Fluctuation-dissipation dynamics in the early Universe

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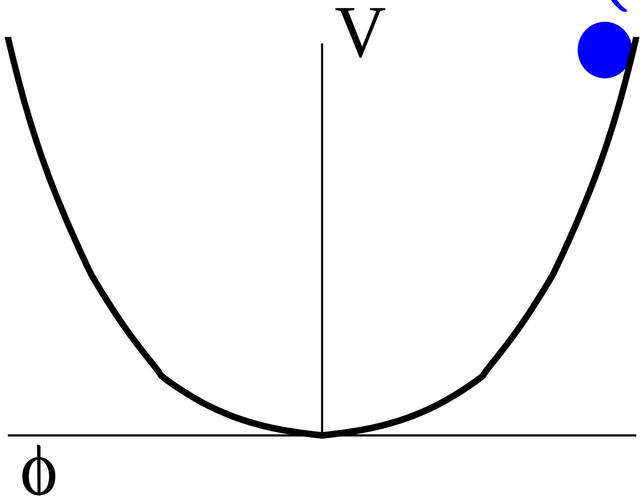
Overview

- Review warm inflation
- Density fluctuations at finite temperature
- Quantum field theory calculation - dissipation, radiative corrections, thermalization
- $\lambda\phi^4$ model warm inflation predictions compared to Planck results
- Other applications of fluctuation-dissipation dynamics in cosmology

Scalar field ("inflaton") dynamics



$$\rho = \frac{\dot{\phi}}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2R^2}$$

$$p = \frac{\dot{\phi}}{2} - V(\phi) - \frac{(\nabla\phi)^2}{6R^2}$$

- Cold inflation: Just Choose $V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Potential energy dominated $3H\dot{\phi} \gg \ddot{\phi}$, "slow-roll"

- Warm Inflation: Υ dominates

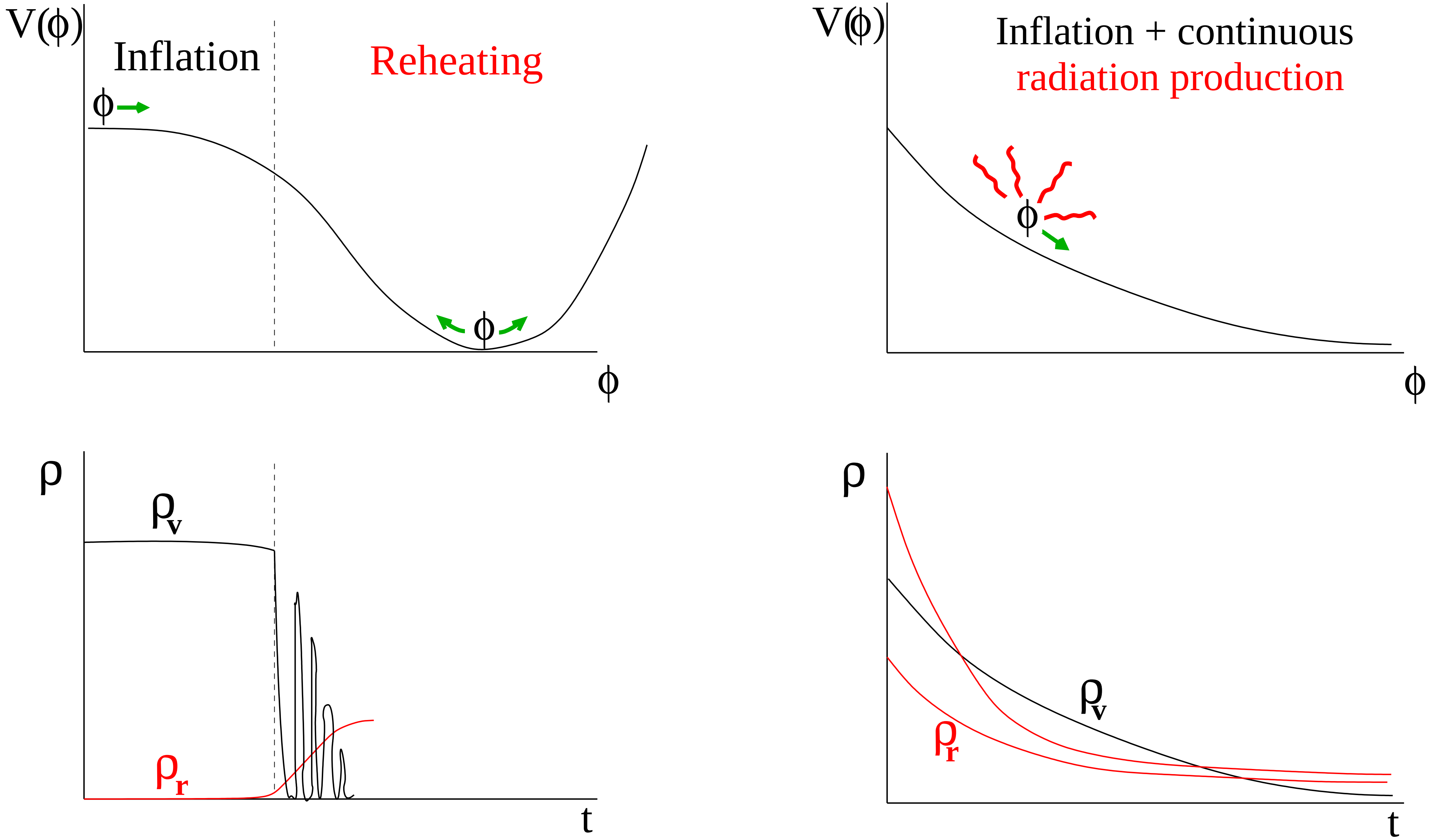
$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + V'(\phi) = 0$$

Slow-roll now means $\Upsilon\dot{\phi} \gg 3H\dot{\phi}, \ddot{\phi}$, overdamped

Two basic inflation pictures

Cold Inflation

Warm Inflation



Energetics

- Consider GUT scale inflation: $M \sim 10^{15}$GeV

$$\Rightarrow \qquad \rho_v \sim M^4 \sim 10^{60}\text{GeV}^4 \quad \Rightarrow \quad \text{H} \sim 10^{10}\text{GeV}$$

  T $>$ H requires > 1 part in $\sim 10^{20}$ of $\rho_v \to \rho_r$
  (influences structure formation)
  T $>$ 1GeV requires > 1 part in $\sim 10^{60}$ of $\rho_v \to \rho_r$
  (makes reheating unnecessary)

- Inflation pictures
  1. Cold inflation: basic assumption is no dissipation
  2. Warm inflation: radiation production inherent [AB, (1995)]

- Theoretical consideration
  Equipartition Hypothesis of Statistical Mechanics: *Scalar field should distribute its energy evenly amongst all degrees of freedom.*

  Dynamical Question: *Will relevant time scales during inflation prohibit the minute' radiation production given above?*

Nature of the fluctuations

Thermal/quantum fluctuations leave "ripples" in early universe

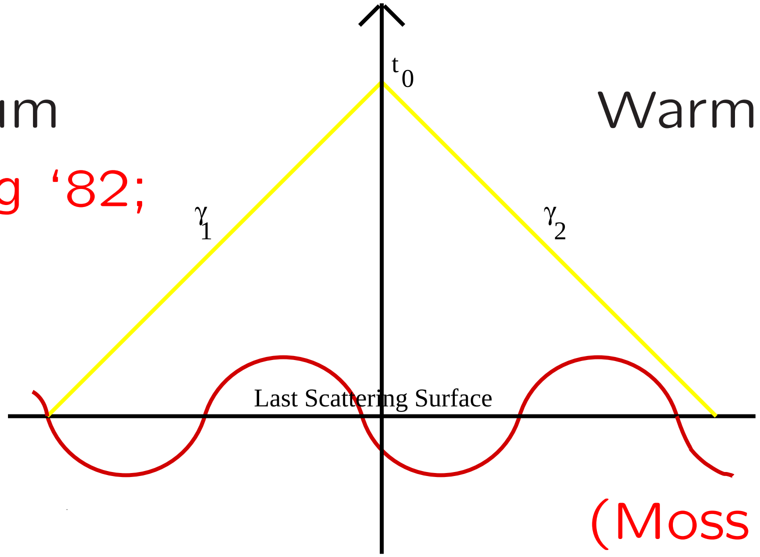
- imprinted on CMB at Last Scattering
- amplified by gravity to make structure

Cold inflation: Quantum

(Guth, Pi '82; Hawking '82;

Starobinsky '82;

Bardeen *et. al.* '83)



Warm Inflation: Thermal

$$\delta\phi = \frac{k_F^{1/2} T^{1/2}}{2\pi}$$

$$\Upsilon < 3H : k_F = H$$

(Moss '85, AB,Fang '95)

$$\delta\phi = \frac{H}{2\pi}$$

$$\Upsilon > 3H : k_F = \sqrt{\Upsilon H}$$

(AB '99)

$$\frac{\delta\rho}{\rho} = \frac{H\delta\phi}{\dot{\phi}}\Bigg|_{\text{HORIZON EXIT}}$$

Scalar field (Φ) QFT dynamics

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi)$$

$$\mathcal{L}_I = -\Phi \sum_{j=1}^{N_\psi} h_j \bar{\psi}_j \psi_j, \; -\frac{1}{2}\sum_{j=1}^{N_\chi} g_j^2 \Phi^2 \chi_j^2 + \cdots$$

- Decompose into background φ and fluctuations ϕ: $\Phi = \varphi + \phi$
- Equation of motion for $\varphi(t)$ from quantum effective action:

$$\frac{\delta \Gamma}{\delta \varphi} = \ddot{\varphi}(t) + 3H\dot{\varphi}(t) + \frac{\partial V}{\partial \varphi} + \int d^4x' \Sigma_R(x - x')\Delta\varphi = 0$$

Adiabatic evolution:

$$\Delta\varphi = \dot{\varphi}(t - t') + \cdots$$

Yielding a friction term at leading order

Closed Time Path approach - Goal

Compute Observable

$$\langle \hat{O}(t) \rangle \equiv \frac{\mathrm{Tr}(\hat{\rho}(t)\hat{O})}{\mathrm{Tr}(\hat{\rho}(t))}$$

Thermal initial state at $T^<$:

$$\rho(T^<) = \exp(-\beta H) = U(T^< - i\beta, T^<)$$

($U(t,t') \equiv \exp[-iH(t-t')]$, time evolution operator)

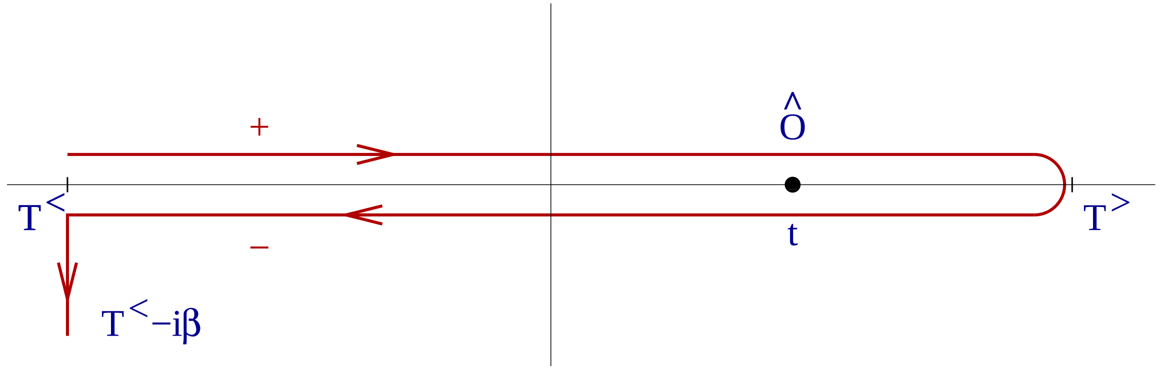
Thus

$$\langle \hat{O}(t) \rangle = \frac{\mathrm{Tr}[U(T^< - i\beta, T^<)\, U(T^<, t)\, \hat{O}\, U(t, T^<)]}{\mathrm{Tr}[U(T^< - i\beta, T^<)]}$$

Also add large positive time $T^>$

$$\langle \hat{O}(t) \rangle = \frac{\mathrm{Tr}[U(T^< - i\beta, T^<)\, U(T^<, T^>)\, U(T^>, t)\, \hat{O}\, U(t, T^<)]}{\mathrm{Tr}[U(T^< - i\beta, T^<)]}$$

Closed Time Path approach - Method



Can express as a path integral

recall $U(t,t') \equiv \int \mathcal{D}\Phi \, \mathrm{exp}\left(i \int_{t'}^{t} d^4x \mathcal{L}[\Phi] \right)$

$$Z[J^+, J^-, J^\beta] \;=\; \mathrm{Tr}[U(T^< - i\beta, T^<; J^\beta) U(T^<, T^>; J^-) U(T^>, T^<; J^+)]$$

$$= \int \mathcal{D}\Phi^+ \mathcal{D}\Phi^- \mathcal{D}\Phi^\beta \, \mathrm{exp}\left(i \int_{T^<}^{T^>} d^4x [\mathcal{L}^{J^+}[\Phi^+] - \mathcal{L}^{J^-}[\Phi^-]] + i \int_{T^<}^{T^< - i\beta} d^4x \mathcal{L}^{J^\beta}[\Phi^\beta] \right)$$

e.g. scalar field theory:

$$\mathcal{L}^J[\Phi] = \frac{1}{2}[\partial_\mu \Phi \partial^\mu \Phi - m^2\Phi^2] - \frac{\lambda}{4!}\Phi^4 + J\Phi$$

