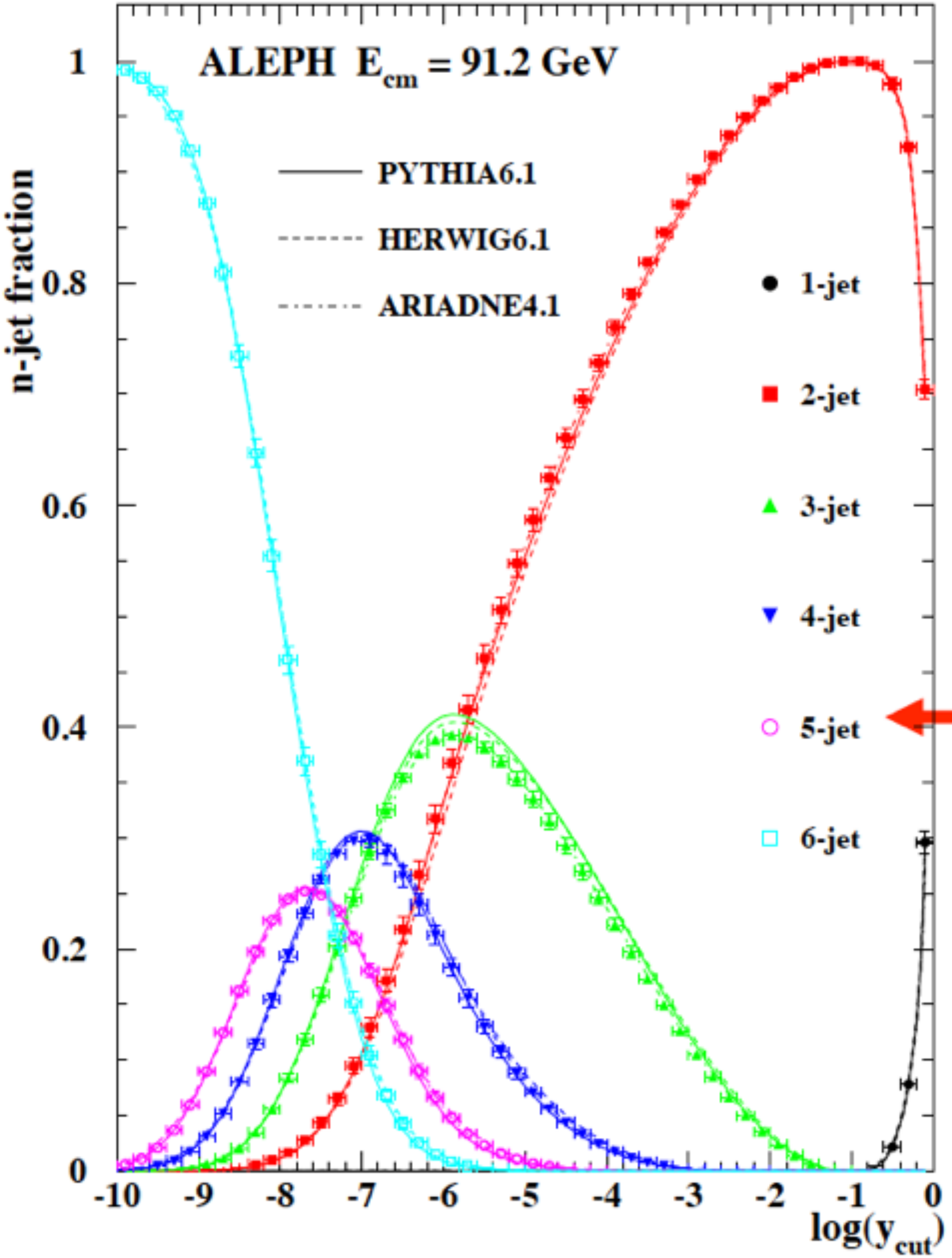


← NLO, i.e. $\alpha_s^2 + \alpha_s^3$

[Nagy Trocsanyi '99, Kosower Weinzierl '99]
[Campbell Cullen Glover '99]

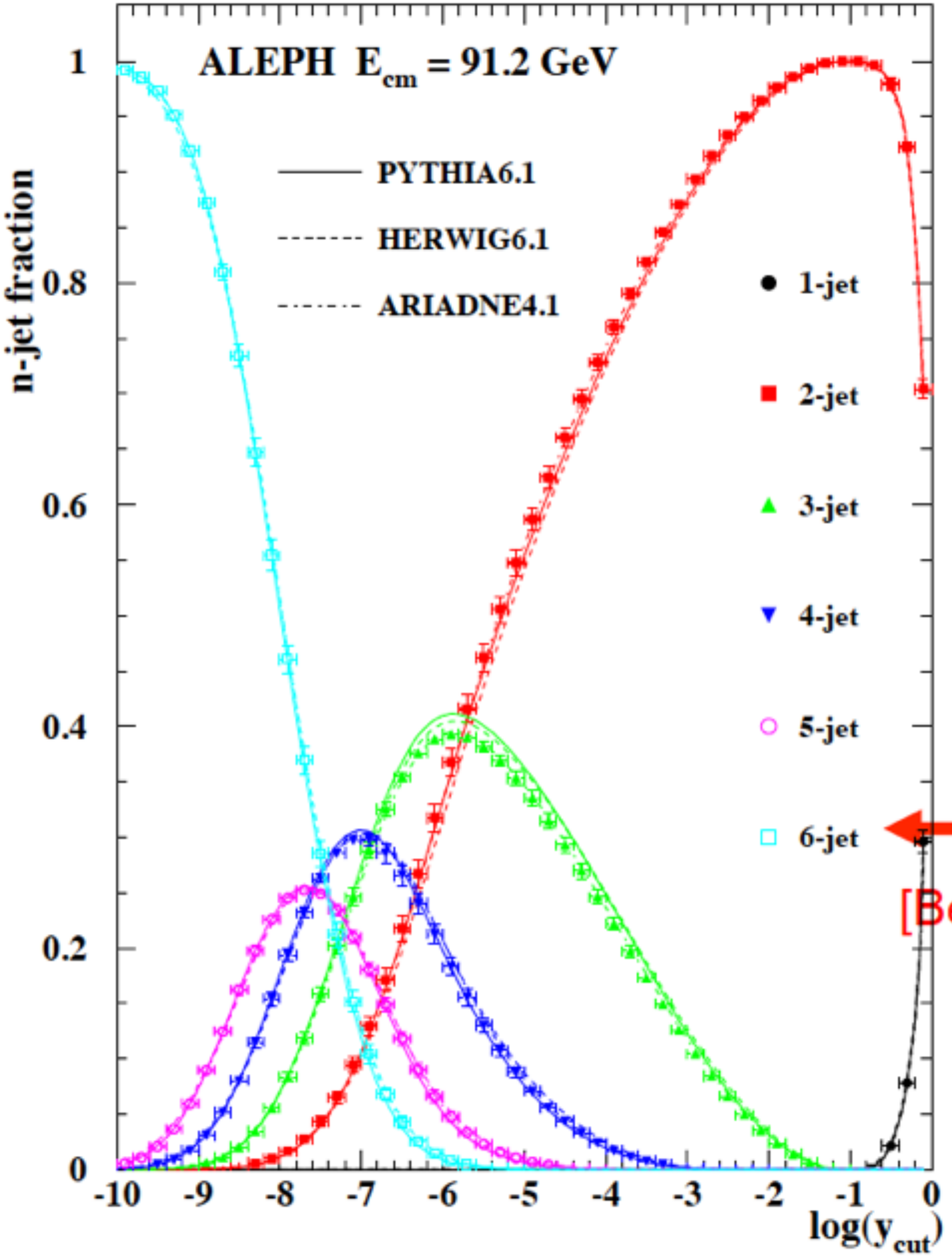
Fixed-order for $e^+e^- \to Z/\gamma^* \to$ hadrons



NLO, i.e. $\alpha_s^3 + \alpha_s^4$

[Frederix Frixione Melnikov Zanderighi '10]

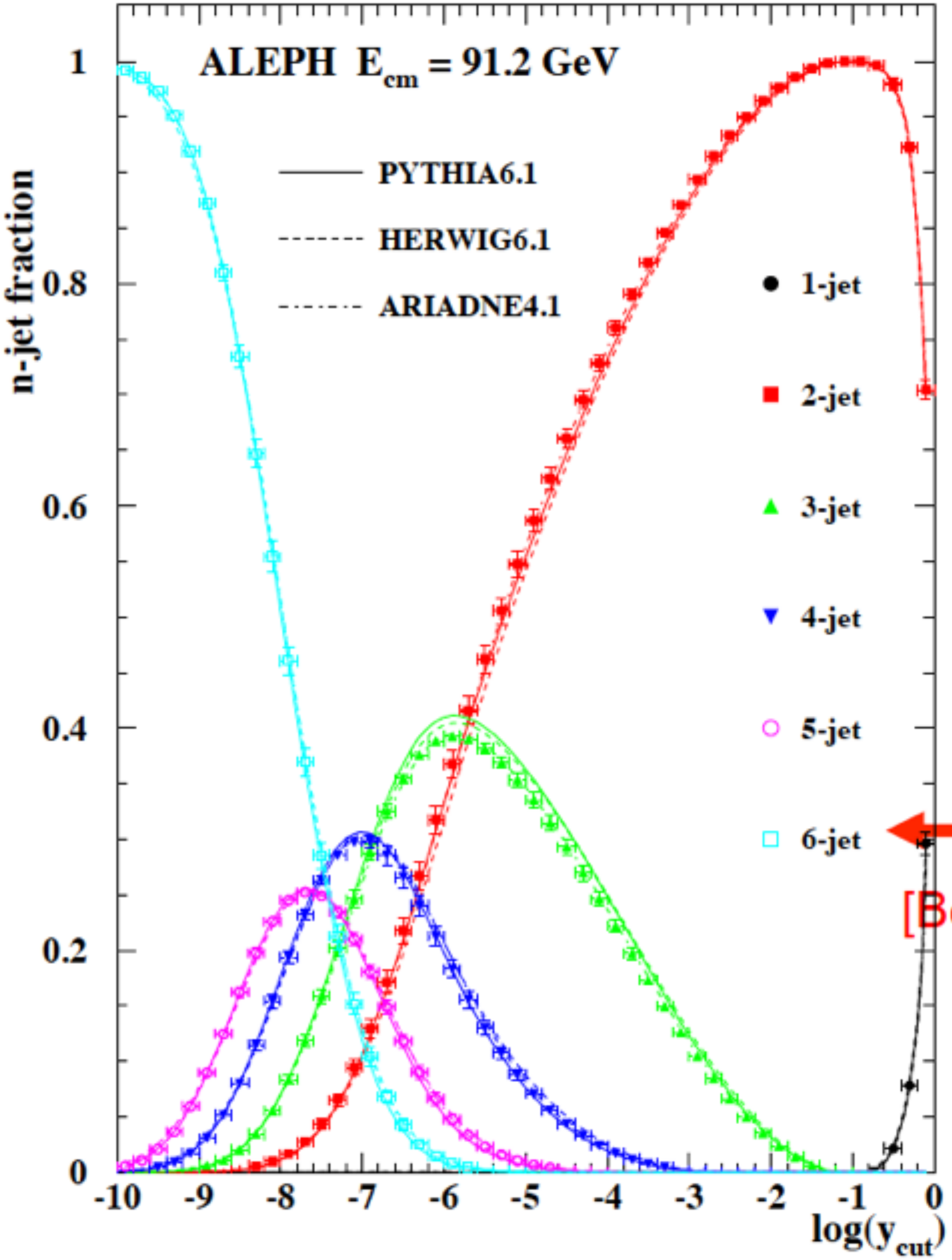
Fixed-order for $e^+e^- \to Z/\gamma^* \to$ hadrons



NLO for 6 and 7 jets (leading color only)

[Becker Goetz Reuschle Schwan Weinzierl '12]

Fixed-order for $e^+e^- \to Z/\gamma^* \to$ hadrons



The “clean” events at e+e- collisions allows one to perform measurements of these quantities quite close to the parton-level definition, into the deep IR regime where a good theory control can be achieved within perturbation theory. In these regimes fixed-order predictions must be supplemented with an all-order treatment of the dominant terms in the perturbative expansion

NLO for 6 and 7 jets (leading color only)

[Becker Goetz Reuschle Schwan Weinzierl ‘12]

Factorisation of amplitudes in the IR

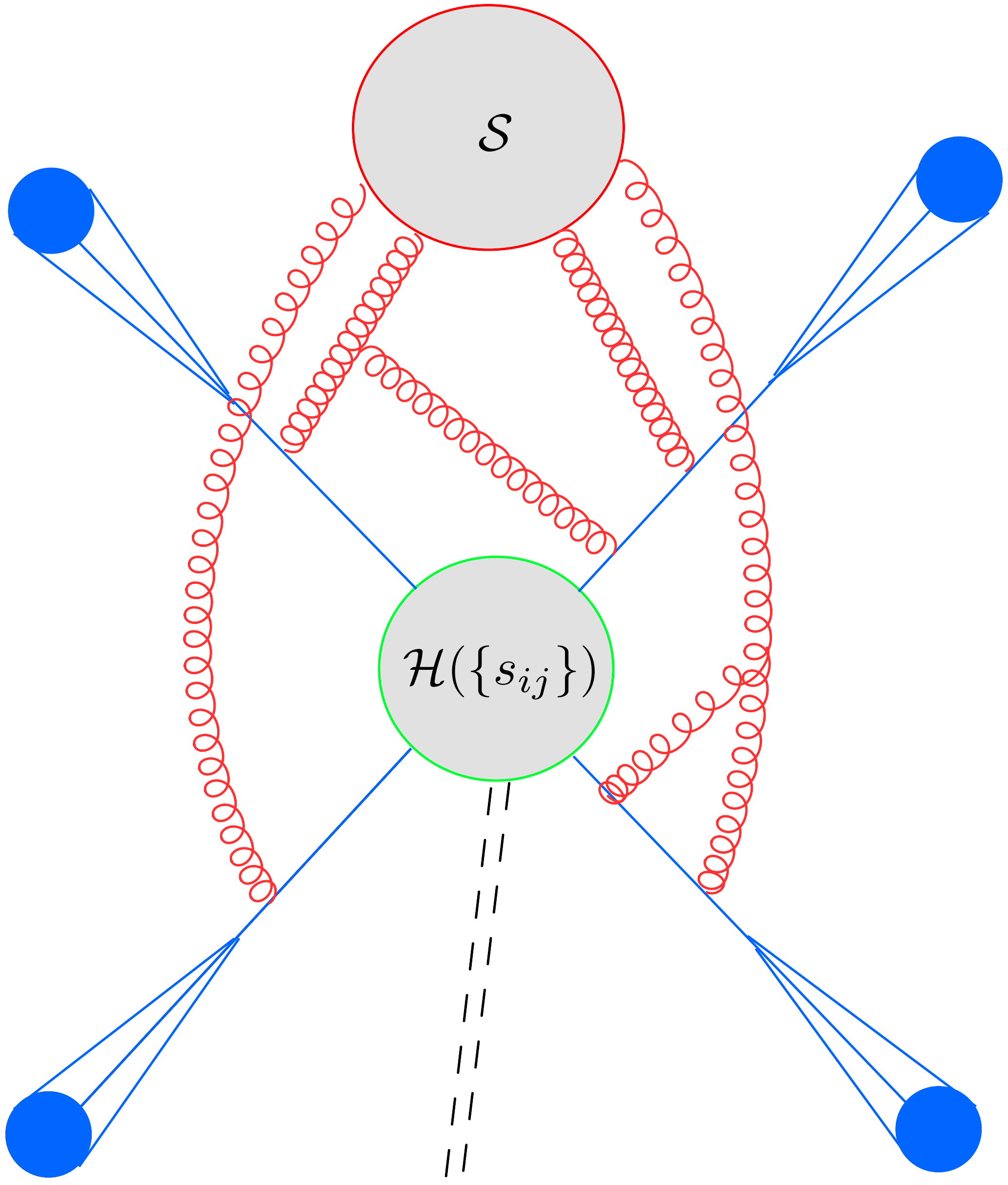
- Consider a IRC observable $V = V(\{\tilde{p}\}, k_1, ..., k_n) \leq 1$ in the Born-like limit $V \to 0$

