Bell violation in the Sky

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Abstract: In this work, we have studied the possibility of setting up Bell's inequality violating experiment in the context of cosmology, based on the basic principles of quantum mechanics. First we start with the physical motivation of implementing the Bell's inequality violation in the context of cosmology. Then to set up the cosmological Bell violating test experiment we introduce a model independent theoretical

Key references

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Outline

- Introduction.
- Bell's inequality in QM.
- Cosmological Bell violating setup.
- Role of new particles (Bell pairs).
- Analogy with axion fluctuations from String Theory.
- Role of isospin breaking interactions.
- Role of spin.
- Bottom lines and future prospects.

Introduction

- According to the inflationary paradigm, primordial fluctuations seen in CMB were produced by quantum mechanical effects in the early universe.

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- An important problem in theoretical physics is to find compelling evidence for their quantum nature.
- In other words, one would like to rule out alternative scenarios where the fluctuations originated through classical statistical mechanics during an inflationary phase.
- Such fluctuations provide seed for large scale structure formation.
- The fluctuations that we observe at present are classical in nature.

- During the QM interpretation of the required fluctuations, highly entangled QM wave function of the universe plays significant role.

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- In the context of QM, Bell experiment is described by the measurement of two non-commutating physical operators which are associated with two distinctive locations in the space-time (Alice 's and Bob's location).

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- In the context of QM, Bell experiment is described by the measurement of two non-commutating physical operators which are associated with two distinctive locations in the space-time (Alice 's and Bob's location).
- Similarly in the context of primordial cosmology, one can also perform cosmological observations on two spatially separated + causally disconnected places upto the epoch of reheating (after inflation).

- Apart from the success in its observational ground it is important to point that for all such observations it is impossible to measure the value of associated canonically conjugate momentum.

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- Consequently, for these observables it is impossible to measure the imprints of two non-commuting operators in primordial cosmology.
- In QM, to design such setup one needs to perform repeated measurement on the same object (here it is the same quantum state of the universe).
- In such a case one can justify the appearance of each and every measurement through a single quantum state.

- In the context of cosmology, one can similarly consider two spatially separated portions in the full sky which exactly mimics the role of performing repeated cosmological Bell's inequality violating experiment via the same QM state.

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- Here one can choose the appropriate properties of two spatially separated portions in the full sky to setup Bell's inequality violating setup in cosmology.
- If it is possible to connect a link between non-commuting cosmological observables and a classical probability distribution originated from inflationary paradigm, then it is possible to setup a Bell's inequality violating setup in cosmology.

Bell's inequality in QM

- Concept of reality = Real properties of microscopic objects determines the results of quantum mechanical experiments,
- Concept of locality = Reality in one place is not affected by experiments done at the same time at a distant place.

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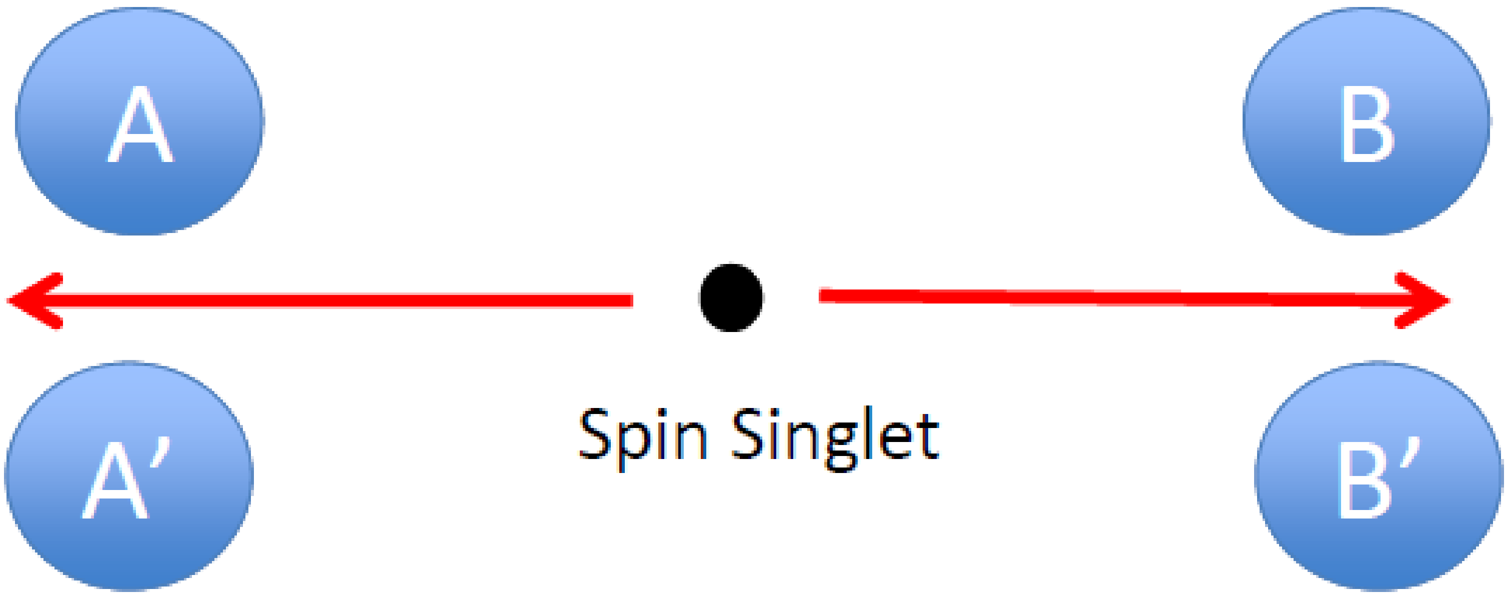
Bell's inequality=No local hidden variable theory is compatible with quantum mechanics.- *John Stewart Bell (1964).*

