

# MAGNETIC SUPERSYMMETRY BREAKING

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Based on :

W. Buchmuller, M. Dierigl, E. D & J. Schweizer,  
arXiv:1611.03798 [hep-th]

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# Outline

- 1) Higher-dimensional completions of the Standard Model
- 2) Magnetic compactifications
- 3) Effective field theory
- 4) Quantum corrections, Wilson lines as goldstone bosons
- 5) Conclusions

# 1) Higher-dimensional completions of the Standard Model

- Structure of Standard Model points towards **Grand Unification** (matter content, gauge group, unification of gauge couplings, neutrino masses)
- Strong theoretical arguments for **supersymmetry** at high scales (gravity, string theory)
- **Extra dimensions** unavoidable in string theory



Extra dims. do address:

- Unification of all forces (including at TeV energies)
- Holographic solutions to the hierarchy problem (RS)
- New ways to **break SUSY**: Scherk-Schwarz
- New models of **inflation**
- Sometimes **higher-dim. symmetries** protect quantum corrections in a way invisible from 4d.

Ex: Internal comp. of a gauge field protected by **gauge symmetry**

$$\delta m_0^2 \sim (\text{loop}) \times \frac{1}{R^2}$$

**Orbifolds** are a natural framework to compactify higher-dimensional theories:

- Fermion **chirality**
- Doublet-triplet **splitting** in orbifold GUT's
- Partial or total **supersymmetry breaking**
- **Compactification scale**  $M_c = R^{-1}$  usually defines the GUT/unification scale.
- Scale of supersymmetry breaking  $M_{SUSY}$  usually much smaller.

## 2) Magnetic compactifications



Widely studied in string theory (« **intersecting branes** »), few papers in field theory.

Consider a 6-dim. theory :  $x_0 x_1 x_2 x_3 x_5 x_6$

An internal magnetic field  $F_{56} = B = f$

- break SUSY, due to the magnetic moment coupling

$$H = -\mu \mathbf{B} = -\frac{q}{m} \mathbf{S} \mathbf{B}$$

- Turns KK states  $k_1, k_2$  into **Landau levels**  $n$ , mass

$$\delta M^2 = (2n + 1) |qB| + 2qB \Sigma_{56}$$

where  $\Sigma_{45}$  is the **internal helicity** of particles.



- An internal magnetic field is **quantized**

$$f = \frac{N}{R^2} \sim M_{\text{GUT}}^2, \quad N = \text{integer flux}$$

- Each Landau level is **N times degenerate**.
- Precisely **N chiral fermion zero modes**.

- Starting with a SUSY 6d theory, it is usually said that the effect of the magnetic field is to add a **D-term Fayet-Iliopoulos (FI) term** in 4d

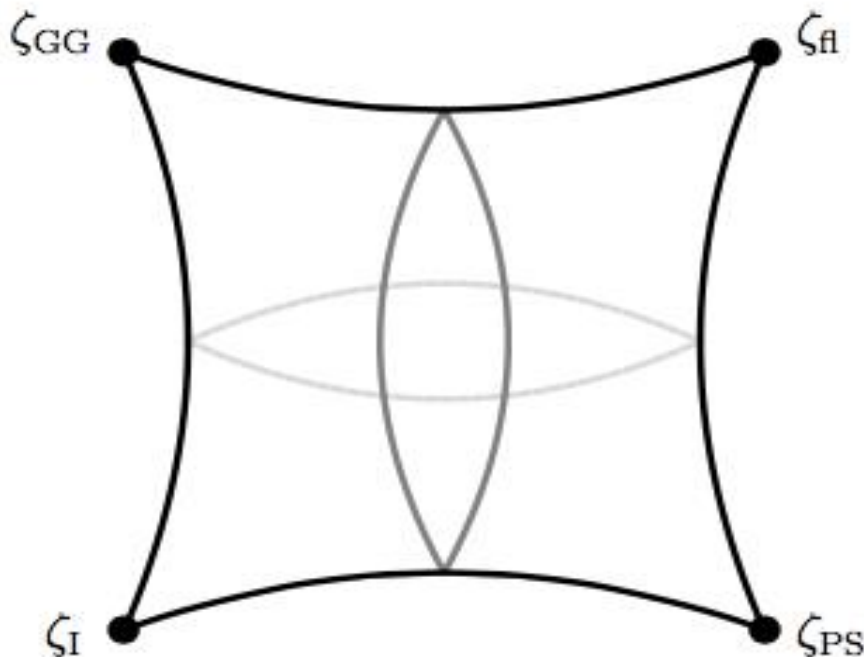
$$D = f \quad \rightarrow \quad V = \frac{1}{2} D^2 = \frac{1}{2} f^2 \sim M_{\text{GUT}}^4$$