- In this limit radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions (singular limit) — QCD squared **amplitudes factorise** in these regimes w.r.t. the Born, up to regular corrections

- Different observables are sensitive to different singular modes which determine the logarithmic structure of the perturbative expansion (e.g. (non) global, hard-collinear logarithms, …)

- In the limit of large logarithms and all-order treatment is necessary - effects often propagate far from the singular limit

$\mathrm{soft\ wide-angle}:\ \alpha_s^n L^m\ \ (m \leq n)$



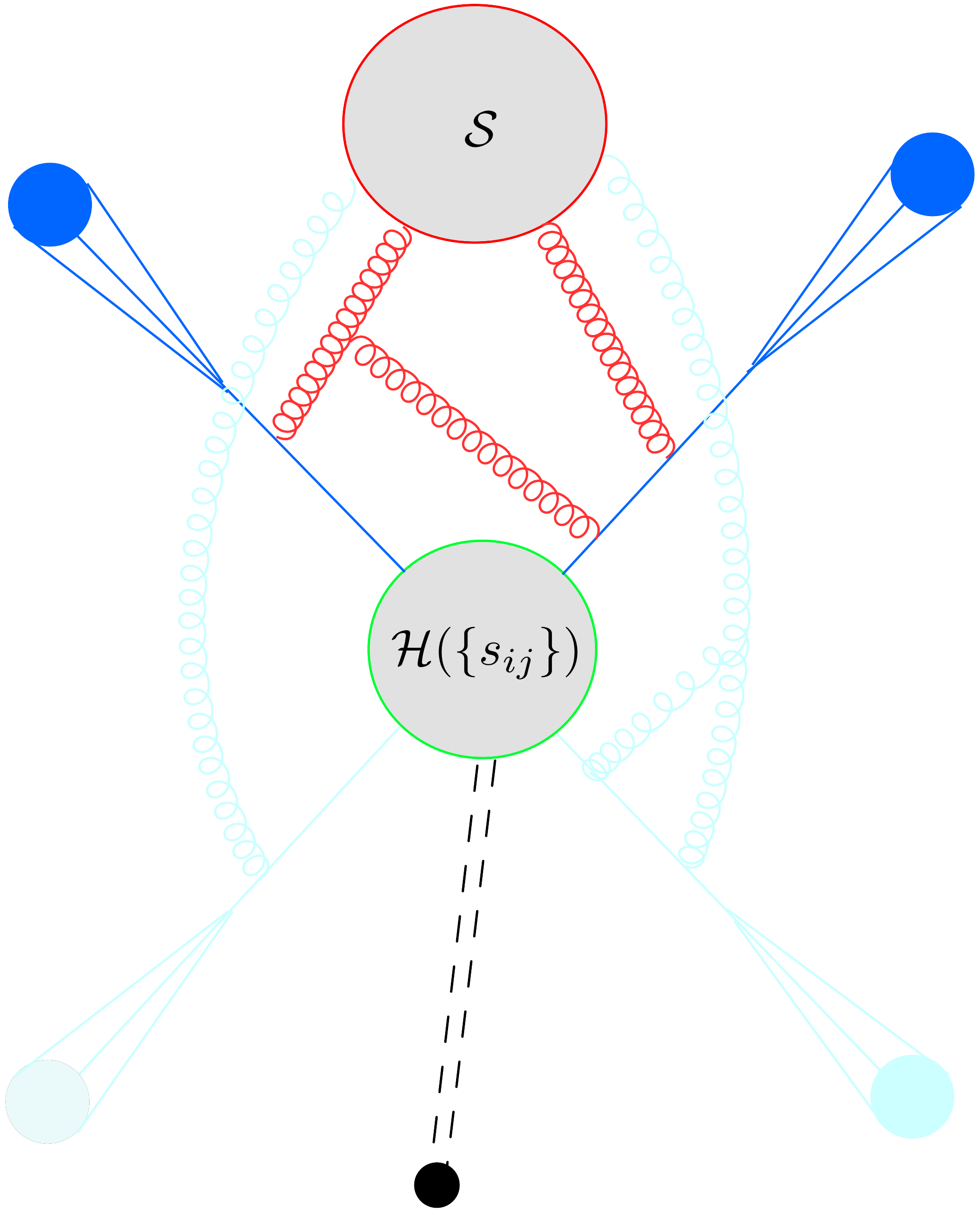
colourless system

$\mathrm{soft-collinear}:\ \alpha_s^n L^m\ \ (m \leq 2n)$

$\mathrm{hard-collinear}:\ \alpha_s^n L^m\ \ (m \leq n)$

Two-emitter processes

- The strong angular separation between different modes ensures they evolve independently at late times after the collision
- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

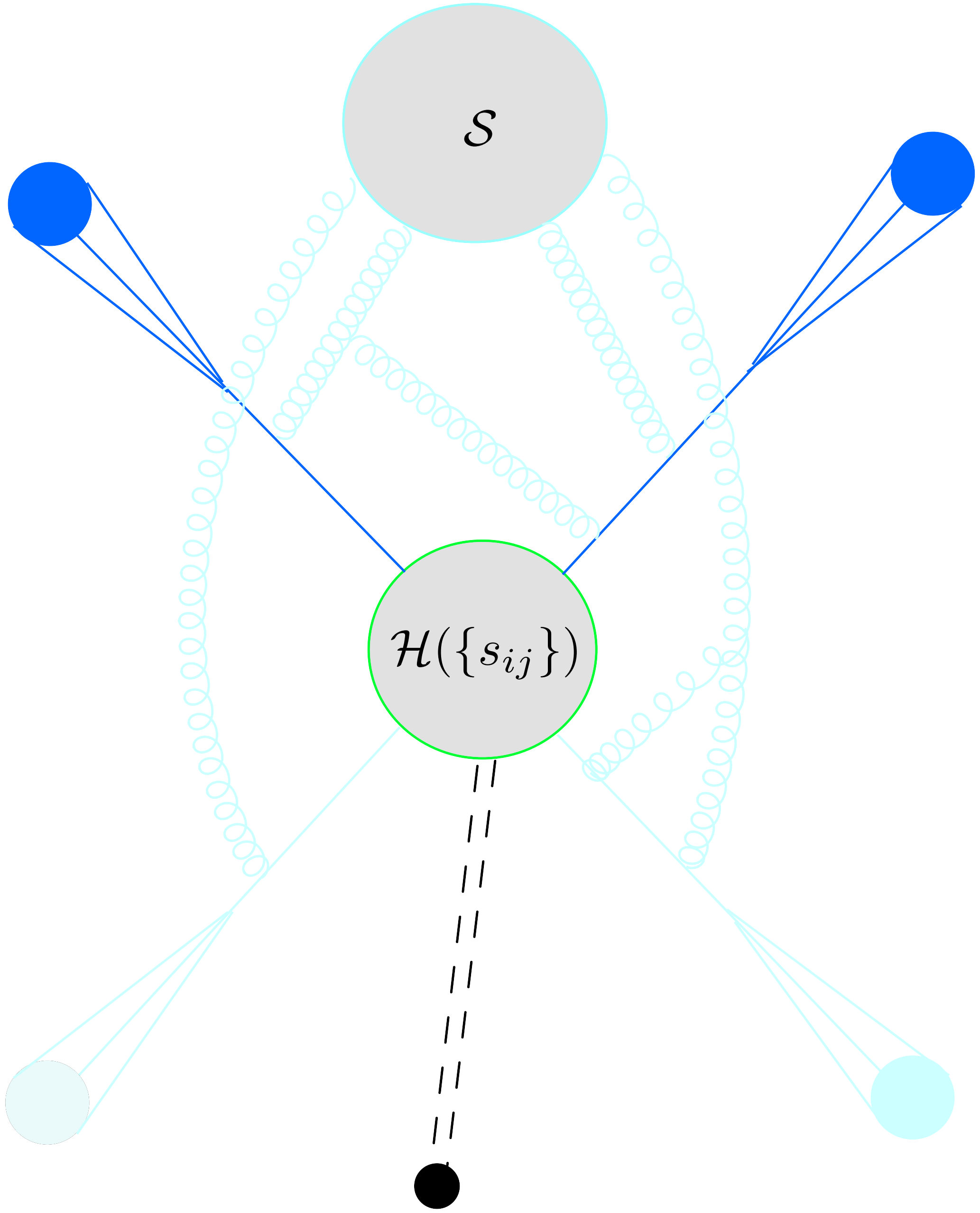


Two-emitter processes

- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

- For continuously global observables in processes with two emitters, colour coherence forces the effect of soft modes exchanged with large angles to vanish
  - Only collinear (soft/hard) modes effectively remain
  - Soft modes can be absent in specific cases

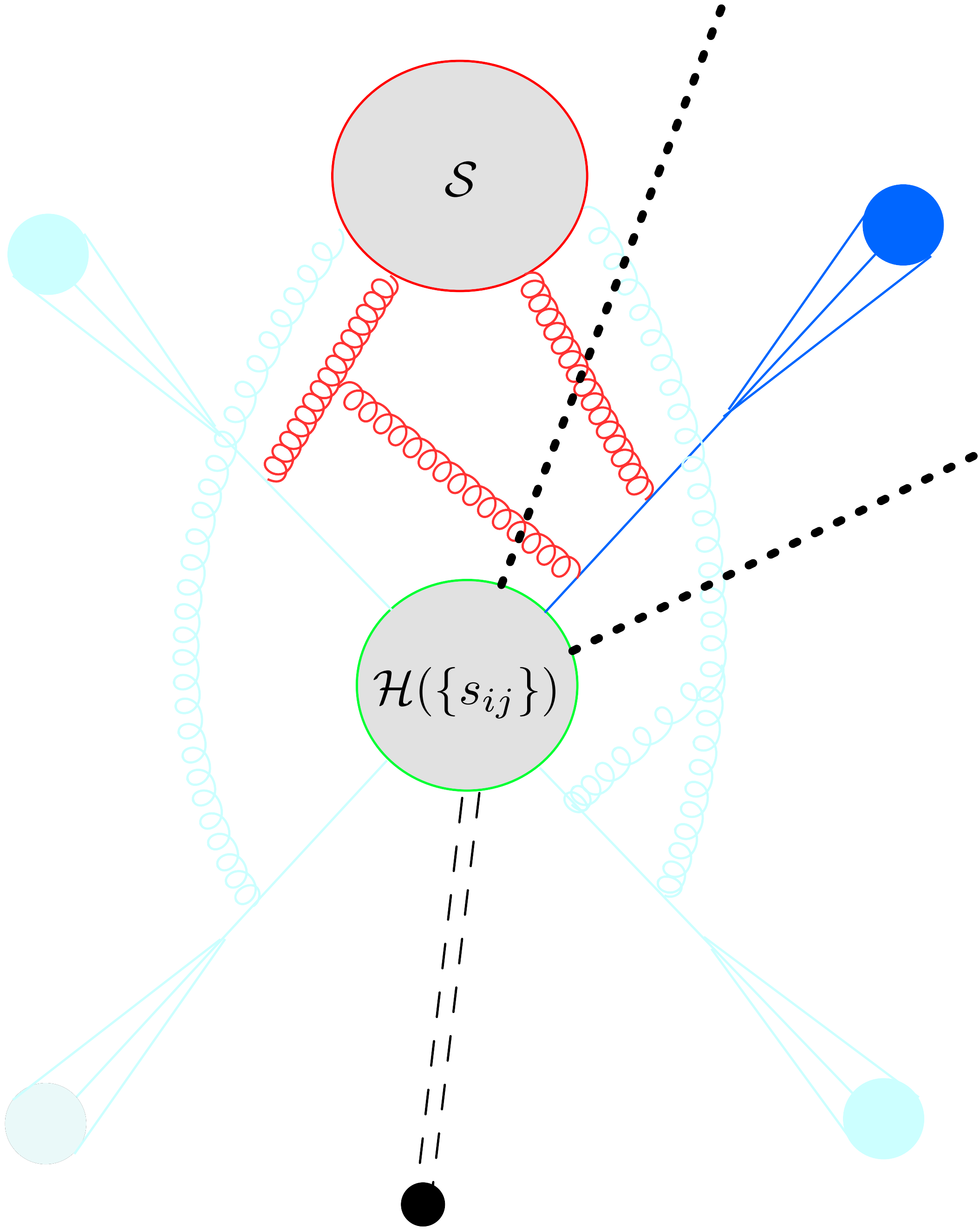


Non-Global observables

- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

- For non-global observables one is always sensitive to the full evolution of the soft radiation outside of the resolved phase-space region
  - In general both soft and collinear modes are present
  - Collinear modes are absent for some observables



[Dasgupta, Salam '01]

Resummation of global observables

- A generic cumulative cross section can be parametrised as

$$\Sigma(v) = \sigma_0 \int \frac{dv_1}{v_1} D(v_1) P(v|v1), \qquad D(v_1) = e^{-R(v_1)} R'(v_1)$$

Probability of emitting the hardest parton v1 = v(k1)

Probability of secondary radiation given the first emission, and the observable's value v

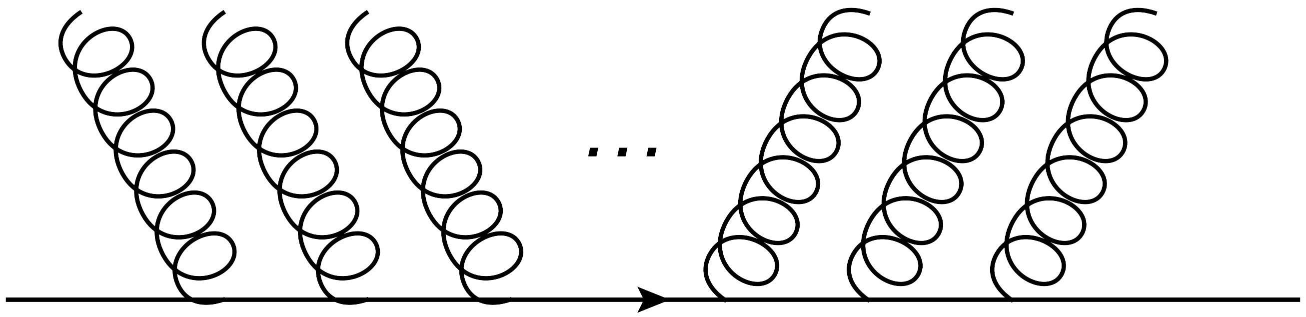
- Under a general property, known as recursive IRC safety (fulfilled by most observables), one can devise an approach to resummation that does not require a factorisation theorem (this applies also to non-global problems).

