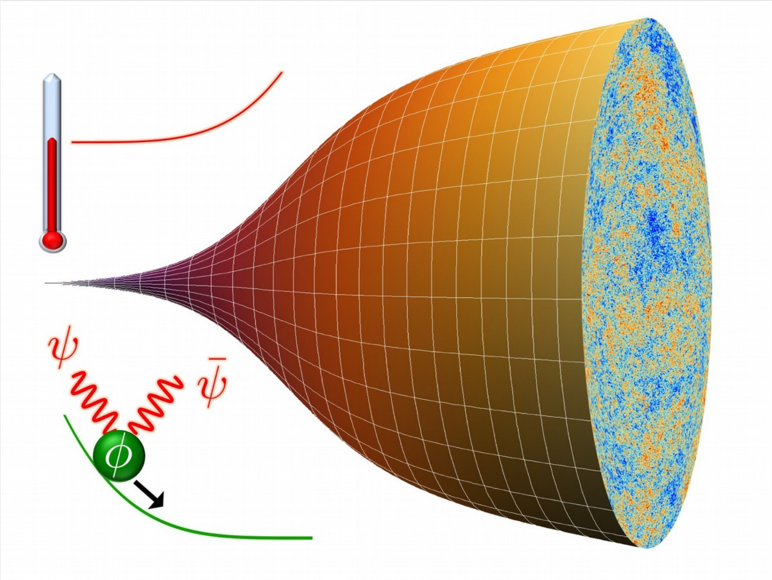
Little Warm Inflation



Cold inflation/Warm inflation

Dissipative coefficient:

High T regime: $\Upsilon(T) = C_T T$

Primordial spectrum: Chaotic models $\lambda \phi^4$

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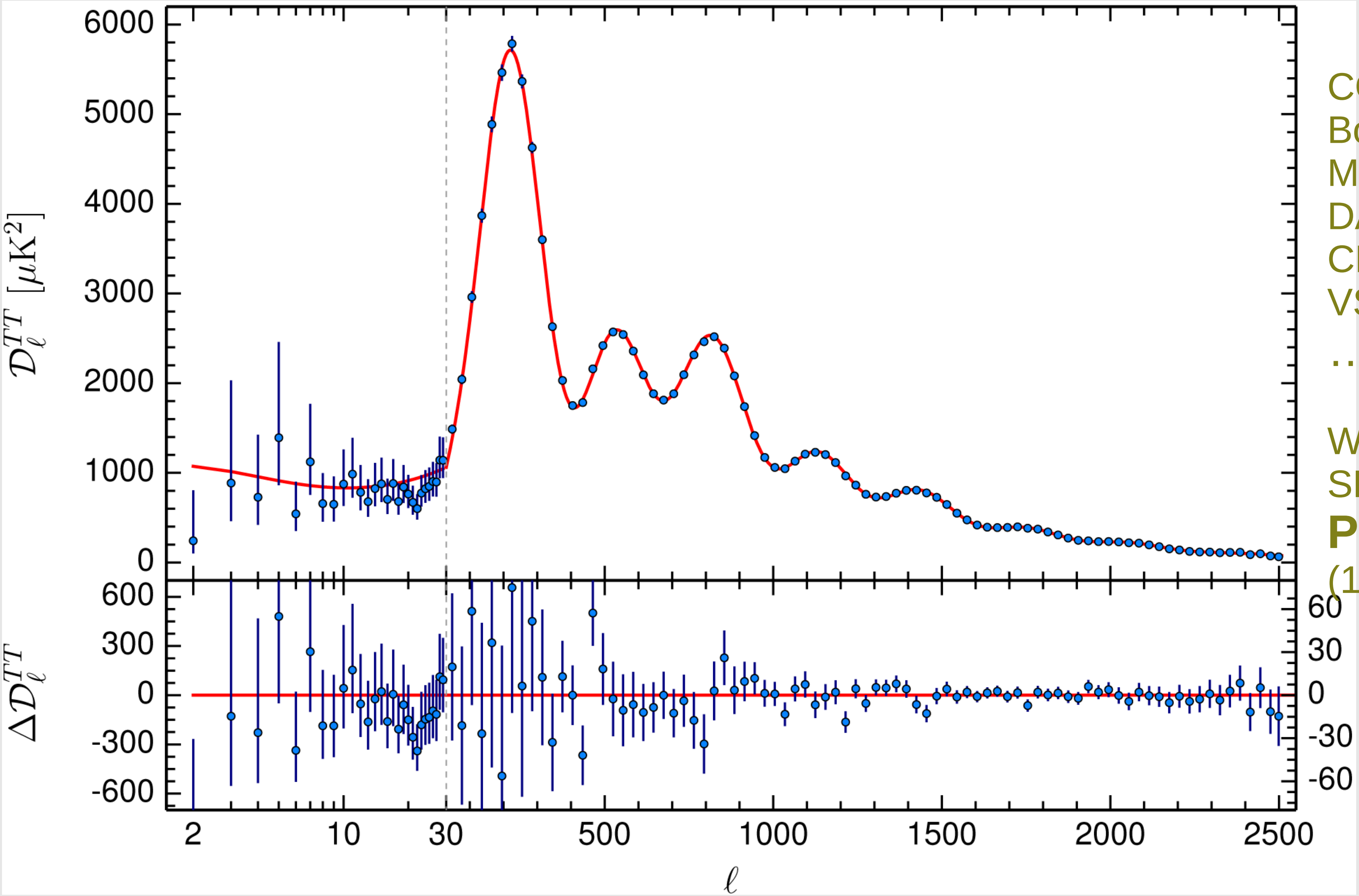
University of Granada

Work done in collab with: A. Berera, R. Ramos, J. Rosa PRL117 (2016) 151301

Cosmic microwave background radiation (CMB)

Spectrum of T fluctuations



COBE (1992)
Boomerang
Maxima
DASI
CBI
VSA
.....

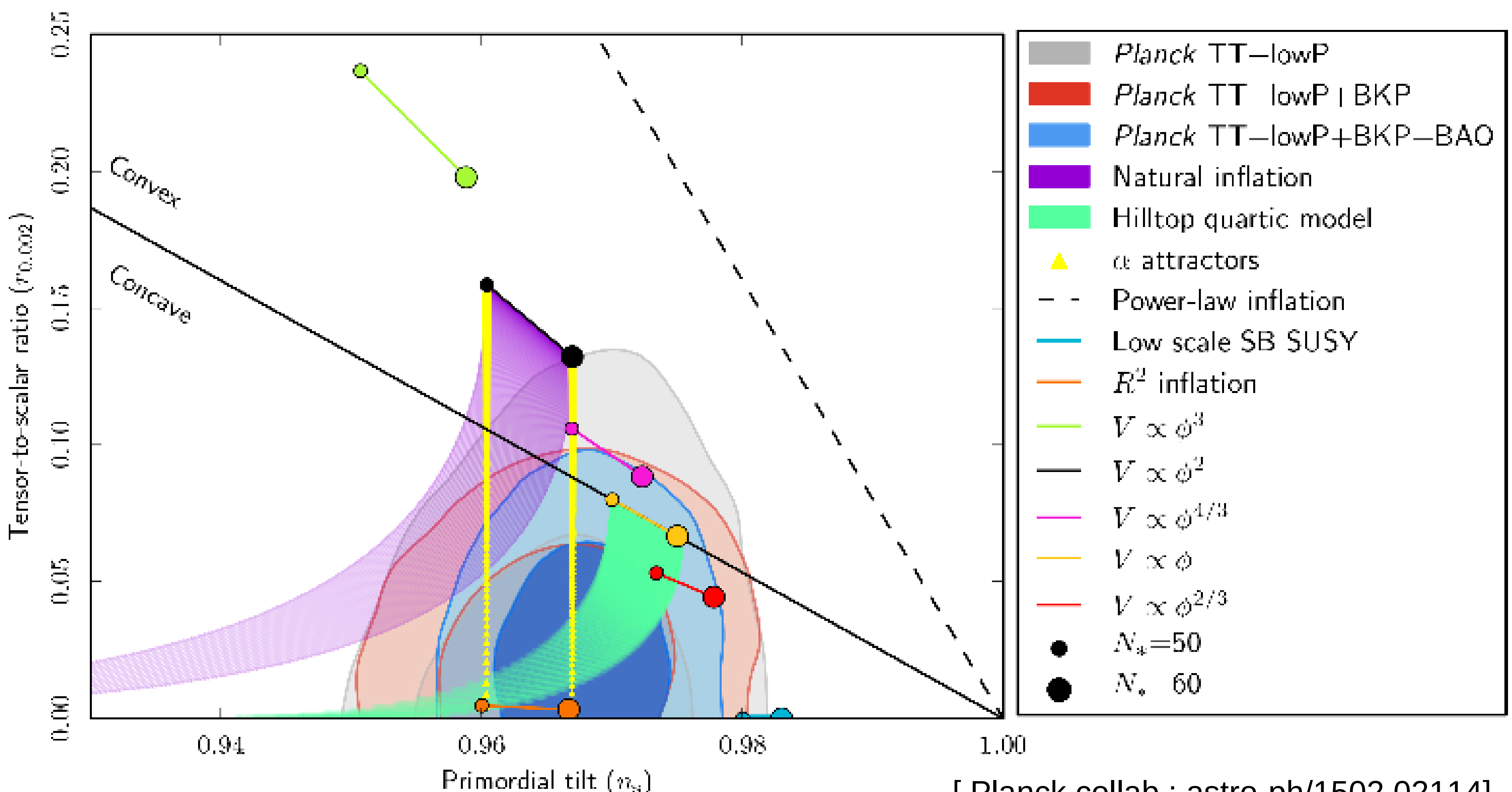
WMAP
SPT
PLANCK
(14/05/09)

[Planck collab.: astro-ph/1502.01589]

Primordial spectrum: ~adiabatic, ~scale-invariant, gaussian?, tensors?