- ◆ Potential relevance to **inflation**

**One example** (Buchmuller, Dierigl, Ruehle, Schweizer, 2015, 2016) :



- SO(10) GUT model in 6d, broken at orbifold fixed points, Standard Model in 4d
- Bulk fields  $45, 16, 16^*, 10's$



$$SO(10) \rightarrow U(1)_A$$



In 4d there are  $N$  16's from **charged** bulk 16-plet with  $N$  flux quanta :

$$16 [SO(10)] \leftarrow 5^k + 10 + 1 [SU(5)] \leftarrow q, l, u^c, e^c, d^c, \langle \otimes \rangle [G_{SM}]$$

- Higgs fields from **uncharged** bulk 10-plets form split multiplets.
- Magnetic flux break SUSY (Bachas;95); « soft » SUSY breaking only for quark-lepton families:

$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15} \text{ GeV})^2$$

$$m_{3/2} \sim 10^{14} \text{ GeV}, \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

However, **not enough**; there is **no mass gap**:  
 soft mass given by the FI term of the same order ( $\propto 1/R^2$ ) as the masses of Landau levels



one needs an **effective theory for the whole tower.**

### 3) Effective field theory



- Consider an abelian 6d SUSY theory compactified on a torus.

N=2 SUSY in 4d before the magnetic flux;

4d Multiplets: **vector**  $(V, \phi)$   
**charged hyper**  $(Q, \tilde{Q})$

6d effective action in superfields: (Marcus, Sagnotti, Siegel ;  
 Arkani-Hamed, Gregoire, Wacker)

$$\begin{aligned}
 S_6 = \int d^6x \left\{ \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left( \partial V \bar{\partial} V + \phi \bar{\phi} + \sqrt{2} V (\bar{\partial} \phi + \partial \bar{\phi}) \right) \right. \\
 \left. + \int d^2\theta \tilde{Q} (\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left( \bar{Q} e^{2gqV} Q + \tilde{Q} e^{-2gqV} \tilde{Q} \right) \right\} \\
 \partial = \partial_5 - i\partial_6, \quad \phi|_{\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}} (A_6 + iA_5)
 \end{aligned}$$



$\phi$  are **internal components of gauge fields** =  
**Wilson lines**

Mode expansions with **flux**:

$$\phi_0|_{\theta=\bar{\theta}=0} = \frac{f}{2\sqrt{2}} (x_5 - ix_6) + \varphi, \quad \varphi = \frac{1}{\sqrt{2}} (a_6 + ia_5)$$

$$Q(x_M) = \sum_{n,j} Q_{n,j}(x_\mu) \psi_{n,j}(x_m) = \sum_{n,j} Q_{n,j}(x_\mu) \frac{1}{\sqrt{n!}} (a^\dagger)^n \psi_{0,j}(x_m),$$

$$\bar{Q}(x_M) = \sum_{n,j} \bar{Q}_{n,j}(x_\mu) \bar{\psi}_{n,j}(x_m) = \sum_{n,j} \bar{Q}_{n,j}(x_\mu) \frac{1}{\sqrt{n!}} (a)^n \bar{\psi}_{0,j}(x_m).$$

where (**harmonic oscillator algebra**)