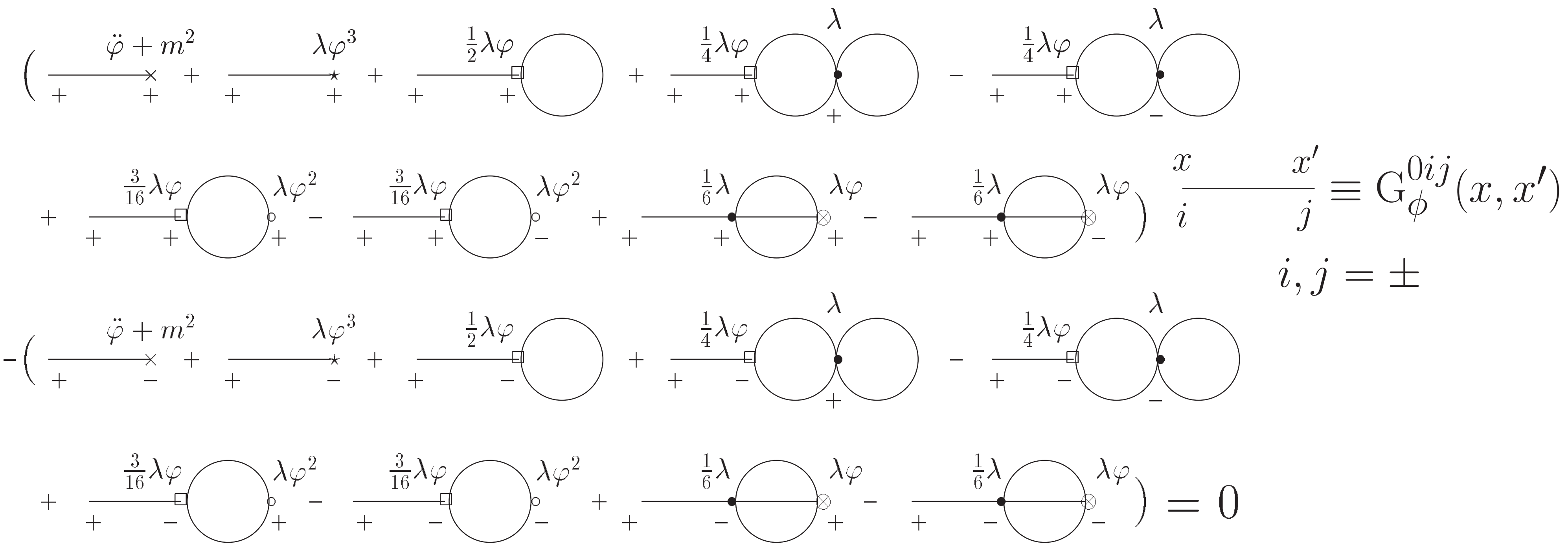
How to obtain the φ-effective EOM

Tadpole Method: demand $\langle\phi\rangle = 0$ and compute

Example: $S = \int d^4x \left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4\right]$, $\quad \Phi = \varphi + \phi \Longrightarrow$

$$S = \int d^4x[-\varphi\frac{1}{2}[\Box + m^2]\varphi - \phi\frac{1}{2}[\Box + m^2]\phi - \phi[\Box + m^2]\varphi - \frac{\lambda}{4!}(\varphi^4 + 4\varphi^3\phi + 6\varphi^2\phi^2 + 4\varphi\phi^3 + \phi^4)]$$

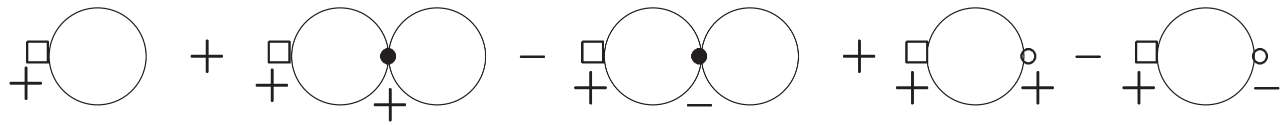
$\langle\phi\rangle = 0$ at lowest nontrivial order $\Longrightarrow$



Tadpole Method (cont.) - Explicit EOM

( ) $= 0 \Longrightarrow$

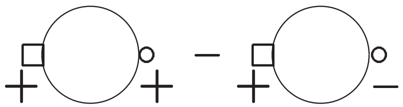
$0 = \int d^4x' G_\phi^{++}(x,x')$ [$\times$ + $\star$ +



$+ \ldots$]

$\Longrightarrow$ [ ] $= 0$, now convert to analytic expression

e.g.



$$= \tfrac{3}{16}\lambda^2\varphi(t') \int d^4y [G_\phi^{++}(x',y)G_\phi^{++}(x',y) - G_\phi^{+-}(x',y)G_\phi^{+-}(x',y)]\varphi^2(t_y)$$

$$= \tfrac{3}{8}\lambda^2\varphi(t') \int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi^2(t_y), \quad \text{etc} \ldots$$

Final Expression:

$$\ddot{\varphi} \;+\; 3H\dot{\varphi} + m^2\varphi + \frac{\lambda}{6}\varphi^3 + \overset{local\ \ 1-loop\ \ V_{eff}}{terms} + \frac{3}{8}\lambda^2\varphi(t')\int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi^2(t_y)$$

$$+ \;\; \frac{1}{3}\lambda^2 \int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi(t_y) = 0$$

Green's functions relations

(AB, Moss and Ramos, PRD 2007)

One-loop effective equation of motion:

$$\left[\Box + M_\phi^2\right]\varphi_c(x) + \frac{\lambda}{3!}\varphi_c^3(x) - \frac{\varphi_c(x)}{2}\int d^3x'\varphi_c^2(\mathbf{x}',t)\mathcal{D}_1(\mathbf{x}-\mathbf{x}',0) - \int d^3x'\varphi_c(\mathbf{x}',t)\mathcal{D}_2(\mathbf{x}-\mathbf{x}',0)$$
$$+\varphi_c(x)\int d^3x'\int_{-\infty}^{t} dt'\varphi_c(\mathbf{x}',t')\dot{\varphi}_c(\mathbf{x}',t')\mathcal{D}_1(\mathbf{x}-\mathbf{x}',t-t') + \int d^3x'\int_{-\infty}^{t} dt'\dot{\varphi}_c(\mathbf{x}',t')\mathcal{D}_2(\mathbf{x}-\mathbf{x}',t-t')$$
$$= \varphi_c(x)\xi_1(x) + \xi_2(x)$$

where defining $\mathcal{C}_i(\mathbf{x}-\mathbf{x}',t-t') = -\frac{\partial}{\partial t'}\mathcal{D}_i(\mathbf{x}-\mathbf{x}',t-t')$

$$\mathcal{C}_1(\mathbf{x}-\mathbf{x}',t-t') = \lambda^2 \mathrm{Im}\left[G_\phi^{++}(x,x')\right]^2 \mathrm{sgn}(t-t') + 4g^4 \mathrm{Im}\left[G_{\chi_j}^{++}(x,x')\right]^2 \mathrm{sgn}(t-t')$$

$$\mathcal{C}_2(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{3}\mathrm{Im}\left[G_\phi^{++}(x,x')\right]^3 \mathrm{sgn}(t-t') + 4g^4 \mathrm{Im}\left[G_\chi^{++}(x,x')G_\phi^{++}(x,x')G_\chi^{++}(x,x')\right]\mathrm{sgn}(t-t')$$

$$\langle\xi_1(x)\xi_1(x')\rangle \equiv \mathcal{N}_1(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{2}\mathrm{Re}\left[G_\phi^{++}(x,x')\right]^2 + 2g^4\mathrm{Re}\left[G_\chi^{++}(x,x')\right]^2$$

$$\langle\xi_2(x)\xi_2(x')\rangle \equiv \mathcal{N}_2(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{6}\mathrm{Re}\left[G_\phi^{++}(x,x')\right]^3 + 2g^4\mathrm{Re}\left[G_\chi^{++}(x,x')^2 G_\phi^{++}(x,x')\right]$$

Fluctuation dissipation theorem - relation

Relation between real and imaginary parts of Green's function leads to generalized fluctuation-dissipation theorem (in Fourier space):

$$\tilde{\mathcal{N}}_1(\mathbf{p},\omega) = 2\omega\left[n(\omega)+\frac{1}{2}\right]\tilde{\Gamma}_1(\mathbf{p},\omega)$$

where $\tilde{\mathcal{D}}_i(\mathbf{p},\omega) = 2\tilde{\Gamma}_i(\mathbf{p},\omega)$

Relates dissipation and noise

In local form, i.e. when $\mathcal{N}(t-t') \equiv \mathcal{N}_0\delta(t-t')$, get familiar expression:

$$N_0 = \tilde{\mathcal{N}}(0) = 2T\tilde{\Gamma}(0)$$

Warm inflation

[AB PRL 75, 1995]

Stochastic Evolution equation includes dissipation and noise (AB and Fang, PRL74, 1912 (1995)):

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3\mathrm{H}\dot{\phi} + \Upsilon\dot{\phi} + \mathrm{V}'(\phi) = \xi$$

Dissipation term leads to radiation production during inflation,

$$\dot{\rho}_r = -4H\rho_r + \Upsilon\dot{\phi}^2$$

Density perturbations:

$$R(k) = \frac{H}{\dot{\phi}}\delta\phi, \quad \delta\phi = \frac{k_F^{1/2}T^{1/2}}{2\pi}$$

Strong dissipative regime:

$$\Upsilon > 3H,\ T > H,\ k_F = \sqrt{\Upsilon H}$$

Weak dissipative regime

$$\Upsilon < 3H,\ T > H,\ k_F = \sqrt{\Upsilon H}(\mathrm{FD}),\ H(\mathrm{thermal})$$

First principles QFT models of warm inflation

Distributed mass model (hep-ph/9809583, PRL 1999) - high-T dissipation, $\mathcal{L}_{int} = -\sum_j g^2(\phi - M_j)^2\chi_j^2$, requires $\sim 10^6$ fields

Monomial models (arXiv:1307.5868) - Low-T dissipation, $\mathcal{L}_{int} = -g^2\phi^2\chi^2 + hM\chi\sigma^2$, requires $\sim 10^6$ fields

Warm little inflaton (arXiv:1604.08838, PRL 2016) - High-T dissipation, $\mathcal{L}_{int} = gM\cos(\phi/M)\overline{\psi}_1\psi_1 + gM\sin(\phi/M)\overline{\psi}_2\psi_2$, requires only 5 fields, very attractive for model building

Yokoyama and Linde (1998) on warm inflation, "...extremely difficult perhaps even impossible..."

This model dispels all doubts on realizing warm inflation from quantum field theory

Inflaton mass

Coupling of inflaton induces χ mass term:

$$m_\chi = g\varphi$$

Thermal corrections to inflaton mass:

$$m_\phi^2 = \begin{cases} g^2 T^2 \,, & m_\chi \ll T \\ g^2 e^{-m_\chi/T} \,, & m_\chi \gtrsim T \end{cases}$$

Problems if inflaton directly coupled to radiation. Can solve if pseudo Nambu-Goldstone boson of a broken gauge symmetry (Warm Little Inflaton)

[AB, Gleiser, Ramos (1998); Yoyoyama,Linde (1998)]

Two-stage dissipation

SUSY protects inflaton potential from radiative corrections

Superpotential:

$$W = F(\Phi) + \frac{g}{2}\Phi X^2 + \frac{h}{2} X Y^2$$

Heavy fields decay into light degrees of freedom [AB,Ramos (2003)]

$$\mathcal{L} = g^2\phi^2\chi^2 + hM\chi\sigma^2$$

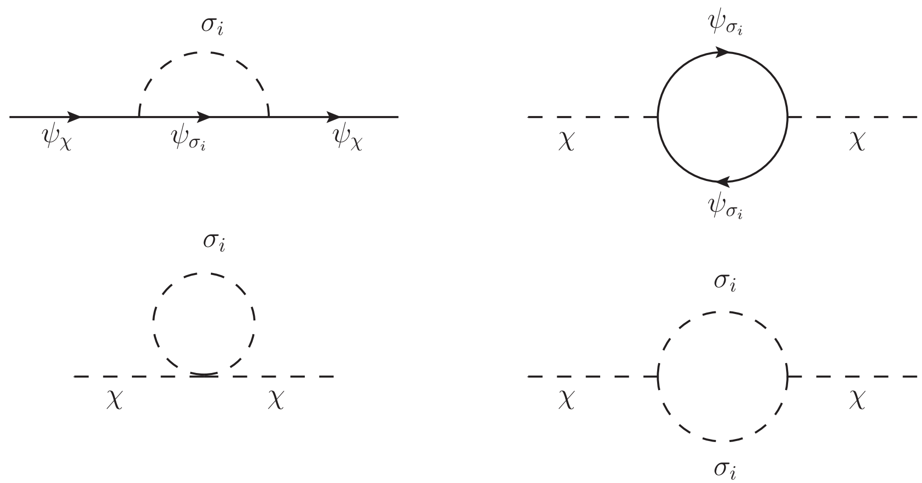
Dissipation produces radiation bath (σ)

Heavy (χ)-fields protect inflaton potential from radiative corrections

Inflaton effective potential

SUSY broken at finite energy density and temperature

[Hall,Moss (2005), Bastero-Gil,AB,Ramos,Rosa (2013)]



$$m_\chi^2 = g^2\varphi^2 \pm \frac{g}{2}\sqrt{V(\varphi)} + \frac{h^2 N_y}{8} T^2$$

$$m_{\psi_\chi}^2 = g^2\varphi^2 + \frac{h^2 N_y}{8} T^2$$

Flatness of potential is protected:

$$V_1(\varphi, T) = \frac{g^2 N_\chi}{32\pi^2} V(\varphi) \log\left(\frac{m_\chi^2}{\mu^2}\right)$$

small radiative corrections for $g^2 N_\chi \lesssim 10$ and $h^2 N_y \lesssim 1$

Dissipation coefficient

One-loop correction to inflaton self-energy:

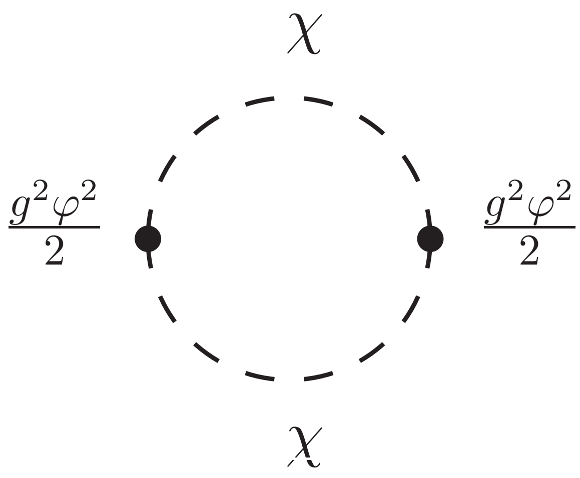
[Moss,Xiong (2008); Bastero-Gil,AB,Ramos (2011)]

$$\Upsilon = \frac{4}{T} g^4 \varphi^2 \int \frac{d^4p}{(2\pi)^4} \rho_\chi^2 n_B (1 + n_B)$$

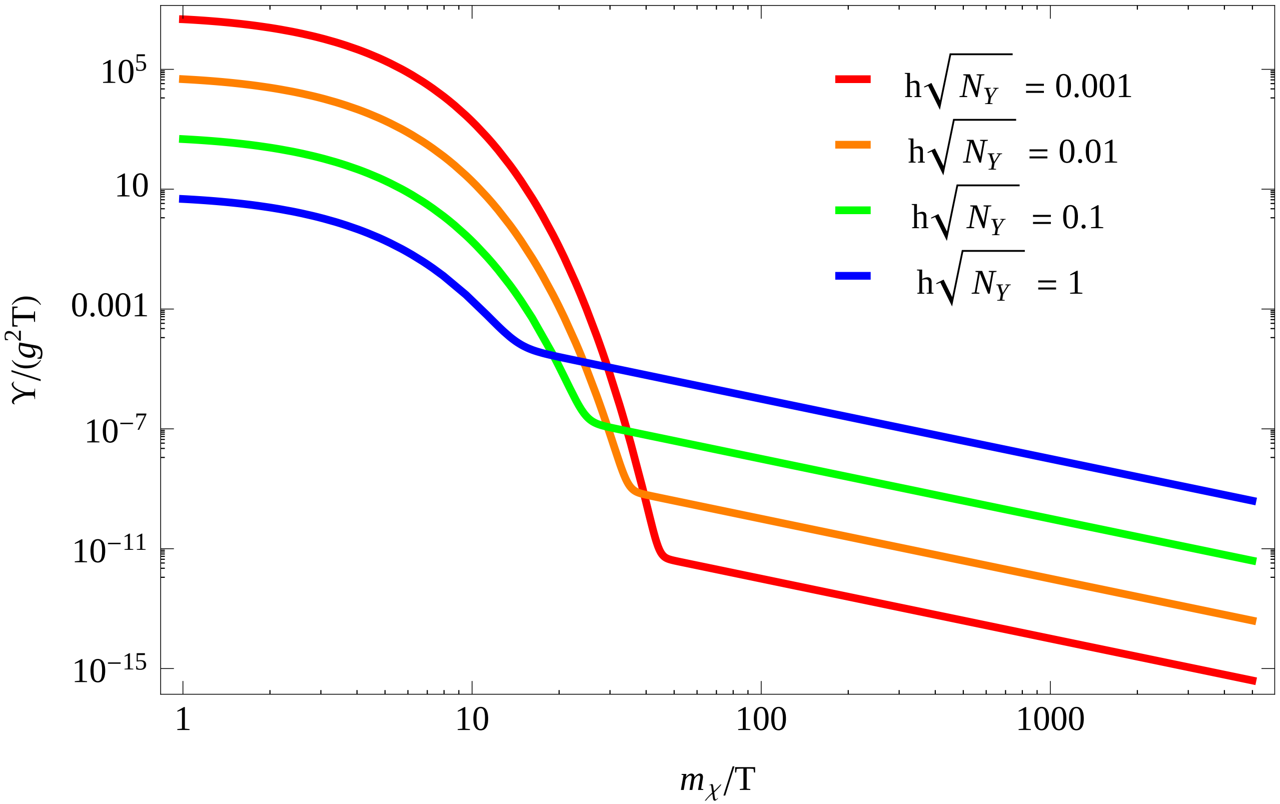
Spectral function (Briet-Wigner):

$$\rho_\chi = \frac{4\omega_p \Gamma_\chi}{(p_0^2 - \omega_p^2)^2 + 4\omega_p^2 \Gamma_\chi^2}$$

where $\omega_p = \sqrt{p^2 + m_\chi^2}$



Dissipative coefficient



$$\Upsilon = 0.02 h^2 N_y N_\chi \frac{T^3}{\varphi^2} + \frac{32}{\sqrt{2\pi}} \frac{g^2 N_\chi}{h^2 N_y} \sqrt{m_\chi T} e^{-m_\chi/T}$$

$(g^2 N_\chi \lesssim 10,\ h^2 N_y \lesssim 1)$

Virtual modes (low-momentum) + on-shell modes (pole)

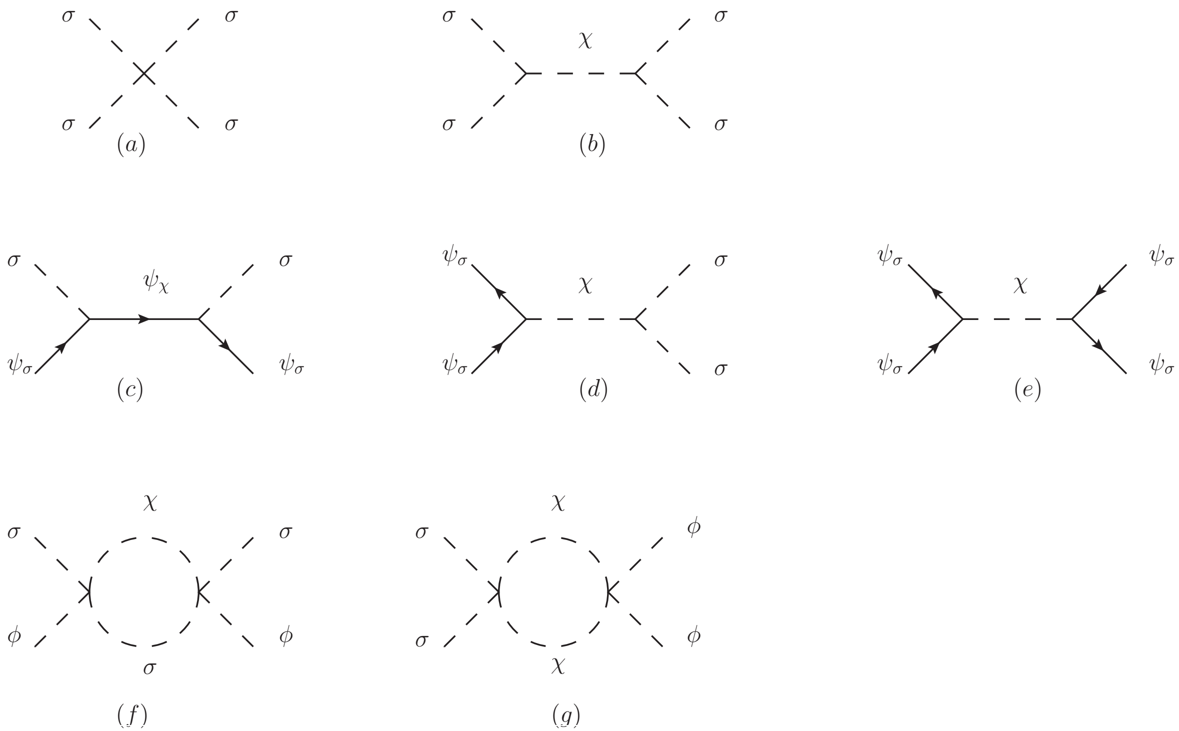
Thermalization

[Bastero-Gil,AB,Ramos,Rosa (2013)]

Decay rates $\Gamma_i > \ H, \ \frac{\dot{\varphi}}{\varphi}$:

$$\chi \longrightarrow \sigma + \sigma, \ \psi_\sigma + \psi_\sigma, \ \sigma + \sigma + \phi$$
$$\chi_R \longrightarrow \chi_I + \phi_I, \ \psi_\chi \psi_\phi$$

Scattering rates $n\sigma_i|v| > H$:



Warm inflation dynamics

Superpotential: $W = W(\Phi) + g\Phi X^2 + hXY^2$

Dissipative coefficient:

$$\Upsilon \approx C_\phi \frac{T^3}{\phi^2}$$

$$C_\phi \equiv 0.02 h^2 N_\chi N_y$$

Slow-roll:

$$\dot{\phi} \approx -\frac{V_\phi}{[3H(1+Q)]} \qquad Q \equiv \Upsilon/(3H)$$

$$4\rho_R \approx 3Q\dot{\phi}^2 \qquad \rho_R = \pi^2 g_* T^4/30$$

$$\epsilon_\phi, \eta_\phi < 1 + Q$$

$Q < (>) 1$ - weak (strong) dissipative regime

Observational consequences

Adiabatic scalar perturbations from thermal fluctuations [Bartrum et al. (2013)]:

$$P_{\mathcal{R}} = \left(\frac{H_*}{\dot{\varphi}_*}\right)^2 \left(\frac{H_*}{2\pi}\right)^2 \left(1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3+4\pi Q_*}}\frac{T_*}{H_*}\right) .$$

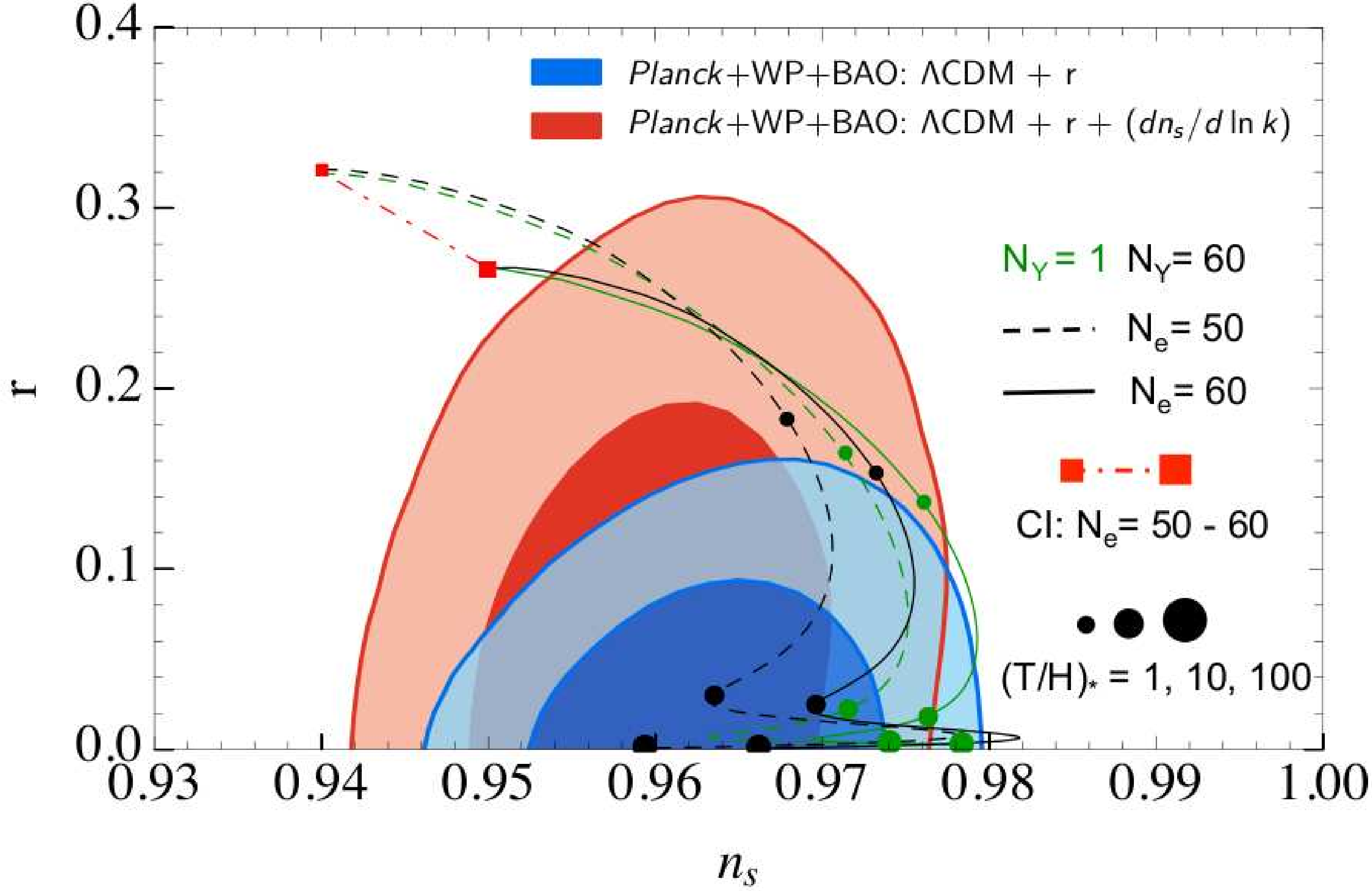
Tensor-scalar ratio [Bartrum,AB,Rosa (2013)]:

$$r = \frac{16\epsilon_\phi}{(1+Q)^{5/2}}\frac{H}{T} < 8|n_t|$$

Presence of radiation during inflation lowers tensor-scalar ratio [Bastero-Gil + AB (2009)]

$\lambda\phi^4$ potential - weak dissipation

[Bartrum. *et al* (2013)]



$\lambda \sim 10^{-14}, \quad 10^{-13} \lesssim g^4 N_\chi \sim 10^{-6}$

Non-gaussianity

[Bastero-Gil, *et al* (2014)]