Primordial spectrum: $P_R = P_R(k_0)(k/k_0)^{n_s-1} \qquad k_0 = 0.002\,\mathrm{Mpc}^{-1}$

Tensor-to-scalar Ratio: $r = P_T/P_R \qquad P_R = 2.2\mathrm{x}10^{-9}$



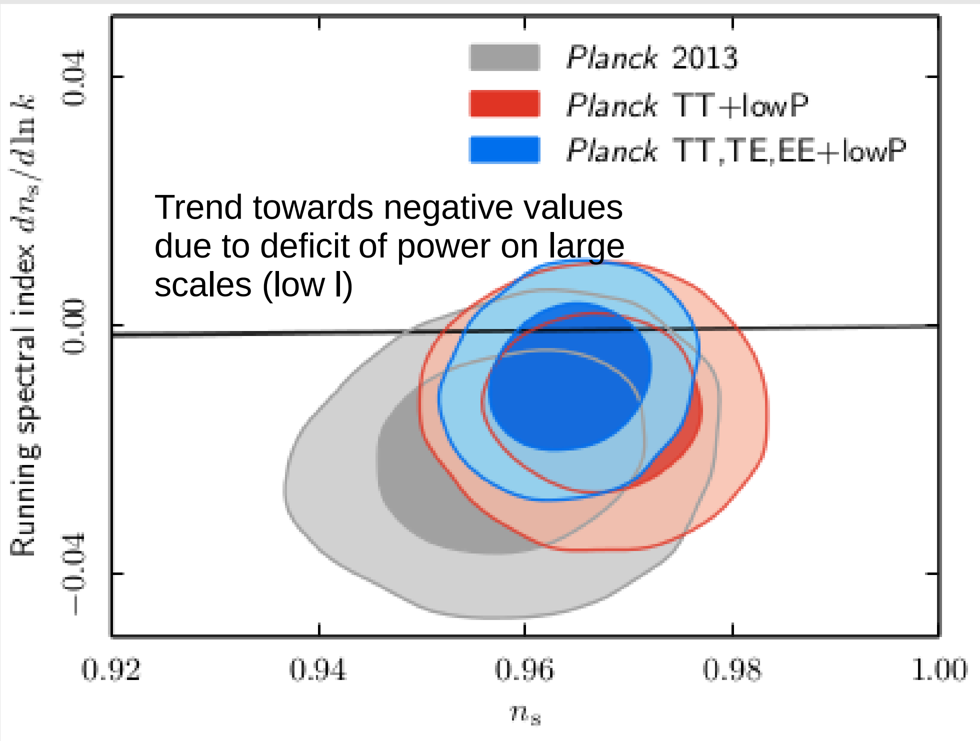
[Planck collab.: astro-ph/1502.02114]

Primordial spectrum: $P_R = P_R(k_0)(k/k_0)^{n_s - 1 + \frac{1}{2}\alpha_s \ln k/k_0 + \cdots}$ $\quad k_0 = 0.05\,\mathrm{Mpc}^{-1}$

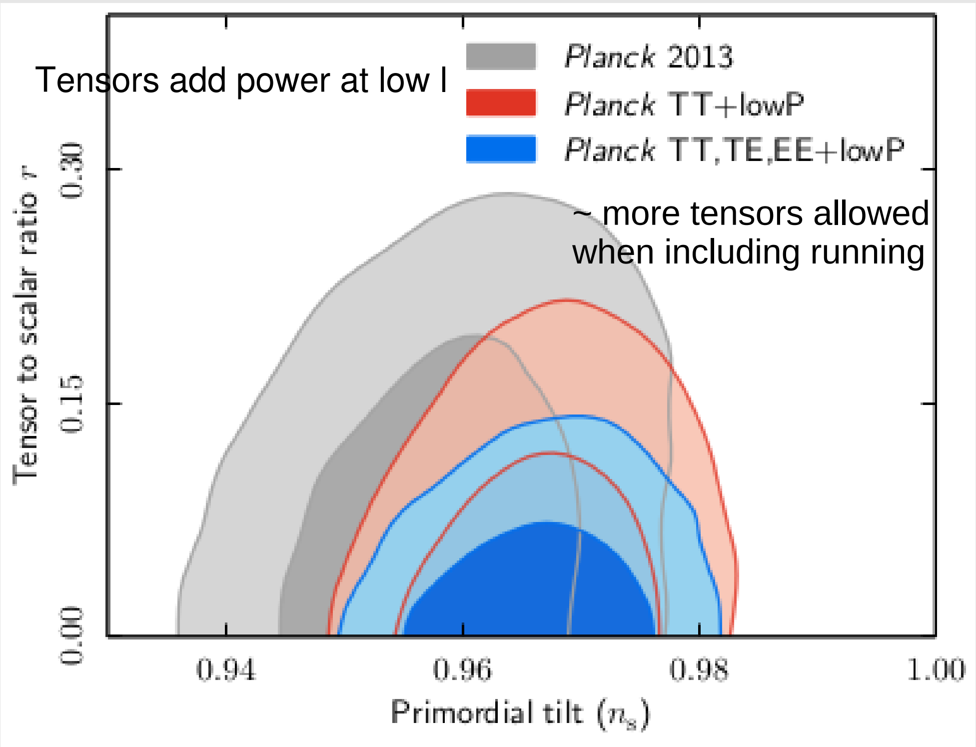
adiabatic, gaussian, scale-invarian spectrum

No evidence for:

non-gaussianity, isocurvature modes or running of the spectral index



$$\alpha_s = \frac{dn_s}{d\ln k} = -0.0057 \pm 0.0071$$



$$\alpha_s = -0.013 \pm 0.01, \qquad r_{0.002} < 0.18$$

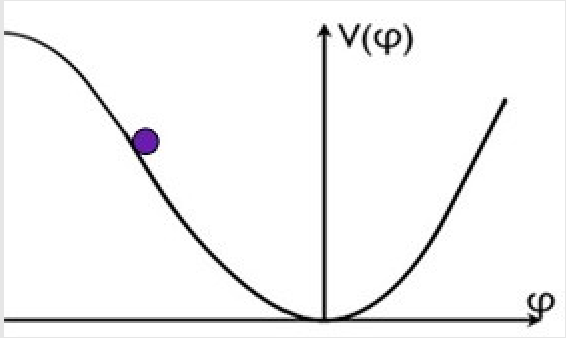
$$[n_t = -r/8 < 0]$$

[Planck collab.: astro-ph/1502.02114]

Slow Roll Inflation

Scalar field rolling down its (flat) potential

$$P = \dot{\varphi}^2/2 - V(\varphi) \approx -V(\varphi)$$ negative pressure



"Flat" potential The curvature and the slope smaller than the (Hubble) expansion rate H

Kinetic energy << potential energy **$H^2 \sim V/3m_P^2$** **Hubble parameter** $(H = \dot{a}/a)$

$(a = \text{scale factor})$

Slow-roll parameters

$$|\eta_\varphi| = m_P^2 \left|\frac{V''}{V}\right| < 1 \qquad \epsilon_\varphi = \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 < 1$$

curvature **slope**

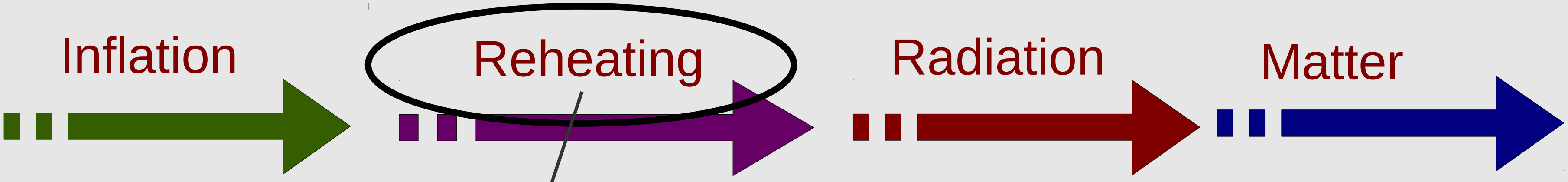
Slow-roll equation

$$\dot{\varphi} \simeq -V'/3H$$

Primordial spectrum

$$P_R \simeq \left(\frac{H}{\dot{\varphi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \qquad n_s = 1 + 2\eta_\varphi - 6\epsilon_\varphi \qquad r = 16\epsilon_\varphi$$

$$V^{1/4} \sim 10^{16}\left(\frac{r}{0.1}\right)^{1/4} \text{Gev}$$



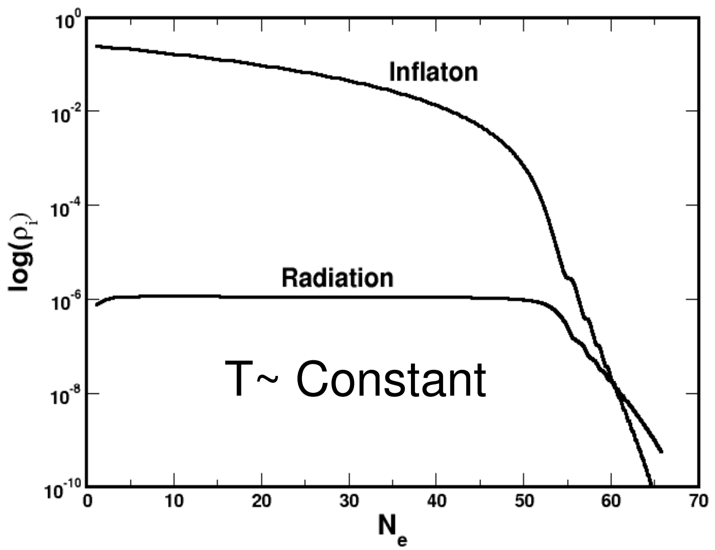
Inflaton interacts with other particles

Interactions with cosmic plasma induce dissipation

$$\ddot{\varphi} + (3H + \Upsilon)\dot{\varphi} + V_{\varphi} = 0$$

"Decay" into light dof= extra friction

"Warm" inflation:



A (small) fraction of the vacuum energy is converted into radiation during inflation

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\varphi}^2$$

"Source term"

Slow-roll:
$$\begin{cases} (3H + \Upsilon)\dot{\varphi} \simeq -V_{\phi} \\ 4H\rho_R \simeq \Upsilon\dot{\varphi}^2 \end{cases}$$

Extra friction term: $Q = \Upsilon/(3H)$ (Particle production versus Hubble friction)

- $Q<<1$, $T<<H$ → Standard **C**old **I**nflation
- $Q< 1$, $T > H$ → **W**eak **D**issipative **R**egime

Standard slow-roll

- $Q>1$, $T > H$ → **St**rong **D**issipative **R**egime

Slow-roll : $3H(1+Q)\dot{\varphi} \simeq -V_{\varphi}(\varphi, T)$, $\rho_r \simeq \frac{3}{4} Q \dot{\varphi}^2$

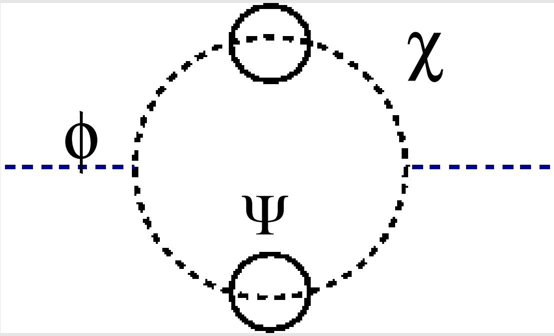
$|\eta_{\varphi}| < (1+Q)$, $\epsilon_{\varphi} < (1+Q)$, $\beta_{\Upsilon} < (1+Q)$, $\delta_T < 1$ **(Thermal corrections)**

$\beta_{\Upsilon} = m_P^2 (\Upsilon_{\varphi} V_{\varphi})/(\Upsilon V)$ $\delta_T = T V_{T\varphi}/V_{\varphi}$

