

# Fragmentation Functions at Belle



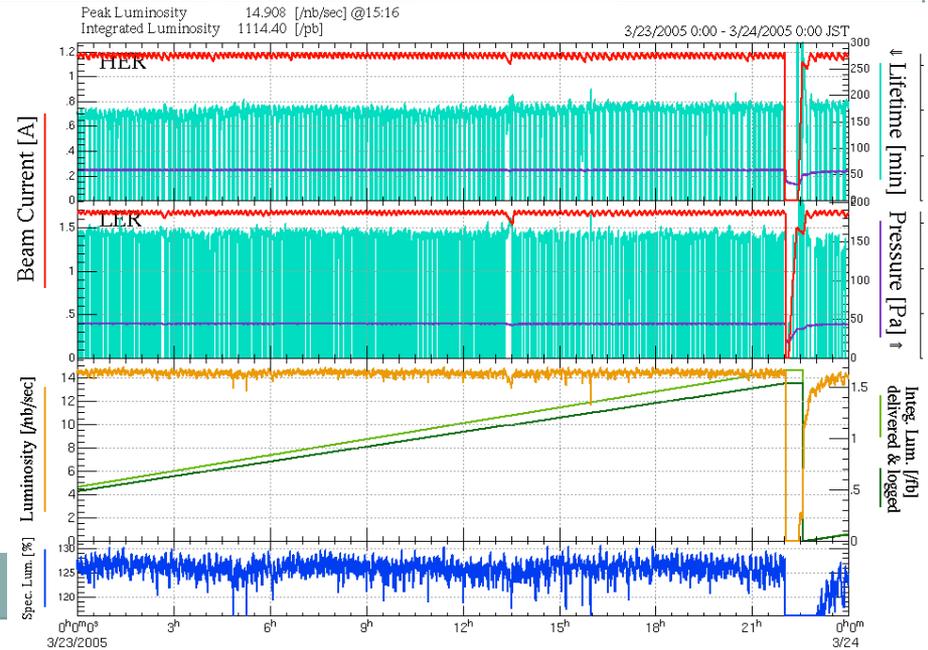
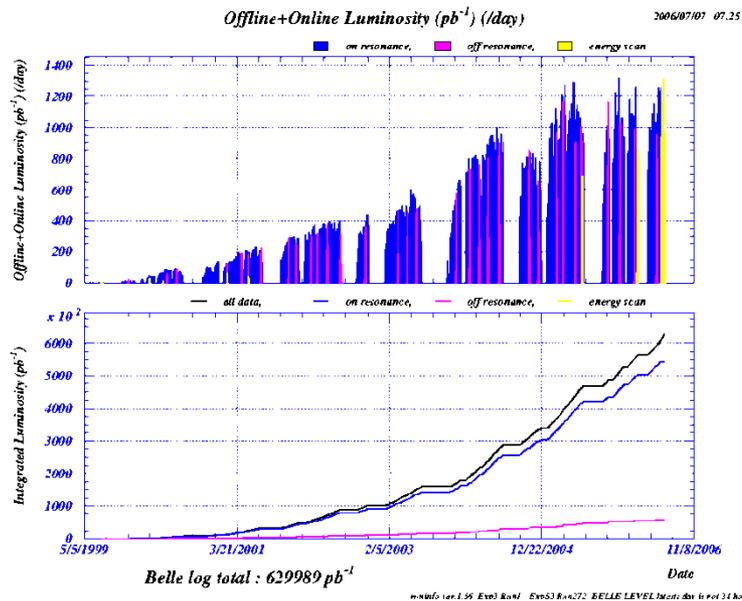
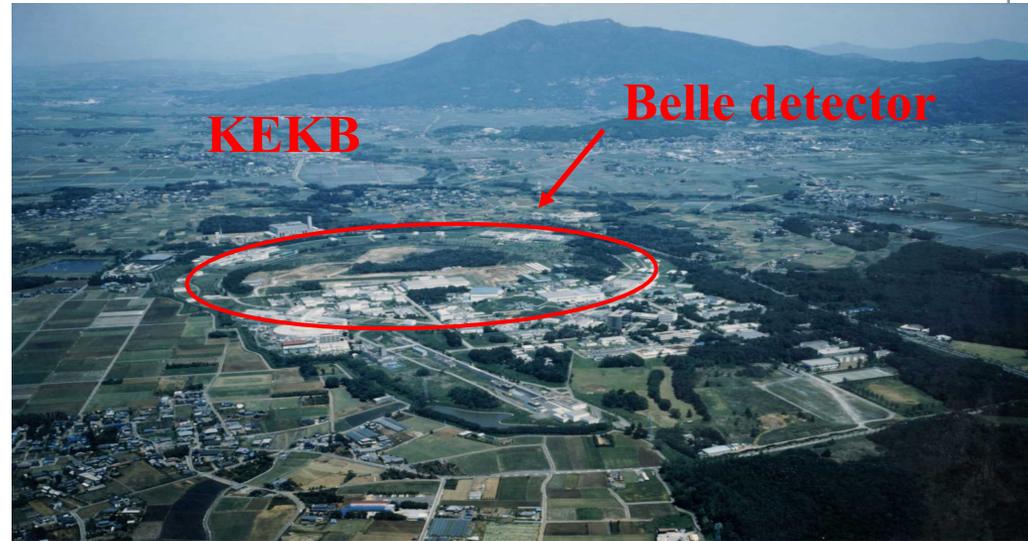
ANSELM VOSSEN



INDIANA UNIVERSITY

# KEKB: $L > 2.1 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ !!

- Asymmetric collider
- $8\text{GeV } e^- + 3.5\text{GeV } e^+$
- $\sqrt{s} = 10.58\text{GeV } (Y(4S))$
- $e^+e^- \rightarrow Y(4S) \rightarrow B \bar{B}$
- Continuum production:  
 $10.52 \text{ GeV}$
- $e^+e^- \rightarrow q \bar{q} \text{ (u,d,s,c)}$
- Integrated Luminosity:  $> 1000 \text{ fb}^{-1}$
- $> 70 \text{ fb}^{-1} \Rightarrow$  continuum



# Belle, a typical e+e- Experiment of generation 2000

•Asym. e<sup>+</sup> (3.5 GeV) e<sup>-</sup> (8 GeV)  
collider:

-√s = **10.58 GeV**, e<sup>+</sup>e<sup>-</sup>

→Y(4S)→B anti-B

-√s = 10.52 GeV, e<sup>+</sup>e<sup>-</sup>→

qqbar (u,d,s,c) 'continuum'

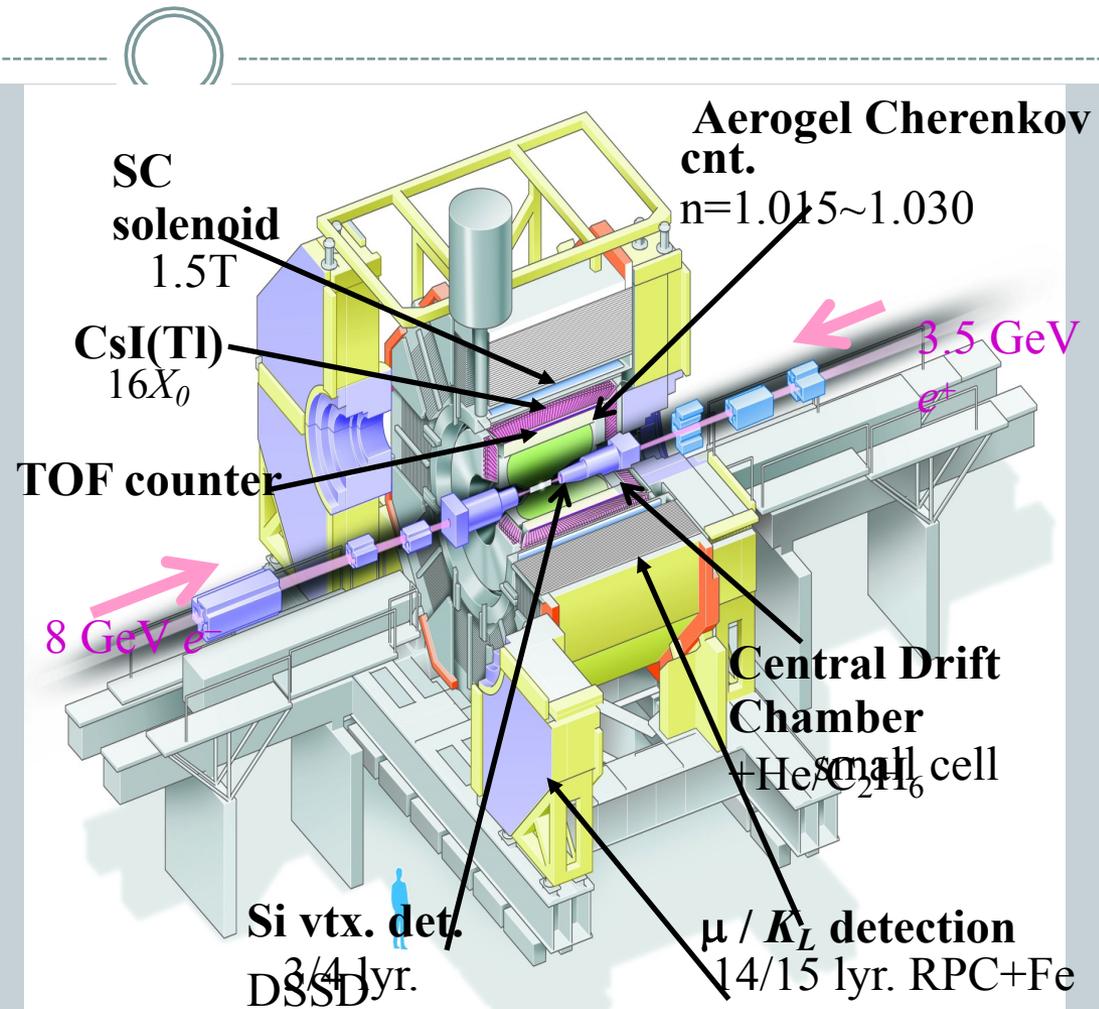
• ideal detector for high  
precision measurements:

- Azimuthally symmetric  
acceptance, high res. Tracking,  
PID: Kaon efficiency ~85%

Available data:

~1.8 \*10<sup>9</sup> events at 10.58  
GeV,

~220 \*10<sup>6</sup> events at 10.52  
GeV

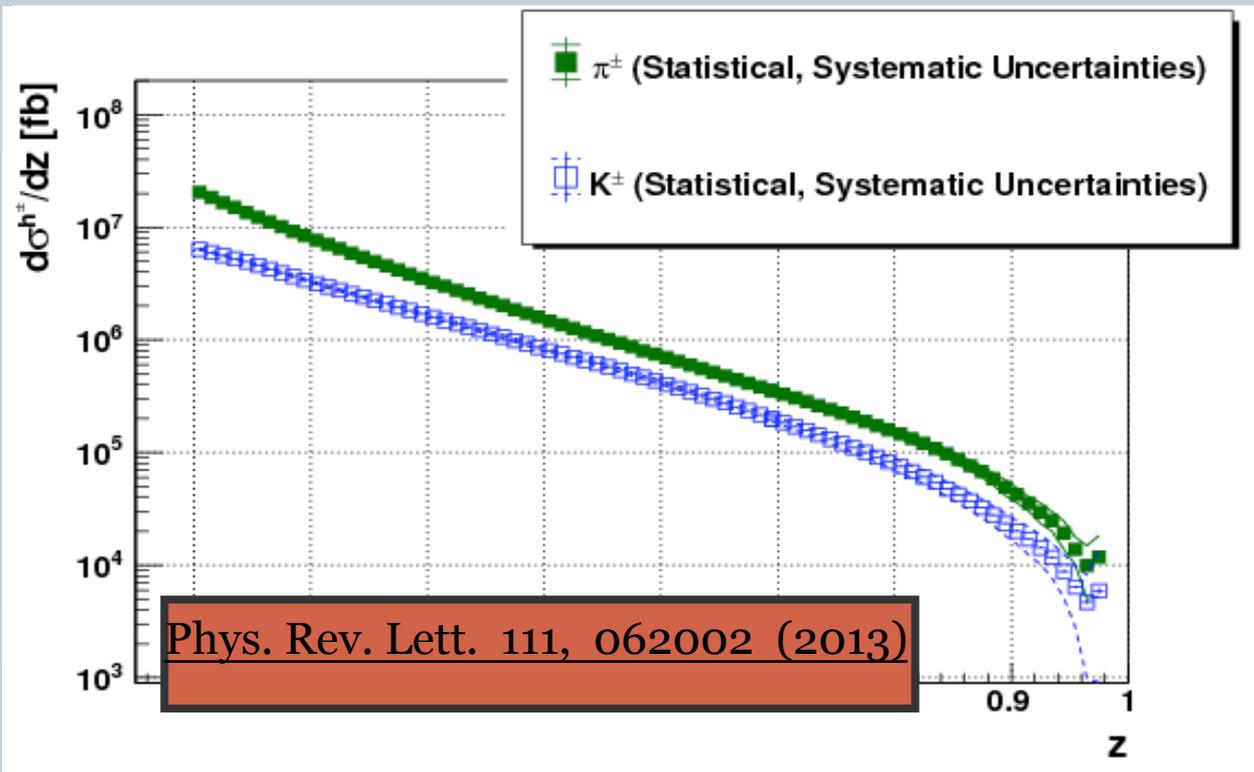


# Cross sections

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$i = \pi, K$

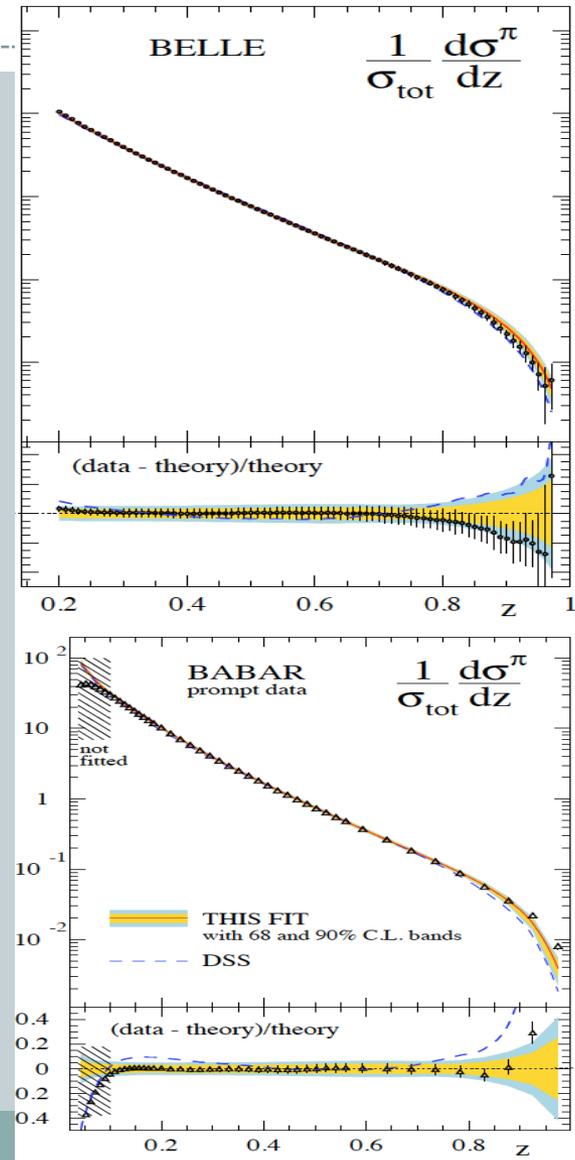
$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$