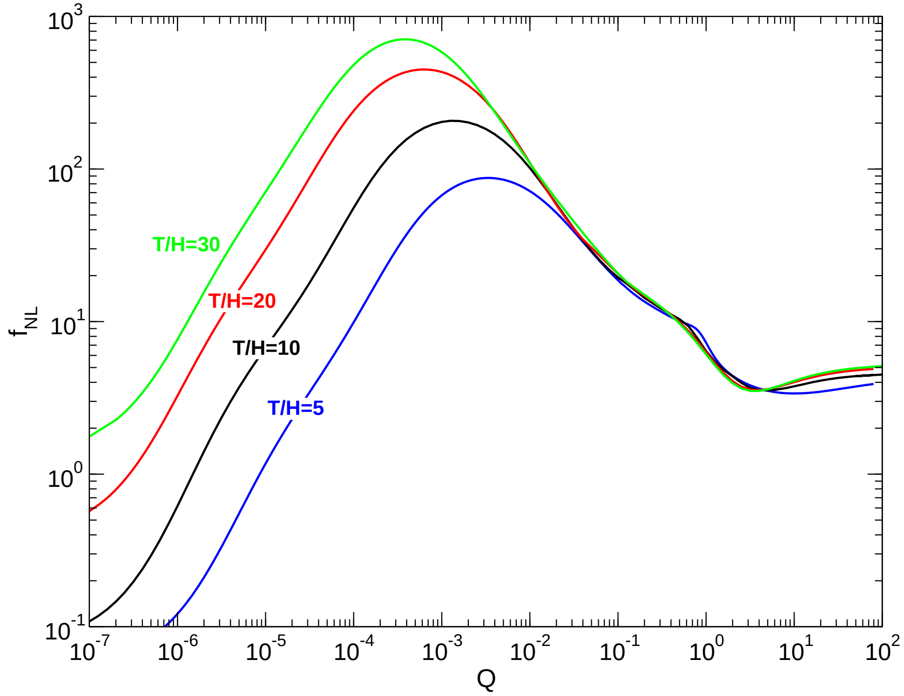
Bispectrum: $B(k_1, k_2, k_3)\delta(\Sigma k) = \sum_{cyc}\langle\Phi_1(k_1, t_f)\Phi_1(k_2, t_f)\Phi_2(k_3, t_f)\rangle$

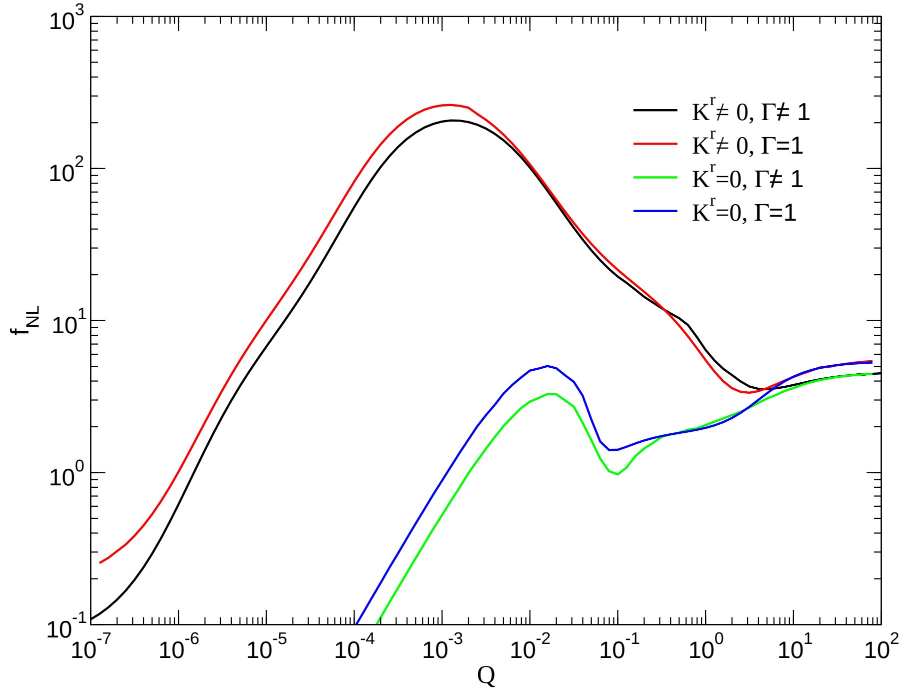
$\Phi_i(k,t)$ gauge invariant variable of order i.

Non-linearity parameter: $f_{NL} = 18B(k,k,k)/5P(k)^2$

Planck bounds: $f_{NL}^{local} = 2.7 \pm 5.8$ $\quad f_{NL}^{equi} = -42 \pm 75$ $\quad f_{NL}^{S} = 4 \pm 33.$

Warm inflation bispectrum shape close to equilateral form





Warm inflation summary

- Dissipative effects and temperature during inflation alter tensor-scalar ratio and tilt
- QFT solutions are perturbative and can require very few fields
- Models exist with large GUTs, branes, extra-dimensions
- In strong dissipative regime there is no η-problem, thus removing a tuning from cold inflation models
- Bulk and shear viscosity affect the radiation bath and density perturbations [Bastero-Gil *et al.* (2014)]
- Control gravition over-abundance [Sanchez *et al.* (2011); Bartrum *et al* (2013)]
- Baryogenesis [Bastero-Gil,AB,Ramos,Rosa (2012)]
- Perturbations always classical, no quantum-to-classical transition issues (possible test Maldacena 2016)

Coupled cosmological stochastic equations

[Bastero-Gil,AB,Moss,Ramos (2014)]

Scalar field + radiation equations with dissipation, viscosity and associated stochastic forces:

$$-\nabla^2\phi(x,t) + \Upsilon\dot{\phi}(x,t) + \Omega_{,\phi} = (2\Upsilon T)^{1/2}\xi^{(\phi)}(x,t)$$

$$\delta\dot{\rho}^{(f)} + (\rho^{(f)} + p^{(f)})\,\nabla\cdot\delta\mathbf{u}^{(\mathbf{f})} + \mathbf{s}_{,\phi}\delta\mathbf{q} = -\delta\mathbf{Q}^{(\phi)}$$

$$\{(\rho^{(f)} + p^{(f)})\,\delta\mathbf{u}^{(f)}\}^{\cdot} + \nabla\delta\mathbf{p}^{(\mathbf{f})} - \eta\nabla^{\mathbf{2}}\delta\mathbf{u}^{(\mathbf{f})} - \left(\zeta + \frac{1}{3}\right)\eta\nabla\nabla\cdot\delta\mathbf{u}^{(\mathbf{f})} = -\delta\mathbf{Q}^{(\phi)} + \nabla\cdot\mathbf{\Sigma}$$

where

$$\delta\mathbf{u}^{(f)} = \nabla\delta\mathbf{v}^{(\mathbf{f})} \qquad \delta\mathbf{Q}^{(\phi)} = \nabla\delta\mathbf{J}^{(\phi)}$$

$$\langle\xi^{(\phi)}(x,t)\xi^{(\phi)}(x',t')\rangle = \delta^{(3)}(x-x')\delta(t-t')$$

$$\begin{aligned}\delta Q^{(\phi)} &= -\delta\Upsilon\dot{\phi}^2 - 2\Upsilon\dot{\phi}\,\delta\dot{\phi} + (2\Upsilon T)^{1/2}\dot{\phi}\,\xi^{(\phi)} + \nabla\cdot\mathbf{P} \\ \delta J^{(\phi)} &= \Upsilon\dot{\phi}\,\delta\phi + \nabla^{-2}\nabla\cdot\dot{\mathbf{P}}\end{aligned}$$

$$\mathbf{P} = -\mathbf{C_P}(\mathbf{2\Upsilon T})^{\mathbf{1/2}}\dot{\phi}\,\nabla^{-\mathbf{2}}\nabla\xi^{(\phi)}$$

$$\langle\Sigma_{ij}(x,t)\Sigma_{kl}(x',t')\rangle = 2T\left(\eta\delta_{ik}\delta_{jl} + \eta\delta_{il}\delta_{jk} + (\zeta - \frac{2}{3}\eta)\delta_{ij}\delta_{kl}\right)\delta^{(3)}(x-x')\delta(t-t')$$

Fluctuation-dissipation in cosmology

[AB (1996), Bastero-Gil *et al* (2014), Bartrum *et al* (2014)]

Treatment of early universe cosmological problems usually classical evolution with some quantum effects superposed, i.e. inflation, reheating, phase transitions etc... Also assume distinct regimes, radiation dominated, cold inflation etc...

This is approximation, should treat as multi-particle problem

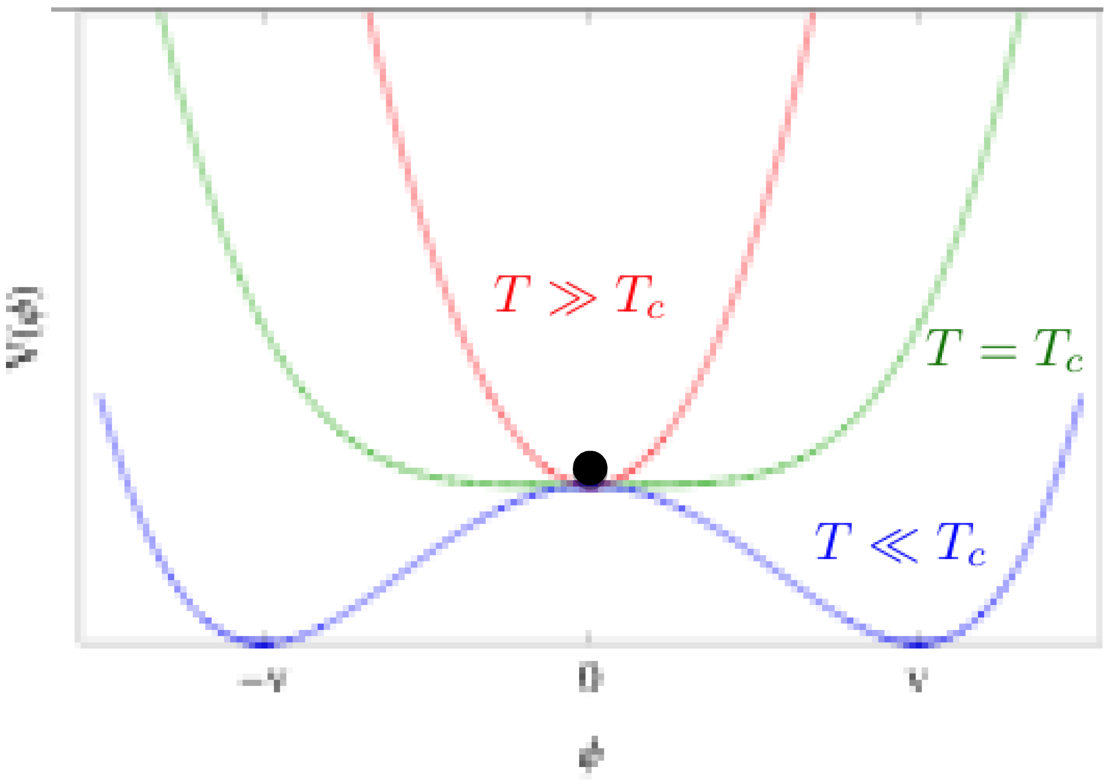
Include effective dynamics from all these degrees of freedom using dissipation terms which by fluctuation-dissipation relations will be accompanied by stochastic forces

Learn from warm inflation that dissipation and stochastic effects do affect observable predictions. Explains Planck data within a well motivated model. An alternative to developing elaborate models which now even involve quantum gravity.

Periods in early universe with FD dynamics, eg. phase transitions, baryogensis, magnetic fields, dark matter, dark energy...

Fluctuation-dissipation dynamics in cosmological phase transition

$$V(\phi, T) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2 + \frac{1}{2}\alpha^2 T^2 \phi^2 + \ldots$$



Fluctuation-dissipation dynamics in cosmoligical phase transition

$$V(\phi, T) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2 + \frac{1}{2}\alpha^2 T^2 \phi^2 + \ldots$$

Phase transition governed by Langevin equation

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = \xi$$

Fluctuation-dissipation relation:

$$\langle \xi(\mathbf{k}, t)\xi(\mathbf{k}', t')\rangle 2(3H + \Upsilon)T\frac{(2\pi)^3}{a^3}\delta^3(\mathbf{k} - \mathbf{k}')\delta(t - t')$$

Dissipation leads to energy and entropy transfer:

$$\dot{\rho}_\phi + 3H\dot{\phi}^2 = -\Upsilon\dot{\phi}^2 \qquad \dot{\rho}_r + 3H(\rho_r + p_r) = \Upsilon\dot{\phi}^2$$

Υ is particle physics models

Higgs coupling to fermions:

$$\mathcal{L} \sim \lambda_e^{ij} \overline{e}_{R,i} \phi^\dagger L_j + \lambda_u^{ij} \overline{u}_{R,i} \phi q_j + \lambda_d^{ij} \overline{d}_{R,i} \phi^\dagger q_j + h.c.$$

High-T/small Higgvalue:

$$\Upsilon_{SM} = \frac{288\zeta(3)T}{\pi^3} \sum_{i=1}^{3} \left(\frac{(\lambda_e^{ii})^2}{(\lambda_e^{ii})^2 + 4(g_1^2 + 3g_2^2)} + frac3(\lambda_u^{ii})^2(\lambda_u^{ii})^2 + 4(g_1^2 + 3g_2^2 +$$

NMSSM scalar singlet gives μ-term $(\mu = g(\phi))$

$$W = g\phi H_u H_d + y_u H_u Q U^c + y_d H_d Q D^c + y_c H_d L E^c$$

dissipation low-T/large field value:

$$\Upsilon = \frac{1}{8\pi} \frac{T^3}{\phi^2} \sum_{ij} \left(3(y_u^{ij})^2 + 3(y_d^{ij})^2 + (y_e^{ij})^2 \right)$$

Υ is particle physics models

GUT SU(5) broken to SM by 24 Higgs:

$$V(24_H) = -\mu^2 \mathrm{Tr}[24_H^2] + a\mathrm{Tr}[24_H^2]^2 + b\mathrm{Tr}[24_H^4] + c\mathrm{Tr}[24_H^3]$$

couples to 5 Higgs:

$$\mathcal{L}_s = -\frac{A^2}{2} 5_H^\dagger 5_H + \frac{B}{4}(5_H^\dagger 5_H)^2 + C 5_H^\dagger 5_H \mathrm{Tr}[24_H^2] + D 5_H^\dagger 24_H^2 5_H + E 5_H^\dagger 24_H 5_H$$

decays into matter fermions:

$$\mathcal{L}_Y = Y_5^{ij} \overline{5}_{Fi} 10_{Fj} 5_H^* + \frac{1}{8} \epsilon_5 Y_{10}^{ij} 10_{Fj} 5_H + h.c.$$

dissipation low-T/large field:

$$\Upsilon_{SU(5)}^{LM} = \frac{0.11}{C^2} \frac{T^7}{\phi^6} \sum_{i,j} \left[10(Y_5^{ij})^4 + 8(Y_{10}^{ij})^4 \right]$$