[Banfi, Salam, Zanderighi '01-'04]

- rIRC safety guarantees:
  - the cancellation of IRC singularities at all orders in the probability $P(v|v1)$
  - all leading logarithms $(\alpha_s^n \ln^{n+1}(1/v))$ exponentiate $\to e^{-R(v)}$
    - multiple-emission effects in $P(v|v1)$ are at most NLL
  - a logarithmic hierarchy in the real emission probability —> At NLL only independent emissions contribute to $P(v|v1)$

Resummation of global observables

- NLL general answer: ensemble of soft-collinear gluons independently emitted and widely separated in rapidity → `CAESAR`



[Banfi, Salam, Zanderighi '01-'04]

$$= \int d\mathcal{Z}[\{R'_{\mathrm{NLL},\ell_i}, k_i\}]\,\Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}\right)$$

$$\mathcal{F}_{\mathrm{NLL}}(v) = \langle\Theta(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v})\rangle$$

- Structure of NNLL corrections more involved: less singular kinematic configurations in the amplitudes and phase space → `ARES`

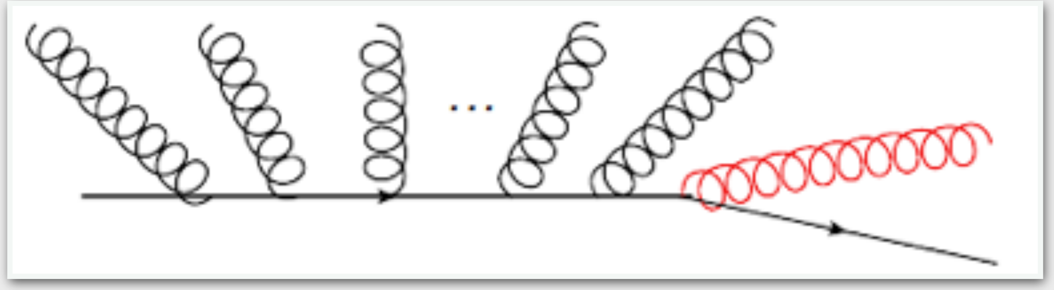
[Banfi, McAslan, PM, Zanderighi '14-'16]

$$\Sigma(v) = \sigma_0 e^{-R(v)}\left[\mathcal{F}_{\mathrm{NLL}} + \frac{\alpha_s}{\pi}\left(\delta\mathcal{F}_{\mathrm{rap}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}}\right)\right]$$

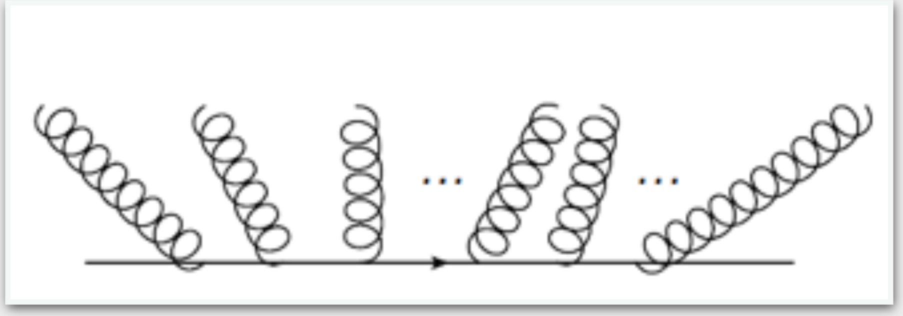
General structure of NNLL (global case)

[Banfi, McAslan, PM, Zanderighi '14-'16]

- (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)
  - correction to the amplitude: hard-collinear corrections
  - correction to the observable: recoil corrections

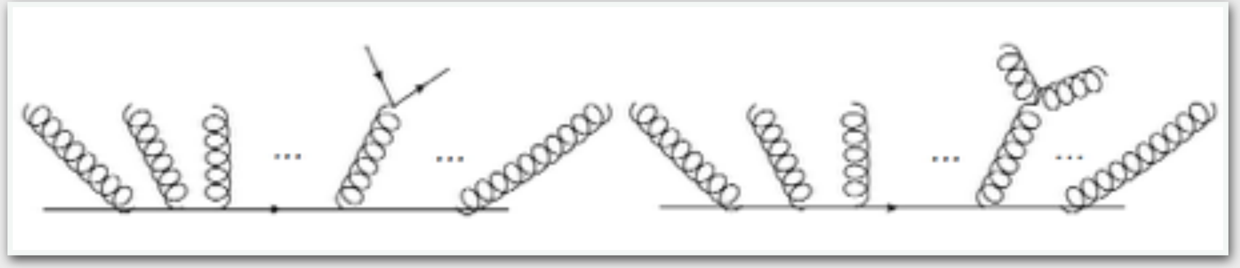


- (at most) one soft-collinear emission is allowed to get arbitrarily close in rapidity to any other of the ensemble (relax strong angular ordering)
  - sensitive to the exact rapidity bounds: rapidity corrections
  - different clustering history if a jet algorithm is used: clustering corrections



- (at most) one soft-collinear gluon is allowed to branch in the real radiation, and the branching is resolved (correction to the CMW scheme for the running coupling)

  - correlated corrections



- (at most) one soft emission is allowed to propagate at small rapidities

  - soft-wide-angle corrections

- Non-trivial abelian correction (~Cf^n, Ca^n) for processes with two emitting legs at the Born level (it simply amounts to accounting for the correct rapidity dependence for one emission) - non-abelian contribution entirely absorbed into running coupling

- Non-abelian structure more involved in the multi leg case due to quantum interference between hard emitters (general formulation at NLL, still unknown at NNLL)

General structure of NNLL (global case)

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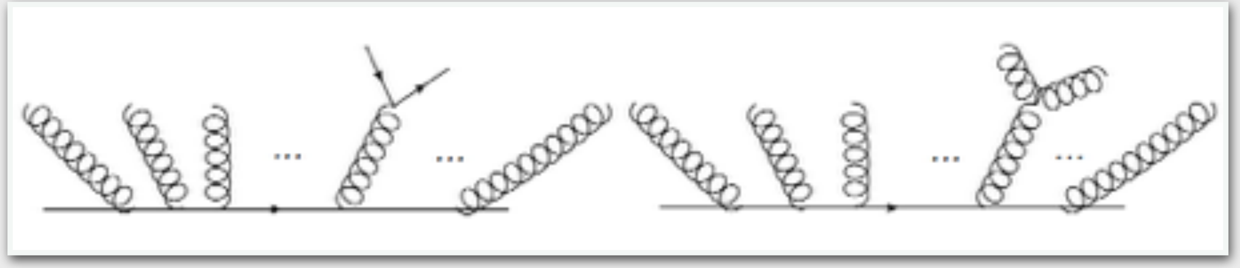
  - different clustering history if a jet algorithm is used: clustering corrections

- use strategy of regions on amplitudes and observable to single out each contribution avoiding double-counting
- all corrections finite in four dimensions -> subtraction of IRC singularities local
- Fast numerical implementation and natural automation for any rIRC safe observable
- Extension to processes with more than 2 legs requires a more general treatment of the soft-wide-angle region
- Systematically extendable to higher orders if necessary



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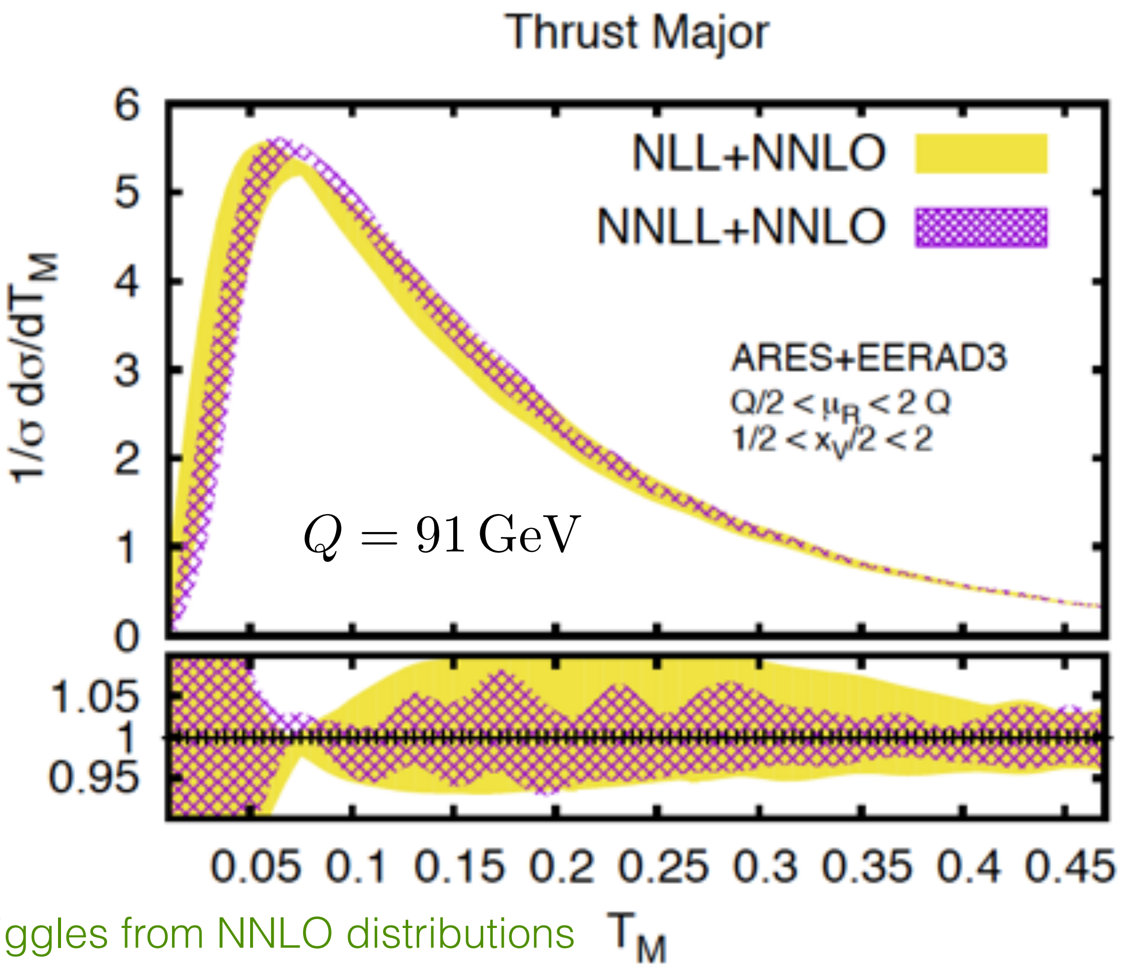
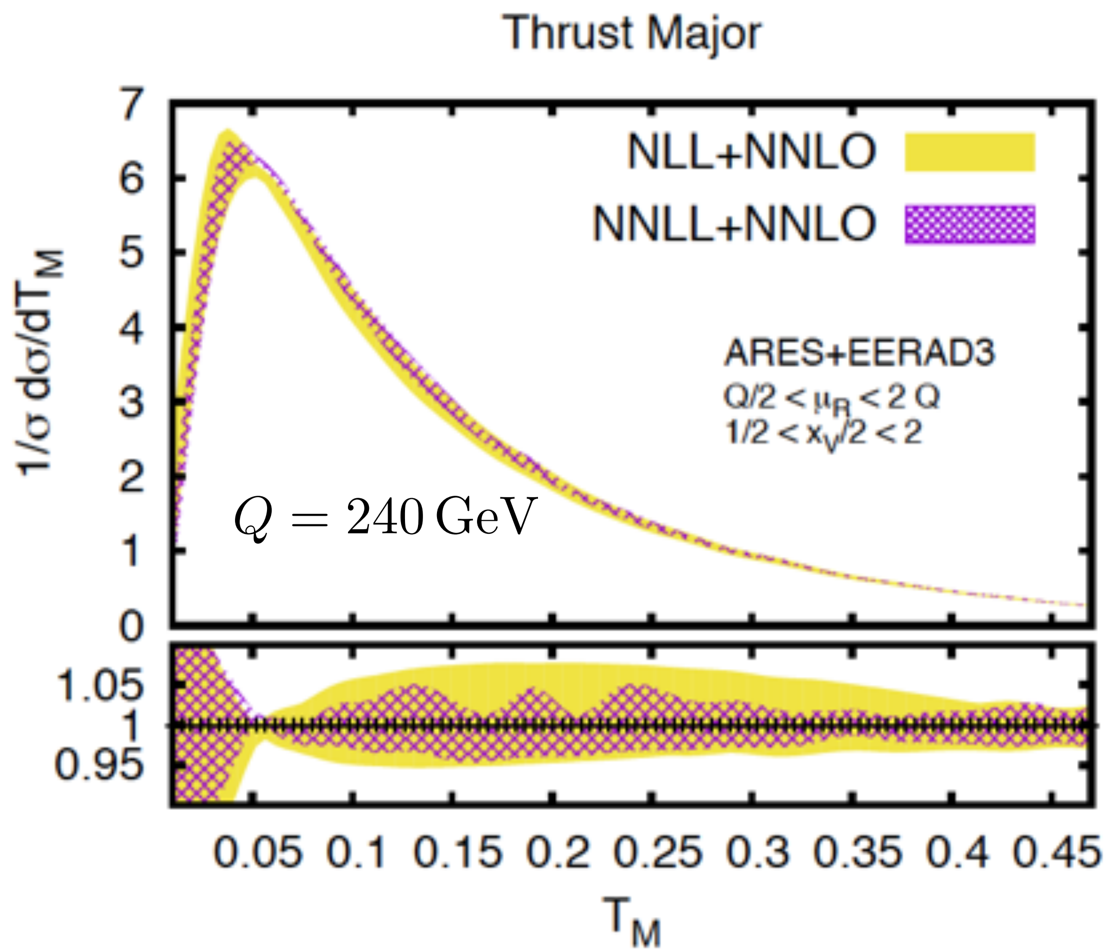
Event Shapes at FCC-ee

- Event shapes originally designed to test the non-abelian nature of QCD and the dynamics of the strong radiation. Possible use beyond this scope:
  - extractions of the strong coupling constant
  - constraining Higgs couplings (e.g. HZ production)
  - new observables ? e.g. q/g discrimination; jet substructure
- Reduction of perturbative uncertainties at these c.o.m. energies (~3%). Small effects become relevant





wiggles from NNLO distributions

Strong coupling constant

World average: [Bethke, Salam, Dissertori '15]

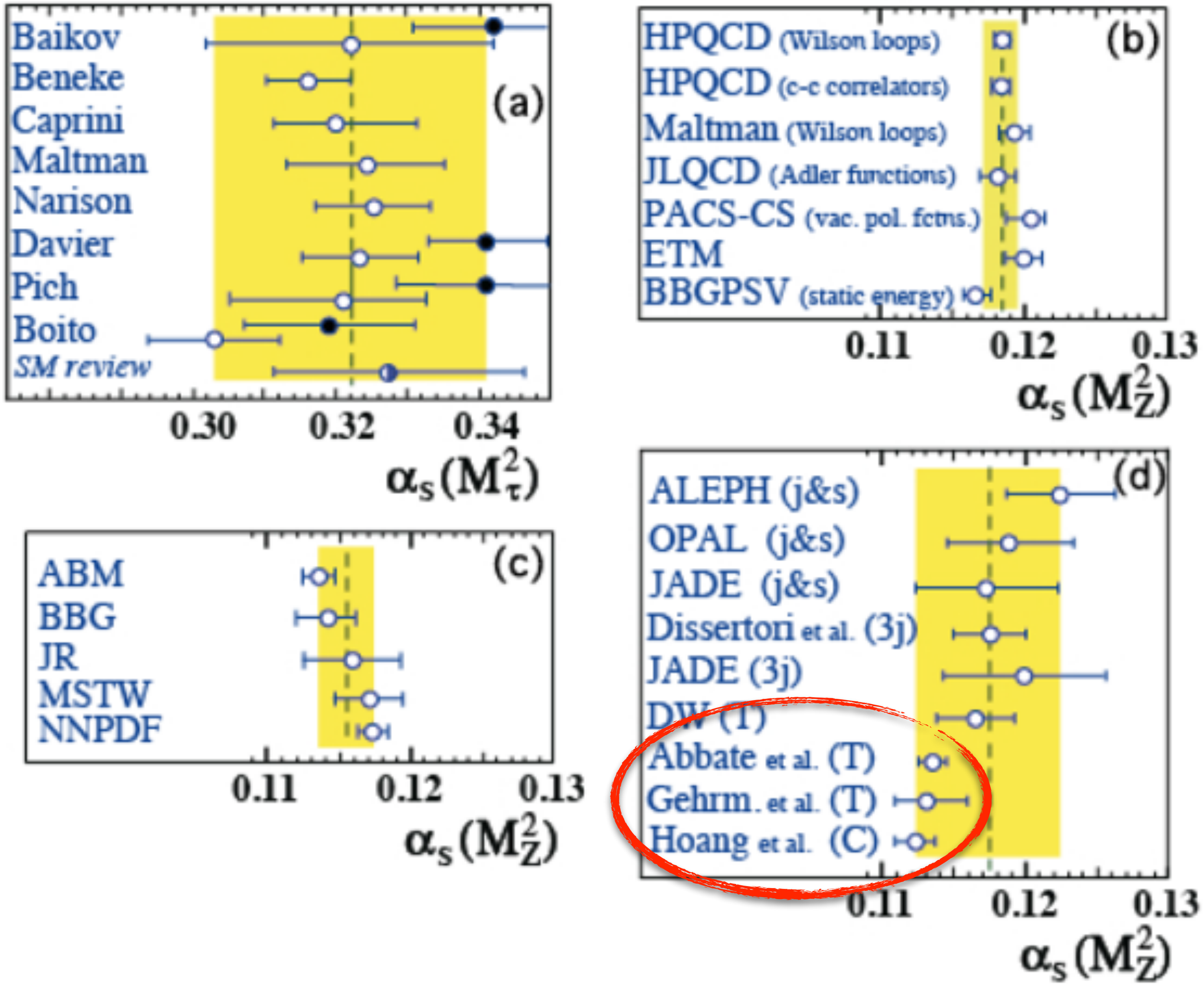
$\alpha_s(M_Z) = 0.1177 \pm 0.0013(1.1\%)$ weighted