# New DSS(E,H-P) Fit

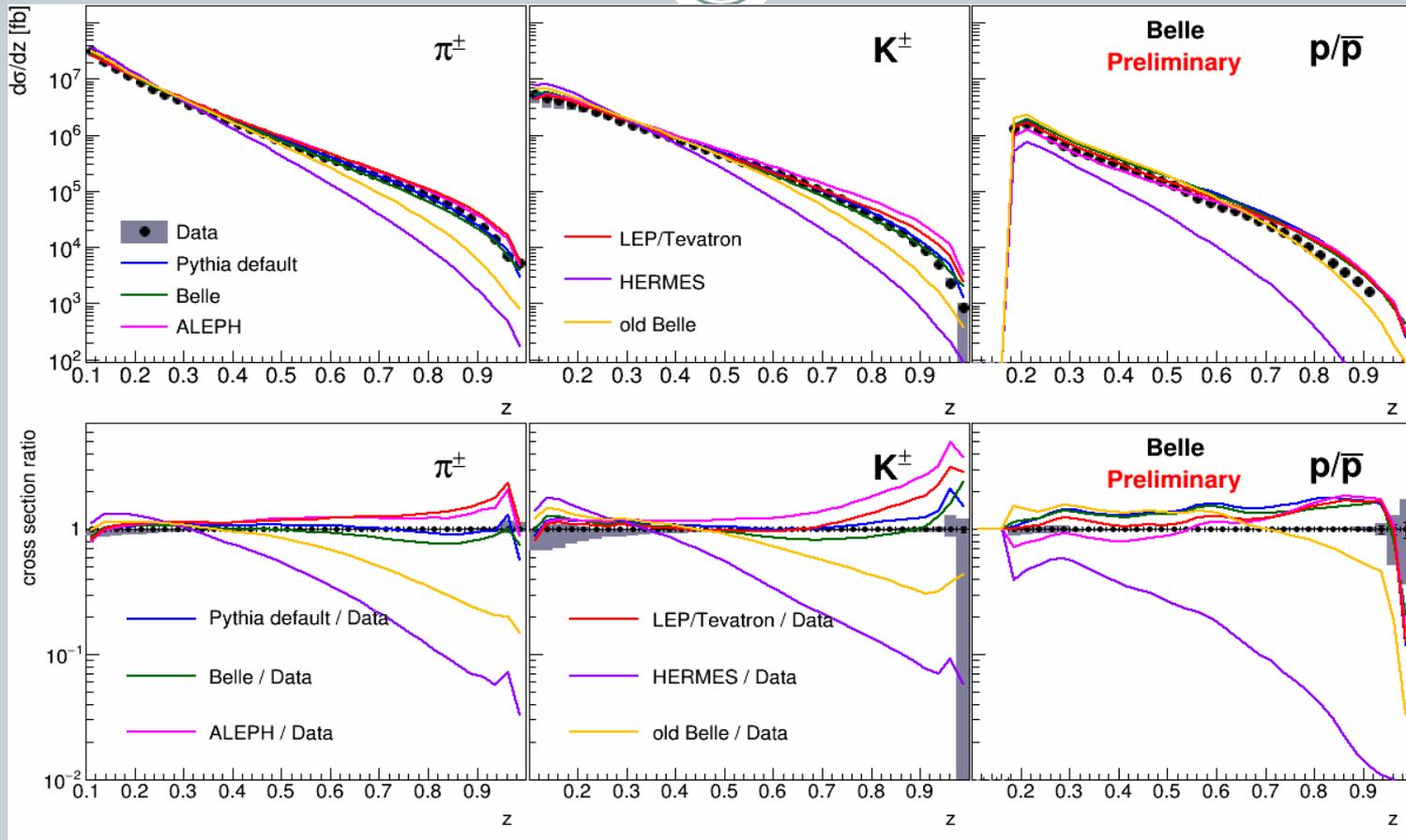
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- Good agreement, however, there seems to be a trend away from the fit for the Belle data at high  $z$
- From DSS:
  - Precise data at high  $z$
  - Some info from scaling violations (Belle vs experiments at  $M_Z$ )
  - Some info on flavor due to charge weighting



# New addition: single protons

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- Default Pythia and current Belle in good agreement with pions and kaons
- Protons not well described by any tune

$$Q = \sqrt{s}$$

$$z = \frac{2E_h}{Q} \approx \frac{E_h}{E_q}$$

# Di-hadrons

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- Single inclusive hadron multiplicities ( $e^+e^- \rightarrow hX$ ) sum over all available flavors and quarks and antiquarks:

$$d\sigma(e^+e^- \rightarrow hX)/dz \propto \sum e_q^2 (D_{1,q}^h(z, Q^2) + D_{1,\bar{q}}^h(z, Q^2))$$

- Especially distinction between favored (ie  $u \rightarrow \pi^+$ ) and disfavored ( $\bar{u} \rightarrow \pi^+$ ) fragmentation would be important
- Idea: Use di-hadron fragmentation, preferably from opposite hemispheres and access favored and disfavored combinations:

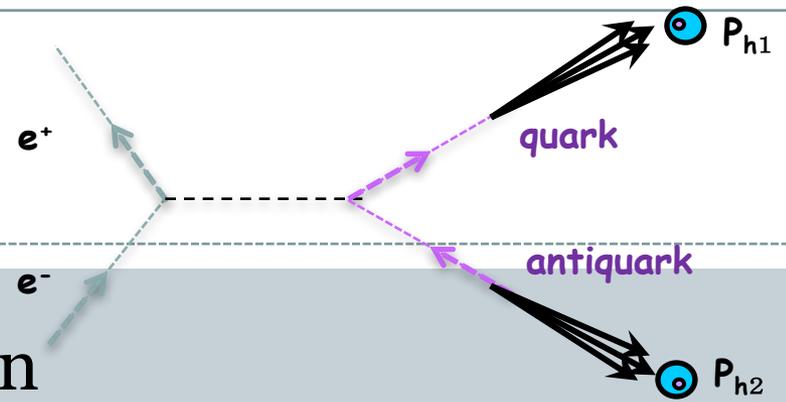
$$u\bar{u} \rightarrow \pi^+ \pi^- X \quad \propto \quad D_{u, fav}^{\pi^+}(z_1, Q^2) \cdot D_{\bar{u}, fav}^{\pi^-}(z_2, Q^2) + D_{\bar{u}, dis}^{\pi^+}(z_1, Q^2) \cdot D_{u, dis}^{\pi^-}(z_2, Q^2)$$

$$u\bar{u} \rightarrow \pi^+ \pi^+ X \quad \propto \quad D_{u, fav}^{\pi^+}(z_1, Q^2) \cdot D_{\bar{u}, dis}^{\pi^+}(z_2, Q^2) + D_{\bar{u}, dis}^{\pi^+}(z_1, Q^2) \cdot D_{u, fav}^{\pi^+}(z_2, Q^2)$$

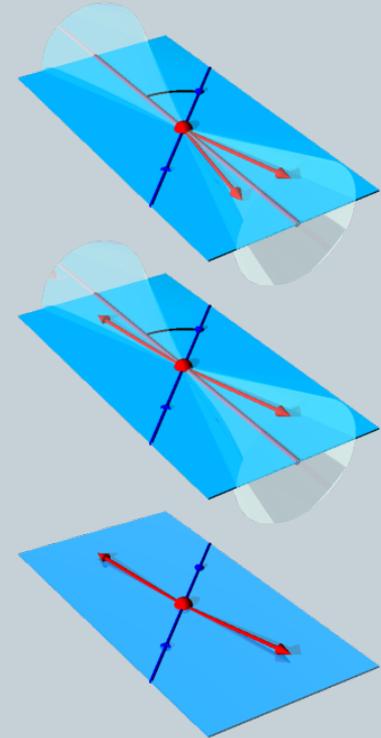
- Also: unpol baseline for interference fragmentation

# Setup

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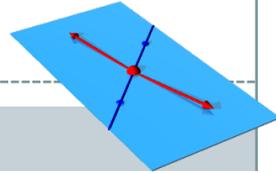
- Generally look at  $4 \times 4$  hadron combinations ( $\pi, K, +, -$ )
  - Keep separate until end: only 6 independent yields
- 3 hemisphere combinations:
  - same hemisphere (thrust  $> 0.8$ )
  - opposite hemisphere (thrust  $> 0.8$ )
  - any combination (no thrust selection)
- $16 \times 16$   $z_1 z_2$  binning between 0.2 - 1



# Results for diagonal $z_1 z_2$ bins

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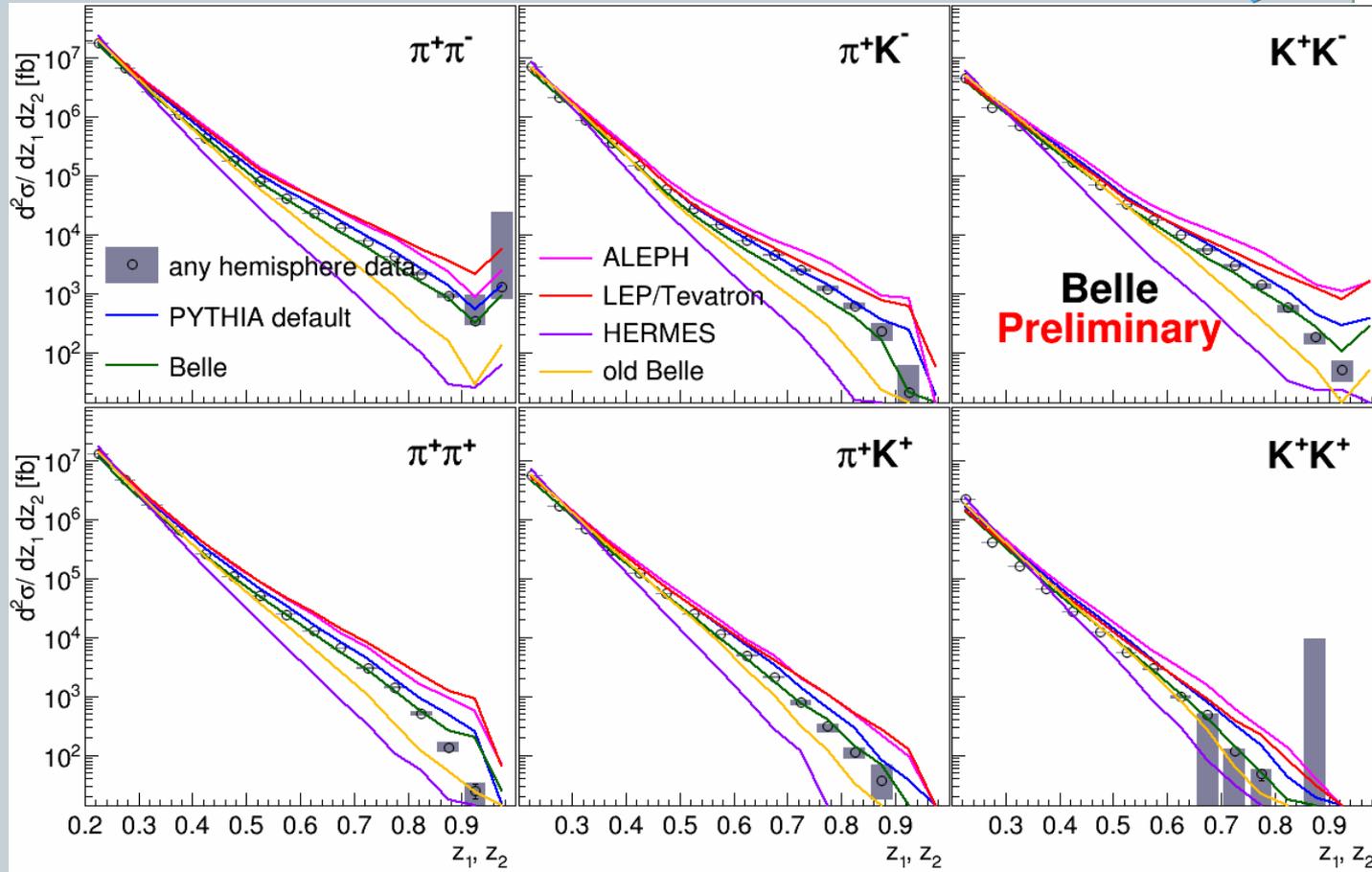
Diagonal  $z_1, z_2$  bins



Low  $z$  dominates  
integral:  
→ Well defined, all  
tunes agree

High  $z$  not well  
measured,  
especially at Belle  
energies:  
→ large spread in  
tunes

Default Pythia  
settings and current  
Belle setting with  
good agreement



# Spin dependent fragmentation

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$$H_{1,q}^{h,\perp}(z, Q^2, k_t)$$

$$H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$$

# Spin dependent fragmentation

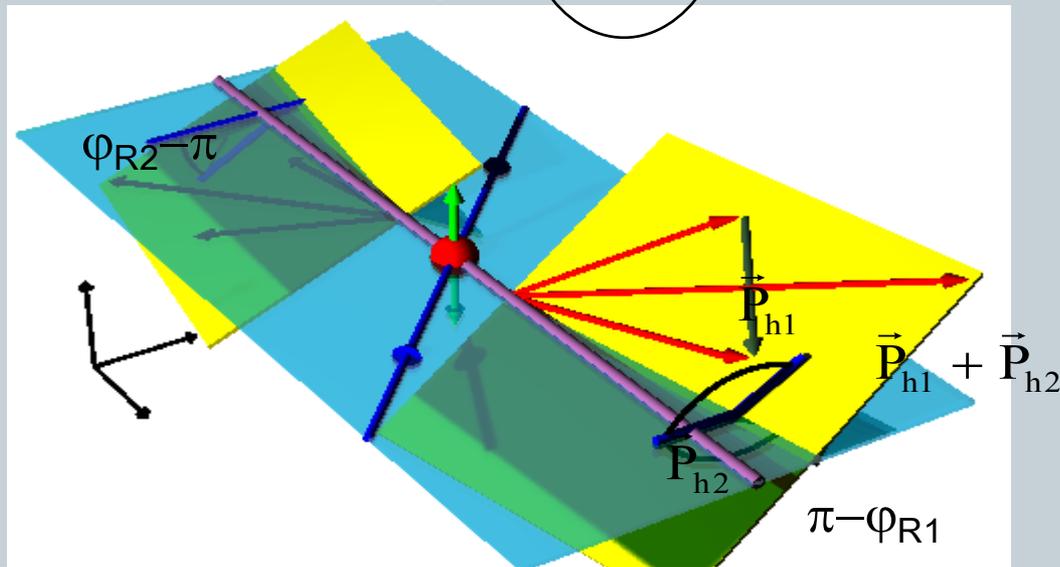
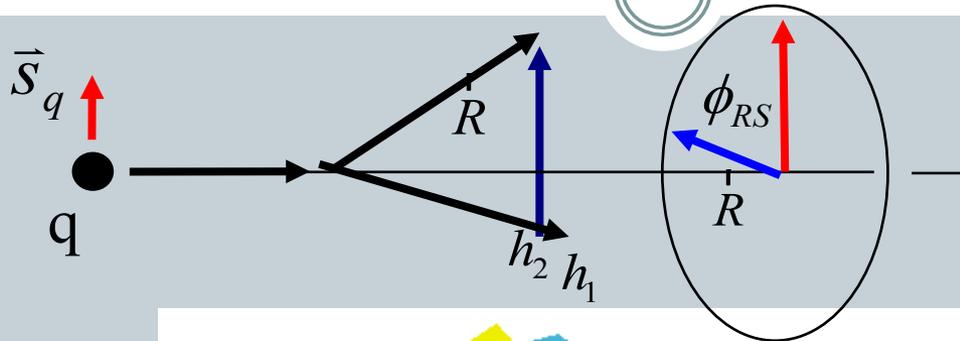
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**See Yesterday's Talk!**

$$H_{1,q}^{h_1, h_2, \Delta} (z, Q^2, M_h)$$

# Di-Hadron Fragmentation

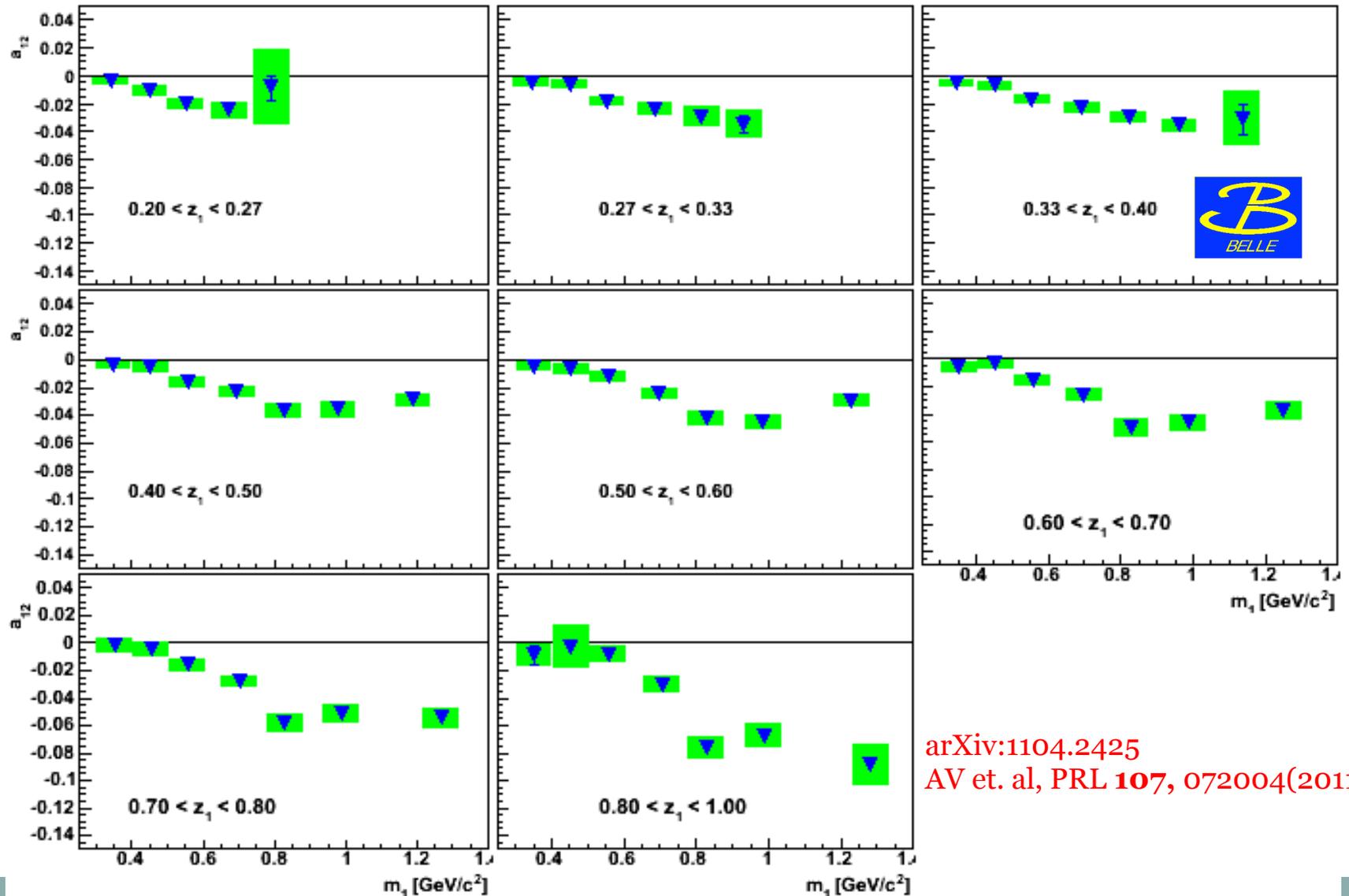
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- No Double Ratio!
- Collinear: Theoretical advantages, e.g. evolution
- Experimentally easier in  $pp$  (no jet reconstruction)