- Q varies during inflation
- Extra friction prolongs inflation → Smaller ϕ values
- Dissipation induces thermal inflaton fluctuations

Interactions & Dissipative coefficient

Low T regime:



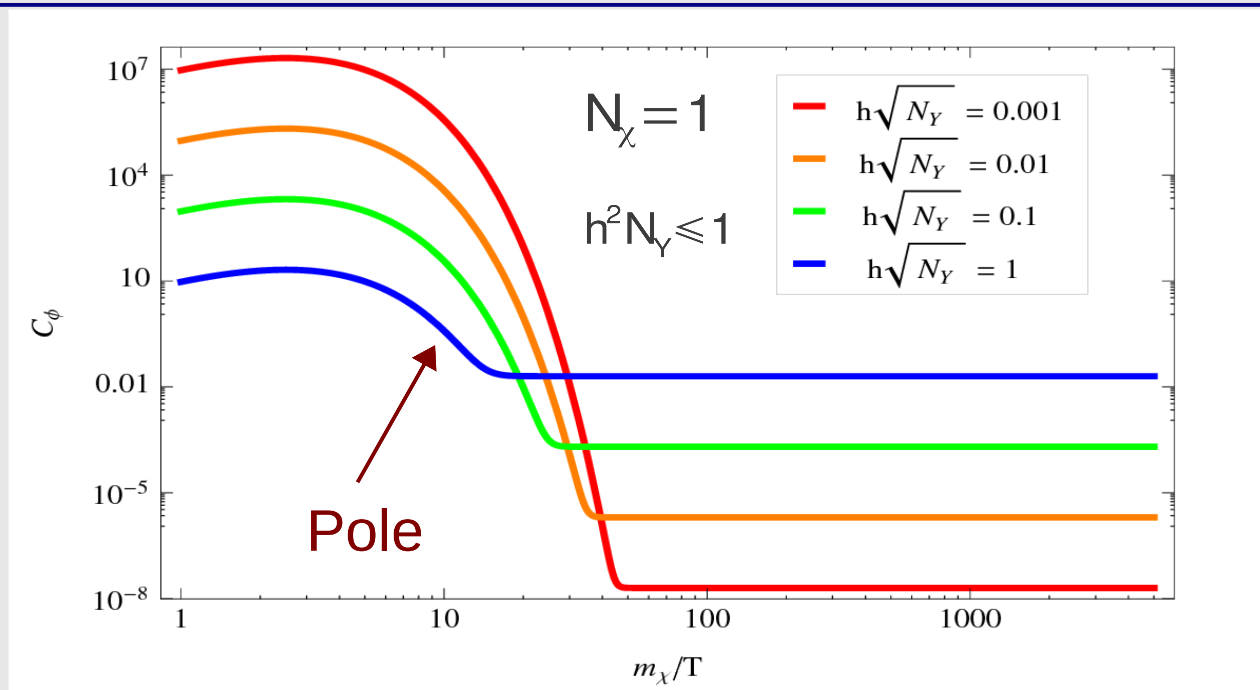
light fermions

$$L = \cdots - \frac{1}{2} m_\varphi^2 \varphi^2 - \frac{g^2}{2} \varphi^2 \chi^2 + h \chi \psi \bar{\psi} + \cdots$$

heavy $m_\chi = g\phi > H, T$

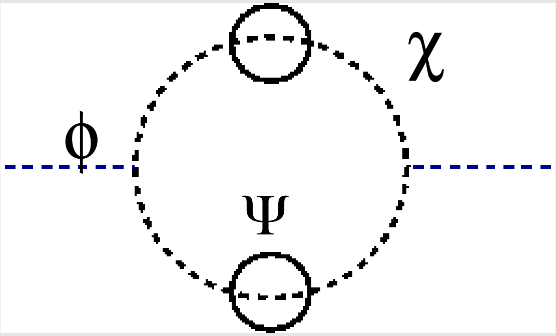
BG, Berera, Ramos & Rosa 2012

$$\boxed{\Upsilon \simeq \frac{32}{\sqrt{2\pi}} \frac{g^2 N_\chi}{h^2 N_Y} (m_\chi T)^{1/2} e^{-m_\chi/T} + 0.02 h^2 N_Y N_\chi \left(\frac{T^3}{\varphi^2}\right) \simeq C_\varphi \frac{T^3}{\varphi^2}}$$



Interactions & Dissipative coefficient

Low T regime:



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Adiabatic approximation:

- Easy to fulfill for not too small values of h

$\Rightarrow$ $T > H$

$$\frac{\Gamma_\chi}{\dot{\varphi}/\varphi} > \frac{\Gamma_\chi}{H} > \left(\frac{\Gamma_\chi}{m_\chi}\right)\left(\frac{m_\chi}{T}\right)\left(\frac{T}{H}\right) > 1$$

$$\dot{\varphi}/\varphi, \quad H < \Gamma_\chi \simeq h^2 m_\chi / (8\pi)$$

Macroscopic Microscopic

- Thermal corrections under control (inflaton coupled to heavy fields) + susy to control T=0 corrections

Getting 50-60 e-fold of inflation typically requires $C_\phi \sim 10^6$

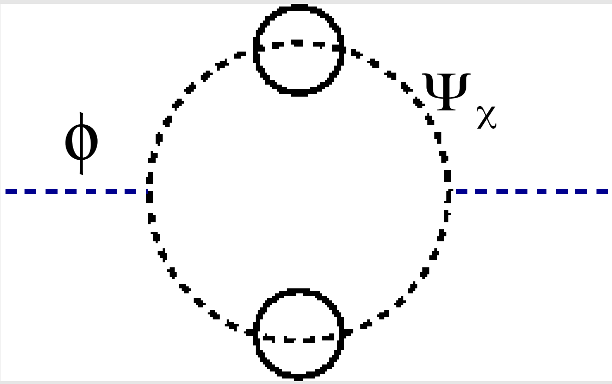
BG, Berera, & Kronberg 2015

Interactions & Dissipative coefficient

High T regime:

light scalar

$$L = \cdots - \frac{1}{2} m_\varphi^2 \varphi^2 - g\varphi \bar{\psi}_\chi \psi_\chi - h\sigma \bar{\psi}_\chi \psi_\chi + \cdots$$



light $m_\Psi = g\phi < H, T$, $g \ll 1$

$$\boxed{\Upsilon \simeq \frac{3}{1 - 0.34 \log h} \frac{g^2}{h^2} T}$$

Linear T coefficient

Adiabatic approximation:

$\Rightarrow$ $T > H$

$\dot{\varphi}/\varphi$, $H < \Gamma_\chi \simeq \frac{\pi}{512} h^4 \left(\frac{T}{H}\right)$

Macroscopic Microscopic

- Small g coupling to keep fermions light
- Not too small h because of adiabatic condition
- How to avoid thermal corrections to inflaton potential due to light fields?

Thermal potential: $$\Delta V_T = -\frac{\pi^2}{90} g_R T^4 + \frac{g^2 \varphi^2}{12} T^2 + \cdots$$

Berera, Gleiser &Ramos PRD'98; Yokoyama & Linde PRD '98

Little Higgs ⟷ Little inflaton

Naturalness problem in the SM (and inflation):

- Scalar field masses are not protected against quadratic radiative corrections by any sym. : why m_h = 125 GeV ? (why the inflaton is light m_ϕ < H ?)

(A) Susy : no. fermions = no. bosons

$$\Delta V_{T=0} \sim \Lambda^2 \mathrm{STr}\, M^2 + \sum_{F,B} (-1)^{2 s_i} (2 s_i + 1) \frac{M^4}{64\pi^2} \ln \frac{M^2}{Q^2} + \cdots$$

$$\Rightarrow \quad \Delta V_T = -\frac{\pi^2}{90} g_R T^4 + \sum_{F,B} \frac{g_i^2 \varphi^2}{12} T^2 + \cdots \quad \Rightarrow \quad \text{Thermal Higgs mass}$$

(B) Little Higgs: Pseudo-Nambu Goldstone boson of a global symmetry
(m_h ~ soft breaking)

Ex: $SU(5) \to O(5)$, $SU(5) \supset [SU(N) x U(1)]_1 x [SU(N) x U(1)]_2$

$f \sim \Lambda/(4\pi) \sim O(\text{few TeV})$

Cancellation of quadratic divergences occurs from particles of the same spin

$$\Rightarrow \quad \Delta V_T = -\frac{\pi^2}{90} g_R T^4 + C\, T^2 + \cdots \quad \Rightarrow \quad \text{No thermal Higgs mass (high T)}$$

[D. Kaplan & M. Schmaltz JHEP 0310; M. Schmaltz JHEP 0408]

Little warm inflation

- Consider a U(1) gauge theory spostaneously broken by two complex Higgs fields

$$\langle \varphi_1 \rangle = \langle \varphi_2 \rangle = M/\sqrt{2}$$

- One Nambu-Goldstone boson is "eaten" by the gauge field, and the other becomes the physical scalar inflaton field

$$\varphi_1 = \frac{M}{\sqrt{2}} e^{\varphi/M}, \qquad \varphi_2 = \frac{M}{\sqrt{2}} e^{-\varphi/M}$$

- Couple the Higgses to charged and singlet Weyl fermions:

$$L = \frac{g}{\sqrt{2}} (\varphi_1 + \varphi_2) \bar{\Psi}_{1L} \Psi_{1R} - i \frac{g}{\sqrt{2}} (\varphi_1 - \varphi_2) \bar{\Psi}_{2L} \Psi_{2R} + hc$$
$$= g M \cos(\varphi/M) \bar{\psi}_1 \psi_1 - g M \sin(\varphi/M) \bar{\psi}_2 \psi_2$$

With interchange symmetry: $\varphi_1 \longleftrightarrow i\varphi_2 \qquad \Psi_{1L,R} \longleftrightarrow \Psi_{2L,R}$

Fermion masses are bounded!!