$$a = \sqrt{\frac{1}{-2qgf}} (iD_5 - D_6)$$

$$a^\dagger = \sqrt{\frac{1}{-2qgf}} (iD_5 + D_6)$$

The final 4d effective action for Landau levels is

FI term



$$\begin{aligned}
 S_4^* = \int d^4x \left[ \int d^4\theta \left( \bar{\varphi}\varphi + \sum_{n,j} (\bar{Q}_{n,j} e^{2ggV_0} Q_{n,j} + \bar{\tilde{Q}}_{n,j} e^{-2ggV_0} \tilde{Q}_{n,j}) + 2fV_0 \right) \right. \\
 + \int d^2\theta \left( \frac{1}{4} \mathcal{W}_0^\alpha \mathcal{W}_{\alpha,0} \right. \\
 \left. \left. + \sum_{n,j} \left( -i\sqrt{-2qgf(n+1)} \tilde{Q}_{n+1,j} Q_{n,j} + \sqrt{2qg} \tilde{Q}_{n,j} \varphi Q_{n,j} \right) \right) + \text{h.c.} \right]
 \end{aligned}$$

Mass terms



One also found the effective action for the **non-abelian case**

## 4) Quantum corrections, Wilson lines as goldstone bosons

Interested in Higgs = internal component of the gauge field.  
6d gauge symmetry could protect its mass ?

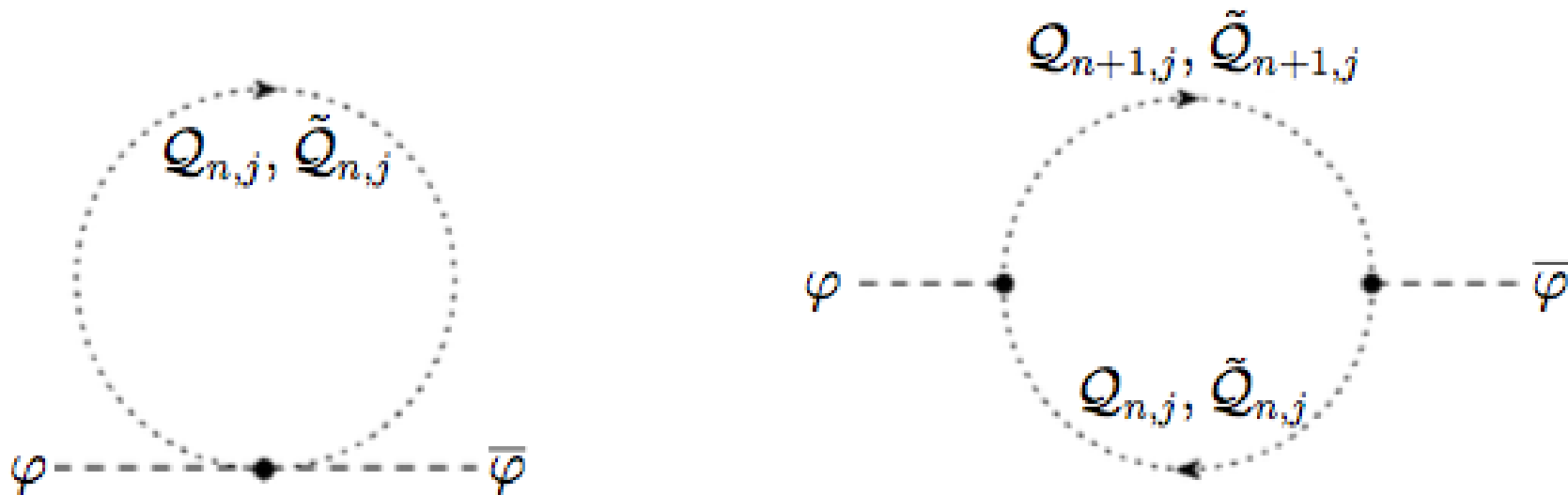
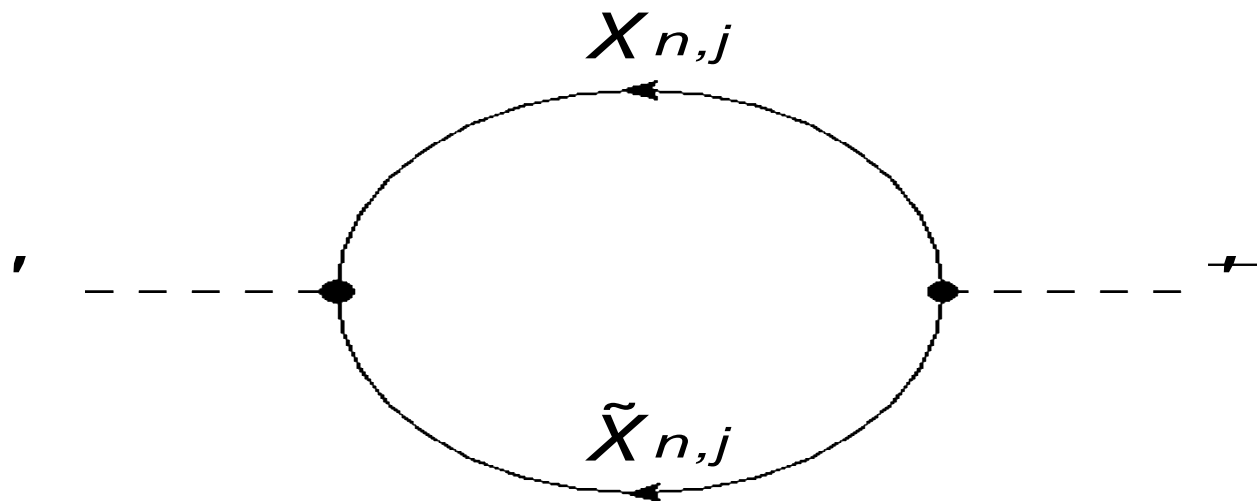