Cosmological phase transition

Simple model interaction: $\mathcal{L} = g\phi\overline{\psi}\psi$

Effective potential

$$V(\phi, T) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2 + \frac{1}{2}\alpha^2 T^2 \phi^2 \exp\left(-\frac{m_{\psi,T}}{T}\right)$$

where $\alpha^2 = g^2 N_F/6$ and $m^2_{\psi,T} = g^2\phi^2 + h^2 T^2$

Dissipative coefficient (high-T)

$$\Upsilon = 11.2 N_F T \exp\left(-\frac{m_{\psi,T}}{T}\right)$$

Higgs-radiation coupled system:

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V'(\phi) \approx 0 \quad \dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

Observational consequences:

Generate small scale perturbations but within horizon today

Small amounts of inflation, so modify inflationary predictions

Dissipative leptogensis

Dynamical field gives neutrino Majorana masses

Dissipation leads to lepton asymmetry due to heavy neutrino excitation and decay

$$\begin{aligned}
\ddot{\phi} &+ (3H+\Upsilon)\dot{\phi}+V'(\phi)=0 \\
\dot{s} &+ 3Hs=\frac{\Upsilon\dot{\phi}^2}{T} \\
\dot{n}_L &+ 3Hn_L=\frac{45\zeta(3)}{2\pi^4}\frac{g_L}{g_*}\frac{\Upsilon\dot{\phi}^2}{T}r_L
\end{aligned}$$

Neutrino masses from seesaw mechanism:

$$m_N=\frac{gv}{\sqrt{2}}\approx 10^{15}y^2\left(\frac{0.1\text{eV}}{m_\nu}\right)\text{GeV}$$

EW Sphalerons convert L into B asymmetry

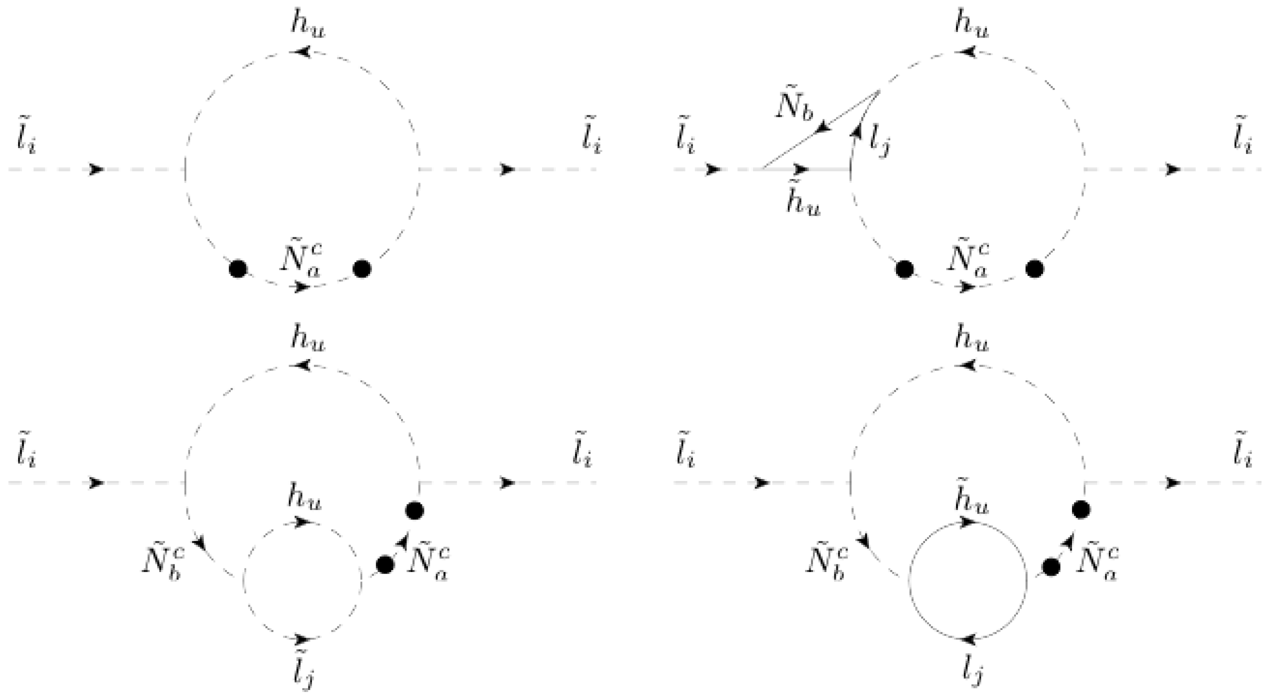
$$\eta_s\approx 10^{-10}\left(\frac{m_\nu}{0.1\text{eV}}\right)^2\left(\frac{m_N}{2\times 10^{15}\text{GeV}}\right)^3\left(\frac{\Delta\phi_i}{v}\right)^2\left(\frac{\epsilon}{0.05}\right)$$

Dissipative leptogensis

SUSY superpotential:

$$W = \frac{1}{2} g_a \Phi N_a^c N_a^c + y_{ai} N_a^c H_u L_i + f(\Phi)$$

Slepton self-energy (1-loop & 2-loops)



Dissipative leptogensis

Can produce baryon asymmetry at lower temperatures

No GUT symmetry restoration required

No real right-handed (s)neutrinos produced

Observational signature

Baryon isocurvature perturbations

Light field during inflation $\langle \delta\phi_i^2 \rangle = H_{inf}^2/2\pi$

Baryon asymmetry depends on initial field displacement
$\eta_s \propto \Delta\phi_i^2/v^2$

Have baryon asymmetry, isocurvature modes
$S_B = \delta\eta_s/\eta_s = 2\delta\phi_i/\Delta\phi_i$

$$B_B^2 = \frac{S_B^2}{P_\zeta^2} \approx \frac{r}{2}\left(\frac{M_p}{\Delta\phi_i}\right)^2$$

These are uncorrelated with adiabatic curvature perturbations

Planck: $B_B < 1.03 \Rightarrow \Delta\phi_i/M_p \gtrsim 0.2\sqrt{r/0.1}$

Conclusion

Warm inflation shown to work for simple models, many new model building opportunities.

Regimes in early Universe governed by fluctuation-dissipation dynamics, eg. phase transitions, inflation, possibly in radiation dominated regime.

Fluctuation-dissipation dynamics offers explanation for many of the unsolved problems of the early Universe, eg. baryon asymmetry, magnetic fields, origin of structure, lower tensor mode, also possibly dark matter and dark energy.

Recognizing the presence of this dynamics and properly accounting for it might be the missing link for completing the dynamical picture of the early Universe.

Two stage dissipative mechanism

(AB and R. Ramos, PRD **63**, 103509 (2001); I. G. Moss and C. Xiong, hep-ph/0603266; Bastero-Gil, AB, Ramos, 1008.1929 [hep-ph])

Basic Lagrangian - inflaton field coupled to heavy field ($> T$) which in turn coupled to light fields ($< T$)

Examples:

$\phi \to \chi \to \psi$ with $\quad m_\chi > 2m_\psi > m_\phi$

$$\mathcal{L}_I = -g_1^2 \phi^\dagger \phi \chi^\dagger \chi - h_2[\chi^\dagger \bar{\psi}_\sigma P_R \psi_\sigma + \chi \bar{\psi}_\sigma P_L \psi_\sigma], \qquad \Upsilon = 0.11 g_1^4 h_2^4 \varphi^2 \frac{T^7}{m_{\psi_\chi}^2}$$

$\phi \to \chi \to y$ with $\quad m_\chi > 2m_y > m_\phi$

$$\mathcal{L}_I = -g_1^2 \phi^\dagger \phi \chi^\dagger \chi - h_1 M[\chi^\dagger \sigma^2 + \chi(\sigma^\dagger)^2], \qquad \Upsilon = 0.026 g_1^4 h_1^4 \varphi^2 \frac{T^3 M^4}{m_\chi^8}$$

$\phi \to \psi_\chi \to \psi_d, y$ with $\quad m_\chi > m_{\psi_y} + m_y > m_\phi$

$$\mathcal{L}_I = -\frac{1}{\sqrt{2}} g_2 \varphi \bar{\psi}_\chi \psi_\chi - h_3[\sigma^\dagger \bar{\psi}_\chi P_R \psi_\sigma + \sigma \bar{\psi}_\chi P_L \psi_\sigma], \qquad \Upsilon = 0.0072 g_2^2 h_3^4 \frac{T^5}{m_{\psi_\chi}^4}$$

Physical picture of two stage dissipative mechanism

(Moss, Graham, PRD78, 123526 (2008); AB, Ramos, PLB607, 1 (2005))

Particle production rate of radiation bath particles due to interactions:

$$\dot{n} = \text{Im}\left[2\int_{-\infty}^{t} dt' \frac{e^{-i\omega(\mathbf{p})(t-t')}}{2\omega(\mathbf{p})}\Sigma_{21}(\mathbf{p},t,t')\right]$$

$\Sigma_{21} = \Sigma_{\sigma,21} + \Sigma_{\psi_\chi,21}$ is the sum of the σ and ψ_σ self-energies

Noting $\rho_r = \int \frac{d^3p}{(2\pi)^3}\omega(\mathbf{p})n$ implies:

$$\Upsilon = \frac{\dot{\rho}_r}{\dot{\varphi}^2} = \frac{1}{\dot{\varphi}^2}\int \frac{d^3p}{(2\pi)^3}\left[\omega_\sigma(\mathbf{p})\dot{n}_\sigma + \omega_{\psi_\sigma}(\mathbf{p})\dot{n}_{\psi_\sigma}\right]$$

Interaction generic in inflation models

- $g^2\phi^2\chi^2$ generic to inflation models (for reheating)
- for $g \gtrsim 10^{-3}$ require SUSY for flat potential
- Minimal SUSY model with this interaction
  $W = \sqrt{\lambda}\Phi^3 + \frac{g}{\sqrt{2}}\Phi X^2 + \frac{h}{\sqrt{2}} X Y^2$

  $(\Phi = \phi + \theta\psi + \theta^2 F,\ X = \chi + \theta\psi_\chi + \theta^2 F_\chi,\ Y = y + \theta\psi_y + \theta^2 F_y)$

  $$\Longrightarrow \mathcal{L}_{int} \sim \frac{1}{4}g^2|\phi|^2|\chi|^2 + \frac{1}{4}g\phi\psi_\chi\psi + \frac{g}{2}\phi\bar{\psi}_\chi\psi_\chi + \frac{1}{\sqrt{2}}h\chi\bar{\psi}_y\psi_y + \cdots$$

- For $\langle\phi\rangle \equiv \varphi \neq 0$ SUSY is broken with $m_{\chi_1} \gg m_{\psi_\chi} \gg m_{\chi_2}$
  ($V_{1-loop} \sim g\lambda\varphi^4 < V_{tree} \sim \lambda\varphi^4$, so flatness preserved)
- $\Longrightarrow$ dissipative mechanism through $\phi \to \chi \to \bar{\psi}_y + \psi_y, 2y, \ldots,$

  just like our toy model, so all results follow

Local limit of noise and dissipation

[AB, Moss and Ramos, (2007)]

$f(t)$ *slowly varying on timescale* τ if Fourier transform $\tilde{f}(\omega)$ satisfies

$$\tilde{f}(\omega) = 0 \quad \text{for} \quad \omega > 2\pi/\tau$$

kernel function $\mathcal{K}(t)$ described as *localized on a timescale* τ *with accuracy* $1-\epsilon$ if Fourier transform $\tilde{\mathcal{K}}(\omega)$ satisfies

$$\left| \frac{\tilde{\mathcal{K}}(\omega) - \tilde{\mathcal{K}}(0)}{\tilde{\mathcal{K}}(0)} \right| < \epsilon \quad \text{for} \quad \omega < 2\pi/\tau$$

$\Rightarrow$ $f(t)$ slowly varying on a timescale τ and $\mathcal{K}(t)$ localized on a timescale τ,

$$\int_{-\infty}^{\infty} K(t-t') f(t') dt' = \tilde{\mathcal{K}}(0) f(t) + \mathcal{O}(\epsilon)$$

$\Rightarrow \mathcal{K}(t) \approx \tilde{\mathcal{K}}(0)\delta(t-t')$

(Note: if $\tilde{\mathcal{K}}(\omega)$ analytic in $\omega \Rightarrow$ kernel can be localised and kernel admits a local derivative expansion. In general, derivative expansion might not exist, even when the kernel is localized in above sense.)