# First measurement of Interference Fragmentation Function

$$a_{12} \propto H_1^{<} * H_1^{<}$$



arXiv:1104.2425  
 AV et. al, PRL **107**, 072004(2011)

# Di-Hadron Asymmetries

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- Di-hadron Cross Section from Boer, Jakob, Radici [PRD 67, (2003) 094003]: Expansion of Fragmentation Matrix  $\Delta$ : encoding possible correlations in fragmentation ( $\mathbf{k}: P_{h1} + P_{h2}$ )

$$\frac{1}{32z} \int dk^+ \Delta(k; P_h, R) \Big|_{k^- = P_h^- / z, \mathbf{k}_T}$$

$$= \frac{1}{4\pi} \frac{1}{4} \left\{ D_1^a(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \not{n}_- - G_1^{\perp a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu n_-^\nu k_T^\rho R_T^\sigma}{M_1 M_2} \gamma_5 \right.$$

$$\left. + H_1^{\triangleleft a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\sigma_{\mu\nu} R_T^\mu n_-^\nu}{M_1 + M_2} + H_1^{\perp a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\sigma_{\mu\nu} k_T^\mu n_-^\nu}{M_1 + M_2} \right\} .$$

$$\langle \cos(2(\phi_R - \phi_{\bar{R}})) \rangle = \sum_{a, \bar{a}} e_a^2 \frac{3\alpha^2}{2Q^2} z^2 \bar{z}^2 A(y) \frac{1}{M_1 M_2 \bar{M}_1 \bar{M}_2} G_1^{\perp a}(z, M_h^2) \bar{G}_1^{\perp a}(\bar{z}, \bar{M}_h^2) .$$

$$\langle \cos(\phi_R + \phi_{\bar{R}} - 2\phi^l) \rangle = \sum_{a, \bar{a}} e_a^2 \frac{3\alpha^2}{Q^2} \frac{z^2 \bar{z}^2 B(y)}{(M_1 + M_2)(\bar{M}_1 + \bar{M}_2)} H_{1(R)}^{\triangleleft a}(z, M_h^2) \bar{H}_{1(\bar{R})}^{\triangleleft a}(\bar{z}, \bar{M}_h^2) .$$

Measure  $\text{Cos}(\phi_{R1} + \phi_{R2}), \text{Cos}(2(\phi_{R1} - \phi_{R2}))$  Modulations!

# Di-hadron Cross Section from Boer, Jakob, Radici

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- $\Delta$ : Fragmentation Matrix, encoding possible correlations in fragmentation
  - $k$ :  $P_{h1} + P_{h2}$
- Helicity dependent correlation of  
Intrinsic transverse momentum with  
Di-hadron plane  $\rightarrow$  Test of TMD framework

from Boer, Jakob, Radici [PRD 67, (2003) 094003]

$$\frac{1}{32z} \int dk^+ \Delta(k; P_h, R) \Big|_{k^- = P_h^- / z, \mathbf{k}_T}$$

$$= \frac{1}{4\pi} \frac{1}{4} \left\{ D_1^a(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \not{n}_- - G_1^{\perp a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu n_-^\nu k_T^\rho R_T^\sigma}{M_1 M_2} \gamma_5 \right.$$

$$\left. + H_1^{\lessdot a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\sigma_{\mu\nu} R_T^\mu n_-^\nu}{M_1 + M_2} + H_1^{\perp a}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \frac{\sigma_{\mu\nu} k_T^\mu n_-^\nu}{M_1 + M_2} \right\}.$$

$$\langle \cos(2(\phi_R - \phi_{\bar{R}})) \rangle = \sum_{a, \bar{a}} e_a^2 \frac{3\alpha^2}{2Q^2} z^2 \bar{z}^2 A(y) \frac{1}{M_1 M_2 \bar{M}_1 \bar{M}_2} G_1^{\perp a}(z, M_h^2) \bar{G}_1^{\perp a}(\bar{z}, \bar{M}_h^2).$$

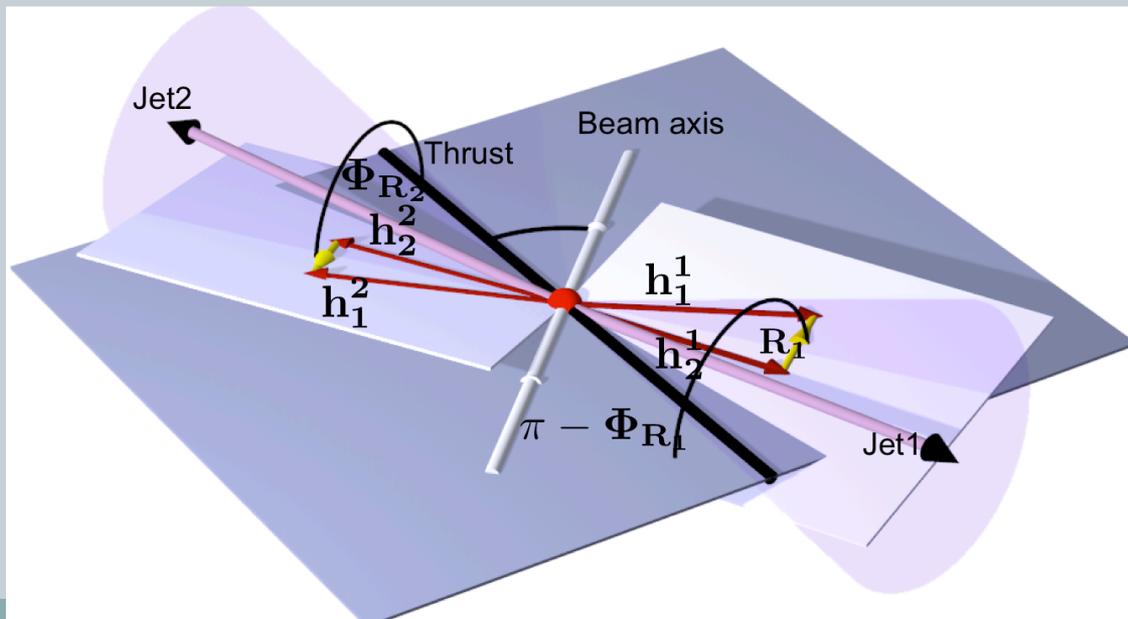
$$\langle \cos(\phi_R + \phi_{\bar{R}} - 2\phi^l) \rangle = \sum_{a, \bar{a}} e_a^2 \frac{3\alpha^2}{Q^2} \frac{z^2 \bar{z}^2 B(y)}{(M_1 + M_2)(\bar{M}_1 + \bar{M}_2)} H_{1(R)}^{\lessdot a}(z, M_h^2) \bar{H}_{1(\bar{R})}^{\lessdot a}(\bar{z}, \bar{M}_h^2).$$

Measure  $\text{Cos}(\phi_{R1} + \phi_{R2}), \text{Cos}(2(\phi_{R1} - \phi_{R2}))$  Modulations

# New: Use Jet Reconstruction at Belle

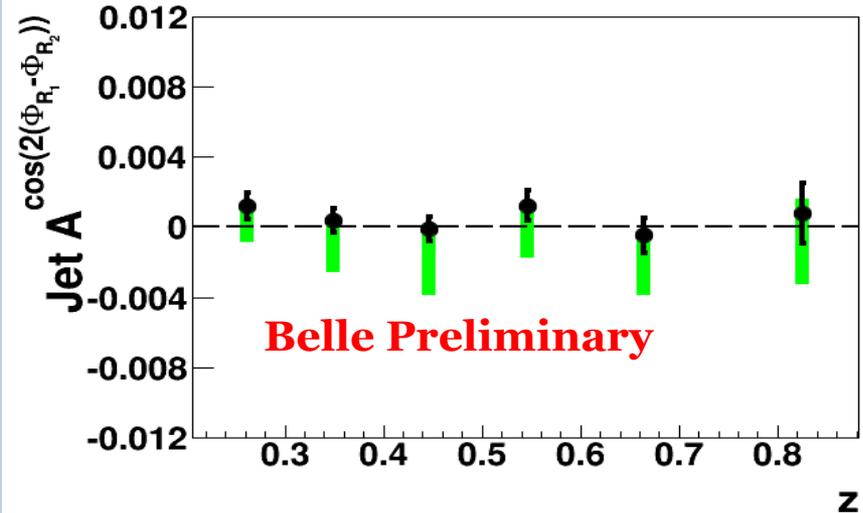
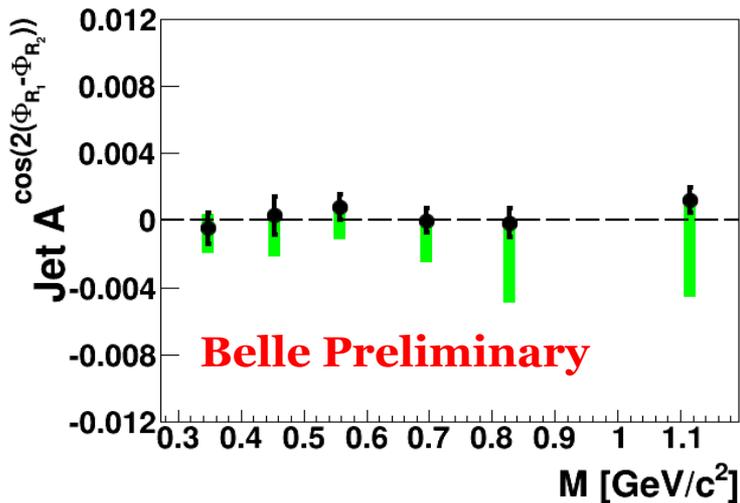
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- Robust vs. final state radiation
- **De-correlate axis between hemispheres**
- We use anti-kT algorithm implemented in fastjet
- Cone radius  $R=1.0$
- Min energy per jet  $2.75 \text{ GeV} \rightarrow$  suppress weak decays
- Only allow events with 2 jets passing energy cut (dijet events)
- Only particles that form the jet are used in the asymmetry calculation
- Thrust cut of  $0.8 < T < 0.95$



# Asymmetries for $\text{Cos}(2(\phi_{R1}-\phi_{R2}))$ ( $G_1^\perp$ ) small

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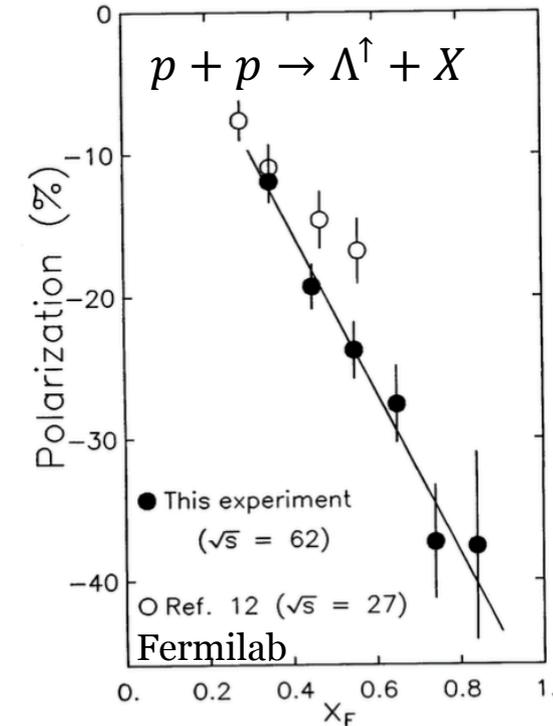
- No evidence of local p-odd effects yet
- Next step: partial wave analysis

# Polarized Hyperon Production

(see Y. Guan @ SPIN2016 & arXiv:1611.06648)

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- Large  $\Lambda$  transverse polarization in unpolarized pp collision **PRL36, 1113 (1976); PRL41, 607 (1978)**
- Caused by polarizing FF  $D_{1T}^\perp(z, p_\perp^2)$ ?
- Polarizing FF is chiral-even, has been proposed as a test of universality. **PRL105,202001 (2010)**
- OPAL experiment at LEP has been looking at transverse  $\Lambda$  polarization, no significant signal was observed. **Eur. Phys. J. C2, 49 (1998)**



ISR data

(Phys.Lett. B185 (1987) 209)

$$x_F = p_L / \max p_L \sim_{LO} x_1 - x_2 \sim_{forward} x_1$$

# Hyperon Production as a tool to study baryon spin structure

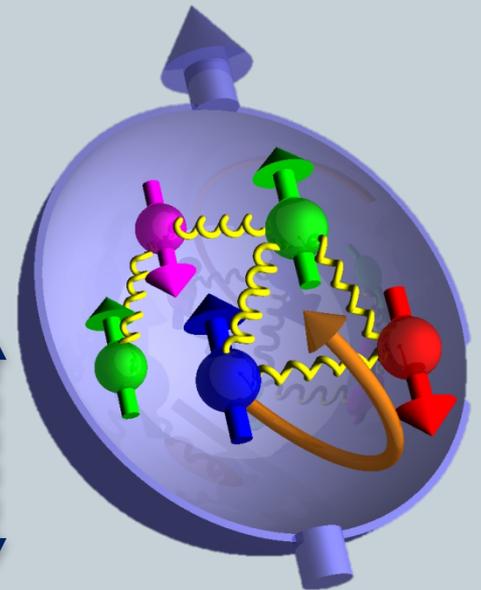


- Lambda polarization allows to study spin-orbit correlation of quarks inside Baryon  $\rightarrow$  counterpart of the Sivers parton distribution function ( $k_T$  dependence of quark distributions in transversely polarized proton)
- A non-vanishing  $D_{1T}^\perp$  could help to shed light on the spin structure of the  $\Lambda$ , especially about the quark orbital angular momentum, a missing part of the spin puzzle of the nucleon.
- Produce Lambda with certain  $p_T$
- Check Transverse Polarization depending on  $p_T$  and flavor
- Analogue of the Sivers effect in the Similar Universality checks (T-odd but not chiral odd) allows to fix sign

$q$

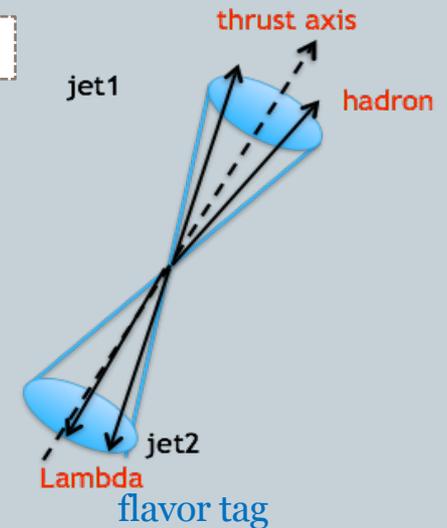
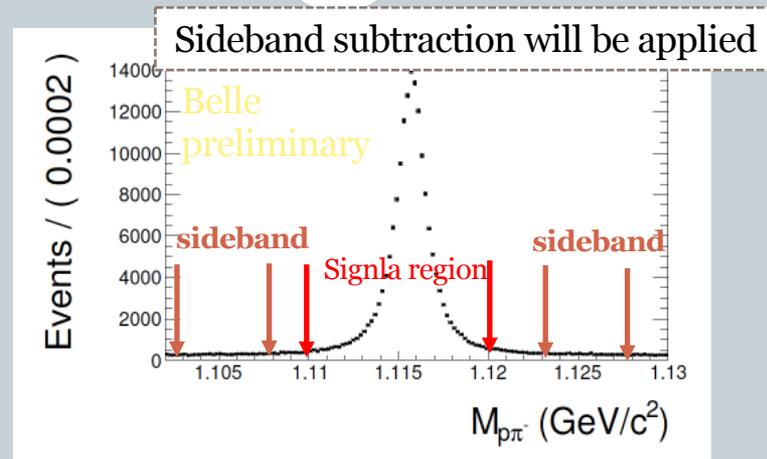