$\alpha_s(M_Z) = 0.1181 \pm 0.0013(1.1\%)$ unweighted

- Large tension between extractions from NNLL(N3LL)+NNLO event shapes and lattice calculations
- At LEP energies issues with high correlation between perturbative and hadronisation corrections from analytic models: $\rho \sim -0.9$

[Dokshitzer, Marchesini, Webber '95; Korchemsky, Sterman '99]

- Thrust and C-parameter very similar (correlated) observables, with nearly the same NP behaviour
- Low values of as are disfavoured by some LHC measurements and recent lattice computations



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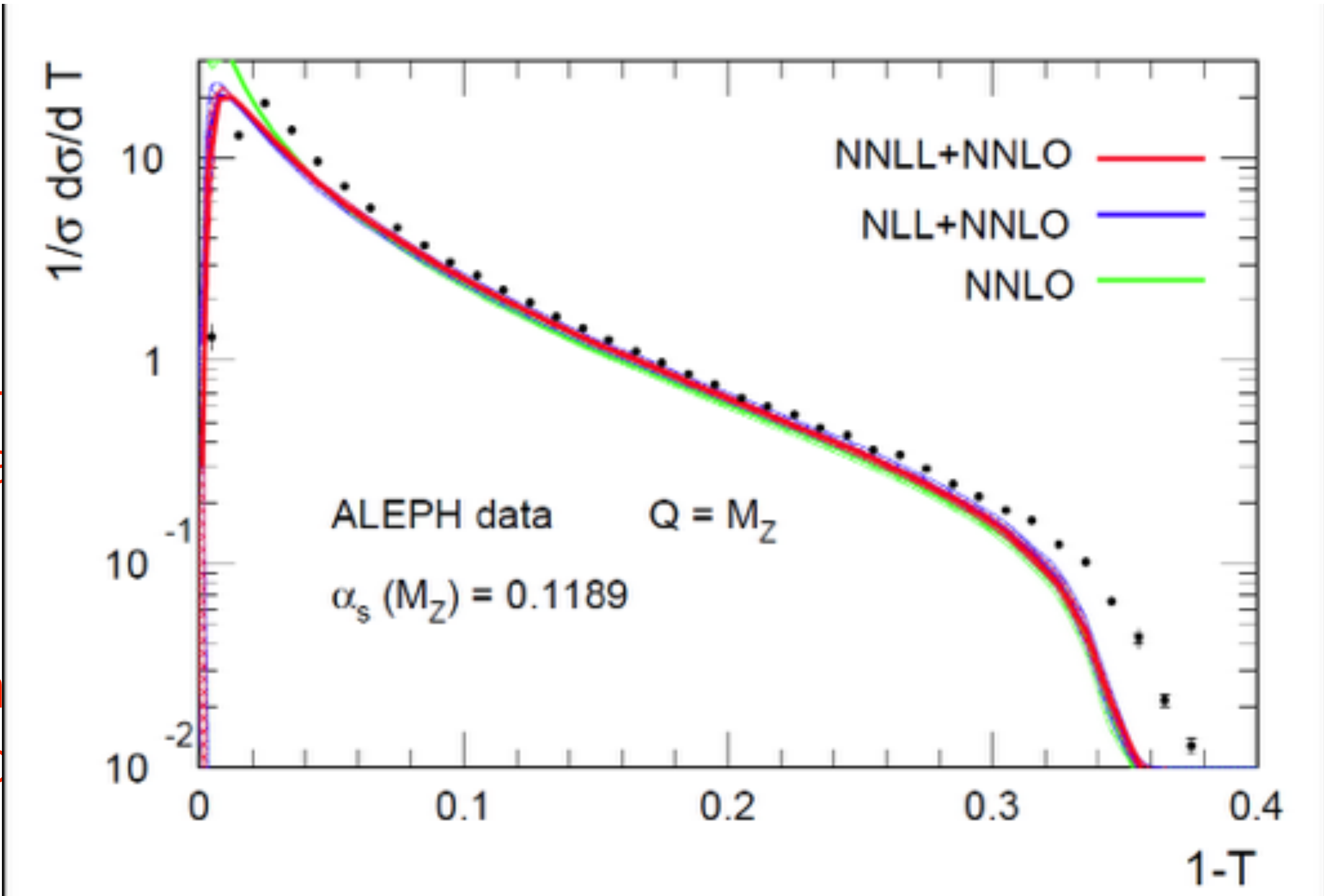
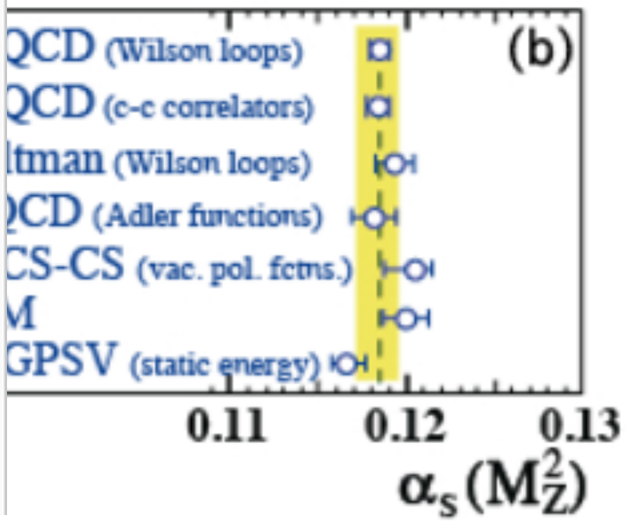


, Salam, Dissertori '15]

$013(1.1\%)$ weighted

$013(1.1\%)$ unweighted



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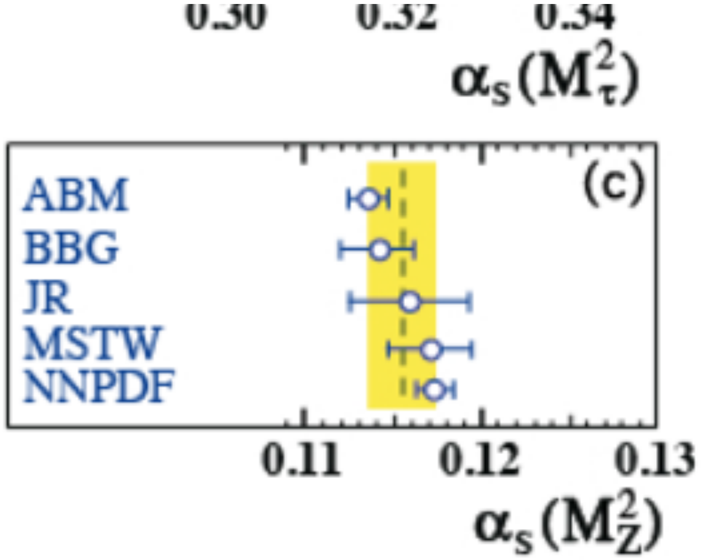

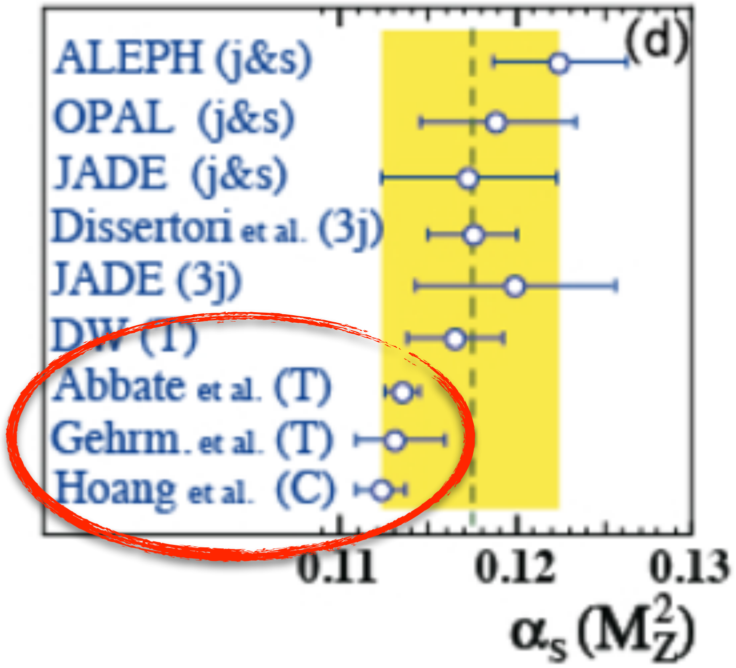

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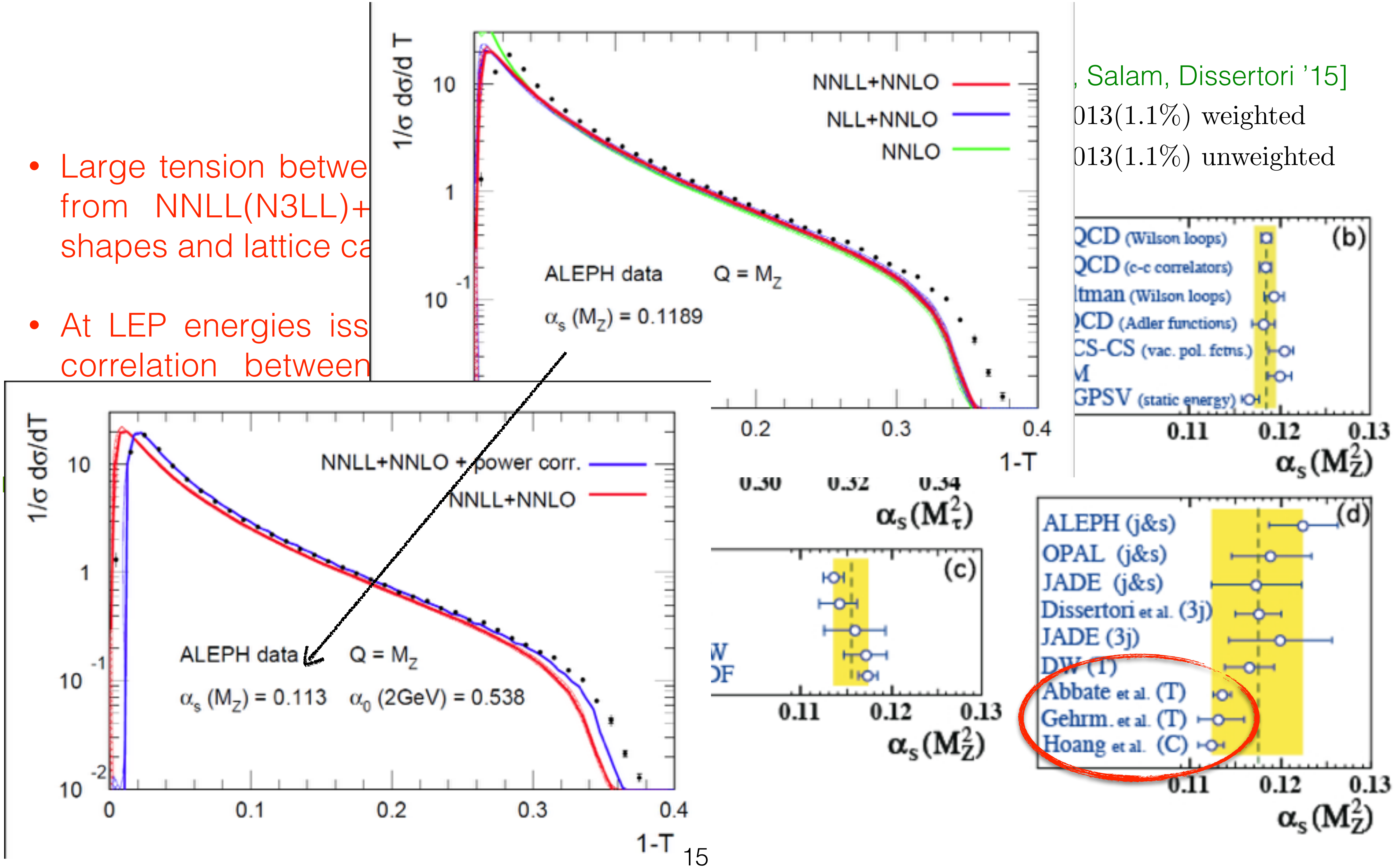

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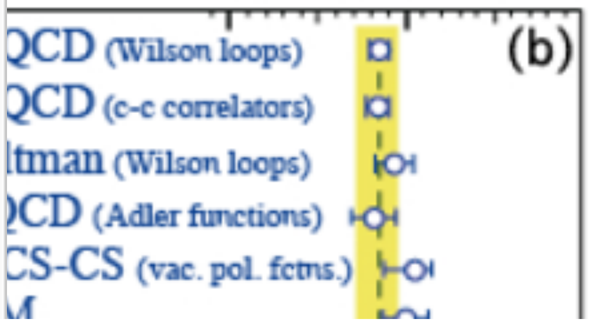