- Light fermions: $gM < T < M$

Little warm inflation

High T regime:

Inflaton a PNGB of a broken U(1) symmetry + pair of fermions + exchange sym.

$$L = \cdots - g M \cos(\varphi/M)\bar{\psi}_1 \psi_1 - g M \sin(\varphi/M)\bar{\psi}_2 \psi_2 - h\,\sigma \sum_{i=1,2} (\bar{\psi}_i \psi_\sigma + \bar{\psi}_\sigma \psi_i) + \cdots$$

light Ψ: $gM < T < M$, $g \ll 1$

Thermal potential:

$$\Delta V_T = -\frac{\pi^2}{90} g_R T^4 + \underbrace{\frac{g^2 M^2}{12} T^2}_{\text{No thermal mass for the inflaton}} + \frac{g^4(\varphi) M^4}{16\pi^2}\left(\log\frac{\mu^2}{T^2} - c_f\right)$$

Light dof

Total energy density:

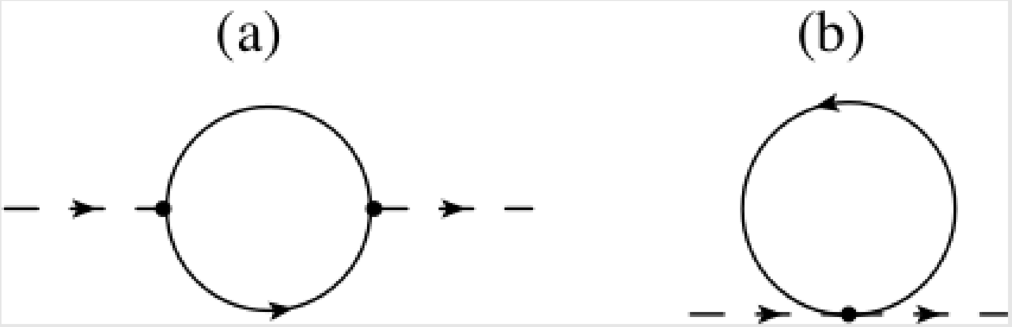
$$\rho_T = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \rho_R \qquad \left[\rho_R = \Delta V_T - T\frac{d\Delta V_T}{dT} = \frac{\pi^2}{30} g_R(\varphi, T) T^4\right]$$

Effective no. of dof:

$$g_R(\varphi, T) \simeq g_R - \frac{5}{2\pi^2}\left(\frac{gM}{T}\right)^2 + \frac{15}{16\pi^4}\left(\frac{gM}{T}\right)^4 \left(3 + \cos\left(\frac{4\varphi}{M}\right)\right)$$

Inflaton self-energy

$$L=-\sum_i \left(m_i+g_i\delta\varphi+\frac{f_i}{2}\delta\varphi^2+\cdots\right)\bar{\Psi}_i\Psi_i$$

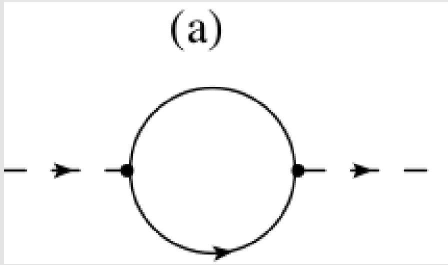


$$\Sigma_\varphi(0)=\sum_i\left(g_i^2+m_i f_i\right)I_T(0)=g^2\left(-\cos\left(2\frac{\varphi}{M}\right)+\cos\left(\frac{2\varphi}{M}\right)\right)I_T(0)=0$$

$$I_T(0)=-\frac{\Lambda^2}{2\pi^2}+\frac{T^2}{6}$$

Cancellations of quadratic divergences and thermal masses!!

Dissipation



Dissipation comes from non-local terms (diagram (a))

No cancellation of dissipative terms:

$$\Upsilon = \frac{4}{T}\sum_i \int \frac{d^3p}{(2\pi)^3} \frac{m_i^2}{\Gamma_i \omega_p^2} n_F(1+n_F) \simeq \frac{3}{1-0.34\log h} \frac{g^2}{h^2} T$$

Linear T coefficient

Decay rate $\Gamma_i = \frac{h^2}{16\pi} \frac{T^2 m_i^2}{\omega_p |p|} F_T(p/T, \omega_p/T)$ $[L = \cdots - h\sigma \sum_i (\bar{\psi}_i \psi_\sigma + \bar{\psi}_\sigma \psi_i) + \cdots]$

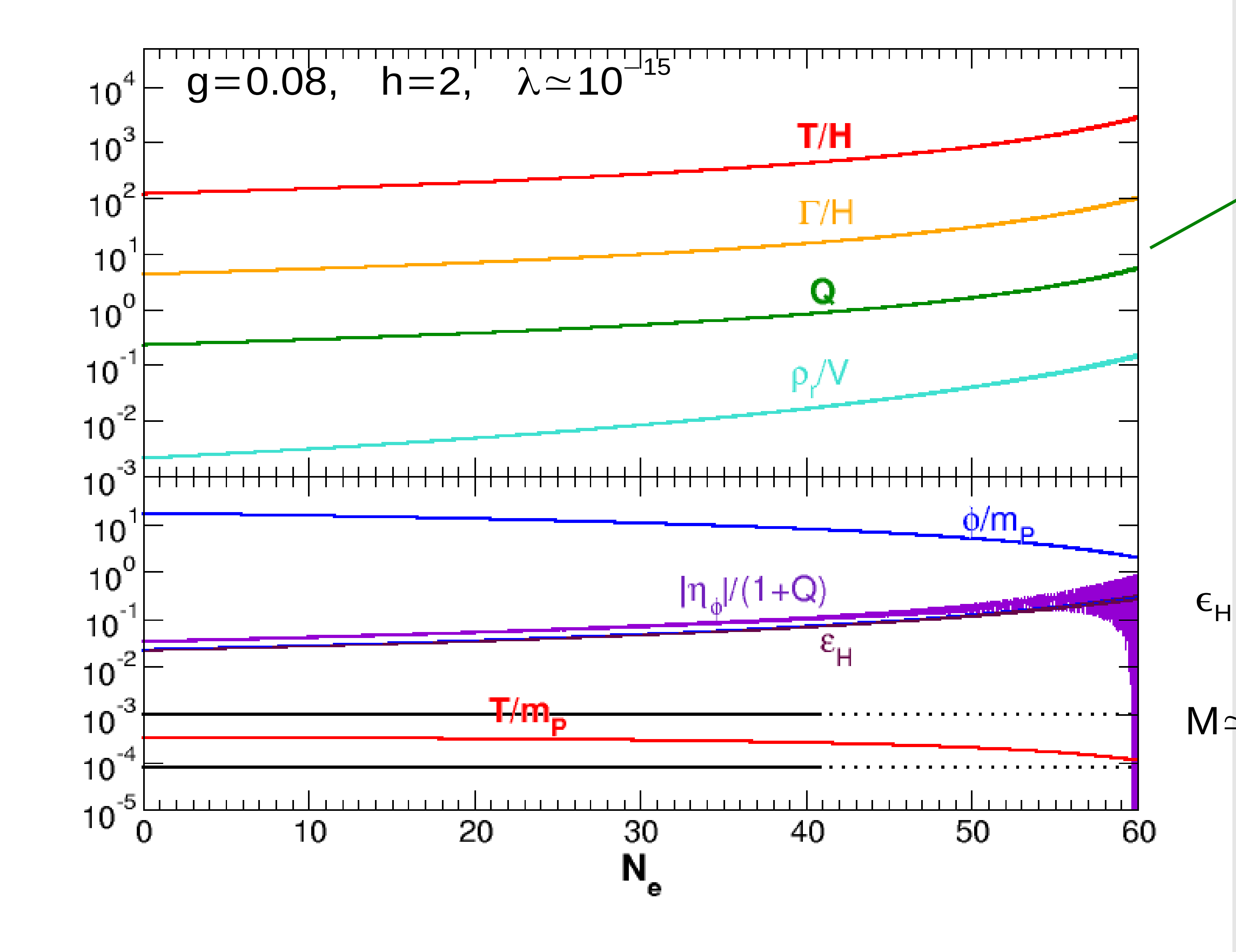
Additional Yukawa interactions with a massless field

Masses $m_i^2 \simeq h^2 T^2/8$

Yokoyama & Linde '98; Moss & Xiong '06; BG, Berera & Ramos '10

Background dynamics

Quartic potential: $V(\varphi)=\frac{\lambda}{4}\varphi^4$



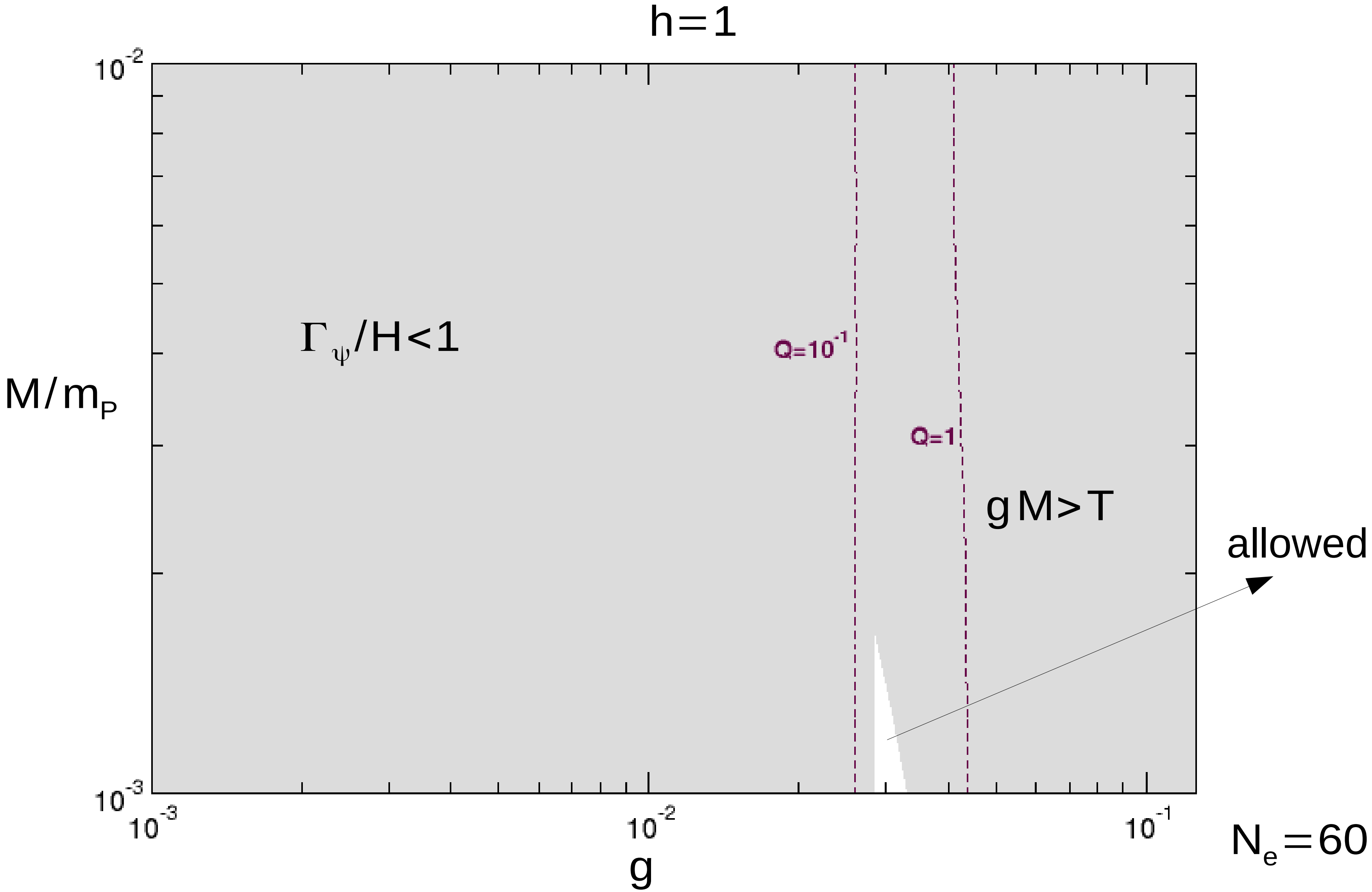
Inflation ends in the strong dissipative regime