$p_T$



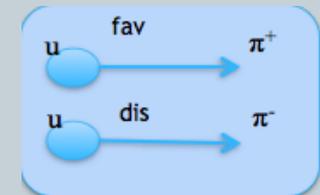
# Lambda Reconstruction

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$\Lambda(u\bar{d}s)$ ;  $\pi^+(u\bar{d})$ ;  $K^+(u\bar{s})$

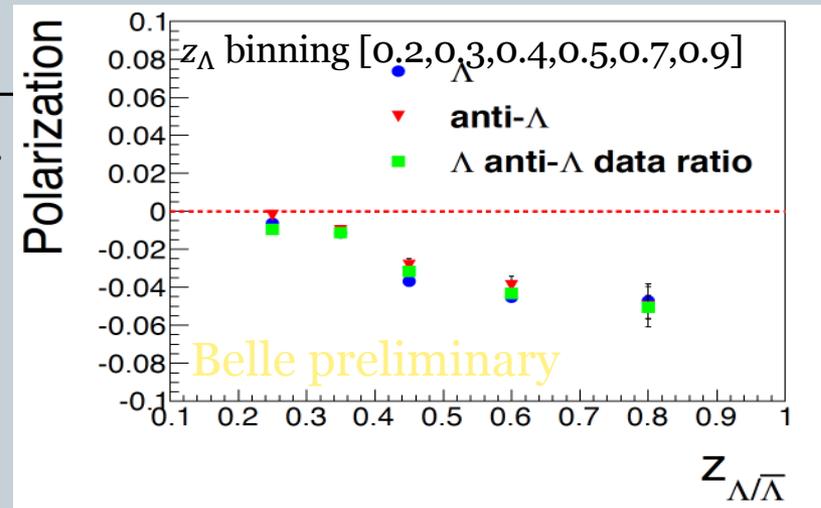
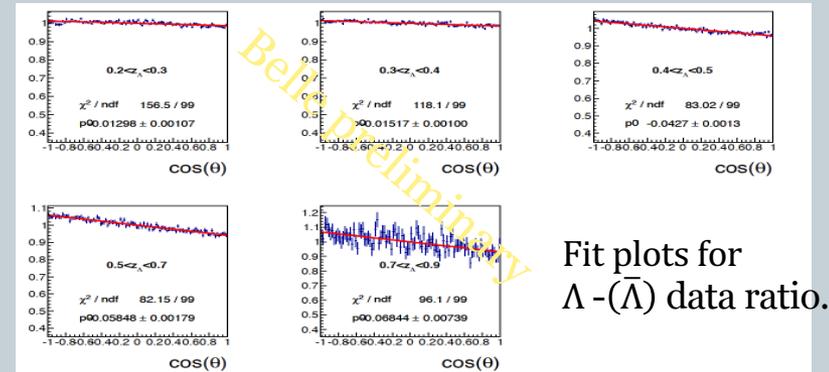
- Signal process  $\Lambda \rightarrow p\pi^- (\bar{\Lambda} \rightarrow \bar{p}\pi^+)$ . Clear  $\Lambda$  peak.
- Detect light hadron ( $K^\pm, \pi^\pm$ ) in the opposite hemisphere  $\rightarrow$  enhance or suppress different flavors fragmenting in  $\Lambda(\bar{\Lambda})$ .



# Fits and Extract polarization

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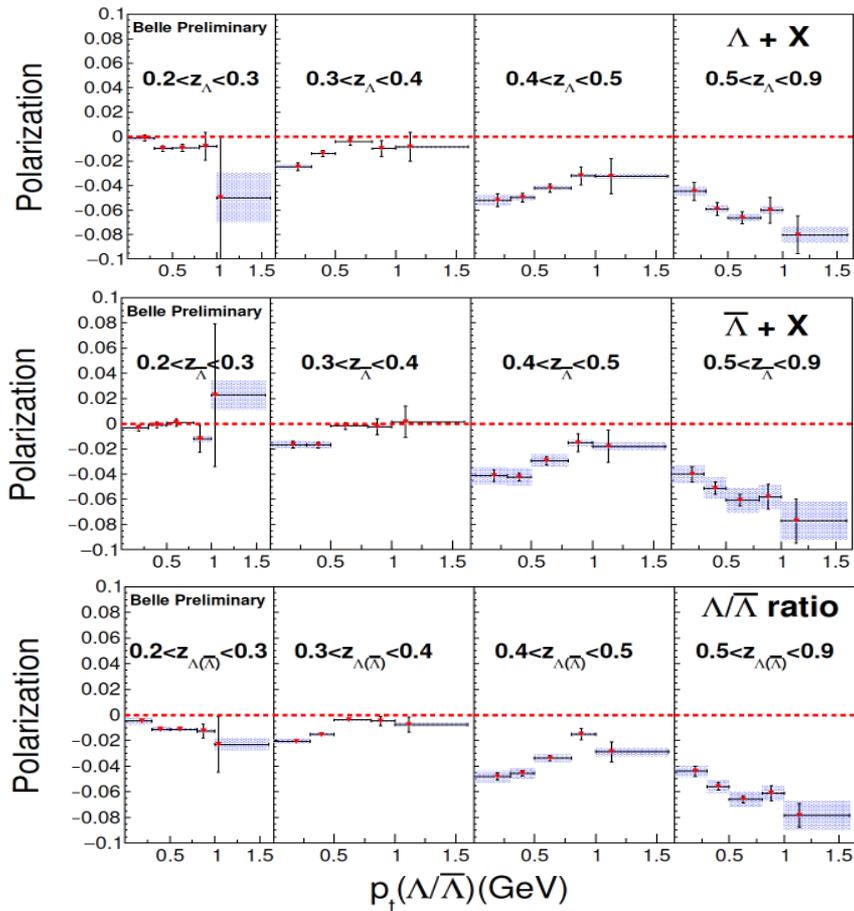
- Fit to the  $\cos\theta$  distributions with  $1 + p_0 \cos\theta$ .
- The polarization of interest:  $p_0/\alpha$ .
- In the data ratio, polarization is obtained via  $p_0/(\alpha_+ - \alpha_-)$ .
- In data ratios, the slope on the  $\cos\theta$  distributions are about two times larger than that in MC-corrected ratios, the  $(\alpha_+ - \alpha_-)$  is also about times larger than  $\alpha_+(\alpha_-)$ .
- Results from MC-corrected ratio and data ratio are consistent with each other.
- Nonzero polarization**, magnitude rises to about  $\sim 5\%$  with  $z_\Lambda = 2E_\Lambda/\sqrt{s}$ .



# Results in thrust frame

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vs.  $(z, p_t)$

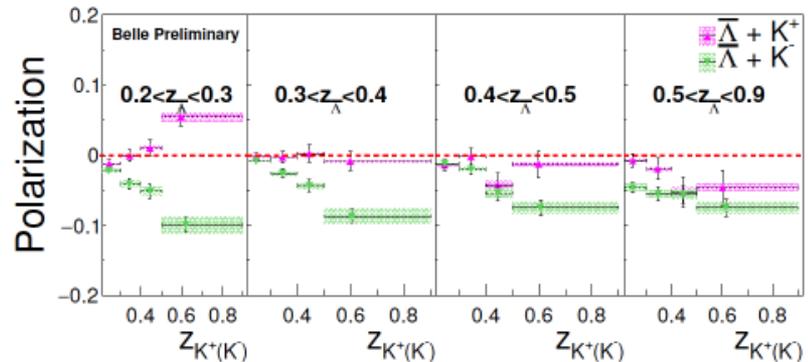
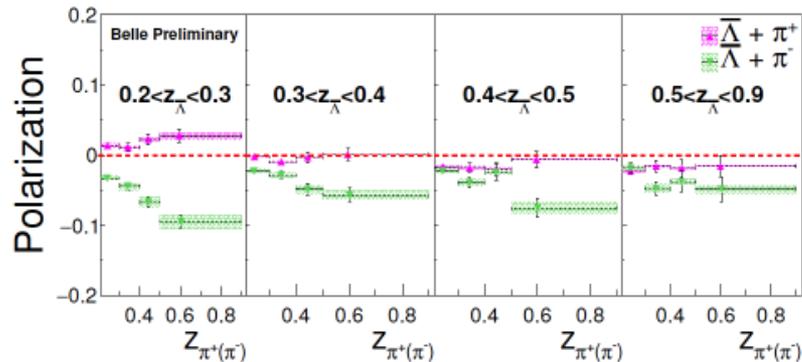
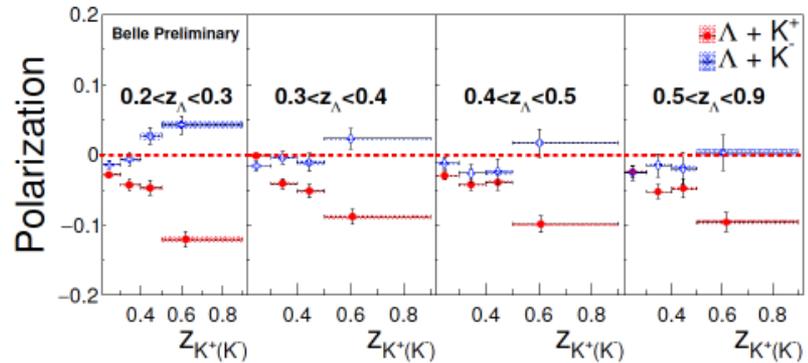
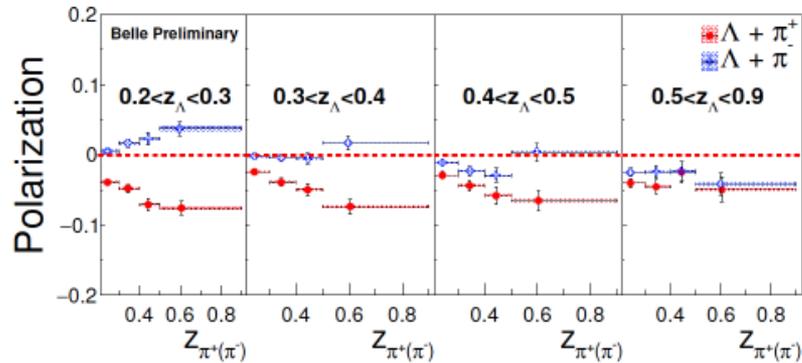


- Four  $z$  bins and five  $p_t$  bins are used:  
 $z_{\Lambda} = [0.2, 0.3, 0.4, 0.5, 0.9]$ ;  
 $p_t = [0.0, 0.3, 0.5, 0.8, 1.0, 1.6]$  GeV
- **Nonzero polarization** was observed. Interesting shape as a function of  $(z_{\Lambda}, p_t)$ .
- The polarization rise with higher  $p_t$  in the lowest  $z_{\Lambda}$  and highest  $z_{\Lambda}$  bin. But the dependence reverses around 1 GeV in the intermediate  $z_{\Lambda}$  bins.
- Results are consistent between  $\Lambda$  and  $(\bar{\Lambda})$  and  $\Lambda - (\bar{\Lambda})$  data ratio.
- Error bars are statistical uncertainties and shaded areas show the systematic uncertainties.

# Results in hadron frame

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vs.  $(z_\Lambda, z_h)$

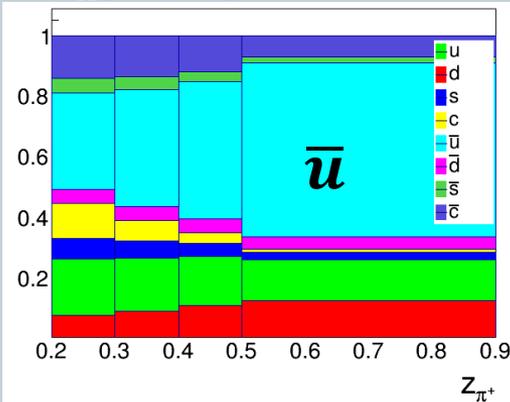


- Similar results with that in the thrust frame.
- Results from charge-conjugate modes are consistent with each other.

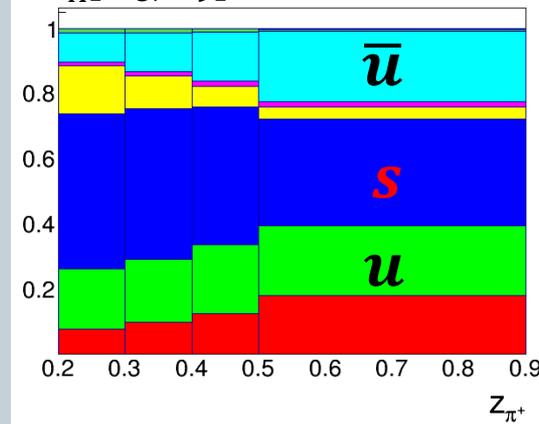
# Quark flavor tag by the light hadron

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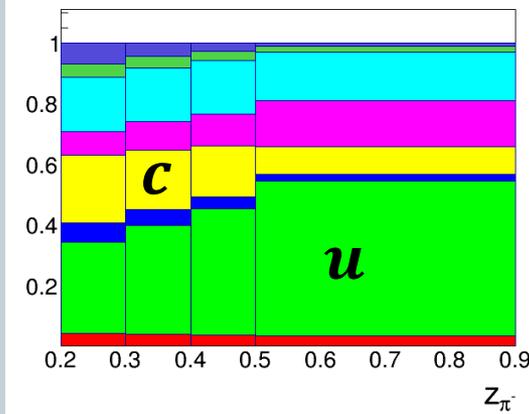
$z_\Lambda[0.2,0.3] \Lambda + \pi^+ + X$



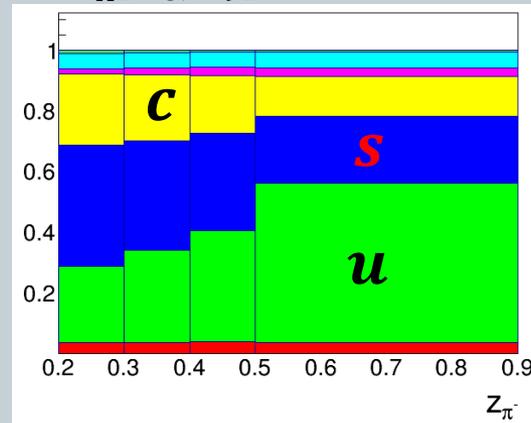
$z_\Lambda[0.5,0.9] \Lambda + \pi^+ + X$



$z_\Lambda[0.2,0.3] \Lambda + \pi^- + X$



$z_\Lambda[0.5,0.9] \Lambda + \pi^- + X$



- An attempt to look at the flavor tag effect of the light hadron, based on MC. (Pythia6.2)
- The fractions of various quark flavors going to the  $\Lambda$ 's hemisphere are shown in different  $[z_\Lambda z_h]$  region.
- MC indicates that the tag of the quark flavors is more effective at low  $z_\Lambda$  and high  $z_h$ . It explains why at low  $z_\Lambda$  and high  $z_h$ , polarization in  $\Lambda + h^+$  and  $\Lambda + h^-$  have opposite sign.