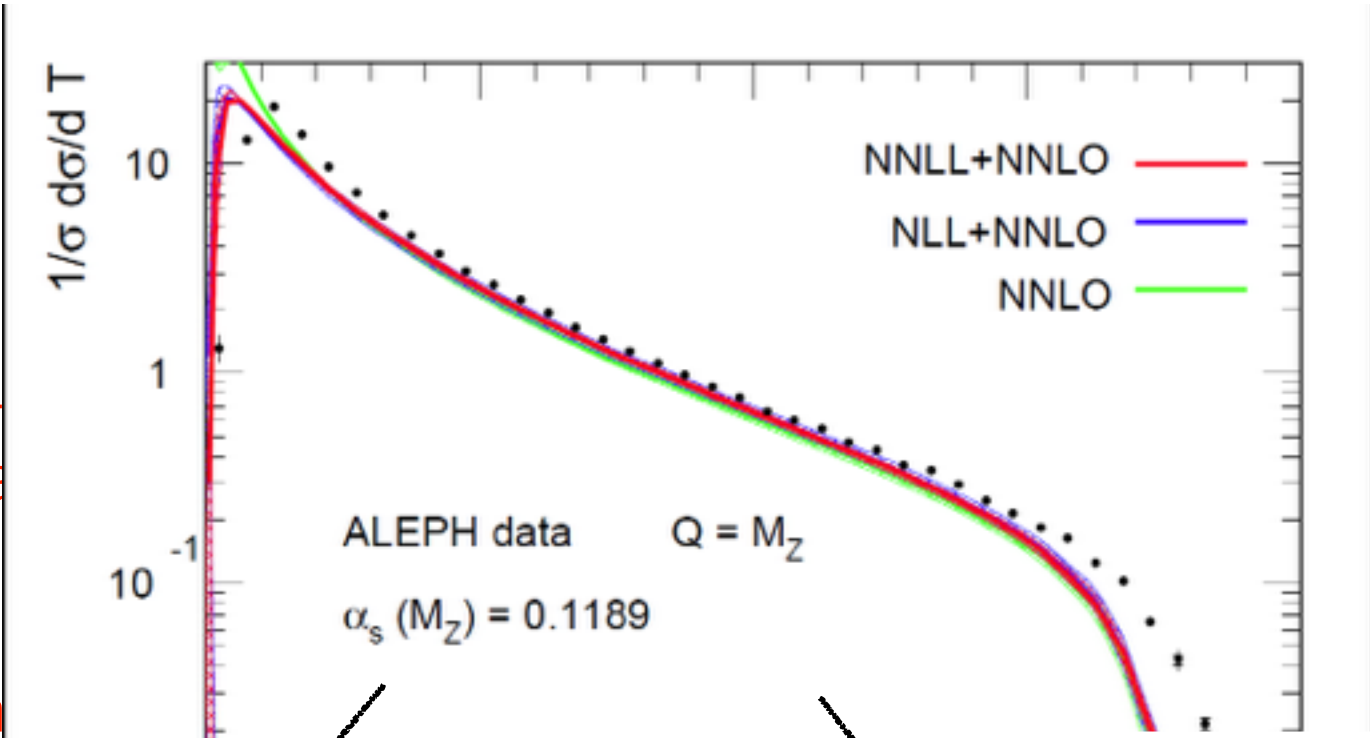

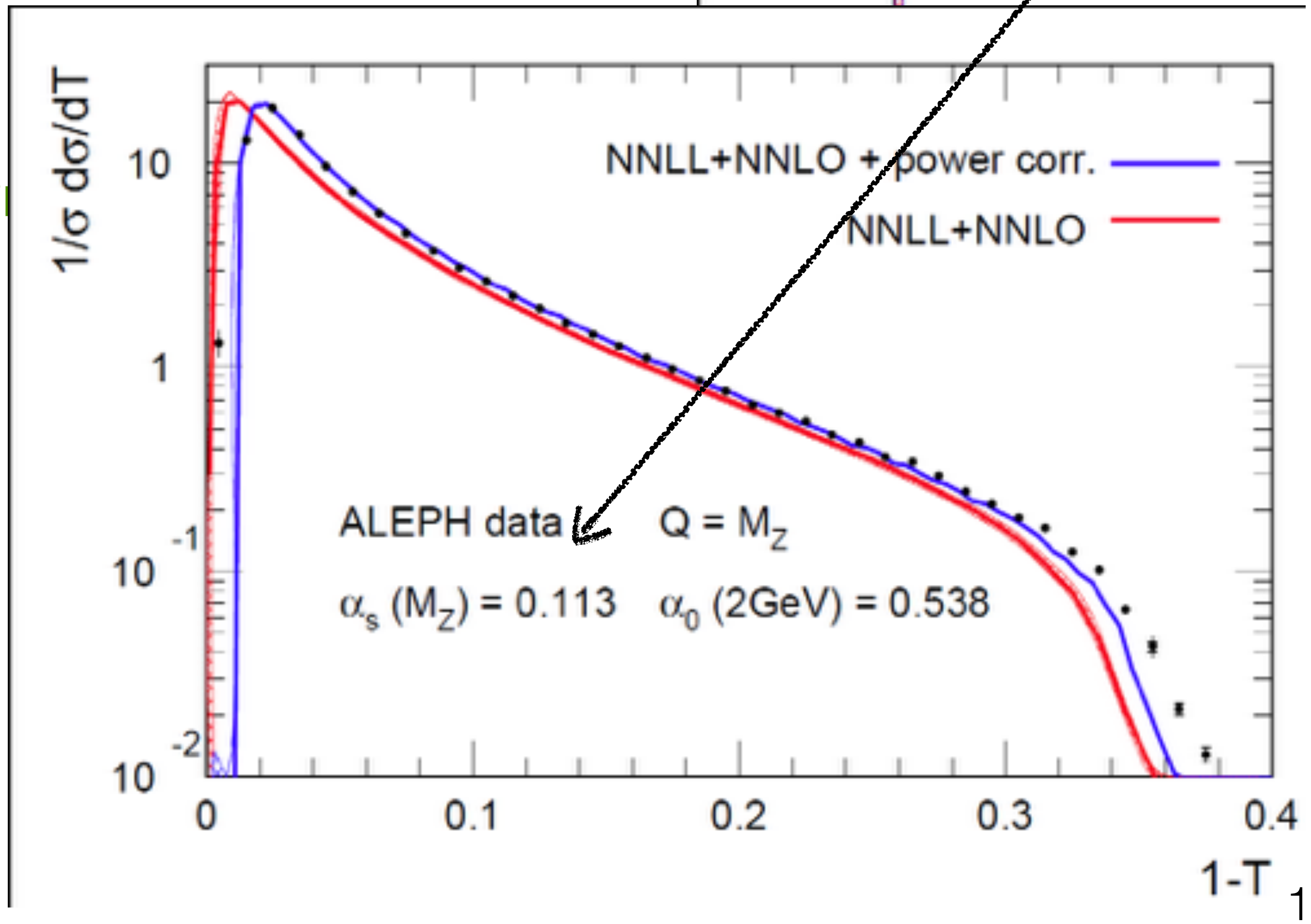

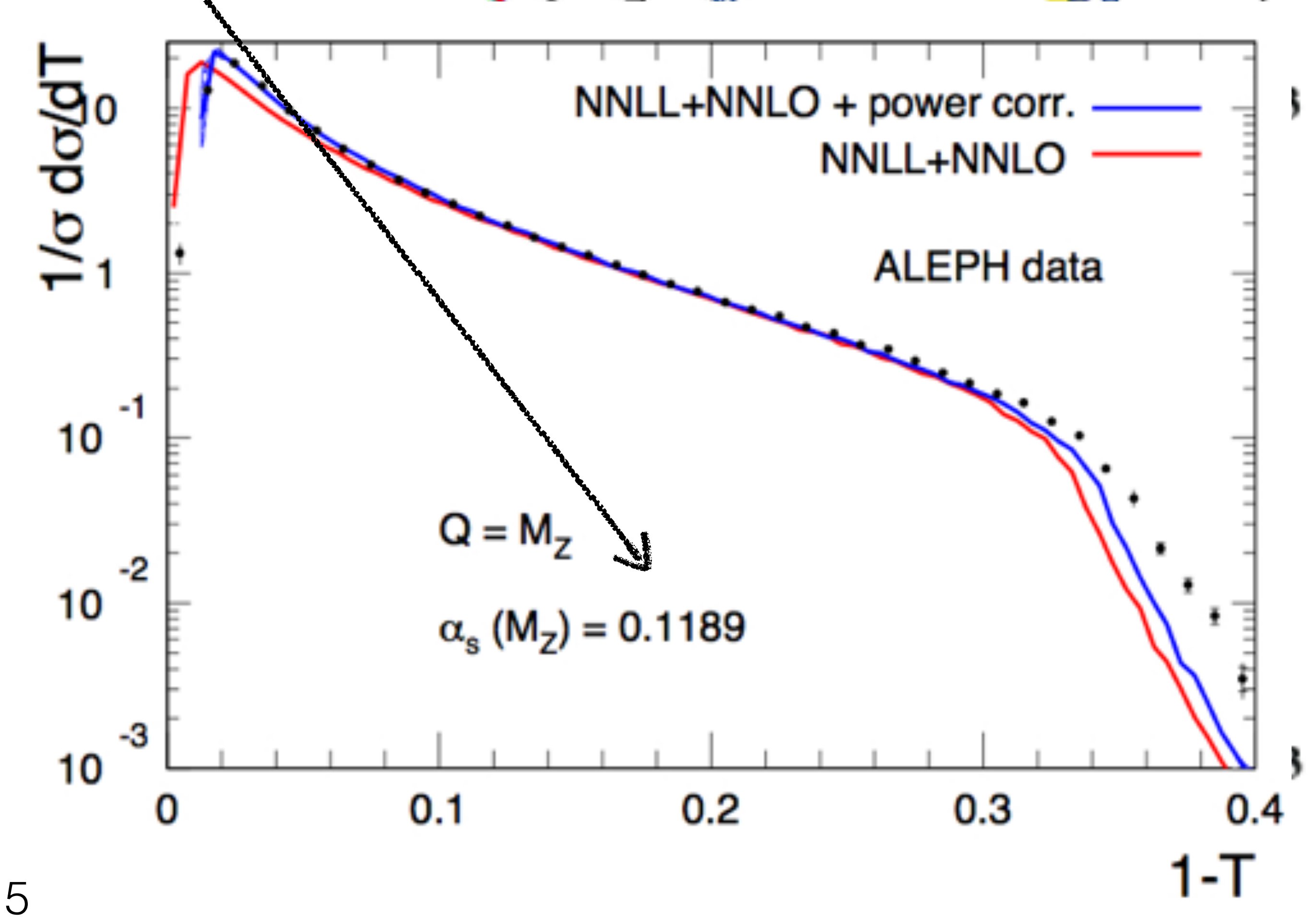

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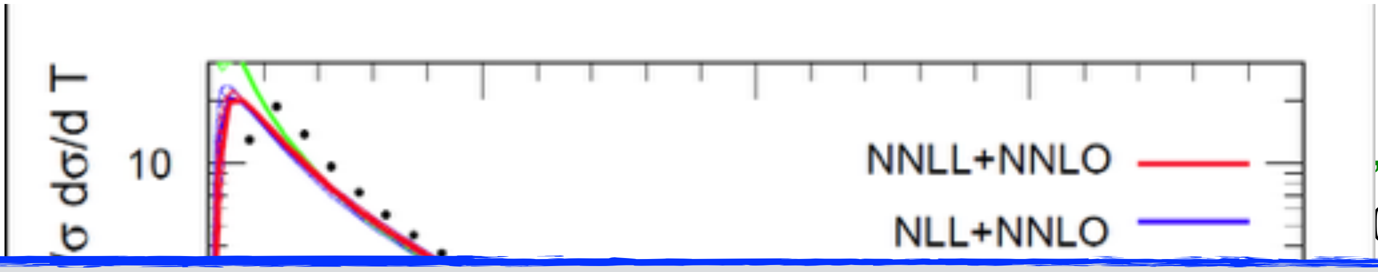

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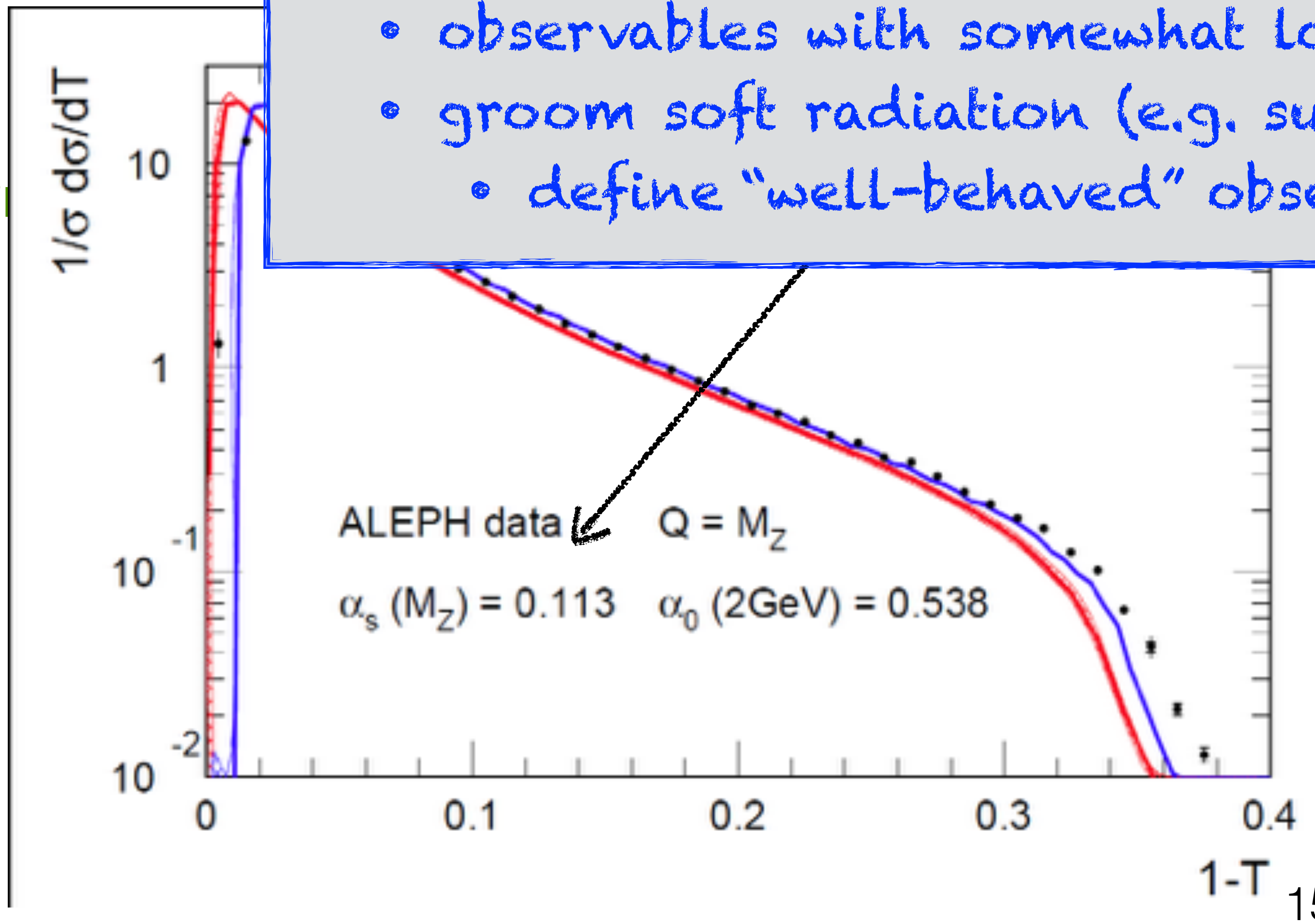
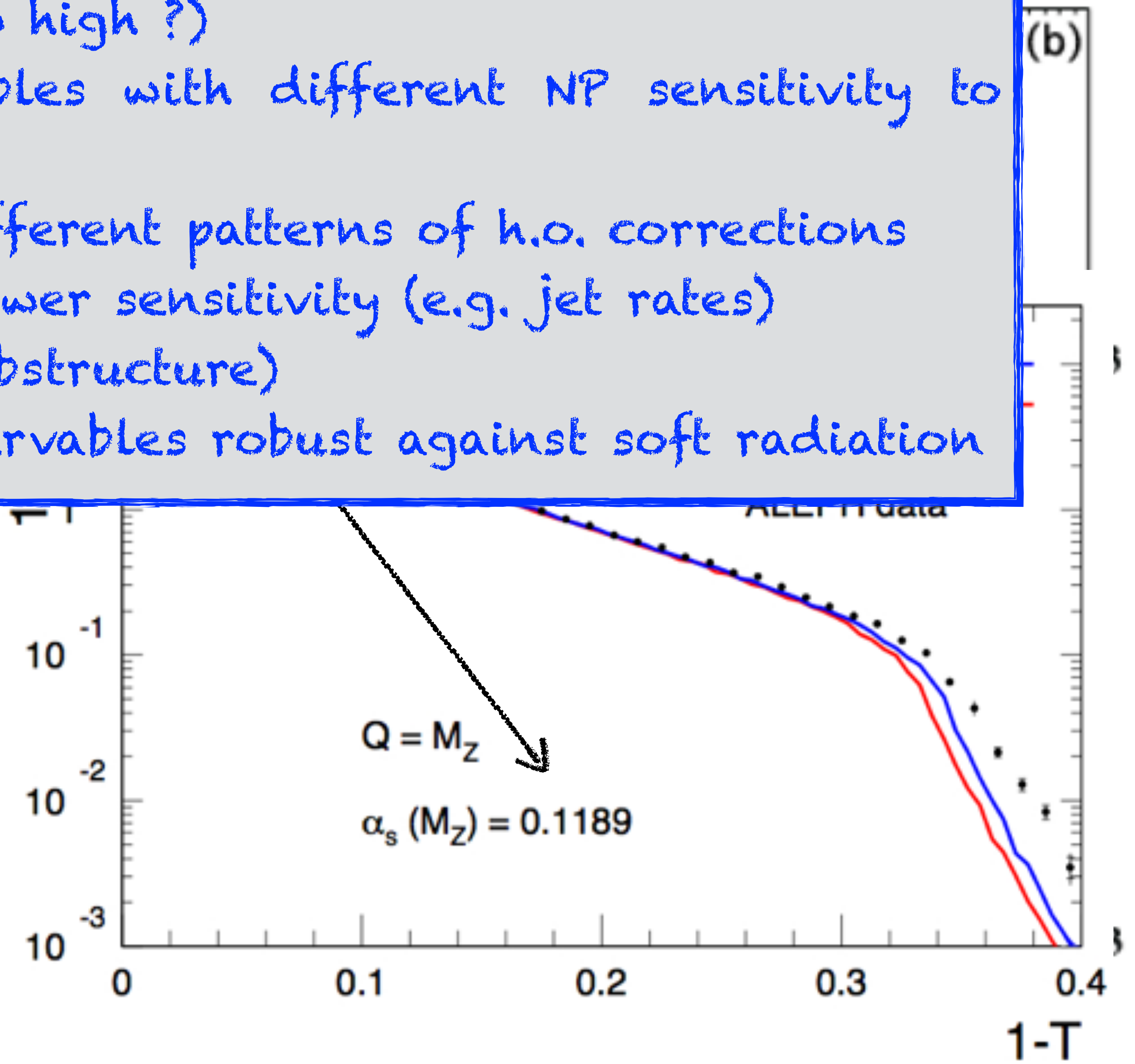
- Large
from
shape