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

$M \simeq 2 \times 10^{15}\,\mathrm{GeV}$

Parameter space: g, h, M

Decay rate: $\frac{\Gamma_\psi}{H} \simeq \frac{\pi}{514} h^4 \frac{T}{H} > 1$ Light fermions: $gM < T < M$



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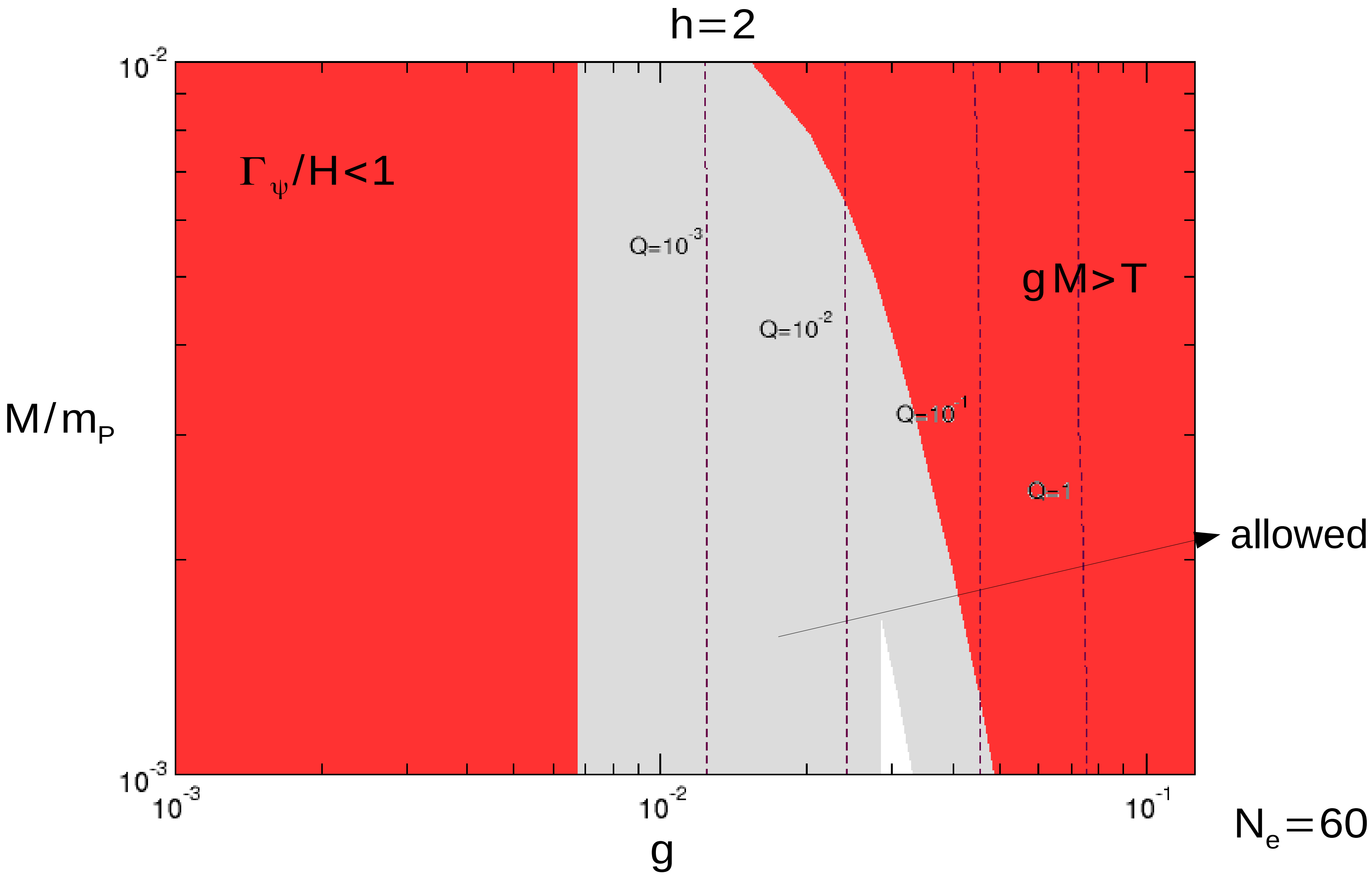

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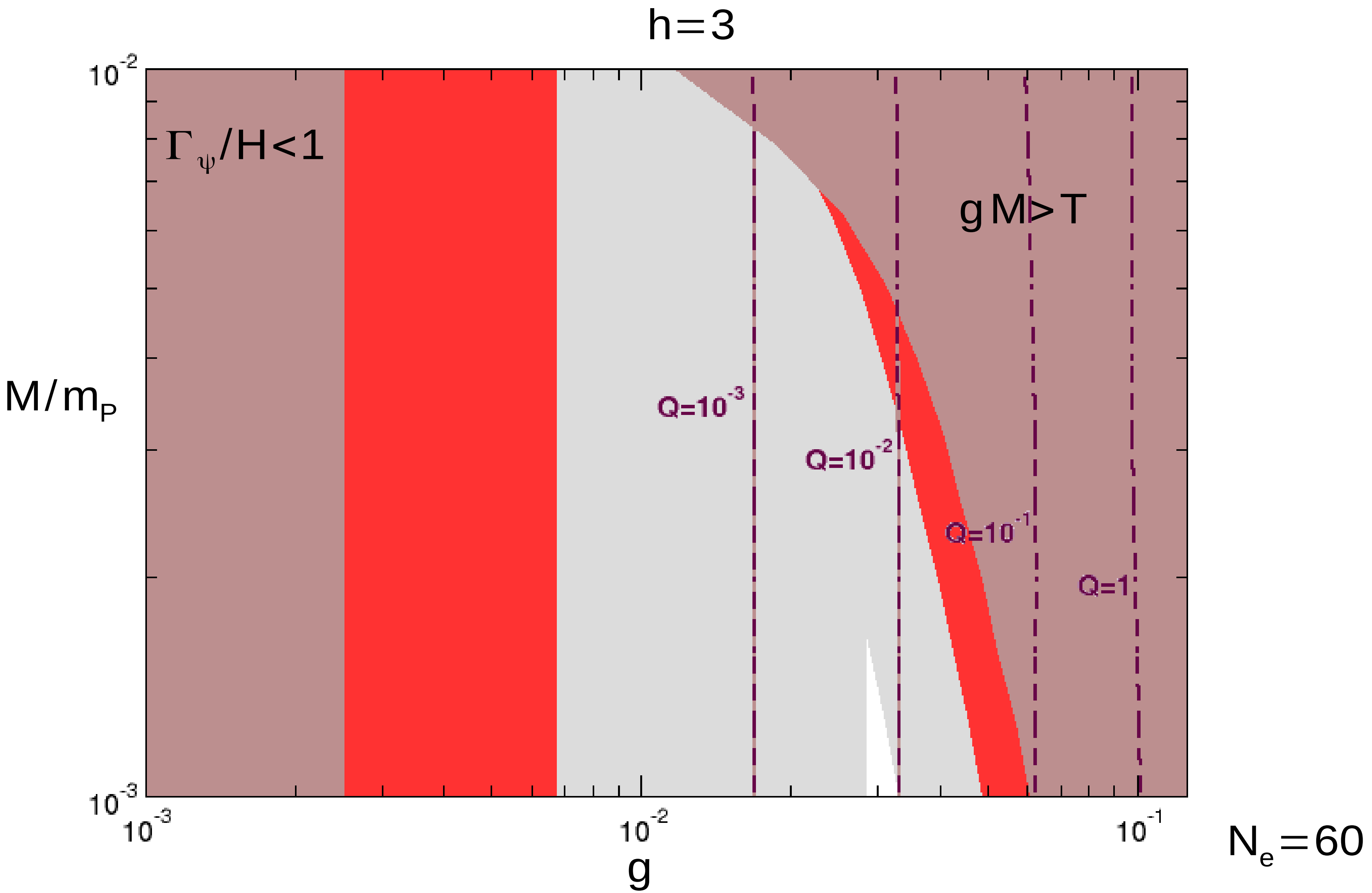


Fluctuations & primordial spectrum: coupled system

Field EOM:

$$\delta\ddot{\varphi}_k^{GI} + (3H + \Upsilon)\delta\dot{\varphi}_k^{GI} + \dot{\varphi}\,\delta\Upsilon^{GI} + \left(\frac{k^2}{a^2} + V_{\varphi\varphi}\right)\delta\varphi_k^{GI} \simeq (2\Upsilon T)^{1/2}\hat{\xi}_k$$

$$\boxed{\frac{\delta\Upsilon^{GI}}{\Upsilon} = \frac{1}{4}\frac{\delta\rho_r^{GI}}{\rho_r} \simeq \frac{\delta T}{T}}$$

Coupled system
inflaton-radiation

Radiation (fluid stress energy-tensor): $T^{\mu\nu}_{rad} = (\rho_r + p_r)u^\mu u^\nu + p_r g^{\mu\nu}$

$$\delta\dot{\rho}_r^{GI} + 4H\delta\rho_r^{GI} \simeq \frac{k^2}{a^2}\Psi_r^{GI} + \dot{\varphi}^2\delta\Upsilon^{GI} + 2\dot{\varphi}\Upsilon\delta\dot{\varphi}^{GI}$$

$$\dot{\Psi}_r^{GI} + 3H\Psi_r^{GI} \simeq -\delta\rho_r^{GI}/3 - \dot{\varphi}\Upsilon\delta\varphi^{GI}$$ Momentum density

(Gauge invariant perturbations: $\delta\varphi_k^{GI} = \delta\varphi - \frac{H}{\dot{\varphi}}\phi$, ϕ :metric perturbation)