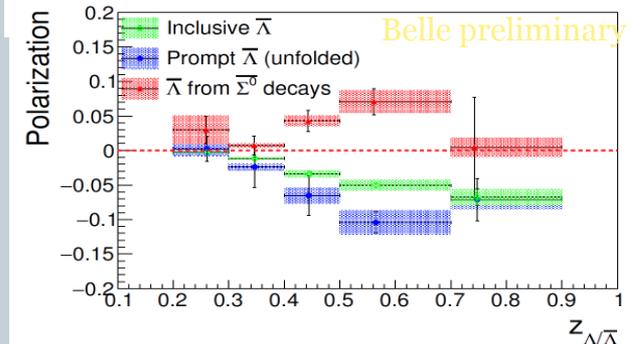
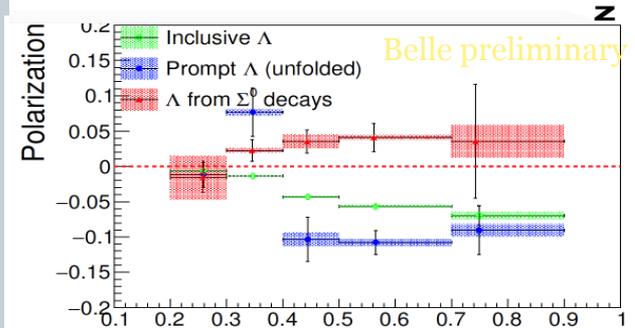
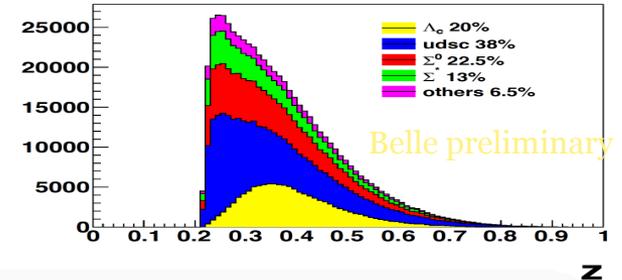
# Background unfolding

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- Non- $\Lambda$  backgrounds are excluded out in the sideband subtraction.
- $\Sigma^*$  decays to  $\Lambda$  strongly, is included in the signal.
- Feed-down from  $\Sigma^0$  (22.5%),  $\Lambda_c$  (20%) decays need to be understood.
- The  $\Sigma^0$ -enhanced ( $\Sigma^0 \rightarrow \Lambda + \gamma$ ) (Br~100%). and  $\Lambda_c$ -enhanced ( $\Lambda_c \rightarrow \Lambda + \pi^+$ ) (Br~1.07%) data sets are selected and studied.
- The measured polarization can be expressed as:

$$P^{mea.} = (1 - \sum_i F_i) P^{true} + \sum_i F_i P_i,$$

- $F_i$  is the fraction of feed-down component  $i$ , estimated from MC.  $P_i$  is polarization of component  $i$ .
- Polarization of  $\Lambda$  from  $\Sigma^0$  decays is found has opposite sign with that of inclusive  $\Lambda$ .



R. Gatto, Phys. Rev. 109, 610 (1958); Phys.Lett.B303,350(1993)

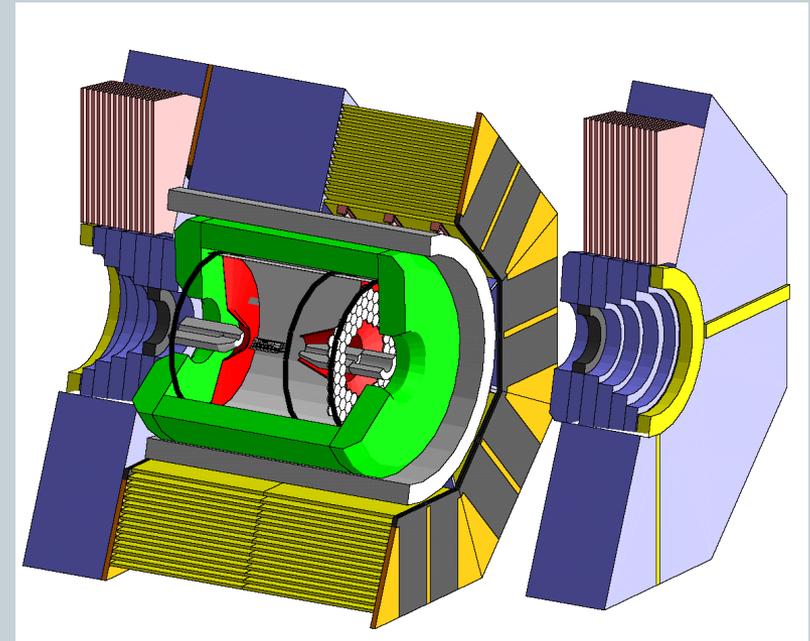
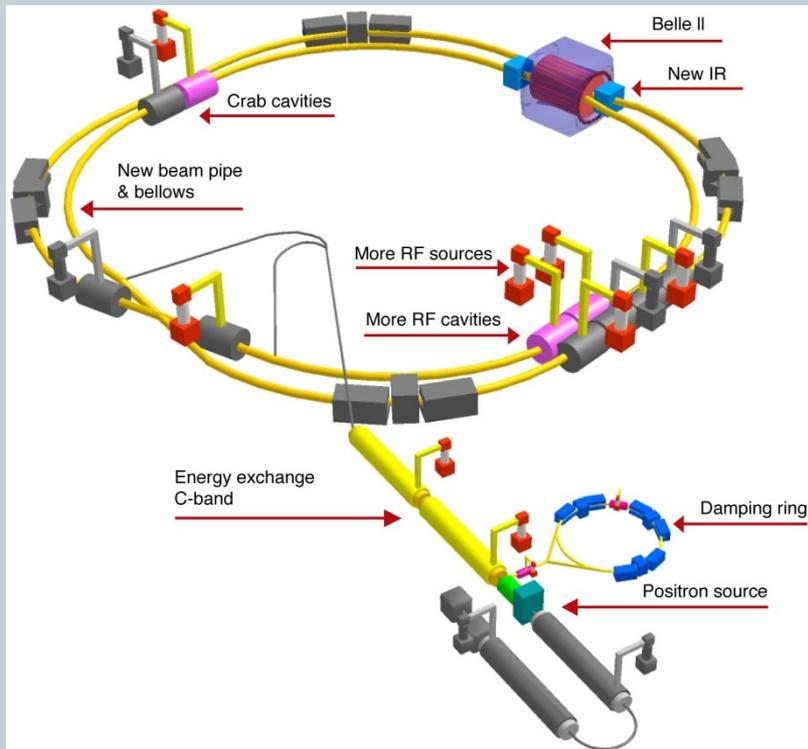
# KEKB/Belle → SuperKEKB,



# Upgrade

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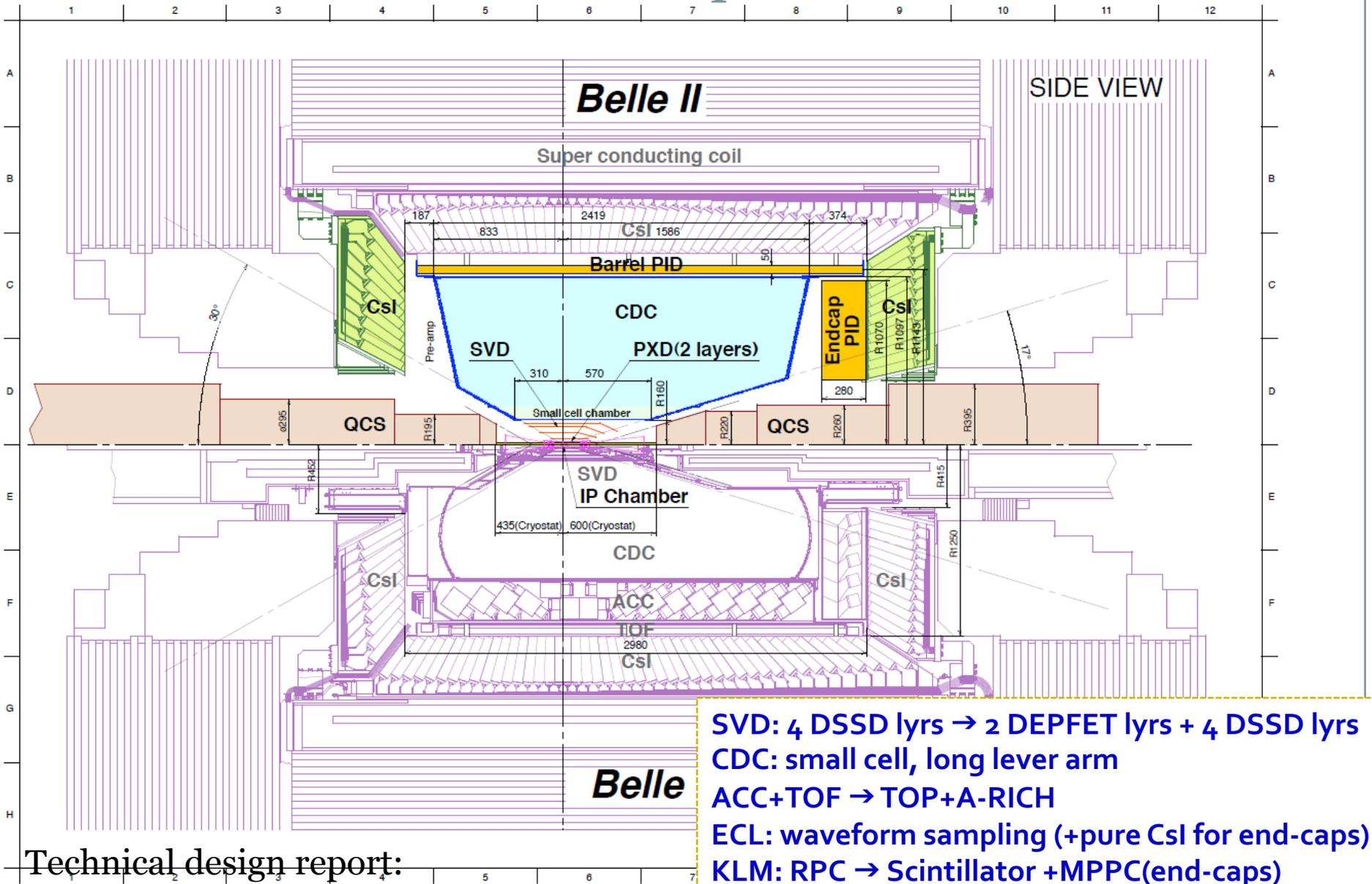
- Aim: super-high luminosity  $\sim 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$  ( $\sim 40 \times$  KEK/Belle)
- Upgrades of Accelerator (Nano-beams + Higher Currents) and Detector (Vtx, PID, higher rates, modern DAQ)
- Significant US contribution



<http://belle2.kek.jp>

Start of commissioning in 2016

# Belle II Detector (in comparison with Belle)



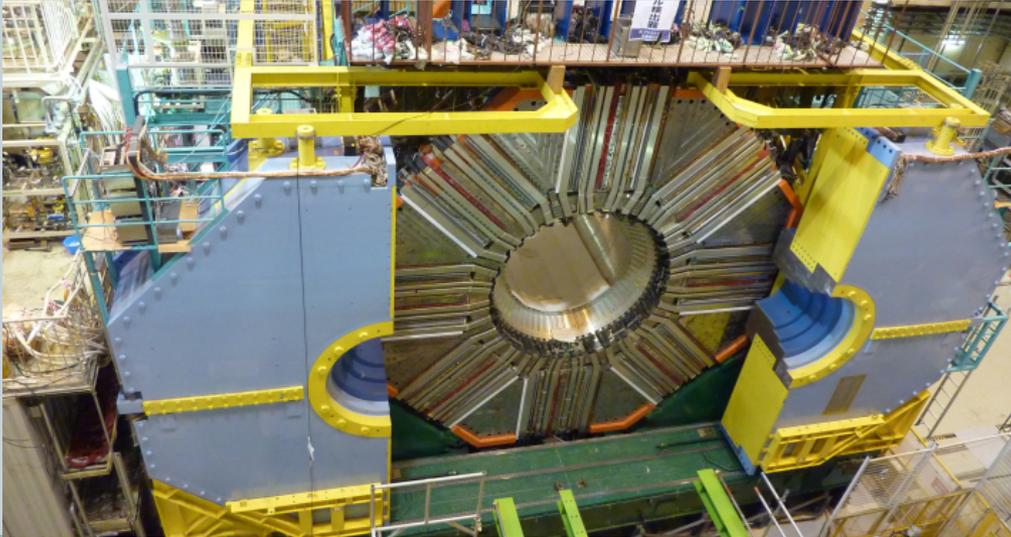
Technical design report:

arXiv:1011.0352

# Status of Belle II Installation

28

Sector Test of KLM  
(B Kunkler from IU)



# Current SuperKEKB/Belle II Schedule

CY2014

CY2015

CY2016

CY2017

CY2018

Super KEKB  
Construction

Startup

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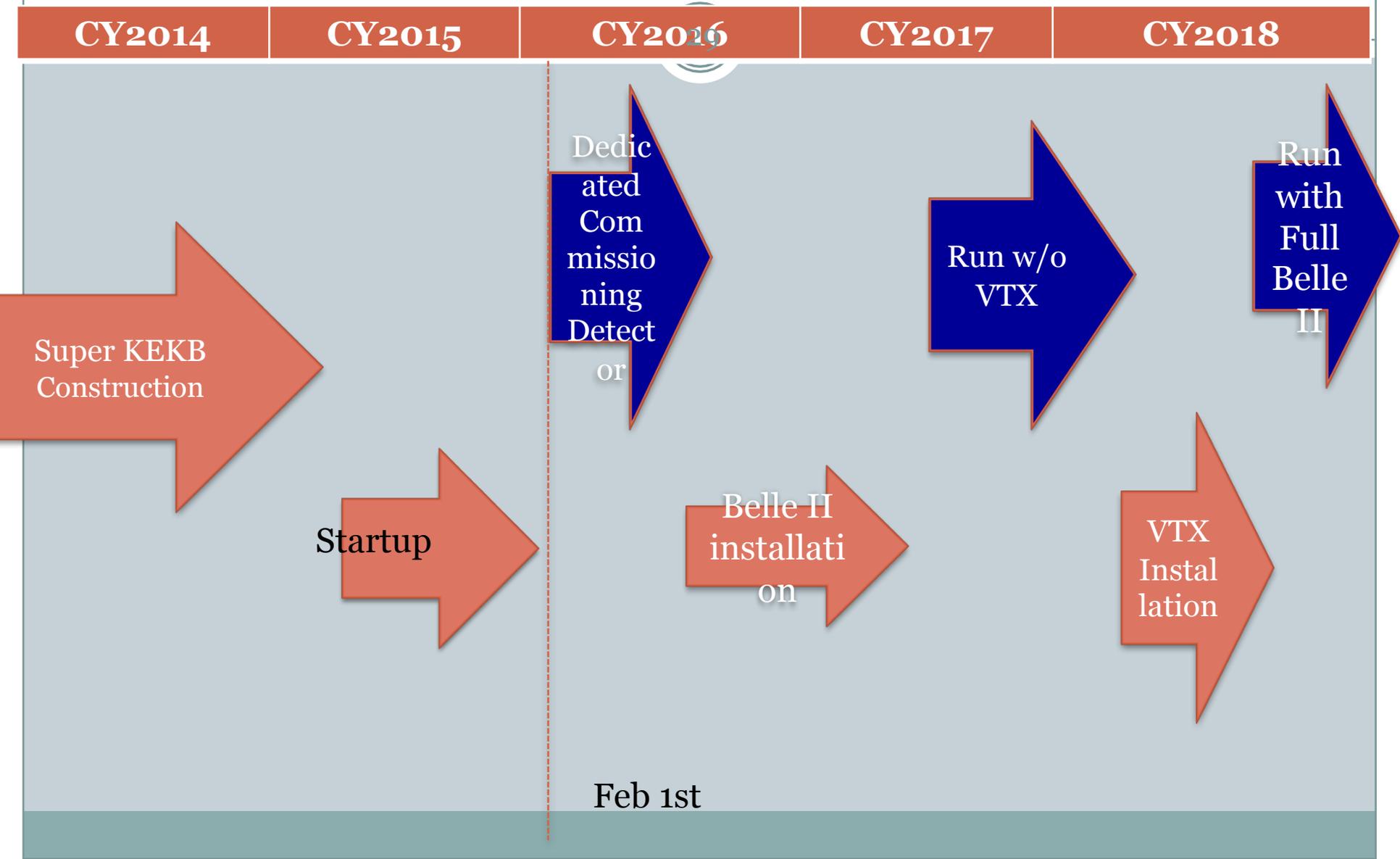
Belle II  
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on

Run w/o  
VTX

VTX  
Instal  
lation

Run  
with  
Full  
Belle  
II

Feb 1st



# Conclusion/Outlook

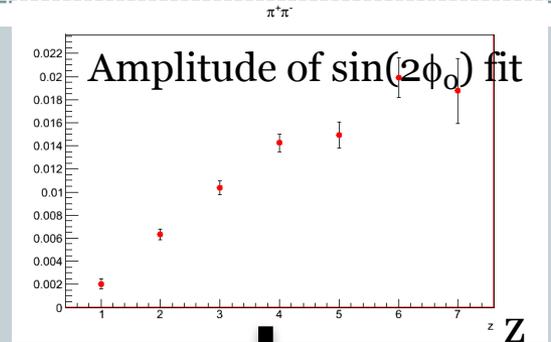
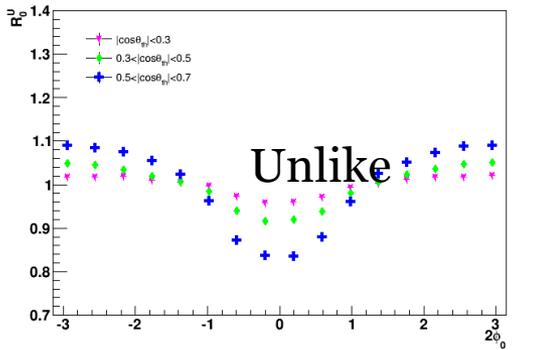
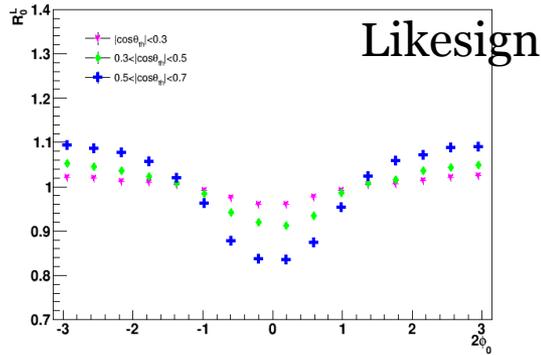