Non-gaussianity

Power spectrum: $\langle R(\mathbf{k_1}) R(\mathbf{k_2}) \rangle = (2\pi)^3 \delta(\mathbf{k_1} + \mathbf{k_2}) P_R(k_1)$

Bispectrum:

$\langle R(\mathbf{k_1}) R(\mathbf{k_2}) R(\mathbf{k_3}) \rangle = (2\pi)^3 \delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_R(k_1, k_2, k_3)$

e.g. local model: $B_R(k_1, k_2, k_3) = -\frac{6}{5} f_{NL} \left(P_R(k_1) P_R(k_2) + perm. \right)$

f_{NL} - Nonlinearity parameter

WMAP bounds:

$-10 < f_{NL}^{local} < 74 \quad (k_3 \ll k_1 \sim k_2); \quad -214 < f_{NL}^{equil} < 266 \quad (k_3 \sim k_1 \sim k_2)$

Planck $(f_{NL} \sim O(10))$

standard inflation models $f_{NL} \sim 0(\epsilon, \eta)$ -

- Single field inflation
- canonical kinetic terms
- slow-roll
- initial vacuum state

(Maldacena '03, Acquaviva, *et al.*, '03)

Non-gaussianity in warm inflation

(Gupta, *et al.*, PRD66, 043510 (2002); Moss and Xiong, JCAP 0704, 007 (2007))

$$\ddot{\Phi}(x) + (3H + \Upsilon)\dot{\Phi}(x) + \Upsilon a^{-2} v_\alpha \partial_\alpha \Phi(x) - a^{-2}\partial^2\Phi(x) + V_\Phi = \xi(x)$$

2nd order term

V - scalar velocity perturbation

Bispectrum

$$B_r^v(k_1, k_2, k_3) \approx 18L(Q) \sum_{cyclic} P_R(k_1)P_R(k_2)\left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right) k_1 k_2$$

$$\Rightarrow \quad L(Q) \equiv \ln(1 + Q/14)$$

Relation for f_{NL}:

$$-15L(Q) < f_{NL}^v < \frac{33}{2} L(Q)$$

$$Q \sim 100 \Rightarrow |f_{NL}| \sim O(30)$$

Observational tests of inflation

- Spectra of energy density fluctuations (scalar spectra)
- Spectra of gravitational waves (tensor spectra)
- Non-gaussian deviations
- Isocurvature fluctuations
- Present day cosmological constant
- Particle Spectra

EXPERIMENTS

- CMB - COBE, MAP, Planck, Boomerang, Maxima
- Redshift surveys - Sloan, 2df ...
- Big Bang Nucleosynthesis
- Supernovae 1A
- Hubble Space Telescope

Summary of WMAP data

WMAP 7 year data and angular power spectra:

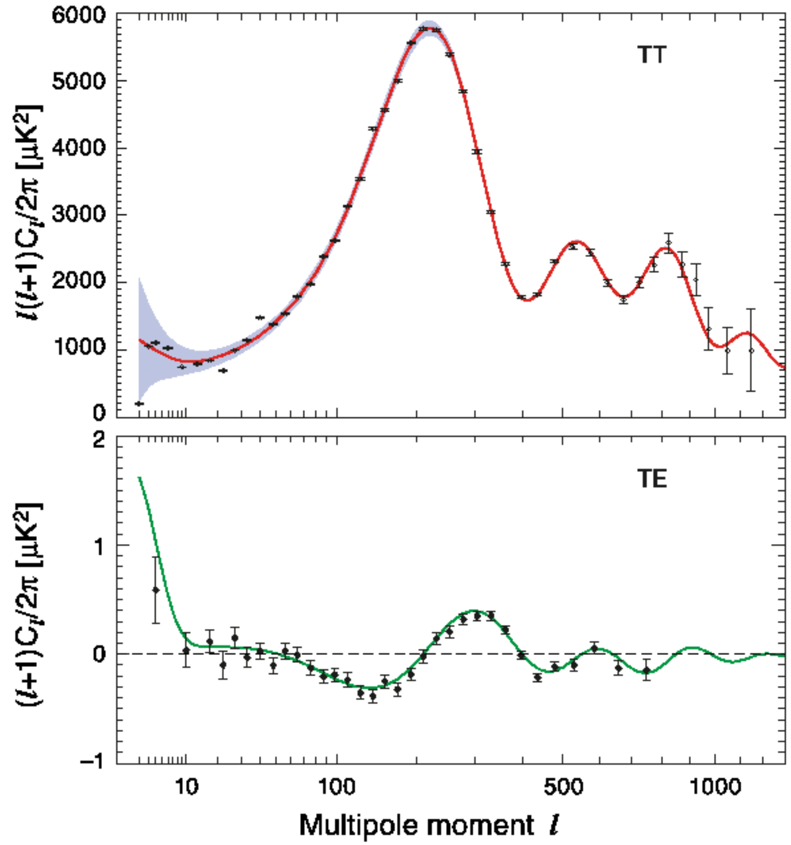
Scalar amplitude: $A_S = (2.43 \pm 0.11)^{-9}$

Scalar spectral index: $n_S = 0.963 \pm 0.014$

Running of spectral index:
$dn_S/d\ln k = -0.034 \pm 0.026$

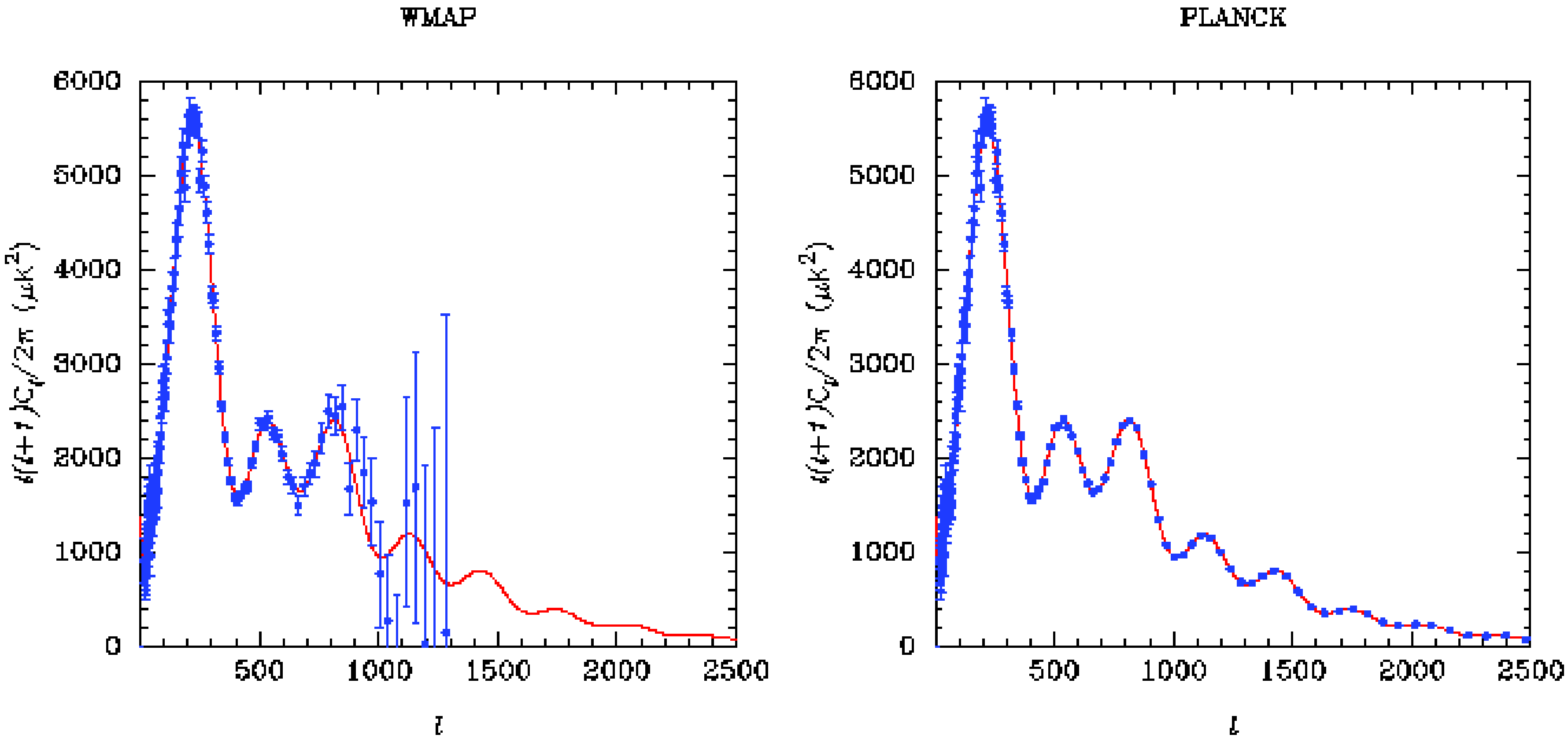
Tensor to scalar ratio: $r < 0.36(95\%\text{CL})$

Nongaussianty: $f_{NL}^{loc} = 32 \pm 21$ (68% CL)
Consistent with Gaussian within 95% CL



Yadav and Wandelt, 2008 found in WMAP 3 year data
$27 < f_{NL}^{loc} < 147$ $(95\%\ CL)$ with rejection of $f_{NL}^{loc} = 0$ at 2.8σ

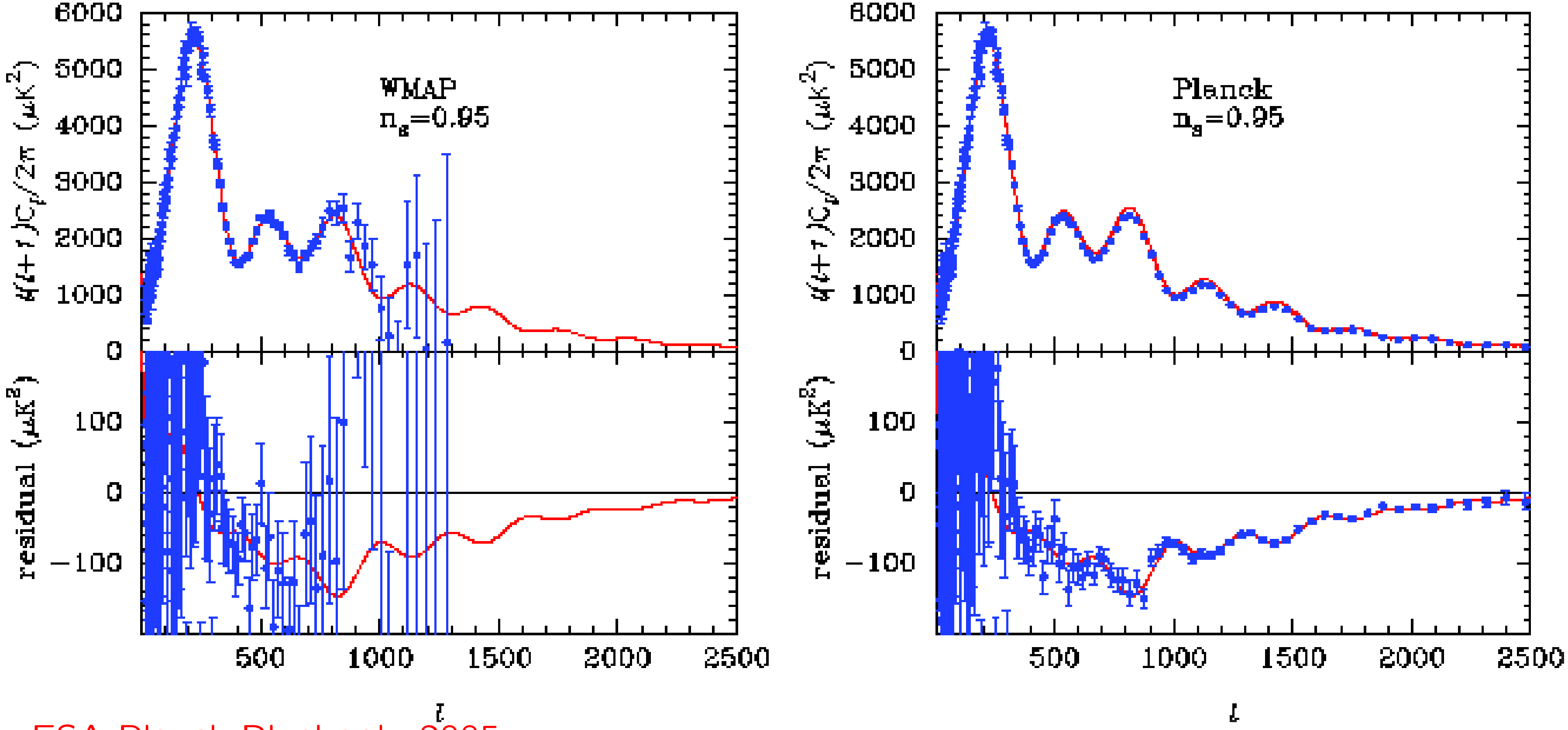
What Planck can achieve



ESA Planck Bluebook, 2005

CMB power spectrum for concordance ΛCMB model (red line) compare WMAP to (projected) Planck data

Distinguishing models



ESA Planck Bluebook, 2005

Solid red lines are concordance ΛCDM model with spectral index $n_S = 0.95$ and 1

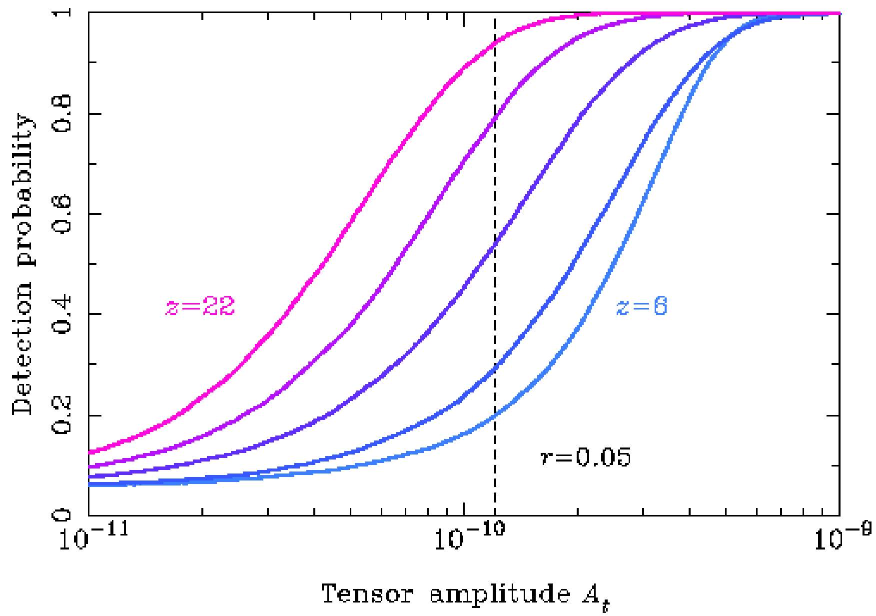
WMAP has difficulty distinguishing between the models vs. Planck can distinguish very well