- At LE
correl

- Viable improvements:
  - higher collider energies (how high ?)
  - combined fit using observables with different NP sensitivity to resolve the degeneracy (?)
    - observables might have different patterns of h.o. corrections
  - observables with somewhat lower sensitivity (e.g. jet rates)
  - groom soft radiation (e.g. substructure)
    - define "well-behaved" observables robust against soft radiation

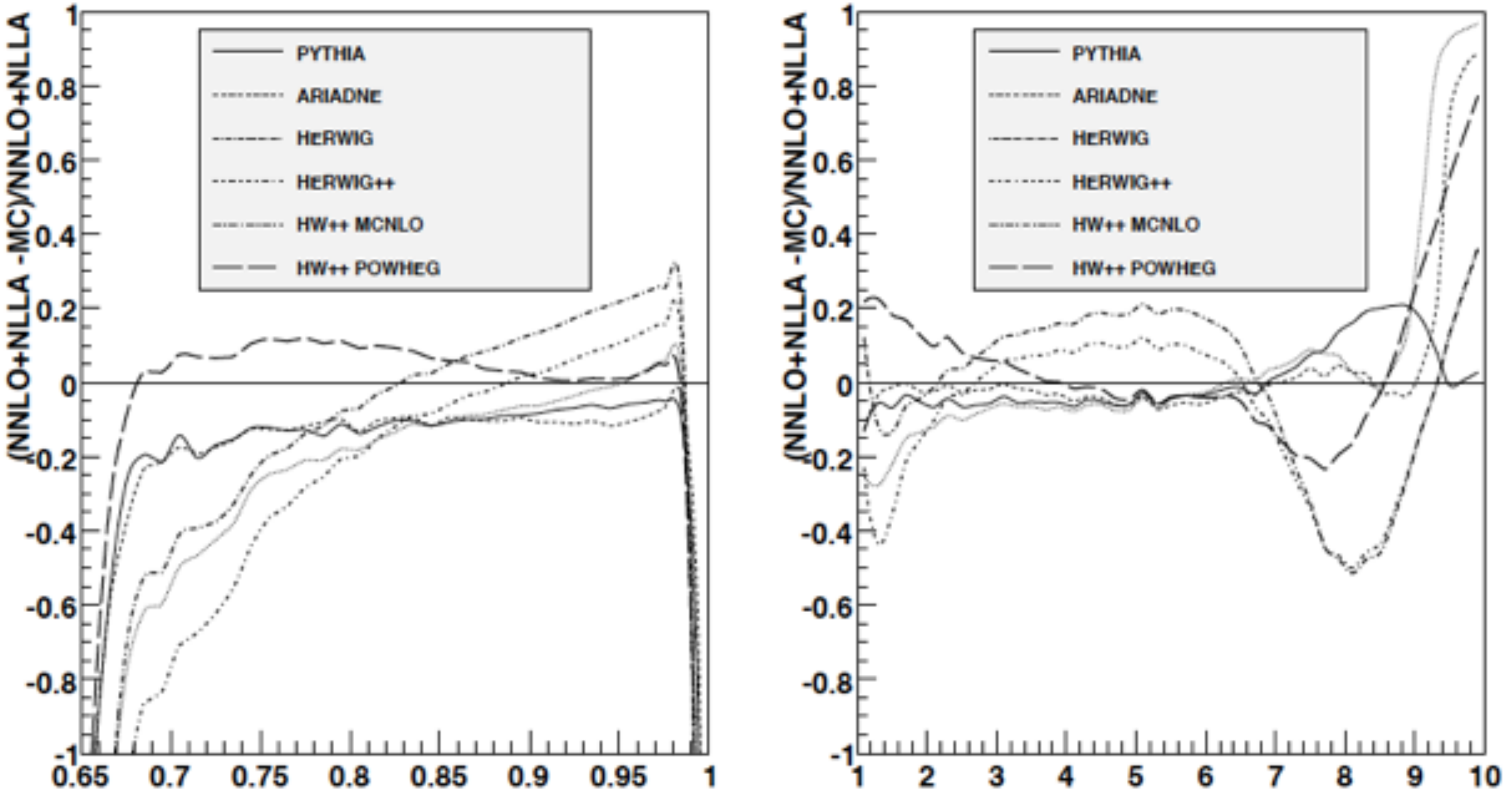


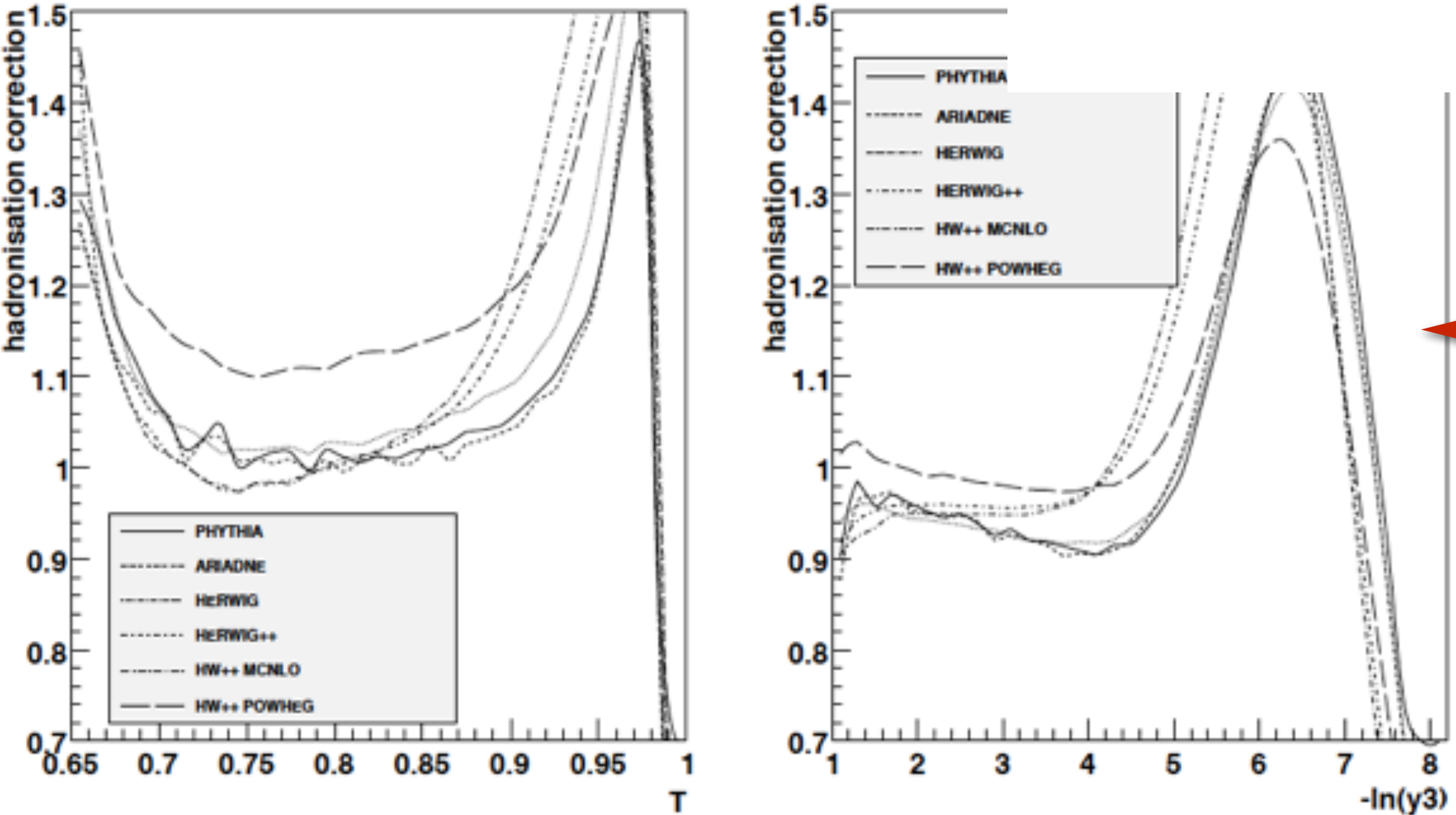


The two-jet rate

e.g. Y3 (Durham) v. T at MZ from [Dissertori et al. '09]

Difference between MCs (parton) and perturbative calculations more stable for Y3.
This fact can be used to rely on MC for full hadronisation correction (also to be tested with recent predictions and generators…good perturbative convergence for Y3), if subleading NP corrections are sizeable





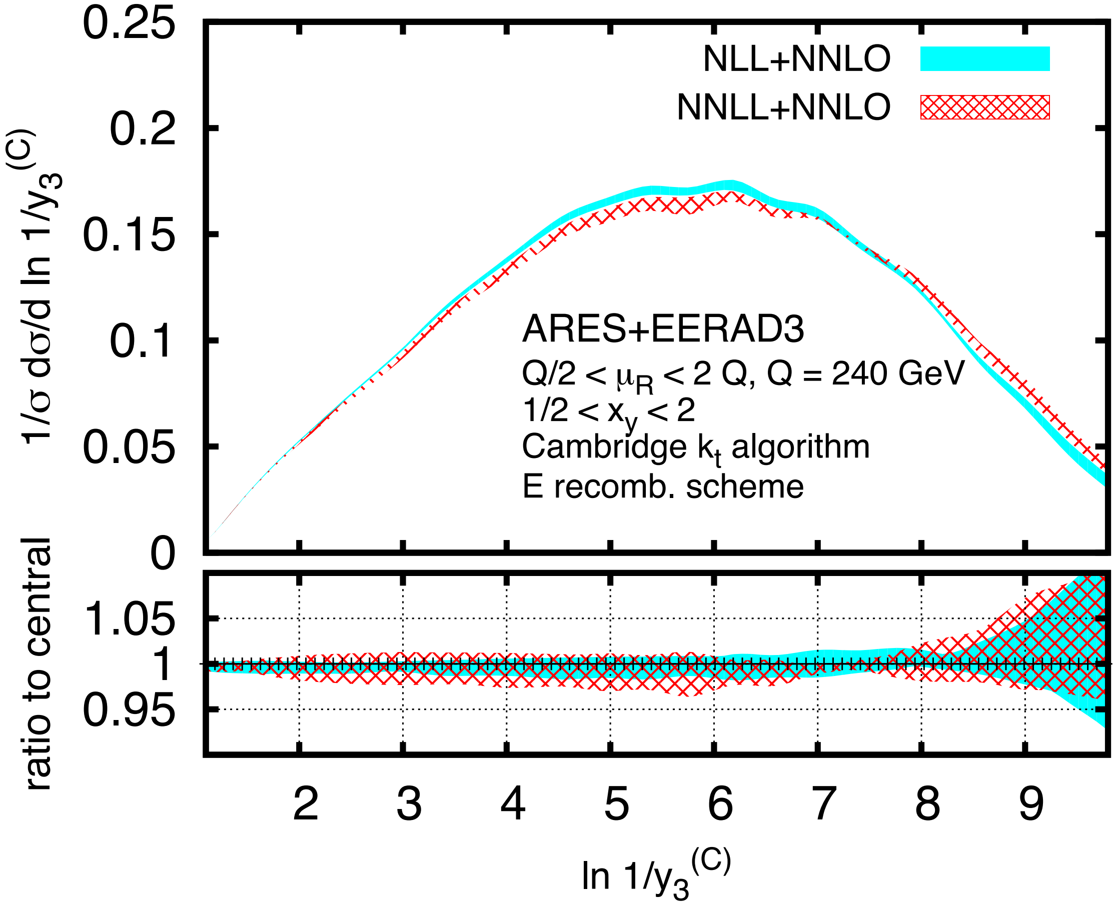
Hadronisation effects less important near the Sudakov peak, smaller spread between generators.
Choice of the fit range should be done accordingly.

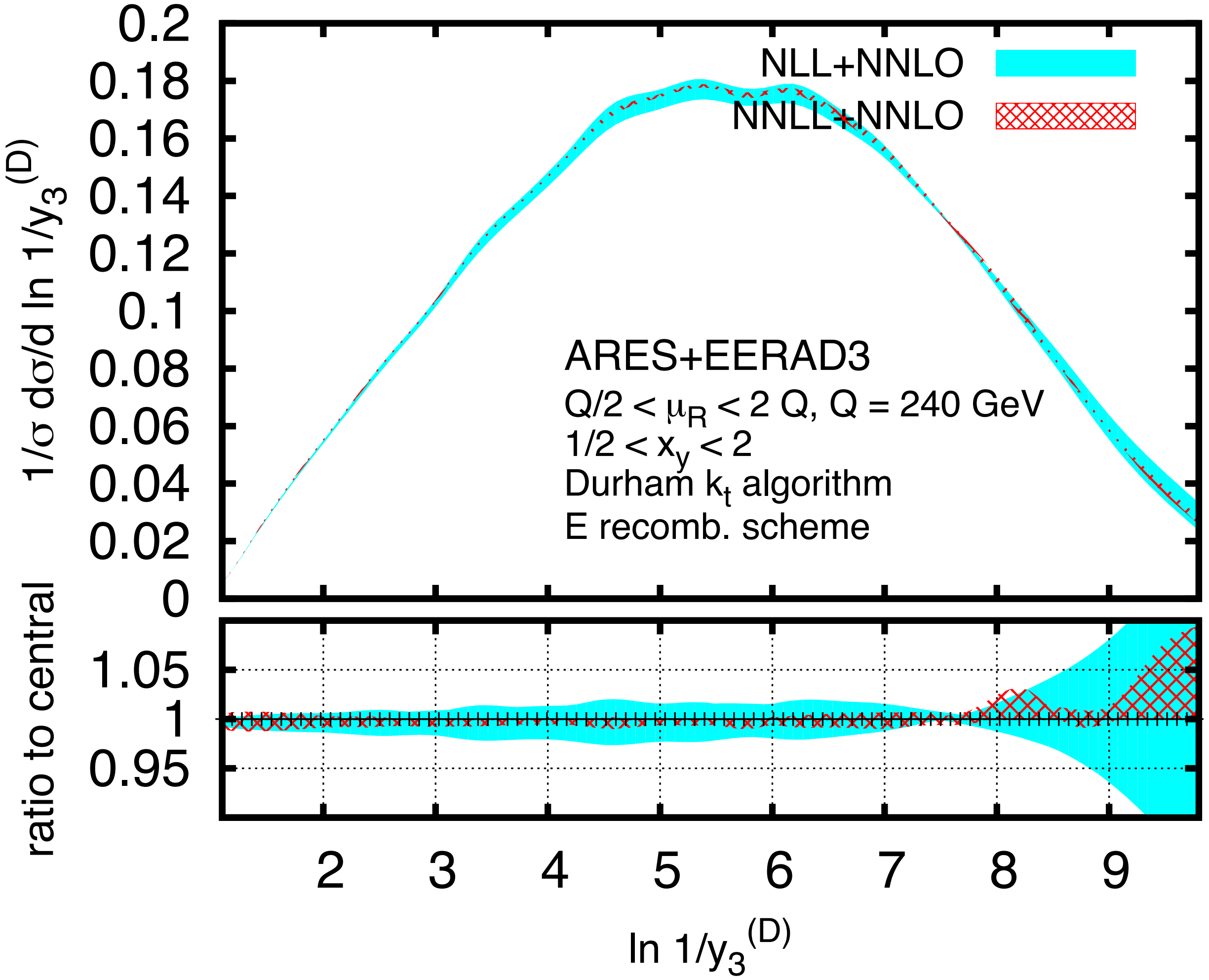
The two-jet rate: jet algorithms at NNLL

NLL -> [Banfi, Salam, Zanderighi '01]
NNLL -> [Banfi, McAslan, PM, Zanderighi '16]

- NNLL+NNLO available for "measure-like" 3-jet resolution definitions:
  - NNLL corrections for Cambridge kt and AO Durham are sizeable due to sensitivity of gluon splitting.