- Hadronization Studies in  $e^+e^-$  provide a complimentary access to non-perturbative QCD
- Exciting new results with respect to polarized and polarizing FFs
- $k_T$  dependent FFs on the horizon
- Belle II will provide ample new opportunities
- **Analysis underway**
  - Back-to-back hadrons to extract  $p_T$  dependence of  $D_1$
  - $p_T$  dependence with respect to thrust axis
  - Di-hadron FFs with mass dependence
  - Di-hadron Collins and di-hadron modulations including kaons

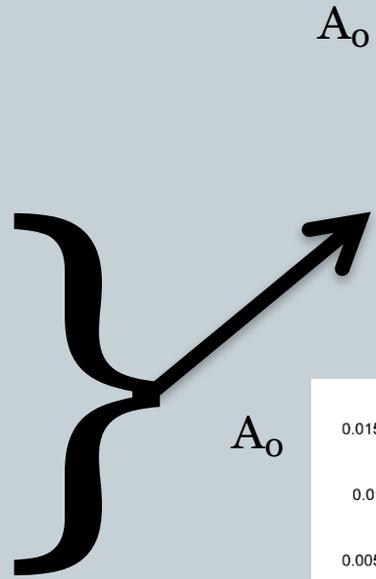
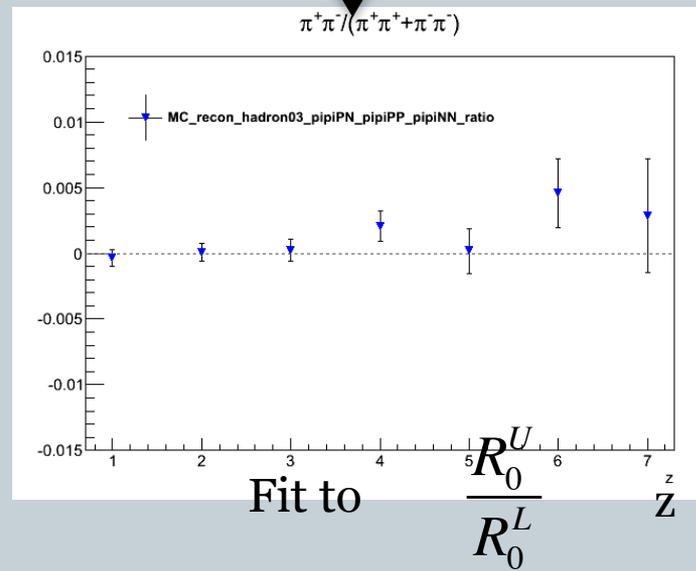


# Use of Double Ratios

32



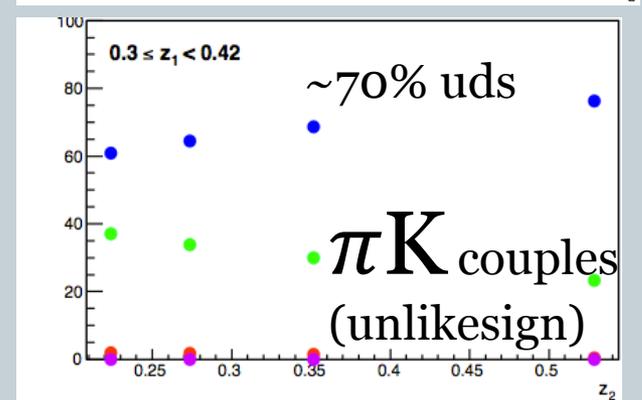
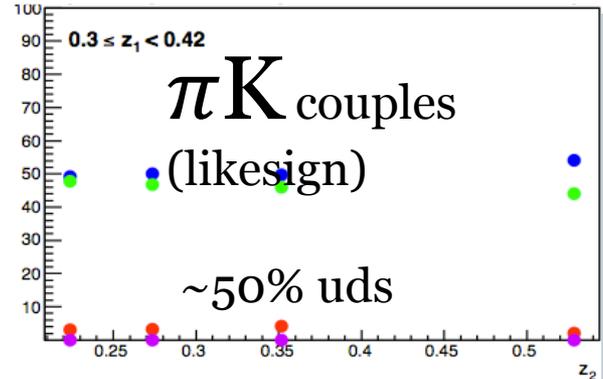
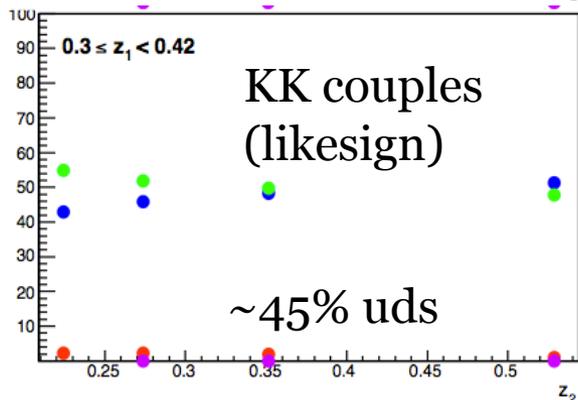
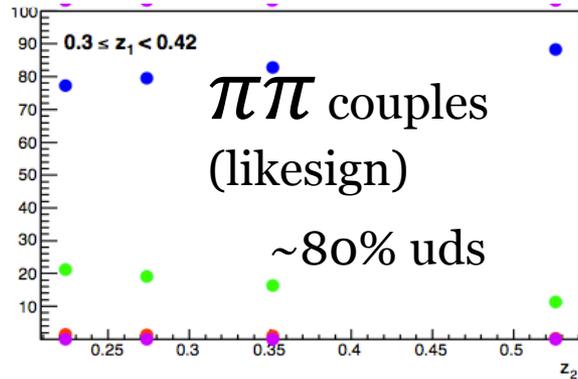
Use of "Double Ratios"



- False asymmetries due to Acceptance and QCD radiation
- Charge independent
- **Open question: Smearing correction/Unfolding in Thrust/Jet method non-trivial in ratio**

# Significant Charm to contribution to UDS

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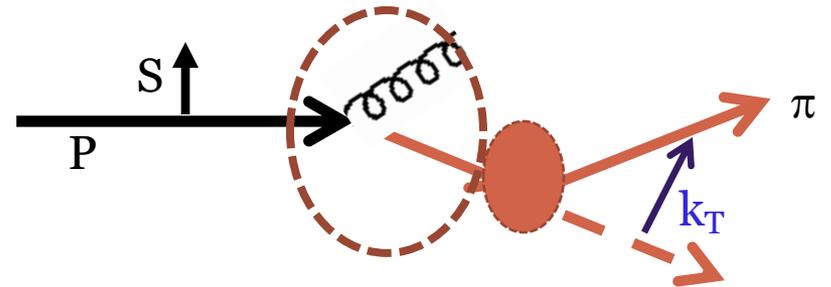
- Current Charm correction methods not satisfying
  - MC (unclear uncertainties)
  - From Data (D-tagged samples): Bias phase space (e.g. selection of decay modes with low momentum pions)
- Need state-of-the-art vertex detection  $\rightarrow$  Belle II

# Transverse momentum dependence

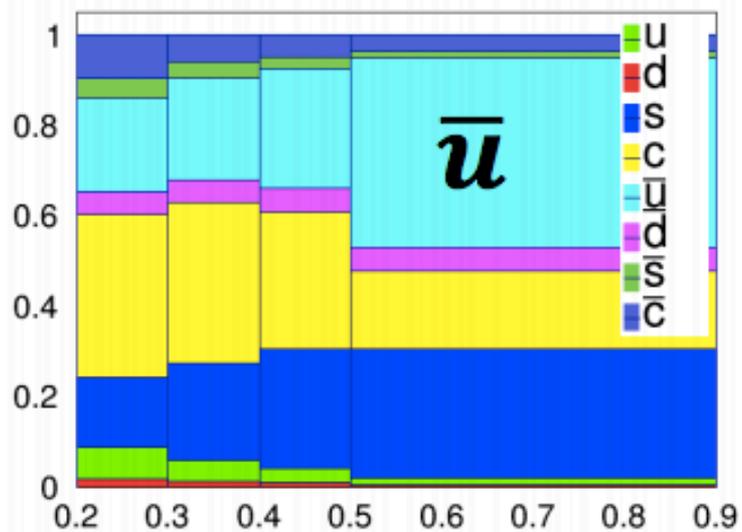
34

AKA UN-INTEGRATED FFS

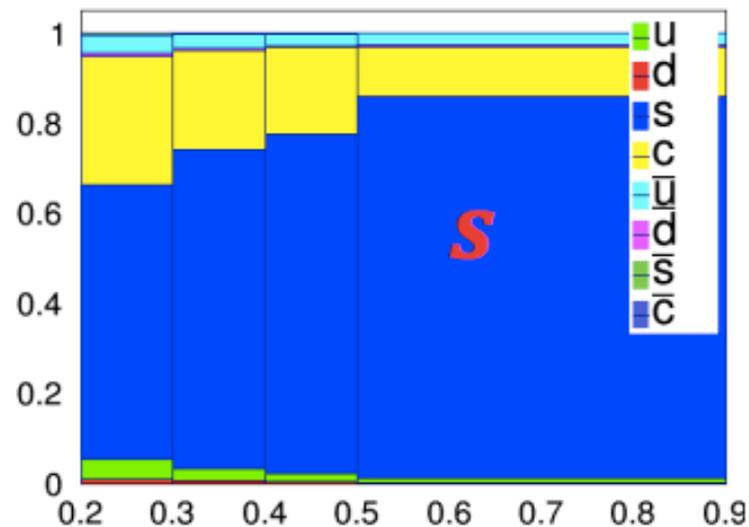
$$D_{1,q}^{h}(z, Q^2, k_t)$$



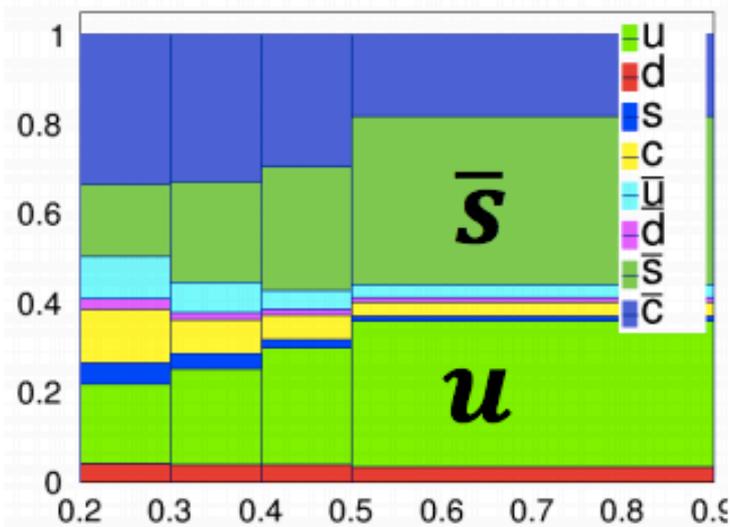
$z_{\Lambda}[0.2,0.3] \quad \Lambda + K^+ + X$



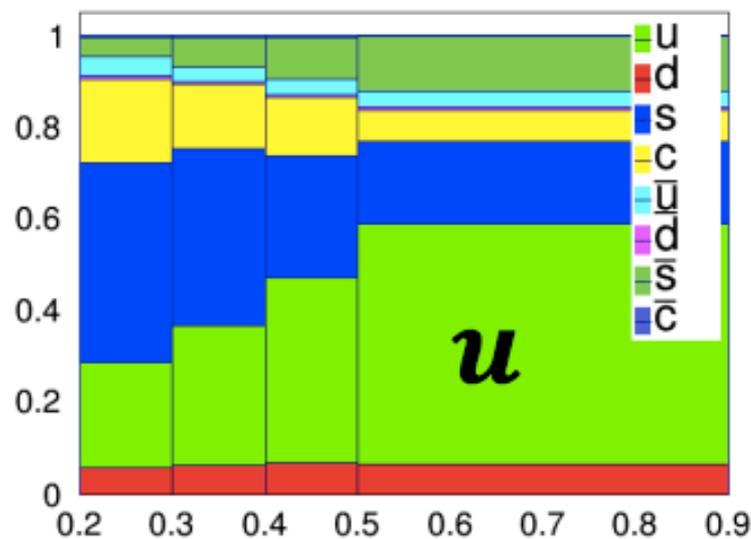
$z_{\Lambda}[0.5,0.9] \quad \Lambda + K^+ + X$



$z_{\Lambda}[0.2,0.3] \quad \Lambda + K^- + X$

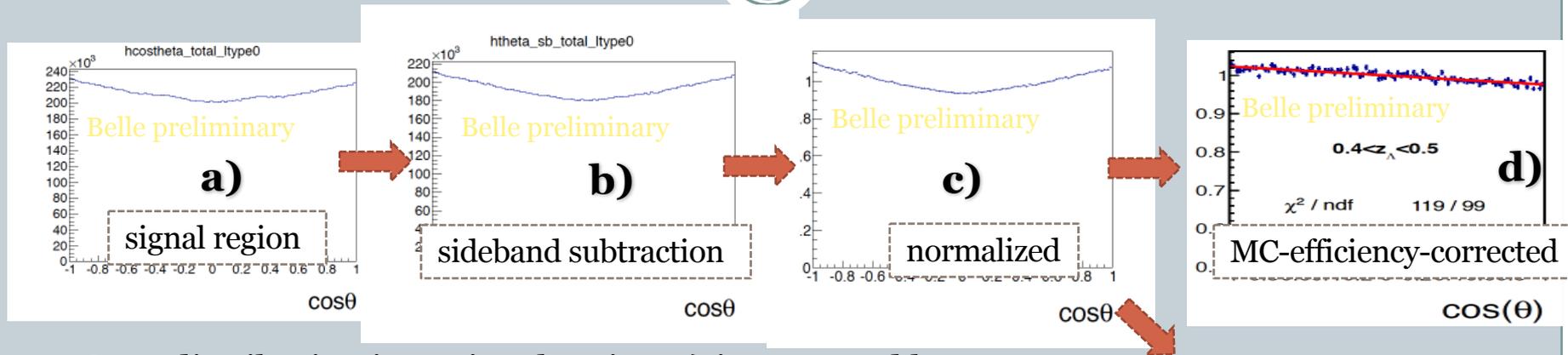


$z_{\Lambda}[0.5,0.9] \quad \Lambda + K^- + X$

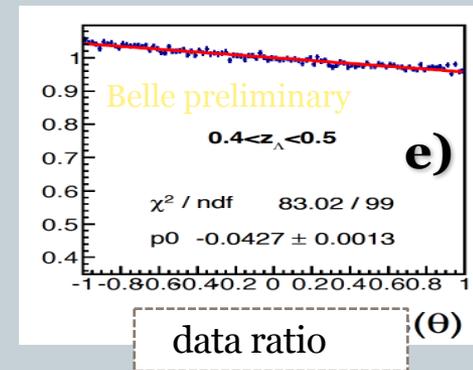


# Analysis flow

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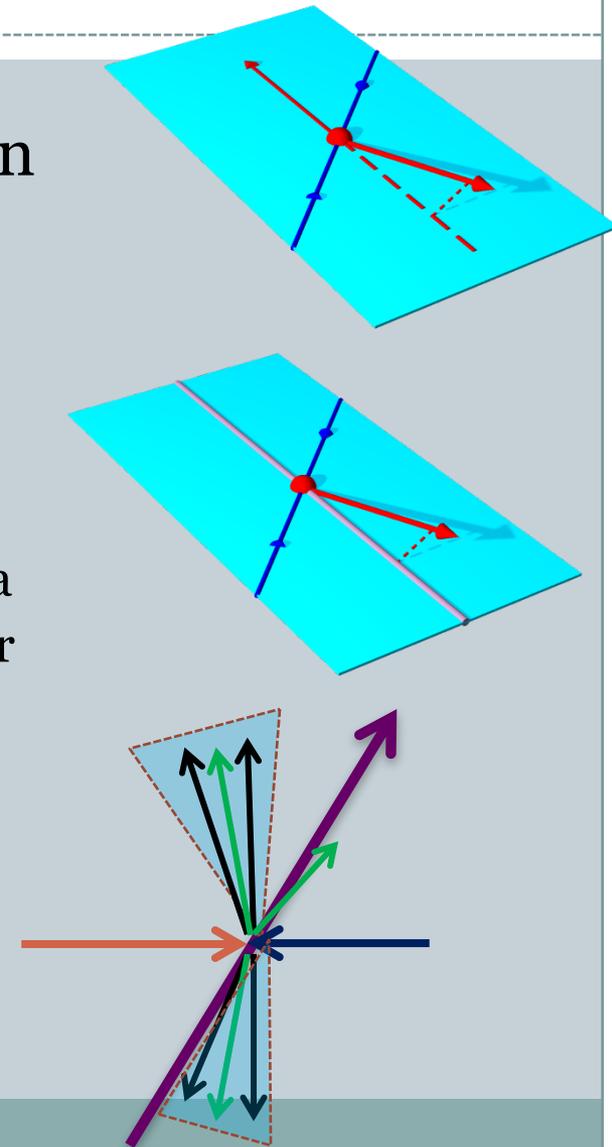
- $\cos\theta$  distribution in  $\Lambda$  signal region **a)** is corrected by sideband subtraction  $\rightarrow$  **b)**
- Normalized by itself, as shown in **c)**.
- The shape **c)** is divided by the corresponding shape from MC, so that we obtain the efficiency-corrected curve **d)**.
- Or **c)** shape of  $\Lambda$  events is divided by that from anti- $\Lambda$  events if we assume efficiency is independent on charge, that is **e)**, this is called data ratios.
- We fit **d)** and **e)** to get the polarization of interest.