Parameter forecast for Planck

PARAMETER FORECASTS FOR WMAP AND PLANCK

Parameter	Input Value	June'03	June'03 +2dF	*WMAP*$_4$	*Planck*	*WMAP*$_4$ ACT/SPT
Flat+weak priors						
ω_b	0.2240	0.00095	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.011	0.007	0.0039	0.0016	0.0035
n_S	0.9570	0.026	0.024	0.0125	0.0045	0.0080
τ	0.108	0.059	0.056	0.020	0.005	0.021
+running						
ω_b	0.2240	0.00162	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.0158	0.007	0.0039	0.0016	0.0035
$n_S(k_n)$	0.9570	0.055	0.024	0.0125	0.0045	0.0080
n_{run}	0.0	0.033	0.029	0.025	0.005	0.0092
τ	0.108	0.112	0.074	0.019	0.006	0.0266

ESA Planck Bluebook, 2005



Experiments	f_{NL} (Bispectrum)	f_{NL} (Skewness)
COBE	600	800
WMAP	20	80
Planck	5	70
Ideal	3	60

Bartolo, *et al.*, 2005

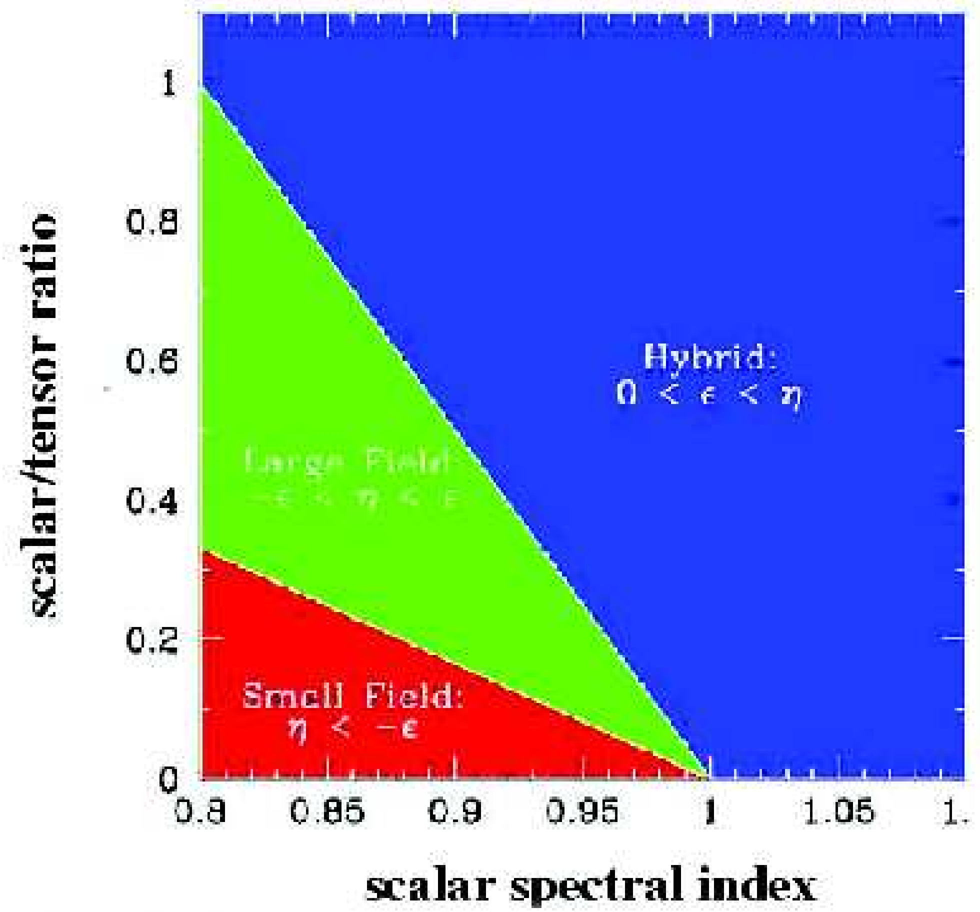
Error on scalar index reduced to half a percent

Tensor-scalar ratio detectable down to ~ 0.05 (optimistic)

If no detection of r in Planck $\Rightarrow$ low energy scale $V^{1/4}$ of inflation,

$V^{1/4} = 3.3 \times 10^{16} r^{1/4}$GeV

Standard inflation models - predictions



Kinney, *et al.*, 2001

Slow roll parameters:

$$\epsilon \equiv \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right) \ll 1$$

$$\eta \equiv \frac{m_{pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

$f_{NL} \lesssim 1$ in all models

$p = 4$ ruled out by WMAP data

If $r < 0.1$ is found by Planck, monomial cold inflation models ruled out

If $f_{NL} \gtrsim 5$ is found by Planck all these simplest of inflation models would be ruled out

Warm inflation models

(AB and Ramos, PLB 607, 1 (2005))

Superpotential:

$$W = W(\Phi) + g\Phi X^2 + hXY^2$$

Dissipative coefficient:

$$\Upsilon \approx C_\phi \frac{T^3}{\phi^2}$$

$$C_\phi \equiv 0.64 h^4 N_\chi N_{decay}^2$$

Slow-roll:

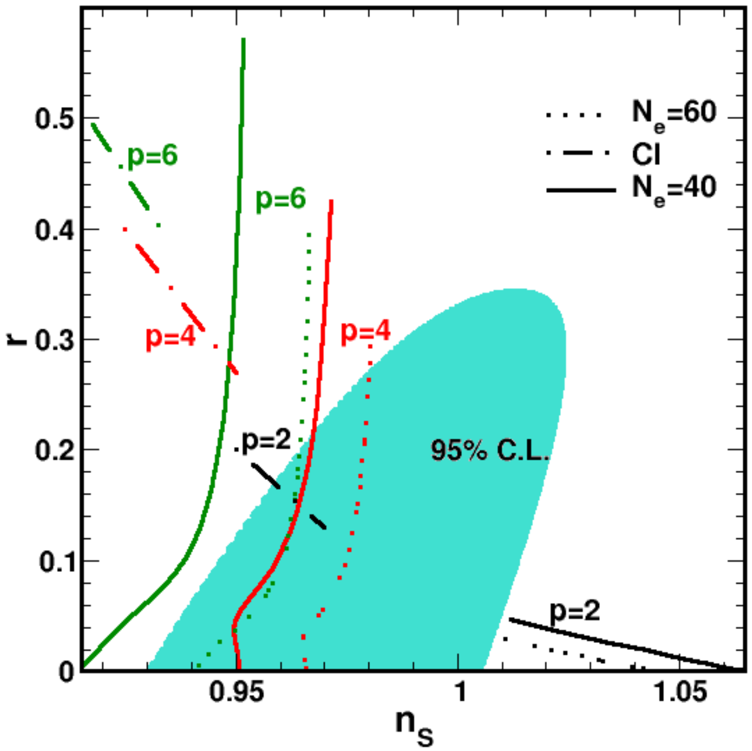
$$\dot{\phi} \approx -\frac{V_\phi}{[3H(1+Q)]} \qquad Q \equiv \Upsilon/(3H)$$

$$4\rho_R \approx 3Q\dot{\phi}^2 \qquad \rho_R = \pi^2 g_* T^4/30$$

Warm inflation models - monomial potential

(Bastero-Gil and AB, Int. J. Mod. Phys A24, 2207 (2009))

$$V = V_0 \left[\frac{\phi}{m_p} \right]^p$$



- $\frac{d(T/H)}{dN_e} > 0$
- Weak DR $\longrightarrow$ Strong DR
- Solves "eta" - problem, $m_\phi > H$
- Solves large ϕ amplitude problem - $\phi < m_P$
- $C_\phi \equiv 0.16 N_\chi N_{decay}^2$
  $\sim 10^6 - 10^8$

$T/H > 1$

Bulk viscosity and background evolution

(Mimoso,Nunes,Pavon PRD73, ‘06; Del Campo, Herrera, Pavon PRD75 ‘07; Del Campo *et al.*, 1007.0103)

Viscous pressure arising from interactions of particles and decay within fluid:

$$\Pi \approx -3\zeta_b H$$

From calculation of ζ_b: $\zeta_b \propto \rho_R^s > 0 \Rightarrow \Pi < 0$

Modified equation of state: $p_R = (\gamma - 1)\rho_R$, $1 \leq \gamma \leq 2$

Evolution of radiation:

$$\dot{\rho}_R + 3H(\rho_R + p_R + \Pi) = \Upsilon\dot{\phi}^2$$

$\Pi < 0 \quad \Rightarrow$ increases the duration of inflation

Presence of bulk pressure also modifies primordial spectrum

Stringy warm inflation

(AB, Kephart, PRL 83, 1084 (1999); Bastero-Gil, *et al.*, 0904.2195 [astro-ph.CO])

- Recurrent problem in embedding inflation in string models is the "eta" - problem, i.e. quantum corrections and SUGRA contributions to inflaton potential ruin required flatness

- Warm inflation solution to the "eta" - problem: large dissipation $\Upsilon \gg H \Rightarrow V'' \gg H^2$, i.e. much bigger than scale for SUGRA corrections.

- Necessary ingredient for warm inflation is large number of fields - naturally available in string theory, i.e. moduli fields, branes, Kaluza-Klein modes come in the hundreds of thousands.

- Example: trapped warm inflation - scalar inflaton field trapped in decaying oscillation about ESP, coupling to other fields leads to warm inflation.

Refining the theory

- Establishing fluctuation-dissipation theorem
- Origin of derivative expansion
- Finite temperature effective potential in SUSY models
- Origin of dissipation from more general non-equilibrium derivation
- Higher loops, resummations etc....
- Apply to more models

Summary of warm inflation

- Treats dynamical effects of inflaton interacting with other fields during inflation

- Model Building
  - Can have inflation models with $m_\phi > H$
  - Particle physics during inflation phase, eg. magnetic fields baryogenesis...

- Observational implications
  - Running spectral index in simple models
  - Blue and red spectra are possible
  - Non-gaussianity at strong dissipation $f_{NL} \sim O(10)$
  - low tensor-scalar ratio $r \ll 0.1$

Green's function $G_\phi^{ij}(x,x')$ - Basic Properties

$$G_\phi(x,x') = \begin{pmatrix} G_\phi^{++}(x,x') & G_\phi^{+-}(x,x') \\ G_\phi^{-+}(x,x') & G_\phi^{--}(x,x') \end{pmatrix} = \begin{pmatrix} i\langle T_+\phi(x)\phi(x')\rangle & i\langle\phi(x')\phi(x)\rangle \\ i\langle\phi(x)\phi(x')\rangle & i\langle T_-\phi(x)\phi(x')\rangle \end{pmatrix} =$$

Fourier space: $G_\phi(x,x') = i\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}.(\mathbf{x}-\mathbf{x}')}\tilde{G}_\phi(\mathbf{q},t-t')$

$$\begin{aligned} \tilde{G}_\phi^{++}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t')\theta(t-t') + G_\phi^{<}(\mathbf{q},t-t')\theta(t'-t), \\ \tilde{G}_\phi^{--}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t')\theta(t'-t) + G_\phi^{<}(\mathbf{q},t-t')\theta(t-t'), \\ \tilde{G}_\phi^{+-}(\mathbf{q},t-t') &= \tilde{G}_\phi^{<}(\mathbf{q},t-t'), \\ \tilde{G}_\phi^{-+}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t') . \end{aligned}$$

Hermiticity: $\tilde{G}_\phi^{>*}(\mathbf{q},t-t') = \tilde{G}_\phi^{>}(\mathbf{q},t'-t)$

Continuity: $\frac{d}{dt}\left[\tilde{G}^{>}(\mathbf{q},t-t') - \tilde{G}_\phi^{>}(\mathbf{q},t'-t)\right]|_{t=t'} = i\delta(t-t')$

Green's Function - equilibrium approximation

Gives lower bound estimate of dissipative effects

(Moss and Xiong, hep-ph/0603266)

Low temperature regime ($m_\chi < T$):

$$G_{\text{equil}}(\mathbf{k},t) = \frac{i}{2(\omega_{\mathbf{k}} - i\Gamma_\chi)} \exp[-i(\omega_{\mathbf{k}} - i\Gamma_\chi)t] + f(\omega_{\mathbf{k}} - i\Gamma_\chi, t) - f(\omega_{\mathbf{k}} + i\Gamma_\chi, t)$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m_\chi^2}$,

$$f(\omega, |\mathbf{k}|, t) = \frac{\exp(i\omega t)}{4\pi\omega} E_1(i(|\mathbf{k}| + \omega)t) - \frac{\exp(-i\omega t)}{4\pi\omega} E_1(i(|\mathbf{k}| - \omega)t)$$

At small time, i.e. $t \sim \tau_\chi = \Gamma_\chi^{-1} \log \frac{m_\chi^2}{\Gamma_\chi^2}$, the behavior same as the exponential decay approximation.

At larger time, power-law decay behavior.