    [Dokshitzer, Leder, Moretti, Webber '97]
  - Uncertainties at FCCee energies at the ~3-4% level





- Durham kt more convergent. Complex structure of NNLL corrections which are moderate in size, small residual perturbative uncertainties.

  [Catani, Dokshitzer, Olsson, Webber '91]
  - Similar situation for Inclusive Durham kt or Flavour kt algorithms

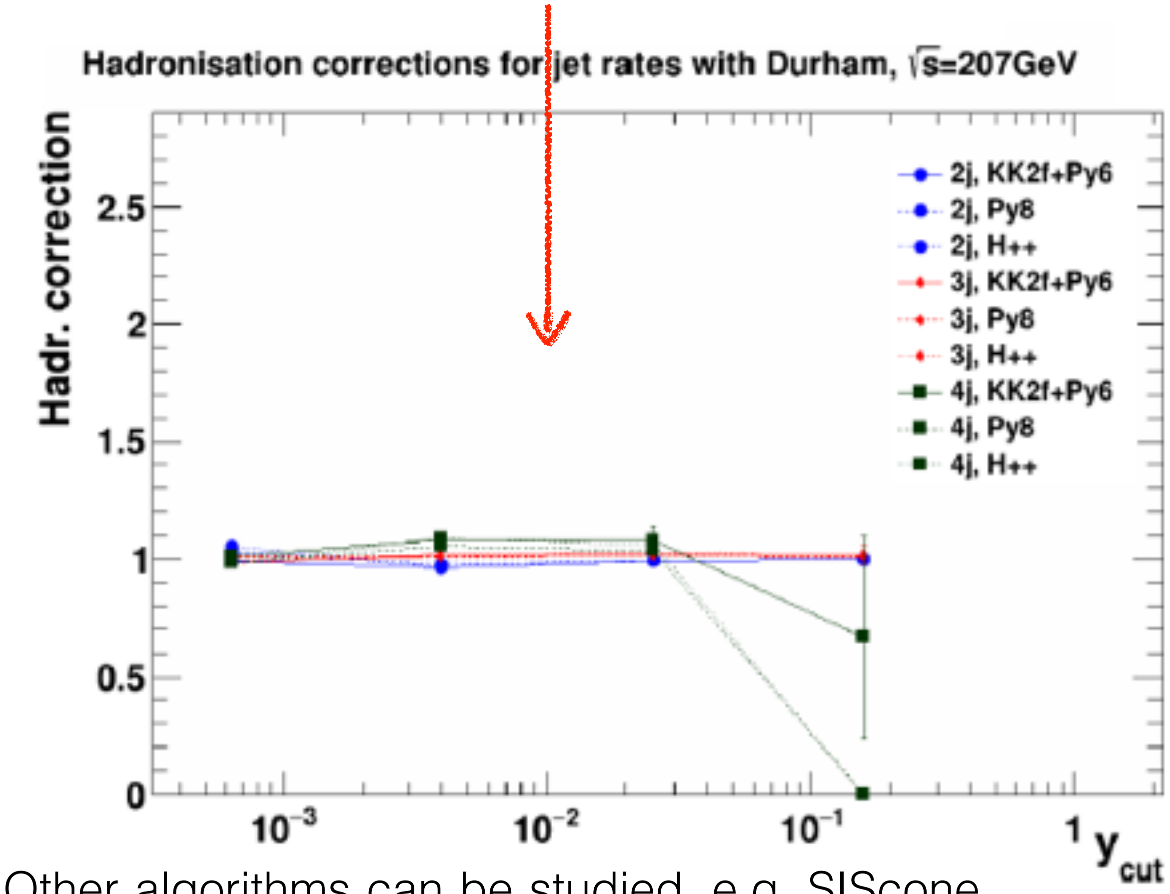
    [Weinzierl '10; Cacciari, Salam, Soyez '11]
    [Banfi, Salam, Zanderighi '06]

The two-jet rate: hadronisation

[Kluth, Verbytskyi]

Slide from S. Kluth's talk at ISMD2016

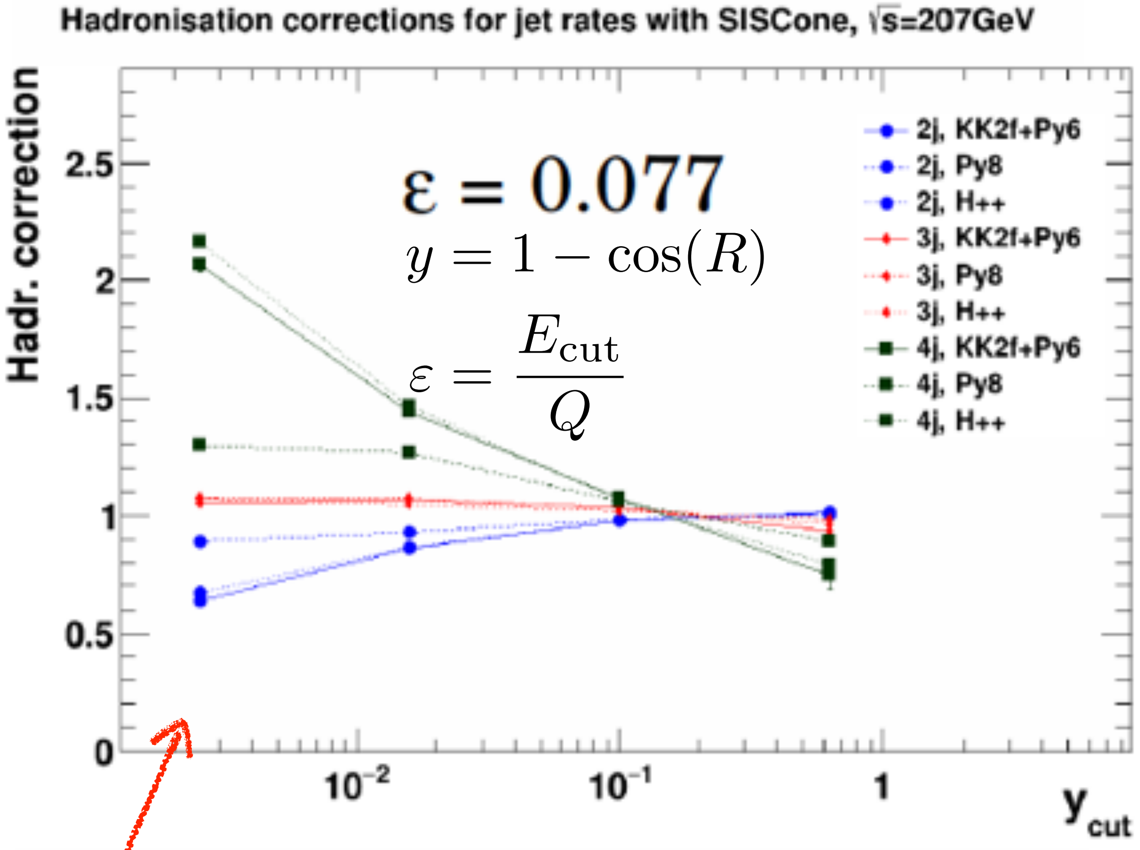
- Important reduction of hadronisation corrections for Durham kt at typical FCCee energies.
- Cambridge also expected to be robust



- Other algorithms can be studied, e.g. SIScone, anti-kt.
- Different definition of the resolution parameter (different scaling, sometimes non-global):
  - Different logarithmic structure

NLL_Σ in [Gerwick, Schumann, Gripaios, Webber '12]

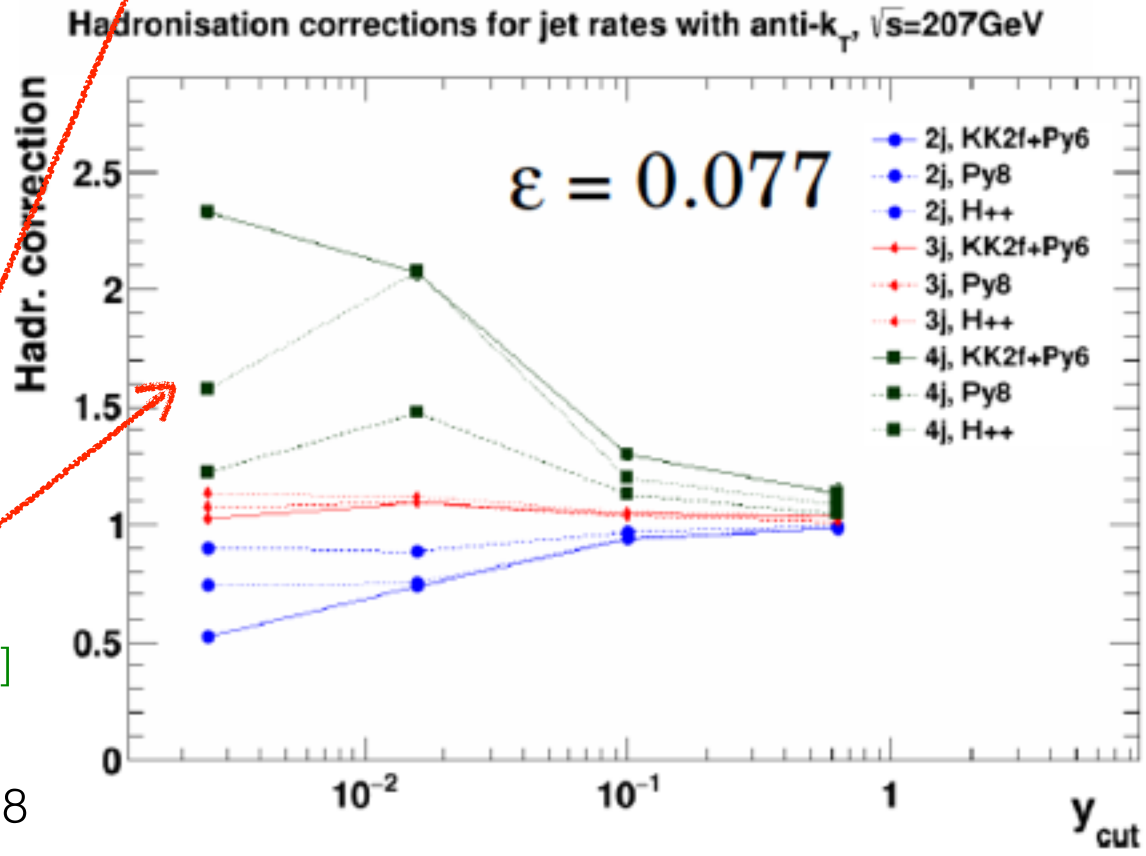
- Non-perturbative corrections still sizeable at FCCee energies