# $k_T$ Dependence of FFs

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- Gain sensitivity into transverse momentum generated in fragmentation
- Two ways to obtain transverse momentum dependence
  - Traditional **2-hadron** FF
    - use transverse momentum between two hadrons (in opposite hemispheres)
    - Usual convolution of two transverse momenta
    - Analysis well underway: First step **shapes** for unidentified charged back-to-back  $\pi+K$
  - Single-hadron FF wrt to **Thrust** or jet axis
    - No convolution
    - Need correction for  $q\bar{q}$  axis
    - Deconvolution of  $q\bar{q}$  axis resolution in  $k_T$  needed



# QCD studies at Belle II

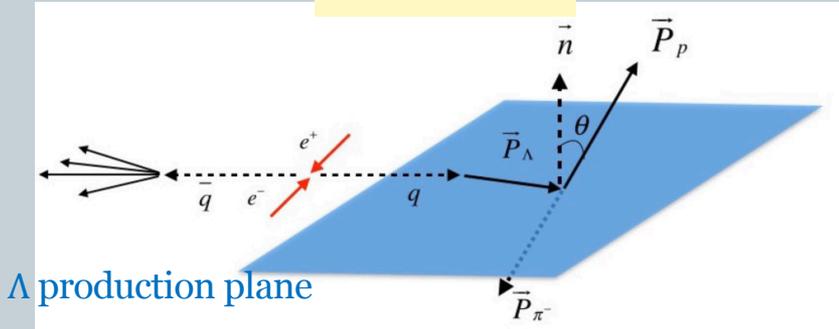


- Precision study of local strong parity violation to probe the QCD vacuum
- Hadronization studies in transverse momentum-spin correlations ( $\Lambda$ )/Fragmentation function
- Precision studies of fragmentation functions needed for JLab12 program
  - Precision
  - Charm suppression
  - Kaon ID

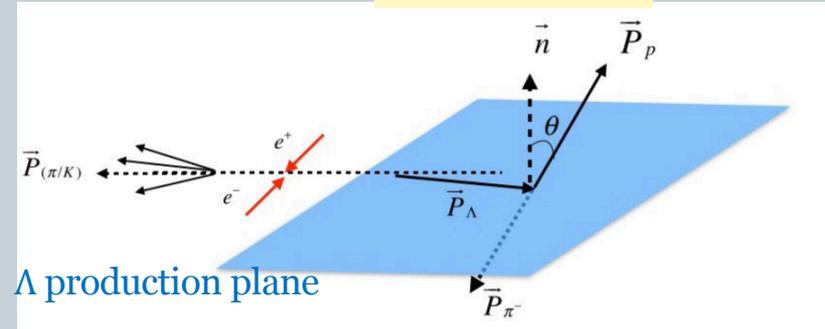
# Reference frames

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Thrust Frame



Hadron Frame



- The reference vector  $\hat{n}$  is perpendicular to the  $\Lambda$  production plane.
- The  $p_t$  is defined as the transverse momentum of  $\Lambda$  relative to thrust axis in thrust frame and to hadron axis in hadron frame
- Give a polarization of  $P$ , the yield of the events follow:

$$\frac{1}{N} \frac{dN}{d\cos\theta} = 1 + \alpha P \cos\theta$$

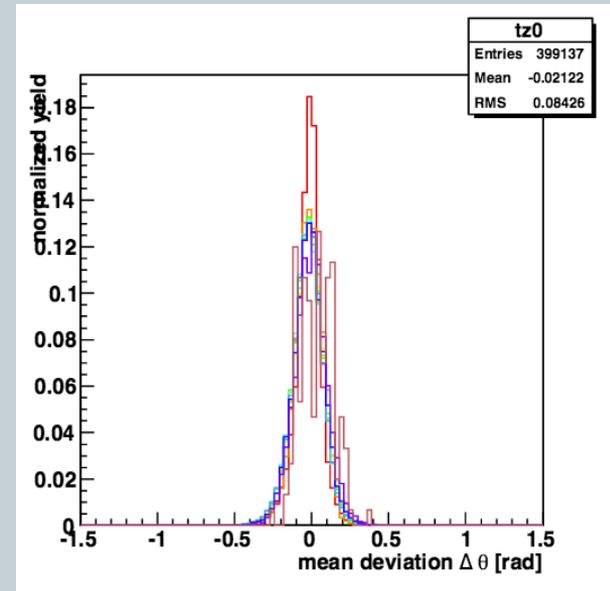
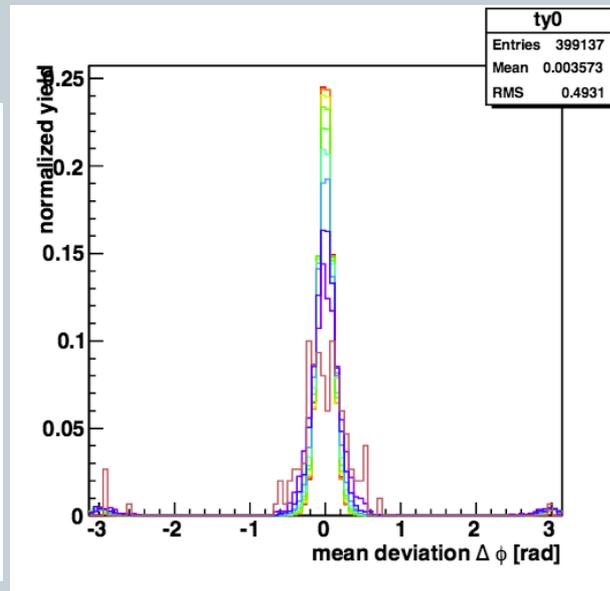
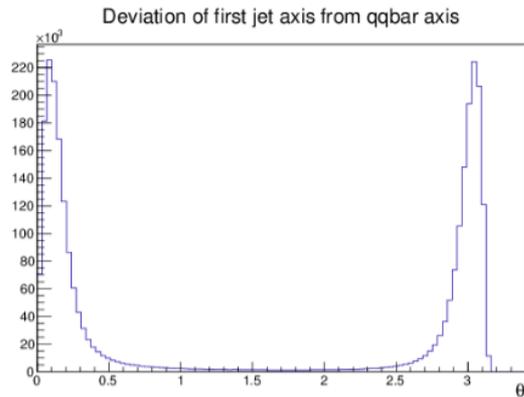
- where  $\alpha$  is the decay parameter:  $\alpha_+ = 0.642 \pm 0.013$  for  $\Lambda$  and  $\alpha_- = -0.71 \pm 0.08$  for  $\bar{\Lambda}$  (PDG).

## kinematic variables

$\Lambda + X$	thrust frame	
variables	$z_\Lambda, p_t$	
$\Lambda + h + X$	thrust frame	hadron frame
variables	$z_\Lambda, z_h, p_t$	$z_\Lambda, z_h, p_t$

# Jet vs Thrust vs q-q bar axis

- Non-negligible resolution effects
- In terms of  $k_T$  multiply by  $\sim z^* 0.5\sqrt{s}$ , e.g. for typical RMS of 0.1 rad RMS  $\sim 100$  MeV RMS for  $p_T$  (Similar as BaBar)

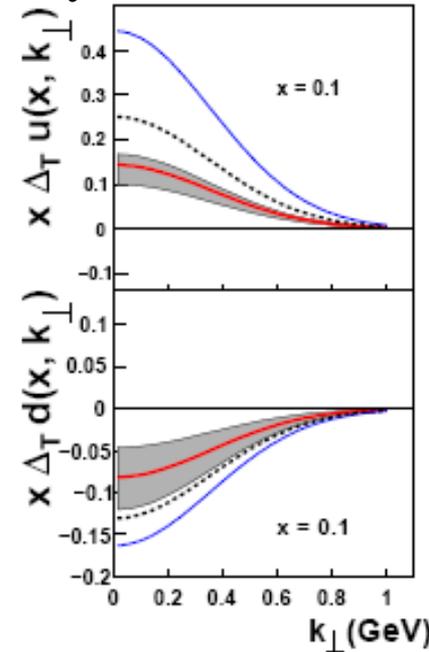
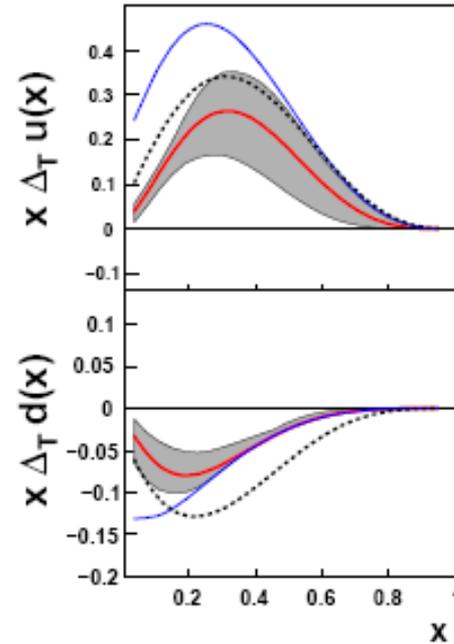
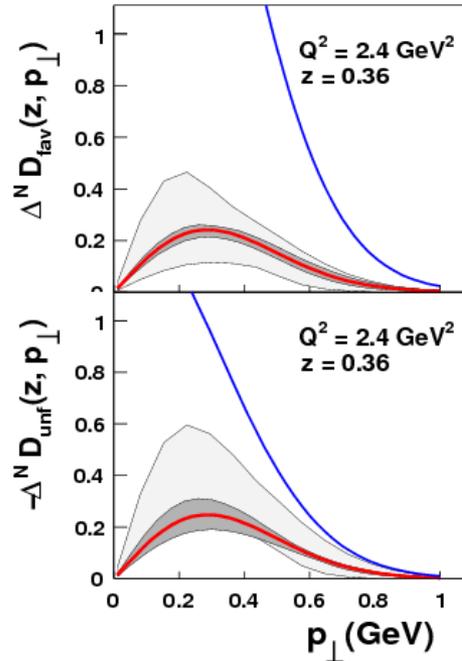
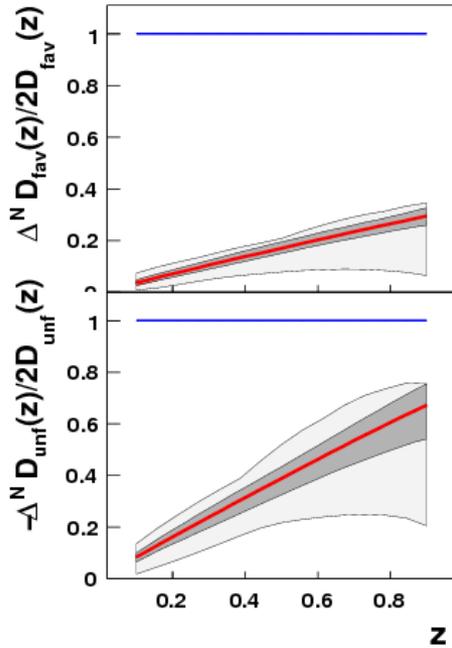


# Global Fit of Collins FF and Transversity (HERMES, COMPASS d, Belle)

Collins function



Transversity

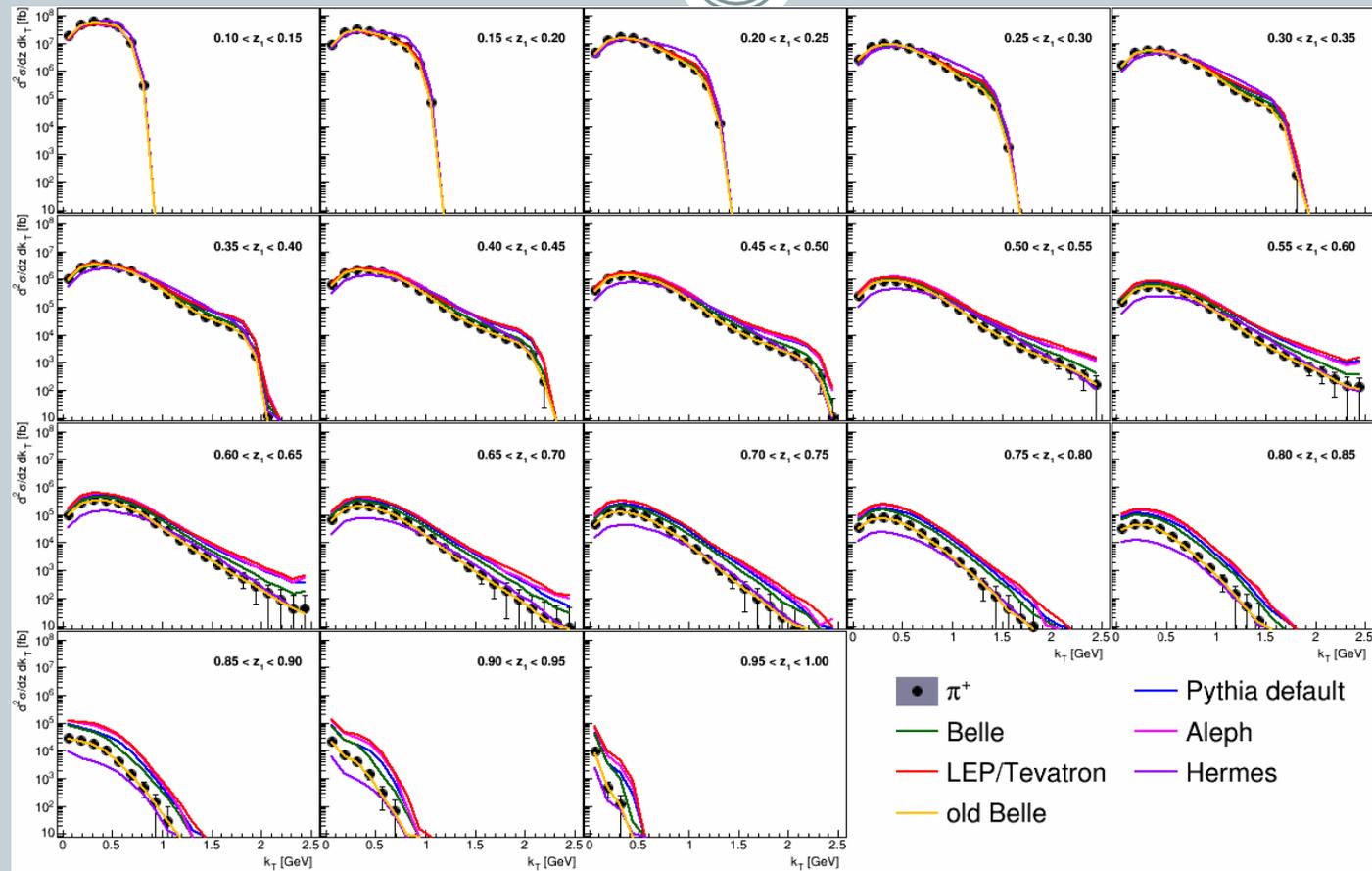


Phys.Rev.D75:054032,2007,  
update in  
Nucl.Phys.Proc.Suppl.191:98-  
107,2009

- Latest SIDIS data not included in FIT
- Open questions:
  - TMD evolution unknown (however from Belle to HERMES no large differences seen)
  - \$K\_t\$ dependence from Assumption (Belle measurements planned)
- Interference FF (IFF) as independent Cross check

# MC example of $k_T$ sensitivities

42

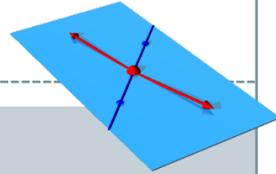


- Jet/Thrust smearing unfolding needs multidimensional unfolding
- Additional Uncertainties due to  $k_T$  description in MC