BG, Berera, Moss & Ramos, '14

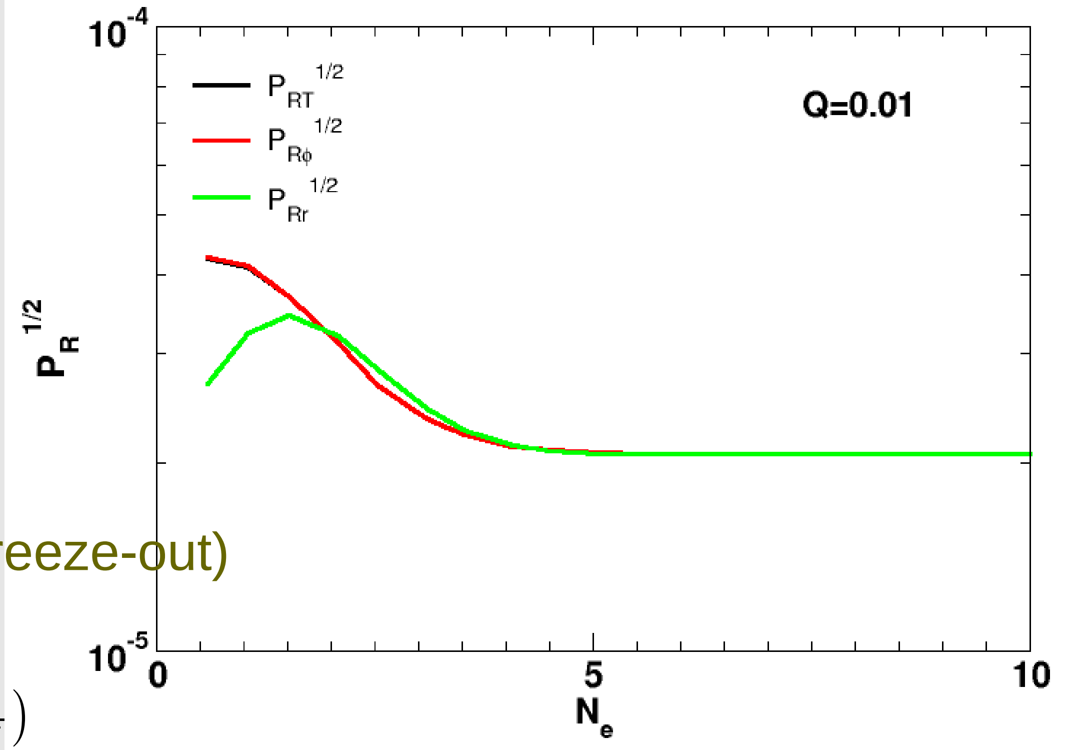
Fluctuations & primordial spectrum: coupled system

Weak dissipative regime ($Q=\Upsilon/H<<1$) : field decoupled from radiation

$$\delta\ddot{\varphi}_k^{GI}+(3H+\Upsilon)\delta\dot{\varphi}_k^{GI}+(\frac{k^2}{a^2}+V_{\varphi\varphi})\delta\varphi_k^{GI}\simeq(2\Upsilon T)^{1/2}\hat{\xi}_k$$

$$P_{\delta\varphi}\simeq\frac{HT}{2\pi}\frac{Q}{\sqrt{1+4\pi Q/3}}$$

Primordial spectrum: $P_R\simeq\left(\frac{H}{\dot{\varphi}}\right)^2 P_{\delta\varphi}$



R is constant after horizon crossing (freeze-out)

$$P_R\simeq(P_R)_{Q=0}(\underbrace{1+2N}+\frac{T}{H}\frac{4\pi Q}{\sqrt{1+4\pi Q/3}})$$

Dissipative processes may maintain a non-trivial distribution of inflaton particles:

$$N\simeq n_{BE}=(e^{k/aT}-1)^{-1}$$

Ramos, da Silva, 1302.3544; BG, Berera, Moss & Ramos, 1401.1149

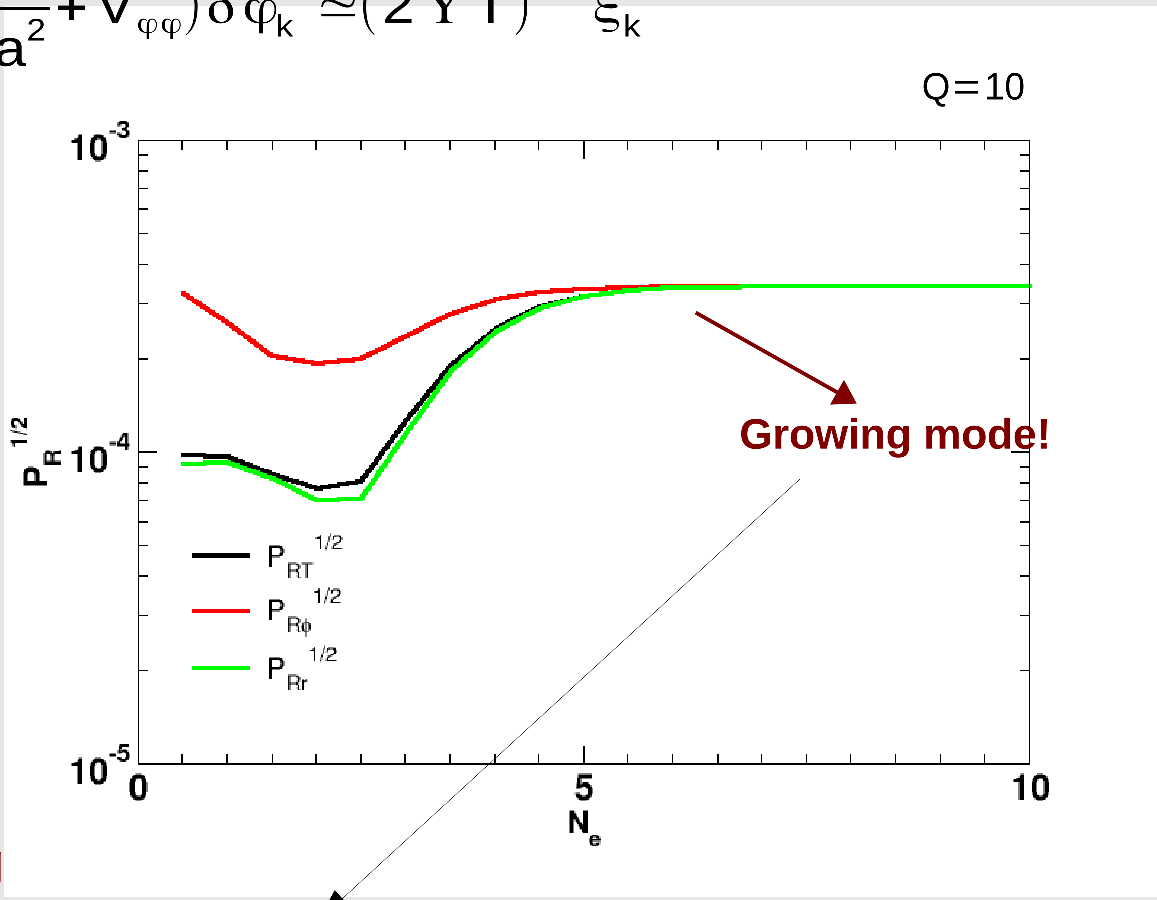
Fluctuations & primordial spectrum: coupled system

Strong dissipative regime ($Q=\Upsilon/H>1$) : coupled system

$$\delta\ddot{\varphi}_k^{GI}+(3H+\Upsilon)\delta\dot{\varphi}_k^{GI}+\dot{\varphi}\,\delta\Upsilon^{GI}+\left(\frac{k^2}{a^2}+V_{\varphi\varphi}\right)\delta\varphi_k^{GI}\simeq(2\Upsilon T)^{1/2}\hat{\xi}_k$$

Primordial spectrum:

$$P_R=\frac{h_\varphi}{h_T}P_{R_\varphi}+\frac{h_r}{h_T}P_{R_r}\simeq P_{R_r}\simeq P_{R_\varphi},\qquad (h_i=\rho_i+p_i)$$

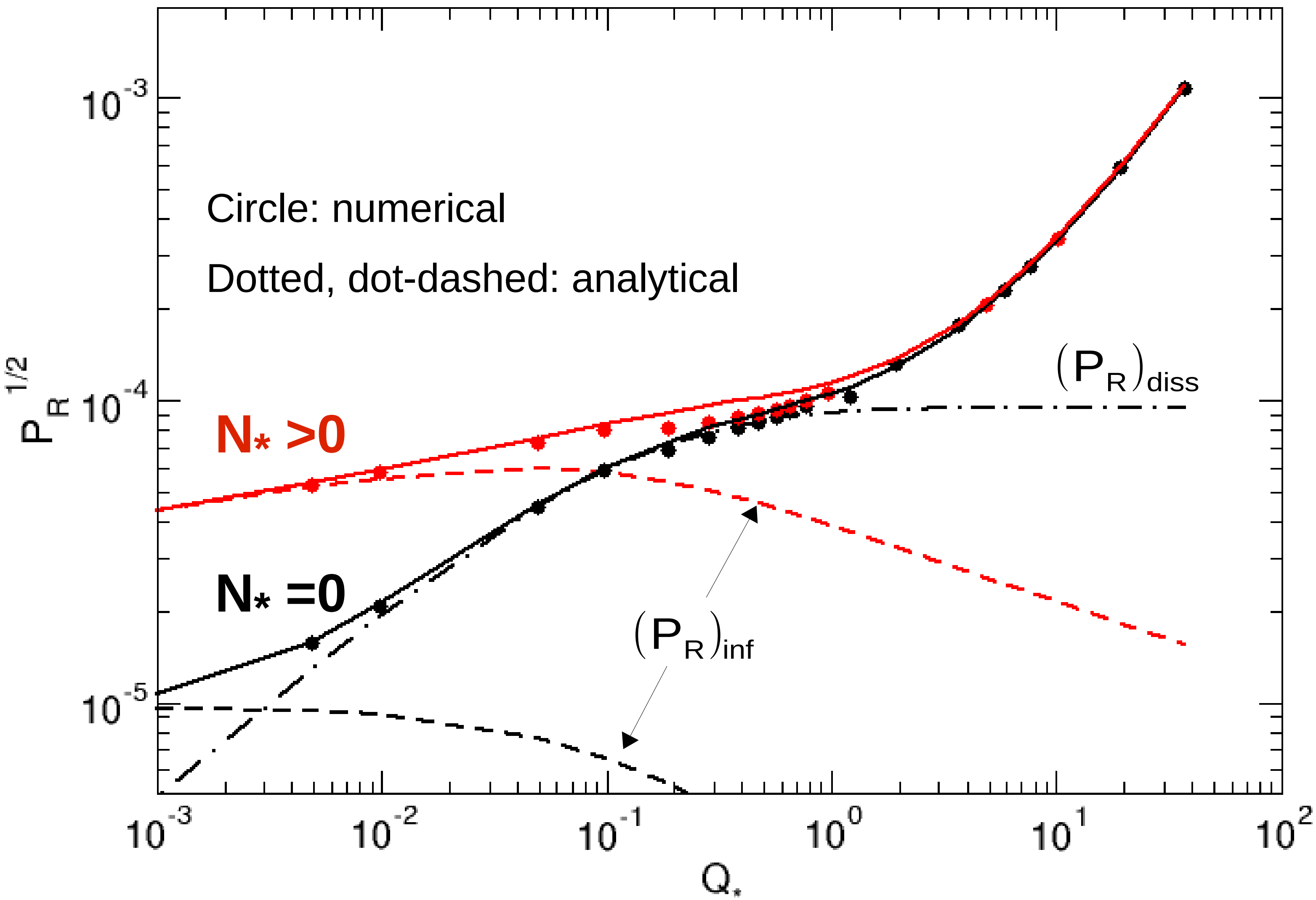


R is constant after horizon crossing

$$P_R\simeq\left(\frac{H}{\dot{\varphi}}\right)^2\left(\frac{H}{2\pi}\right)^2\left(1+2N+\frac{T}{H}\frac{4\pi Q}{\sqrt{1+4\pi Q/3}}\right)\times G[Q],\qquad Q=\Upsilon/(3H)$$