$\varepsilon = 0.077$

$y = 1 - \cos(R)$

$\varepsilon = \frac{E_{\mathrm{cut}}}{Q}$

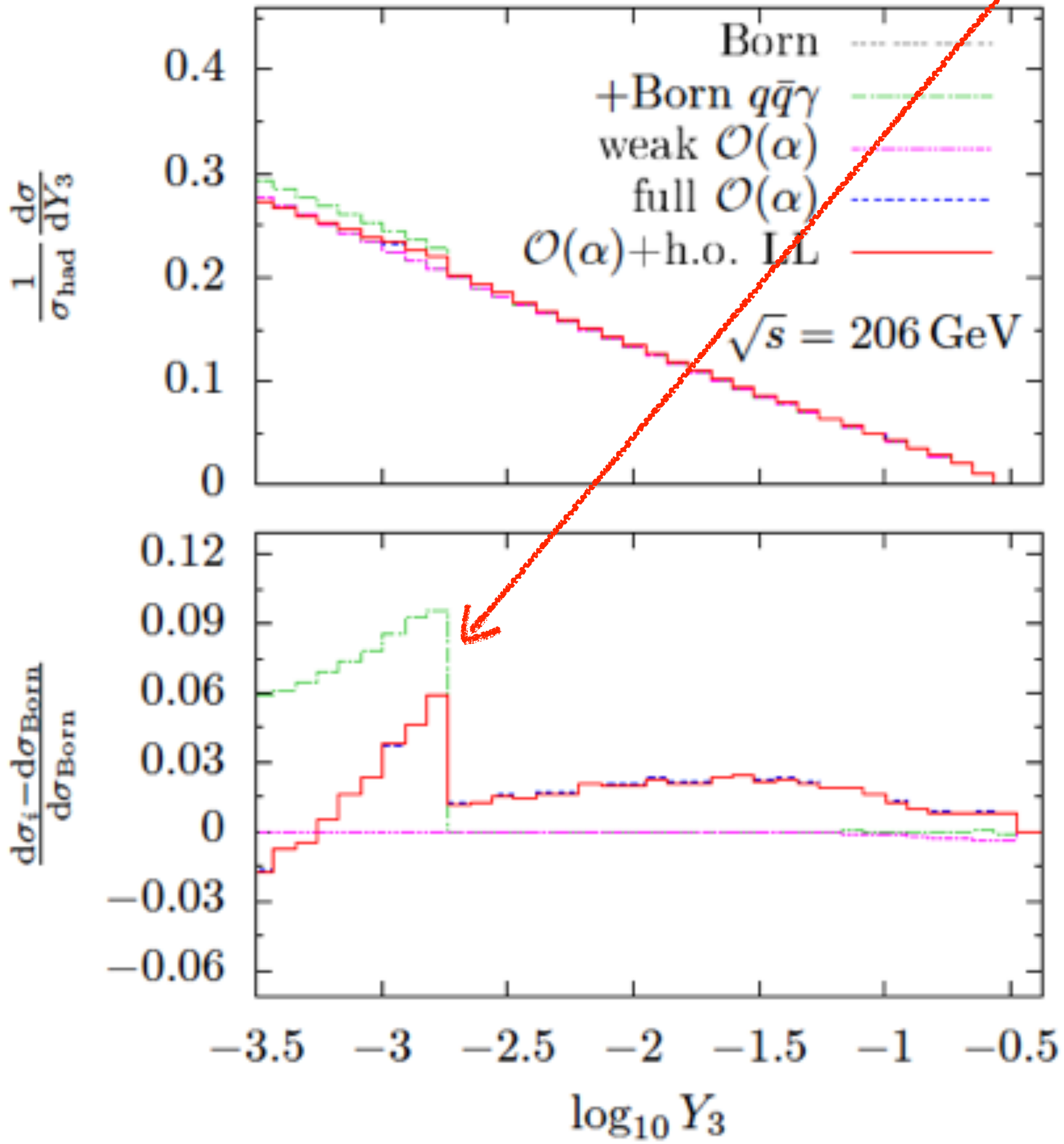


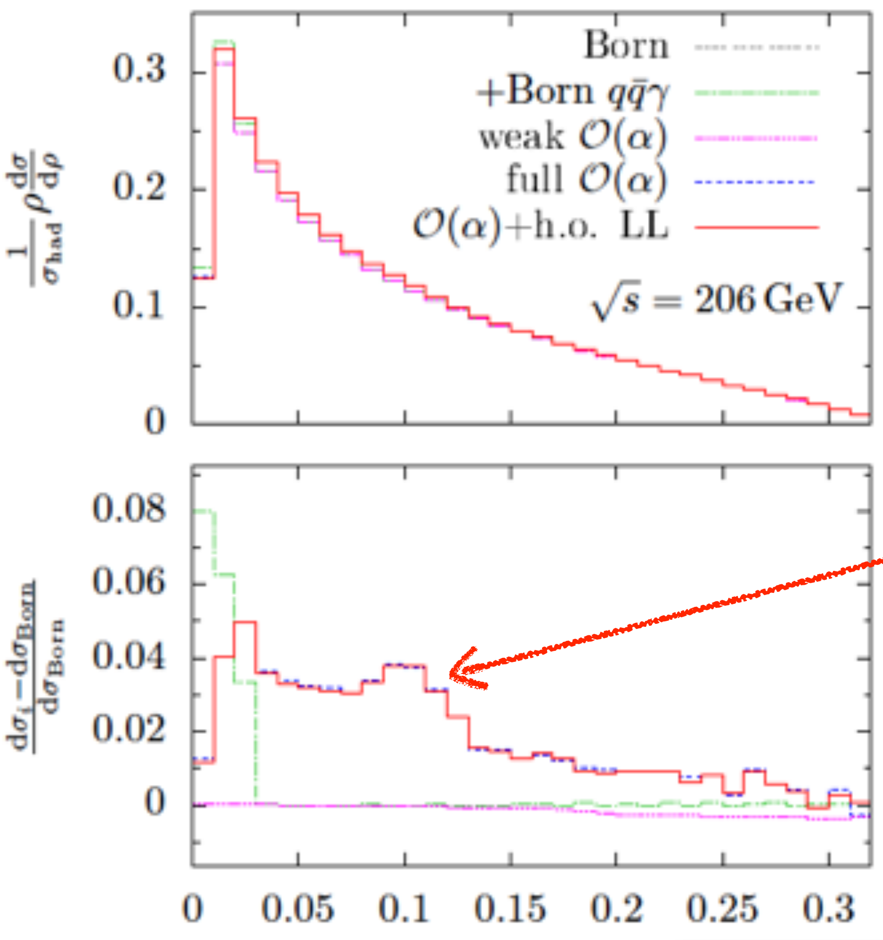
$\varepsilon = 0.077$

EW corrections at NLO

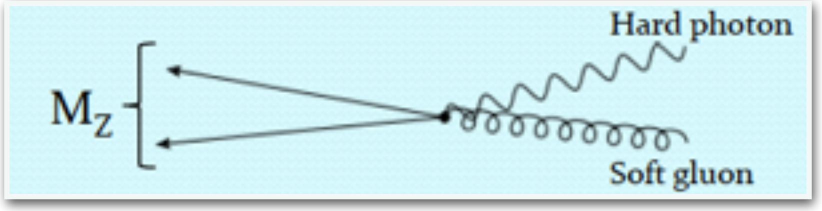
[Denner, Dittmaier, Gehrmann, Kurz ‘10]

- Technically as large as NNLO QCD
  - Weak corrections at the permille level
  - important contamination from ISR and $Z/\gamma^* \to q\bar{q}\gamma$ final state, mainly cancels in shapes
    - photon isolation helps to some extent: contamination in the 2-jet lim.





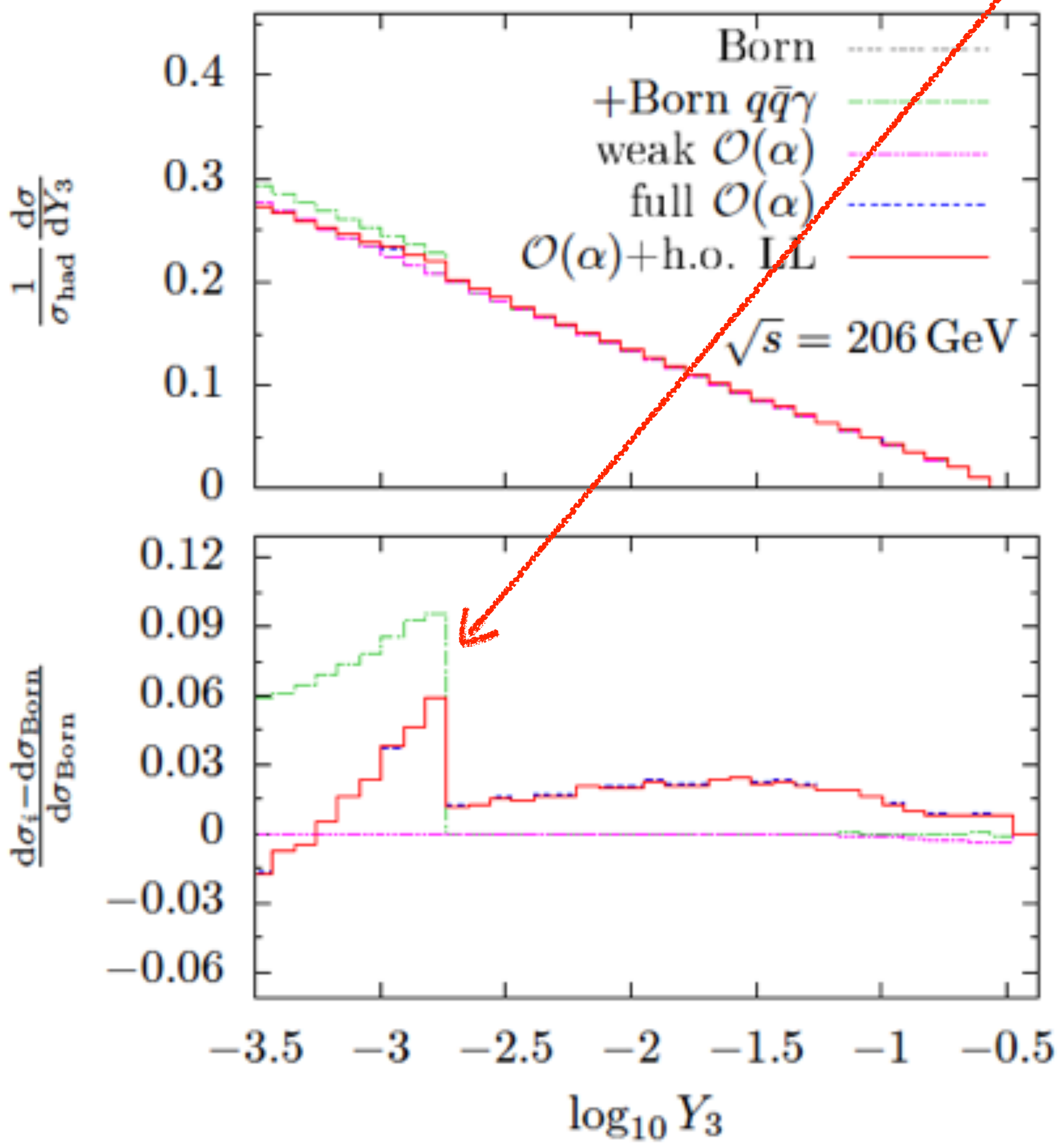
Effect of radiative return still sizeable for some observables at these energies. Depends on the soft scale of the problem. Good QED modelling becomes necessary

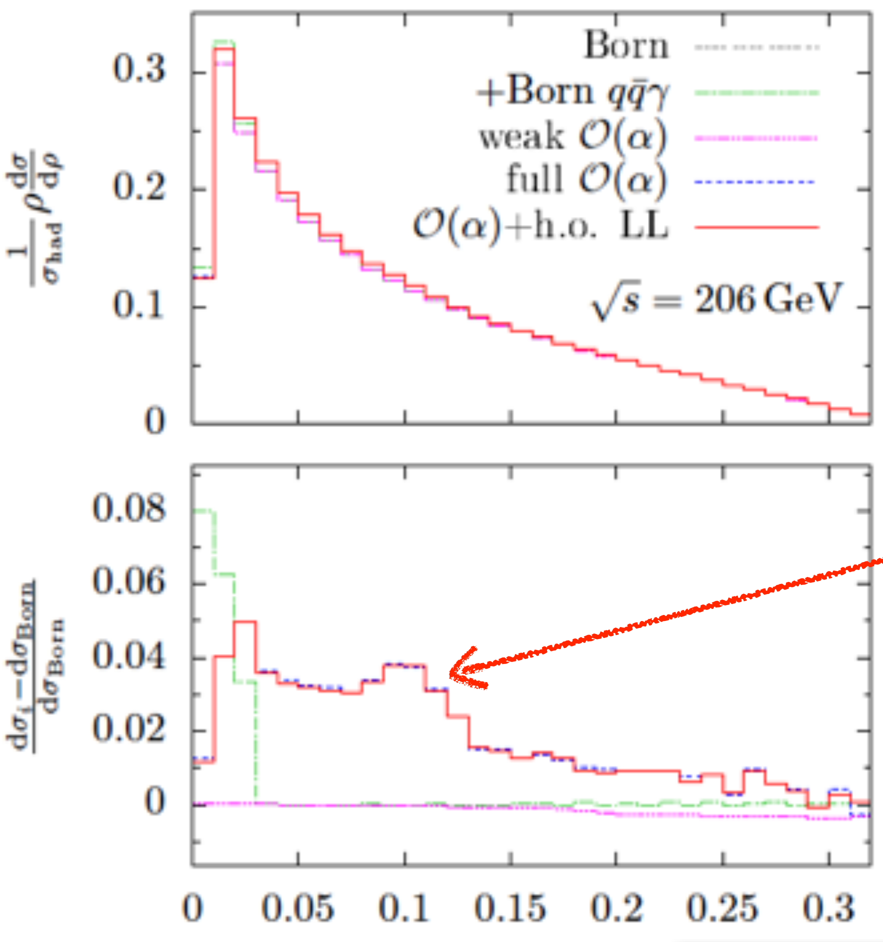


EW corrections at NLO

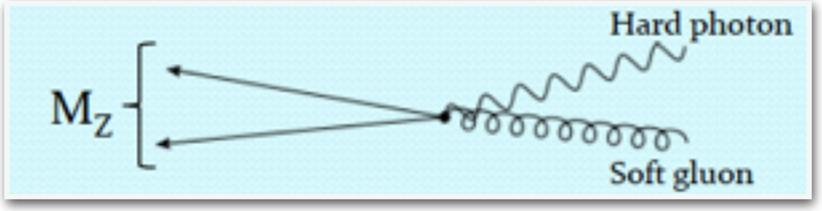
[Denner, Dittmaier, Gehrmann, Kurz ‘10]

- Technically as large as NNLO QCD
  - Weak corrections at the permille level
  - important contamination from ISR and $Z/\gamma^* \to q\bar{q}\gamma$ final state, mainly cancels in shapes
    - photon isolation helps to some extent: contamination in the 2-jet lim.





Effect of radiative return still sizeable for some observables at these energies. Depends on the soft scale of the problem. Good QED modelling becomes necessary



Higgs couplings from event-shapes

- Event shapes could give access to precision measurements of Higgs couplings, e.g. light quarks

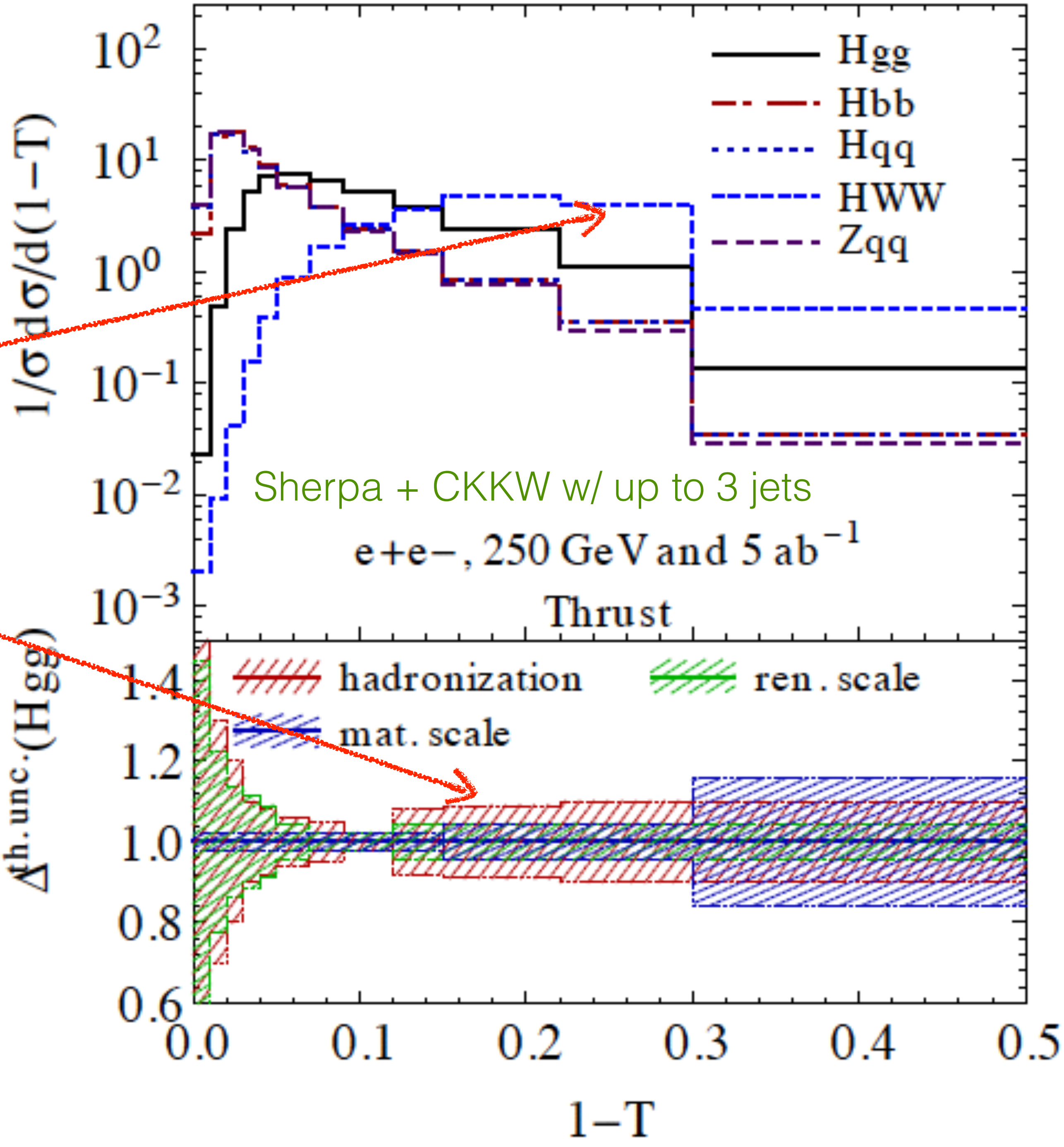
- Measurement of event shapes in Higgs rest frame after reconstructing the Z
    - Good experimental precision
    - Decent background rejection
    - Precise perturbative calculation possible (although had. still large)

- The measurement is model dependent (requires good knowledge of remaining couplings involved)

- Can it improve on LHC bounds ?
    - Use of heavy-flavour tagging and q/g discrimination might help

$e^+e^- \to H\,(\to \text{hadrons})\,Z\,(\to l^+l^-)$

[Gao '16]



Conclusions

- Precise theory predictions for most rIRC safe global 2-jet event shapes available (NN(3)LL+NNLO)

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07-'08] [Weinzierl '09] [Del Duca, Duhr, Kardos, Somogyi, Szor, Trocsanyi, Tullipant '16]

[Becher, Schwartz '08] [Chien, Schwartz '10] [Becher, Bell '12] [Hoang et al. '14] [Banfi, McAslan, PM, Zanderighi '14-'16] [Bell, Hornig, Lee, Talbert '16 (in progress)]

  - very desirable: 3-jet observables (e.g. jet rate) and generic non-global observables, state of the art is NLL (LL for NG + very promising recent progresses)

- Perturbative uncertainty at FCC-ee is reduced to the few-% level across the spectrum

  - This certainly will supplement/improve LEP measurements and understanding of aspects of QCD dynamics.

    - Precise extractions of couplings (alphas, Higgs,…) still limited by hadronisation at these energies (dominant theory uncertainty)

      - A careful choice of the observables to perform these measurements is still necessary, however in some cases perspectives seem promising

    - New insights are required to improve on this, meaning either better understanding of the NP dynamics, or hadronisation-resilient observables with good perturbative performance