# Ratios to opposite charge pion pairs

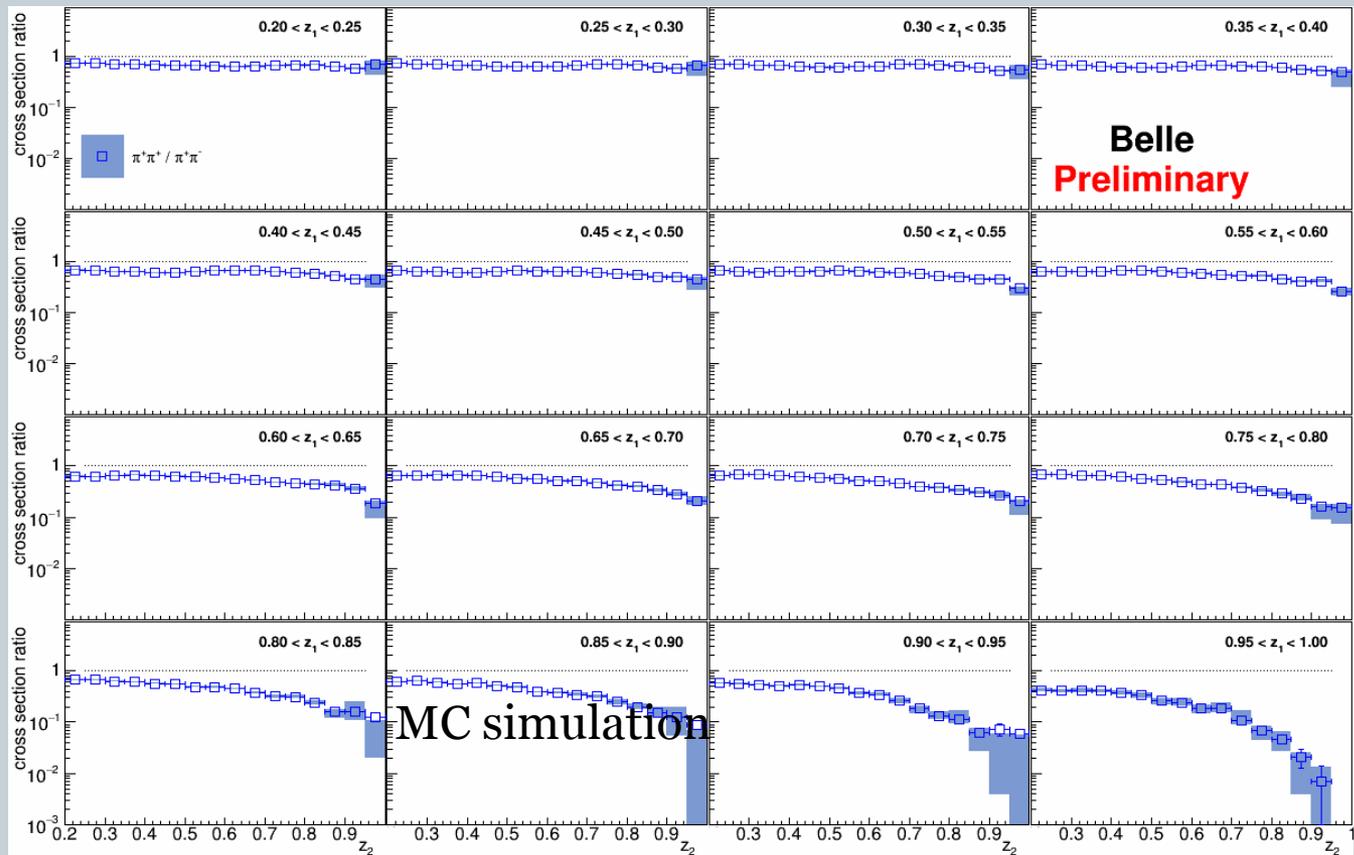
43

$$R \approx \frac{D_{fav}(z_1)D_{fav}(z_2) + D_{dis}(z_1)D_{dis}(z_2)}{D_{dis}(z_1)D_{fav}(z_2) + D_{fav}(z_1)D_{dis}(z_2)}$$



$\pi^+\pi^+$  comparable to  $\pi^+\pi^-$  at low  $z$ , decreasing towards high  $z$ :

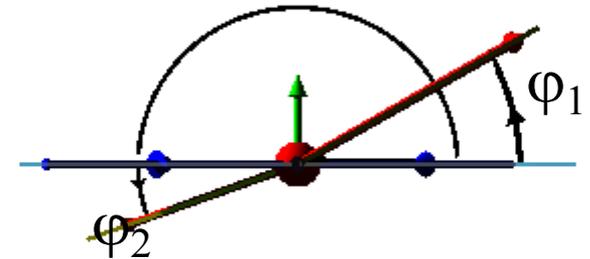
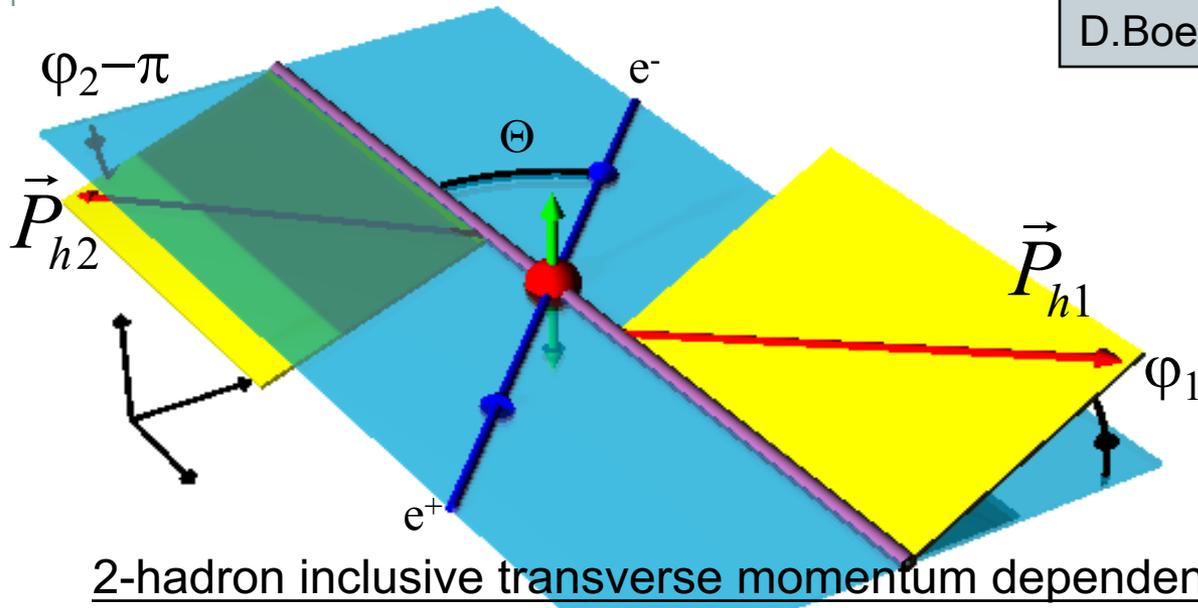
- Favored and disfavored fragmentation similar at low  $z$
- Disfavored much smaller at high  $z$



# Collins fragmentation in $e^+e^-$ : Angles and Cross section $\cos(\phi_1+\phi_2)$ method

$e^+e^-$  CMS frame:

D.Boer: Nucl.Phys. B806 (2009) 23-6



2-hadron inclusive transverse momentum dependent cross section:

$$\frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2q_T} = \dots B(y) \cos(\phi_1 + \phi_2) H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)$$

$$B(y) = y(1-y) \stackrel{\text{cm}}{=} \frac{1}{4} \sin^2 \Theta$$

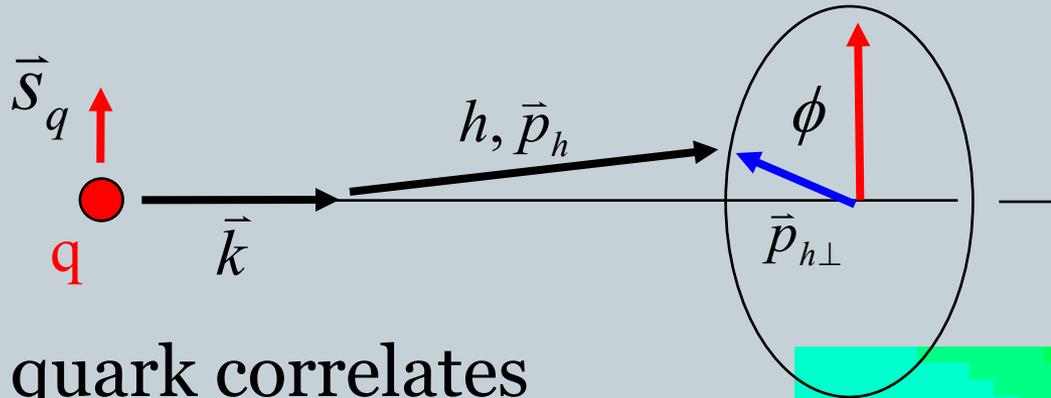
Net (anti-)alignment of  
transverse quark spins

# Collins fragmentation function

J. Collins, Nucl. Phys. B396, (1993) 161

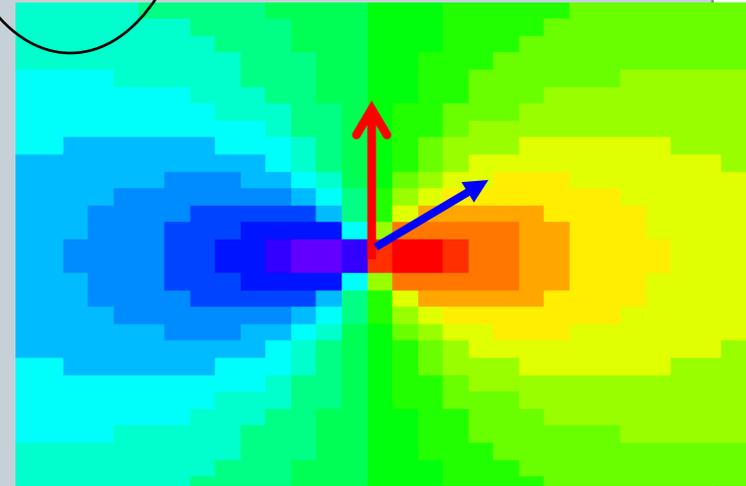
45

$$D_{q\uparrow}^h(z, P_{h\perp}) = D_{1,q}^h(z, P_{h\perp}^2) + H_{1,q}^{\perp h}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$



- Spin of quark correlates with hadron transverse momentum

→ translates into azimuthal anisotropy of final state hadrons



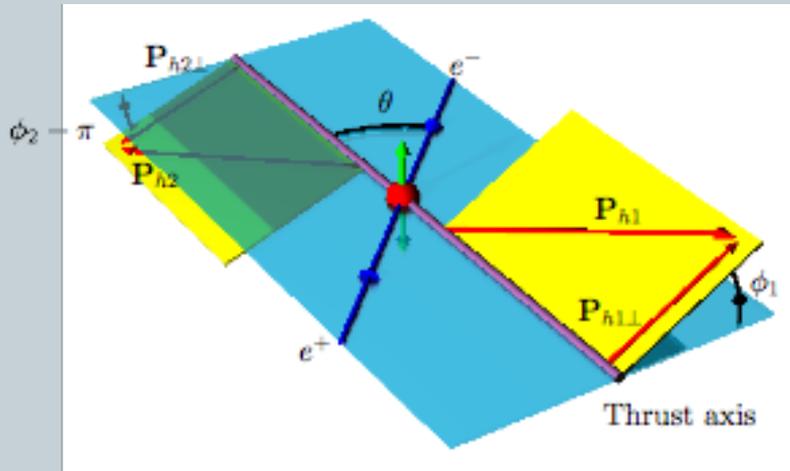
# There are two methods with two or one soft scale

46

D. Boer  
Nucl.Phys.B806:23,2009

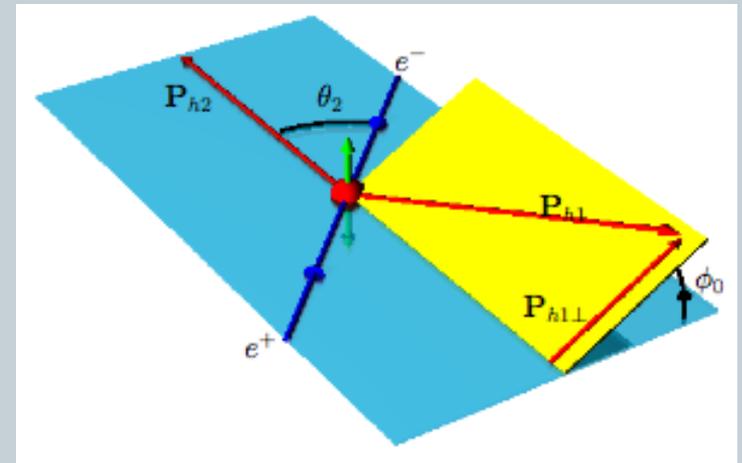
$\phi_1 + \phi_2$  method:

hadron azimuthal angles with respect to the  $q\bar{q}$  axis proxy



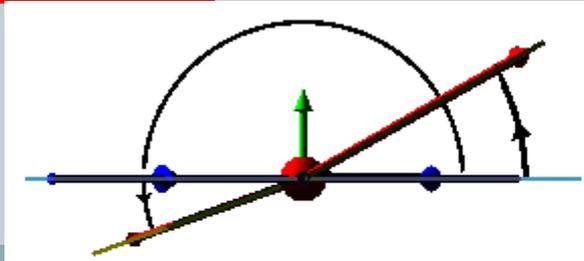
$\phi_0$  method:

hadron 1 azimuthal angle with respect to hadron 2



$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right) \quad \sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right)$$

$$R_{12}^{U/L} = \frac{N(\varphi_1 + \varphi_2)}{\langle N_{12} \rangle}$$



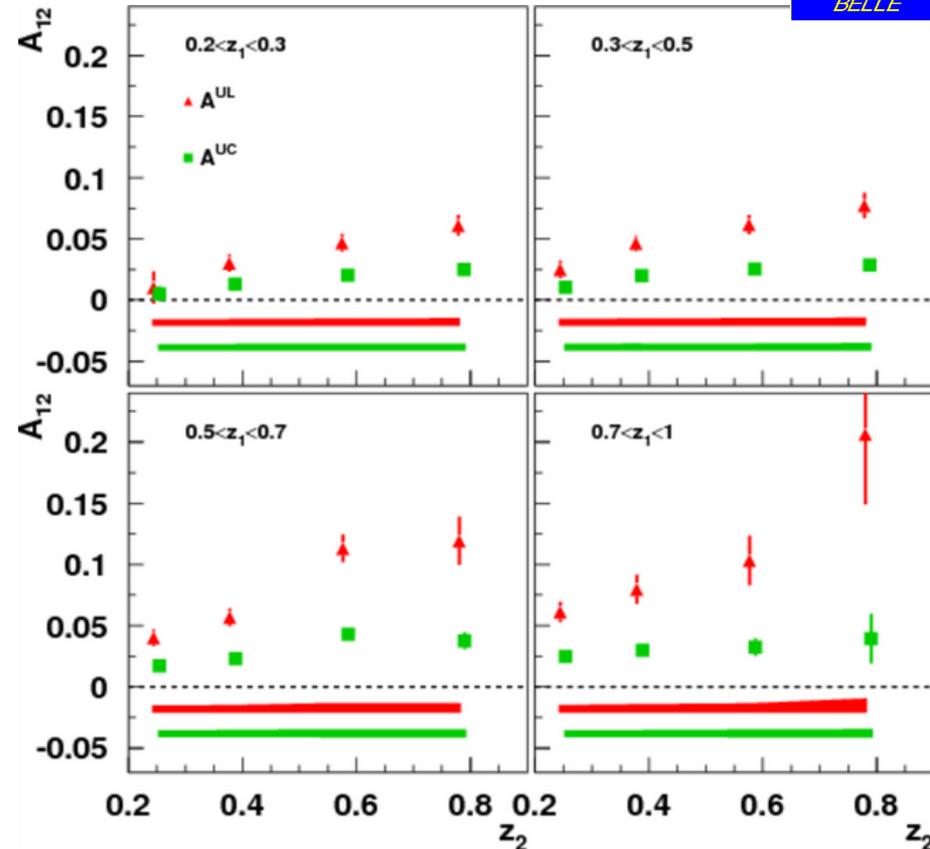
$$R_0^{U/L} = \frac{N(2\varphi_0)}{\langle N_0 \rangle}$$

kT moment or convolution: Important which kT region is sampled

# Belle Collins asymmetries



- **Red points** :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over **like** sign pion pair ratio :  $A^{UL}$
- **Green points** :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over any charged pion pair ratio :  $A^{UC}$
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF



RS et al (Belle), PRL96: 232002  
 PRD 78:032011, Erratum D86:039905