Leads to dissipative coefficient:

$$\Upsilon_{\text{equil}}(\varphi, T) = 4 \times 10^{-2} g^2 h^4 \left(\frac{g\phi}{m_\chi}\right)^4 \frac{T^3}{m_\chi^2}$$

Green's Function - equil. approx. (cont)

(Hosoya and Sakagami, PRD **29**, 2228 (1984);
Berera, Ramos, Gleiser, PRD **58**, 123508 (1998))

High Temperature limit:

$$\Upsilon \approx \frac{132}{\pi T} \varphi^2 N_\chi \ln \left(\frac{2T}{2\mu(T)} \right)$$

where thermal mass

$$\mu(T) = \frac{gT}{\sqrt{12}}$$

Local limit of one-loop kernels

Homogeneous field $\varphi_c(x) = \varphi_c(t)$

$$\tilde{\mathcal{N}}_1(\omega) \simeq g^4 \left(e^{\beta\omega} + 1\right) \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} n(\omega') n(\omega - \omega') \tilde{\rho}_\chi(\mathbf{k}, \omega') \tilde{\rho}_\chi(\mathbf{k}, \omega - \omega')$$

$$\equiv g^4 h^4 \frac{\mathcal{M}^4 T^4}{m_\chi^8} F_1(\beta\omega)$$

$x \equiv \omega/T$

Warm inflation on a computer

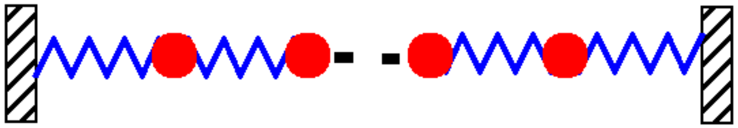
[AB, G. Lacagnina (lattice gauge), C. Verdozzi (condensed matter)]

Purpose:

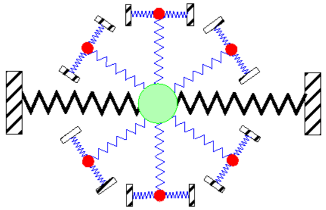
- Study of overdamped motion and its universal features
- study of how equipartition is achieved

Feasible goals:

- Numerical simulations of classical/quantum models from condensed matter:



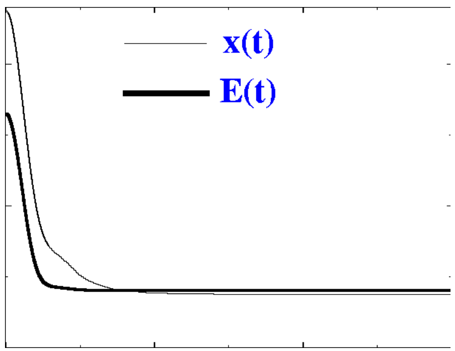
Fermi-Paste-Ulam

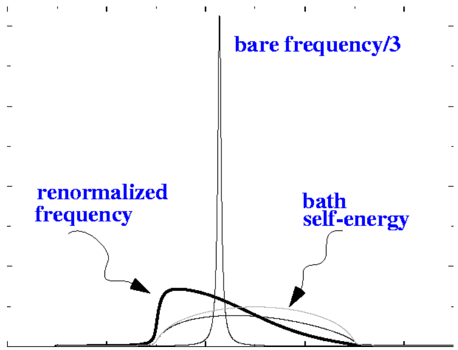


Caldeira-Leggett

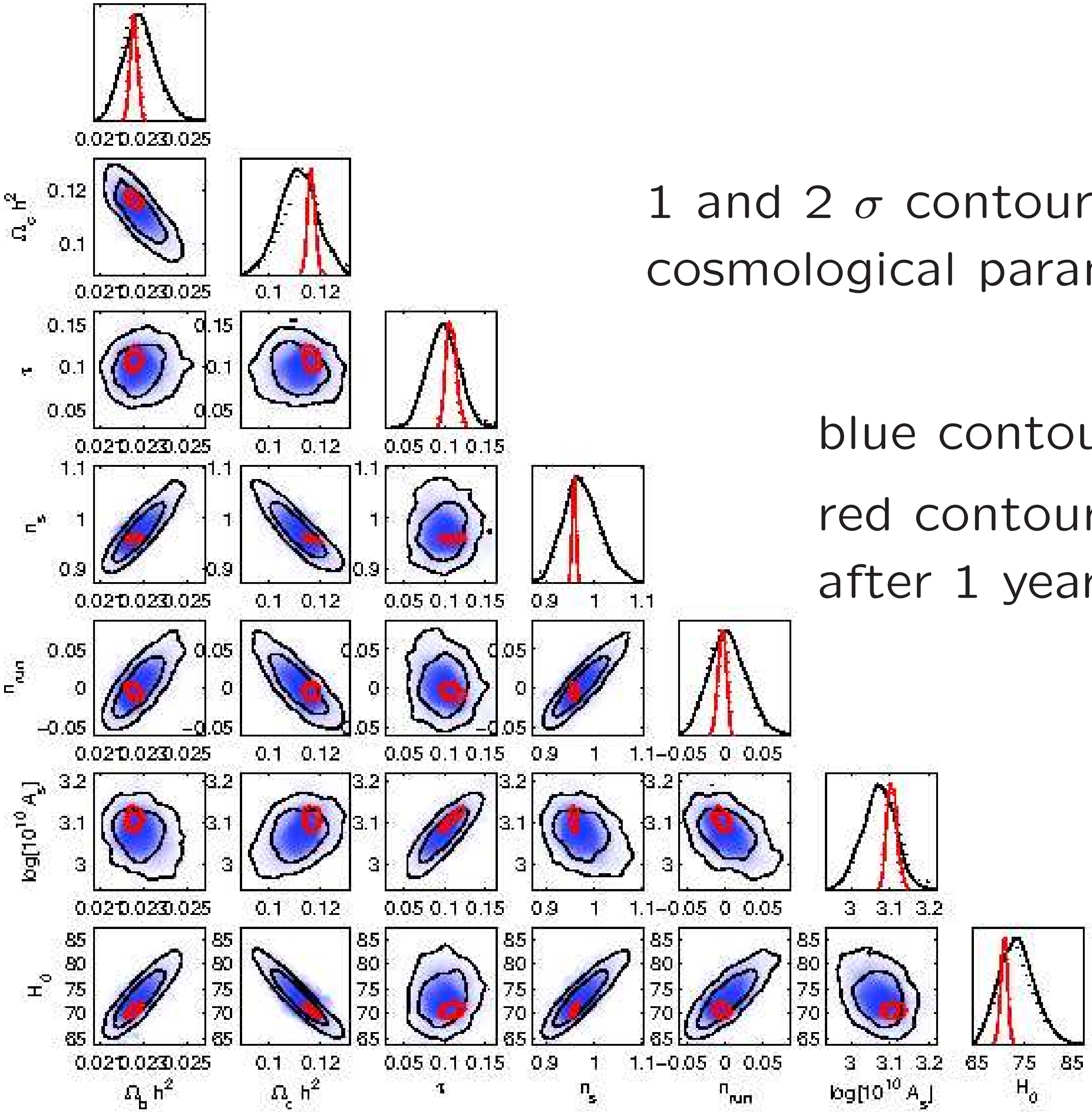
- Simulations of lattice quantum field theory models: Caldeira-Leggett, ϕ^4 ...

Example of overdamping in the classical regime





Improving parameter estimation



1 and 2 σ contour regions for cosmological parameters

blue contours WMAP 4

red contours Planck projected after 1 year of observation

ESA Planck Bluebook, 2005

Worked example - no η-problem

Model: $V = \frac{m^2}{2}\varphi^2$

$\Longrightarrow N_e \approx 2\sqrt{2}\frac{\varphi_0}{m}\frac{\Upsilon}{m_P}$, $T \approx \frac{m^{3/4} m_P^{1/4} \varphi_0^{1/4}}{\Upsilon^{1/4}}$, $\frac{\delta\rho}{\rho} \approx (\frac{\varphi_0}{m})^{3/8}(\frac{\Upsilon}{m_P})^{9/8}$

For $N_e = 60$, $\frac{\delta\rho}{\rho} = 10^{-5} \Longrightarrow \frac{\varphi_0}{m} \approx 6 \times 10^8$, $\frac{\Upsilon}{m_P} \approx 4 \times 10^{-8}$

e.g., $m = 10^9 \text{GeV} \Longrightarrow \frac{H}{m} \approx 0.17$, $\frac{\varphi_0}{m_P} \approx 6 \times 10^{-2}$, $T \approx 10^4 m$

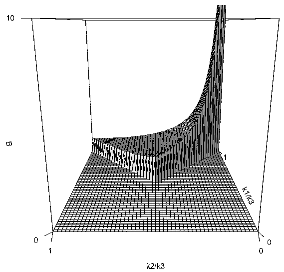
- No η-problem: $m > H$
- No φ amplitude problem: $\varphi < m_P$
- No graceful exit problem: inflation $\rightarrow$ RD automatic
- No quantum-to-classical trans. problem: $\delta\varphi$ classical

Template shapes of bispectrum

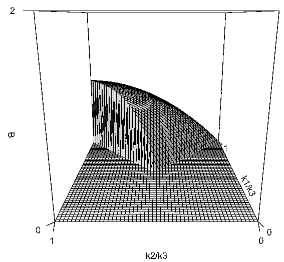
Discriminate models by determining the shape of the bispectrum

Template bispectra:

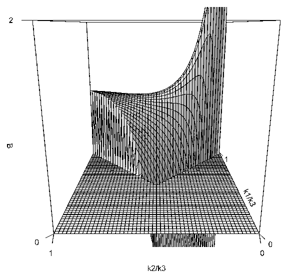
Local, $B_L = \sum_{cyc} k_1^{-3} k_2^{-3}$



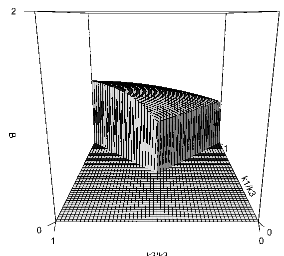
Equilateral, $B_E = \sum_{cyc} -3k_1^{-3}k_2^{-3} - 2k_1^{-2}k_2^{-2}k_3^{-2} + 6k_1^{-1}k_2^{-2}k_3^{-3}$



Strong warm inflation $B_S = \sum_{cyc} k_1^{-3}k_2^{-3}(k_1^{-2} + k_2^{-2})\mathbf{k}_1 \cdot \mathbf{k}_2$



Weak warm inflation $B_W = \sum_{cyc} k_1^{-3}k_2^{-3}\mathbf{k}_1 \cdot \mathbf{k}_2$

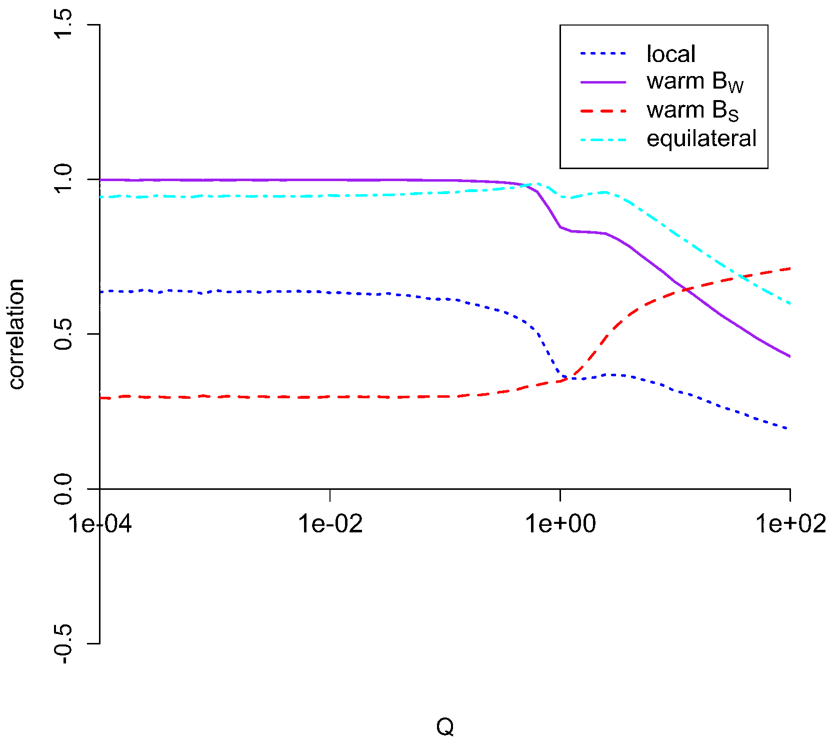


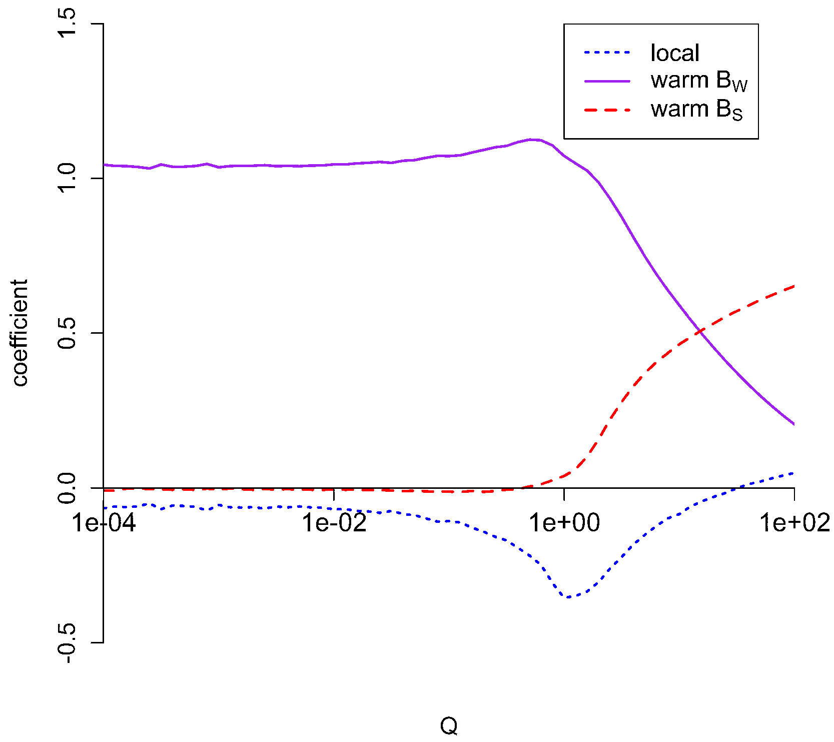
Bispectrum shape

Matching numerical bispectrum to given template, distance function:

$$B_1 \cdot B_2 = \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \frac{B_1(k_1,k_2,k_3)B_2(k_1,k_2,k_3)}{P(k_1)P(k_2)P(k_3)}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3);.$$

$$Q \ll 1 \Rightarrow B_W \quad Q \gg 1 \Rightarrow B_S$$





Warm inflation summary

Excellent consistency with Planck/BICEP2 data is still possible for one of the simplest inflation models by including some thermodynamic considerations

All the good properties of the ϕ^4 inflation model remain in tact:

- Renormalizable - familiar quantum field theory interactions
- Initial conditions - eternal inflation still occurs in the warm regime [AB, Rangarajan (2013)]

General warm inflation picture of interactions, particle production and dissipation during inflation could have relevance in explaining CMB data. Other dynamical realizations and models of warm inflation should be explored