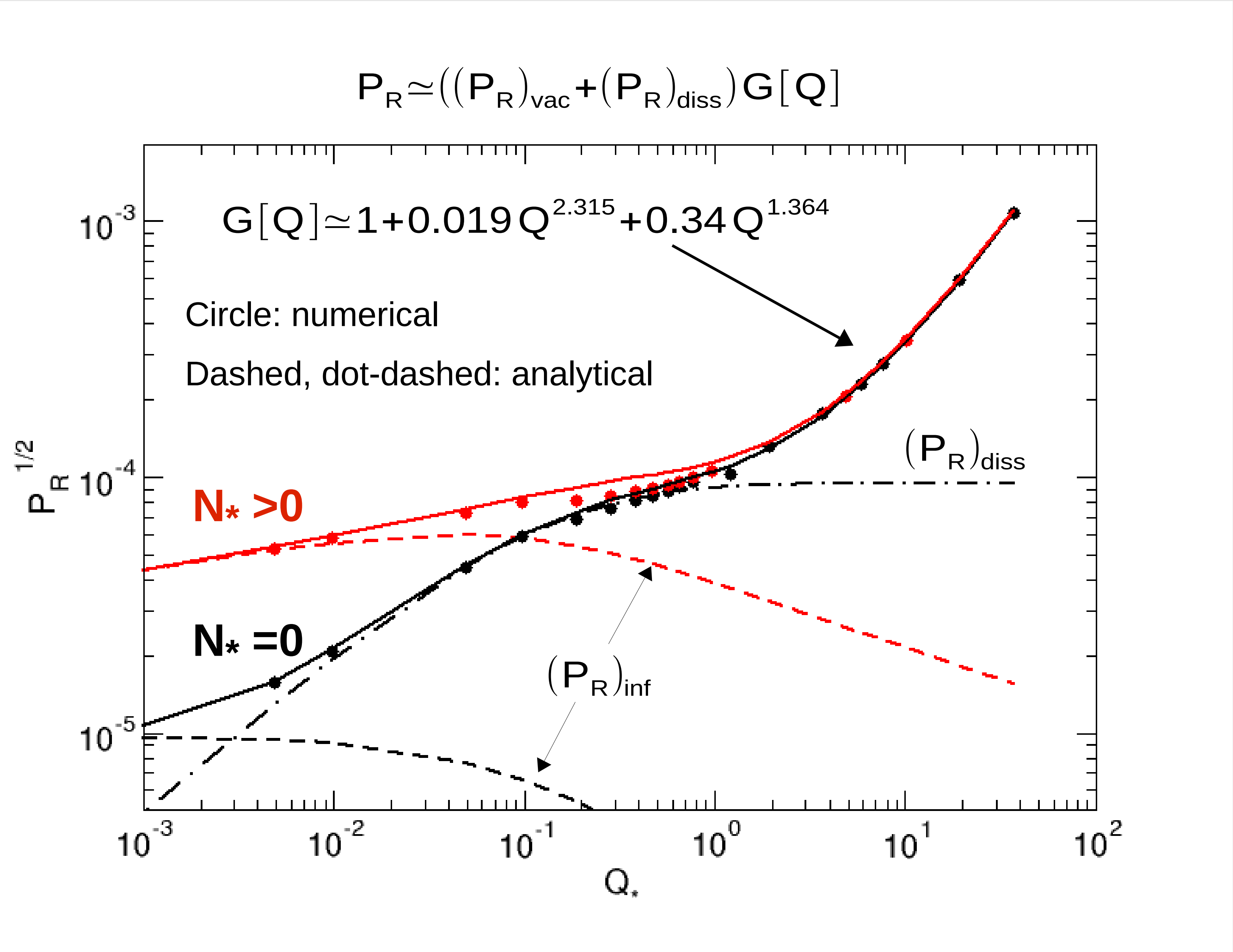
Primordial spectrum

$$P_R \simeq ((P_R)_{vac} + (P_R)_{diss}) G[Q]$$



Chaotic model: $V(\varphi) = \lambda \varphi^4/4, \quad \lambda = 10^{-14}, \ N_e = 50$

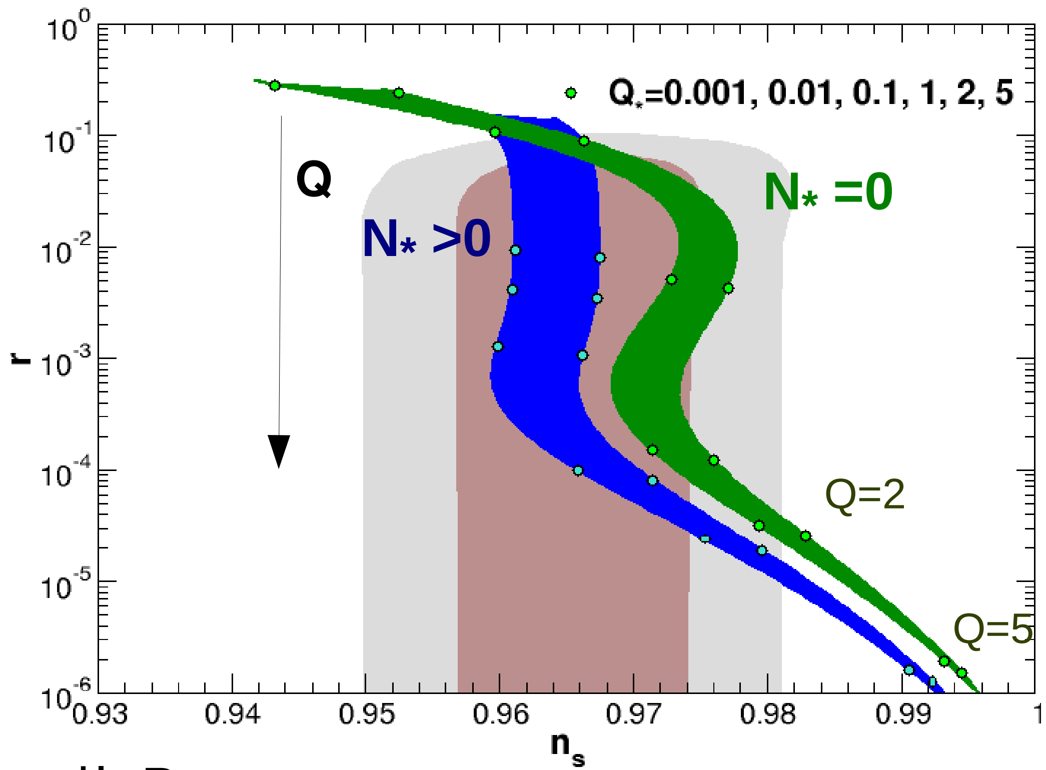
Primordial spectrum



Chaotic model: $V(\varphi) = \lambda \varphi^4 / 4, \quad \lambda = 10^{-14}, \;\; N_e = 50$

Primordial spectrum: quartic chaotic model

$$V(\varphi)=\frac{\lambda}{4}\varphi^{4}, \quad N_e=50-60$$



$$n_s-1=\frac{d\ln P_R}{dN_e}=(n_s-1)_N+(n_s-1)_{diss}+(n_s-1)_G, \quad (n_s-1)_G>0$$

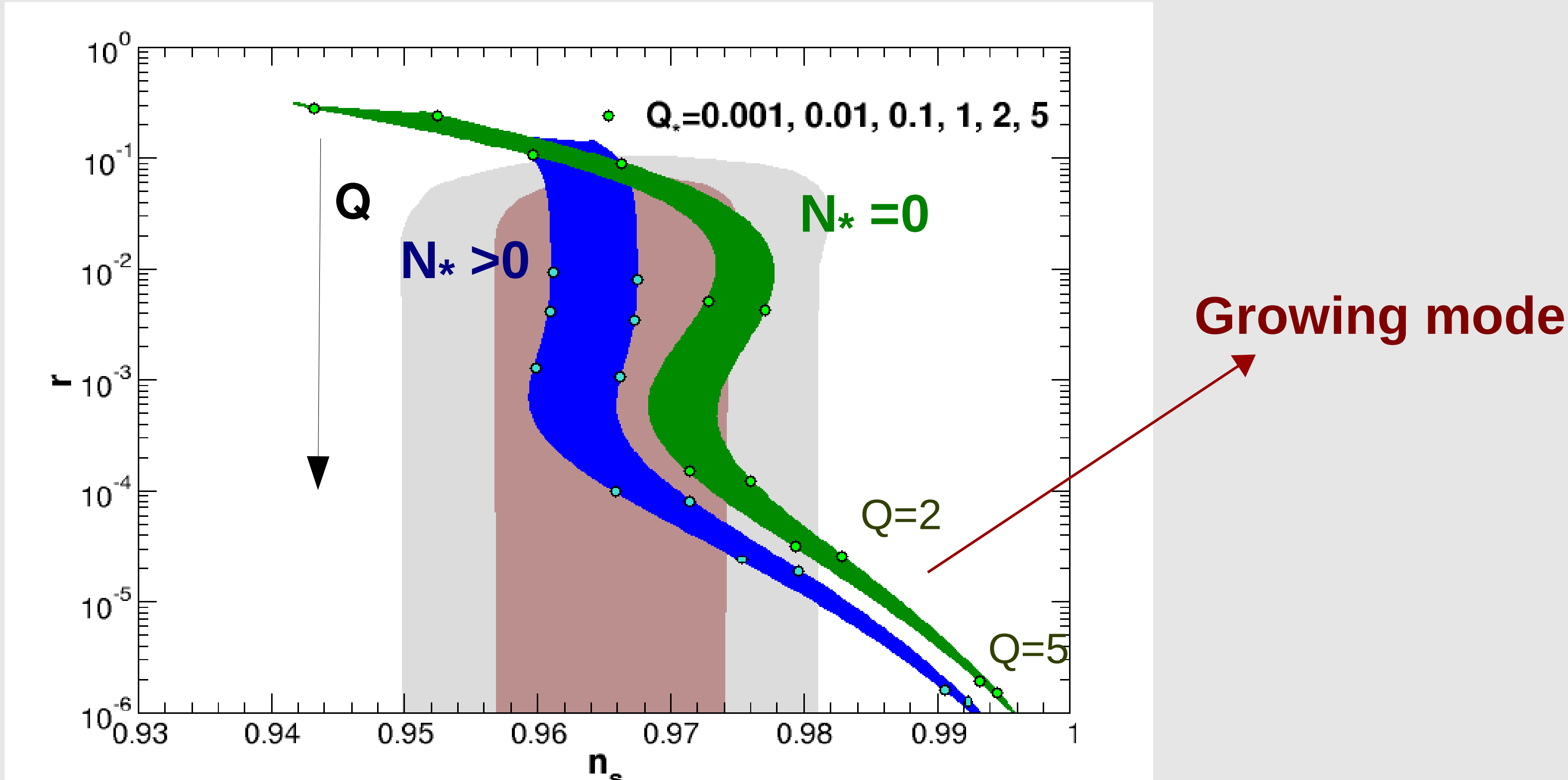
$$r\simeq\frac{16\epsilon_\phi}{(1+2N+\Delta_Q)G[Q]}\leq 16\epsilon_\varphi$$