Figure 3: Bosonic contributions to the Wilson line mass with flux.

Each contribution is **quadratically divergent**: the sum of the whole tower is however **exactly zero** !

$$\begin{aligned}
 \delta m_b^2 &= -4g^2 g^2 |N| \int_0^\infty \frac{d^4 k}{(2\pi)^4} \left[ \frac{n}{k^2 + i(n + \frac{1}{2})} - \frac{n+1}{k^2 + i(n + \frac{3}{2})} \right] \\
 &= -\frac{g^2 g^2}{4\pi^2} |N| \int_0^\infty dt \frac{1}{t^2} \left[ n e^{-i(n + \frac{1}{2})t} - (n+1) e^{-i(n + \frac{3}{2})t} \right] \\
 &= -\frac{g^2 g^2}{4\pi^2} |N| \int_0^\infty dt \frac{1}{t^2} \left[ \frac{e^{i(n + \frac{1}{2})t}}{(e^{it} - 1)^2} - \frac{e^{i(n + \frac{3}{2})t}}{(e^{it} - 1)^2} \right] = 0
 \end{aligned}$$

The same is true for the fermionic contribution



Is there's a **symmetry reason** ?



## Action of charged matter fields invariant under translations

$$S_6 = \int d^6x \left( -D_M \bar{Q} D^M Q \right), \quad D_M Q = (\partial_M + i q g A_M) Q$$

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta A_n = \epsilon^m \partial_m A_n$$

## Symmetries for constant Wilson line background

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta a_n = 0$$

Flux background breaks the symmetries **spontaneously**

$$D_m Q = \left( \partial_m + i q g \left( a_m + \frac{f}{2} \epsilon_{mn} x_n \right) \right) Q, \quad \langle A_m \rangle = \frac{f}{2} \epsilon_{mn} x_n$$

Translational symmetries now **non-linearly realized**  
with Wilson lines as **Goldstone bosons**

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta a_n = \epsilon^m \frac{f}{2} \epsilon_{nm}$$

# Conclusions, Perspectives

- ◆ Strong theoretical arguments for **SUSY** at high **scales**: gravity, string theory
- ◆ Energy scale of **grand unification**  $M_{GUT} \sim 10^{16} \text{GeV}$   
Scale of **SUSY breaking**  $M_{SUSY} ?$
- ◆ **Magnetized** compactifications : high-scale SUSY breaking

$$M_{SUSY} \sim M_{GUT} \sim R^{-1}$$

- ◆ Hope for a **higher-dim. protection** of scalar masses: Higgs mass, inflaton.
- ◆ Various **applications** possible: moduli stabilization, inflation, orbifold GUT's.

# Thank you