Quartic:

$$\boxed{N\neq 0, Q<1: \; n_s\simeq 1-2/N_e, \quad r\simeq 16\epsilon_\varphi\left(\frac{H}{T}\right)\ll 0.1}$$

Primordial spectrum: quartic chaotic model

$$V(\varphi)=\frac{\lambda}{4}\varphi^{4}, \quad N_e=50-60$$



$$n_s-1=\frac{d\ln P_R}{dN_e}=(n_s-1)_N+(n_s-1)_{diss}+(n_s-1)_G, \quad (n_s-1)_G>0$$

$$r\simeq\frac{16\,\epsilon_\phi}{(1+2N+\Delta_Q)\,G[Q]}\leq 16\,\epsilon_\varphi$$

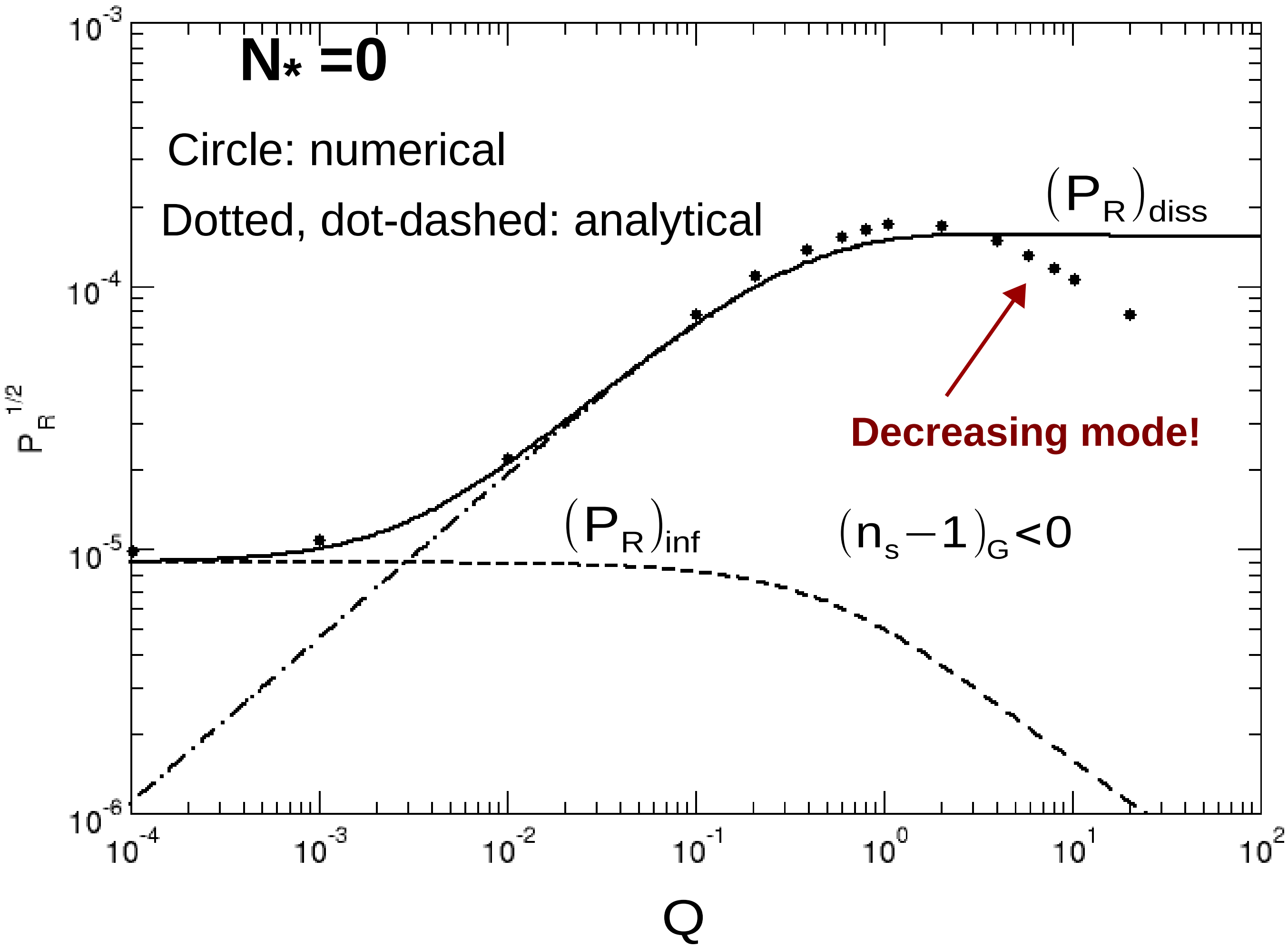
Quartic:

$$N\neq 0, Q<1: \quad n_s\simeq 1-2/N_e, \quad r\simeq 16\,\epsilon_\varphi\left(\frac{H}{T}\right)\ll 0.1$$

Primordial spectrum: Υ = C/T

Light bosons decaying into light dof

$$P_R \simeq ((P_R)_{vac} + (P_R)_{diss}) G[Q]$$



Chaotic model: $V(\varphi) = \lambda\varphi^4/4$, $\lambda = 10^{-14}$, $N_e = 50$

Summary

- Dissipative effects due to decaying fields can be relevant during inflation, and modify the inflationary predictions

- "Low T" regime for dissipation (massive scalar χ decaying into light dof): thermal corrections under control, but required large number of fields $N_\chi \sim 10^6$

- "High T" regime for dissipation (light fermion ψ decaying into light dof): $\Upsilon = C_T\, T$

  Inflaton a PNGB of a broken U(1) symmetry + pair of fermions + exchange sym.

  Light fermions: $gM < T$ + thermal corrections under control + minimal matter content

  $\boxed{\lambda\phi^4 \text{ compatible with data, } Q^* \sim 0.01\text{-}1,\ r \sim 0.1\text{-}10^{-4}}$

- For a T dependent dissipative coefficient, the field and radiation perturbation EOM form a coupled system: Field fluctuations are amplified before freeze-out ($Q < 1$)

  Blue-tilted spectrum for $Q >> 1$

- Non-gaussianity compatible with observations for both weak and strong dissipative regime, with a characteristic shape

Warm inflation &Non-gaussiantiy : T dependent diss. coefficient

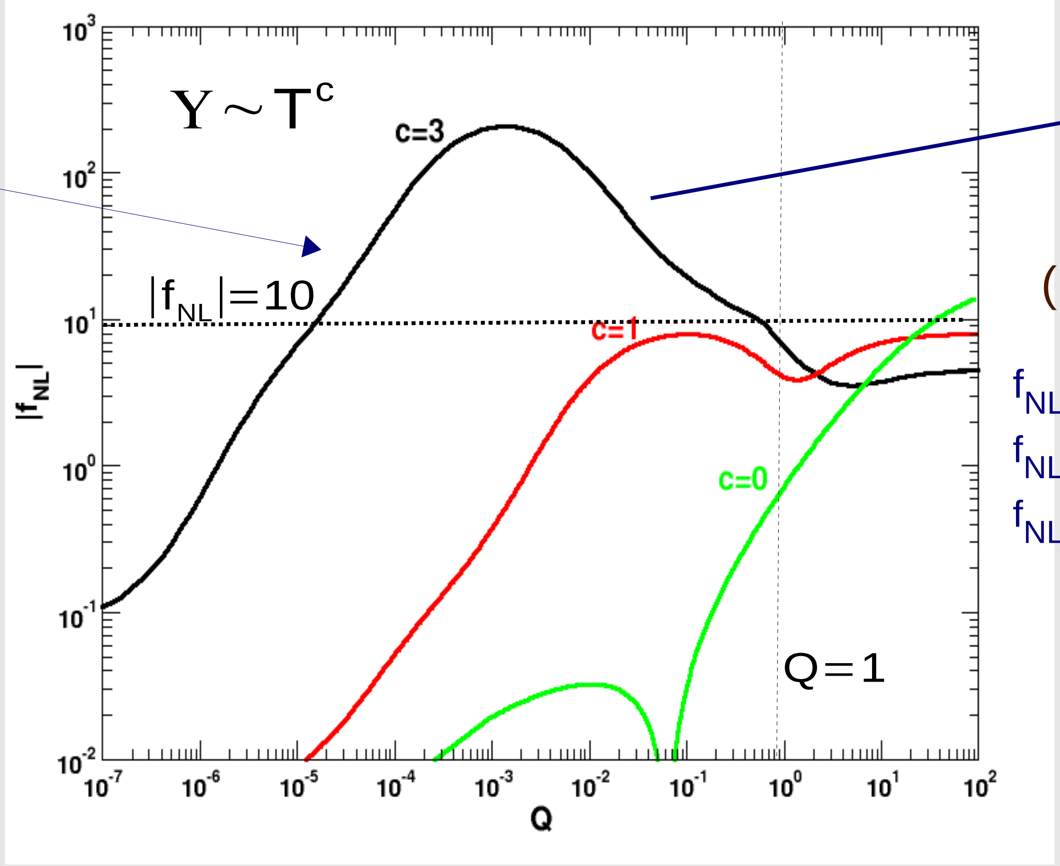
- **Bispectrum:** $B_R(k_{1,} k_{2,} k_3) = \sum_{cyc} \langle R_1(k_1) R_1(k_2) R_2(k_3) \rangle = A_B(k) \bar{B}(k_{1,} k_{2,} k_3)$

shape

$$f_{NL} = \frac{18}{5} \frac{A_B(k)}{P_R(k)^2}$$ Non-linear parameter

$$P_R \simeq ((P_R)_{vac} + (P_R)_{diss}) F[Q]$$

$(P_R)_{vac} > (P_R)_{diss}$



$(P_R)_{vac} < (P_R)_{diss}$

(Planck: f_{NL}<~O(10))

f_{NL}^{local} = 2.7 +- 5.8 $(k_3 << k_1 \sim k_2)$
f_{NL}^{equil} =-42 +- 75 $(k_3 \sim k_1 \sim k_2)$
f_{NL}^{warm} = 4 +- 33

[BG, Berera, Moss, Ramos '14]