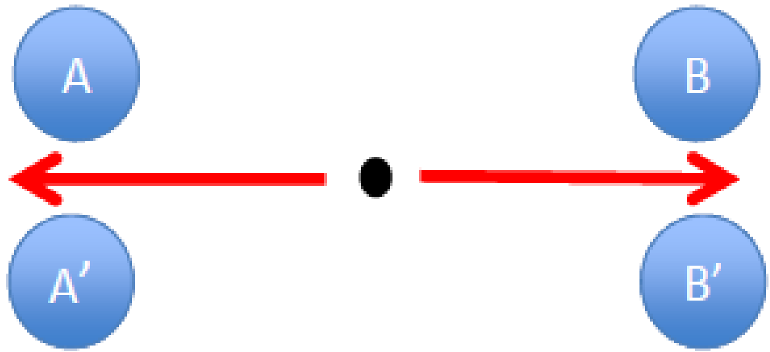
Bell Inequality

Bell 1964



All operators, A, A', B, B' have eigenvalues +1 or -1 .

e.g. $A = \vec{n}.\vec{\sigma}$, $A' = \vec{n}'.\vec{\sigma}$

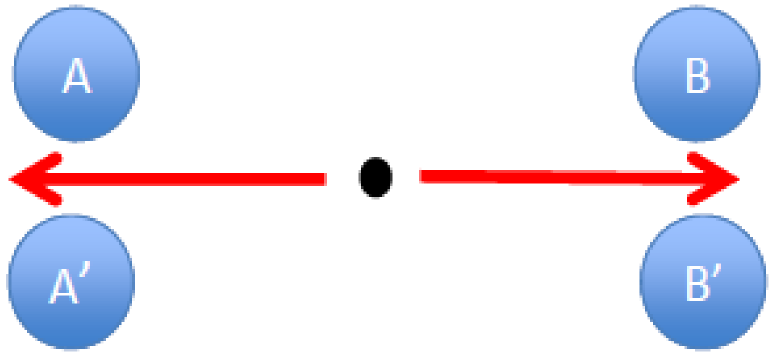


$$C = AB - AB' + A'B + A'B'$$

Clauser, Horne, Simony, Holt, 1969

$$C = A(B - B') + A'(B + B')$$

$$C^2 = 4 + [A, A'][B, B']$$



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$$C^2 = 4 + [A, A'][B, B']$$

$$|C|_{\mathrm{QM,max}} = 2\sqrt{2} > 2 = |C|_{\mathrm{clasical,max}}$$

- If we choose:

$$A = \sigma_x \ , \quad A' = \sigma_y$$

$$B = \sin\theta\sigma_x + \cos\theta\sigma_y \ ,$$

$$B' = \cos\theta\sigma_x - \sin\theta\sigma_y$$

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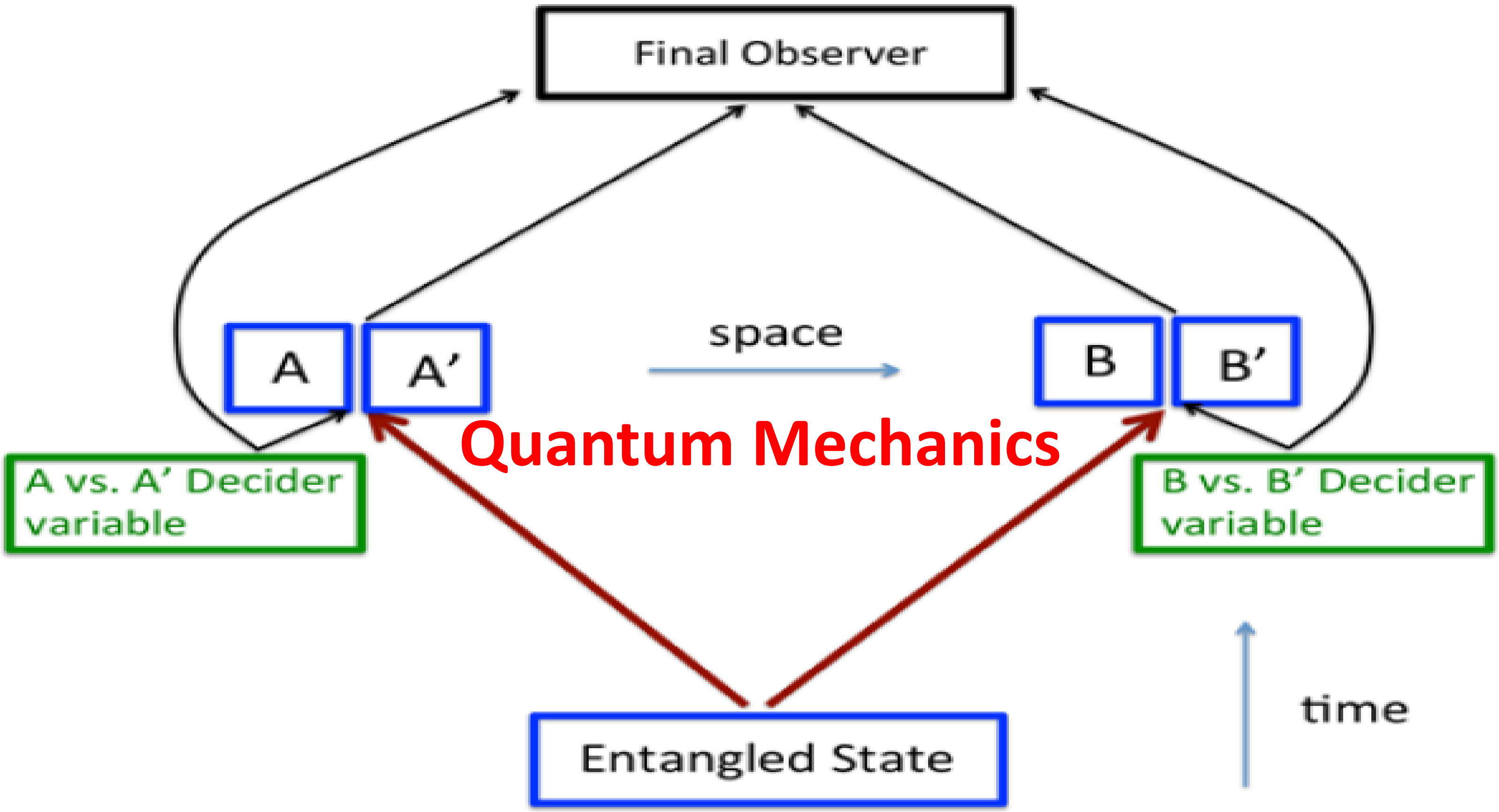
$$C = -2\sqrt{2}\cos(\theta - \tfrac{\pi}{4}).$$

For $\theta=\pi/4$ we get the maximal violation of the Bell's inequality, which contains an extra $\sqrt{2}$ factor.

Set up for a Bell inequality violating experiment. An entangled state is produced in the past and its two parts are transmitted to Alice and Bob who perform measurements on A or A' or B or B'. The choice of experiment (A vs. A') is determined by a local variable, which we can call Alice's "free will" or decider variable. The results of the experiments and the values of the decider variables are classically transmitted to a central observer who computes the statistical averages. All classical communications have been denoted here by black lines.



Cosmological Bell violating setup

- Each Fourier mode of fluctuation can be treated as a time dependent harmonic oscillator.

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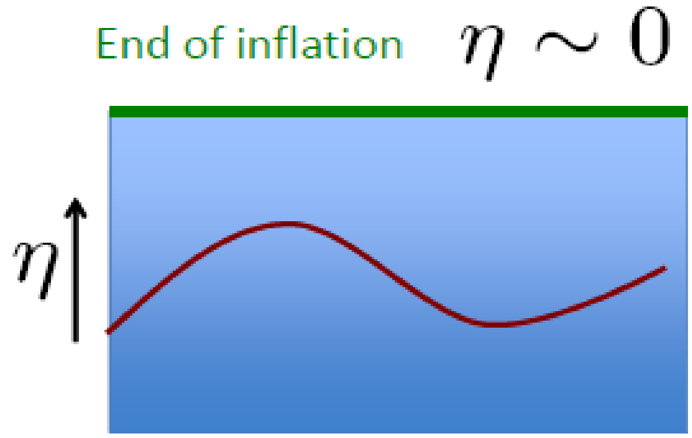
$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2}$$

Comoving coordinates



$$S = \int \frac{d\eta}{\eta^2}\left(|\dot{\phi}|^2 - k^2|\phi|^2\right)$$

$$k^3[\phi(\eta), \eta\partial_\eta\phi] = \eta^3 k^3 \quad \to 0 \ \text{ as } \ \eta k \to 0$$

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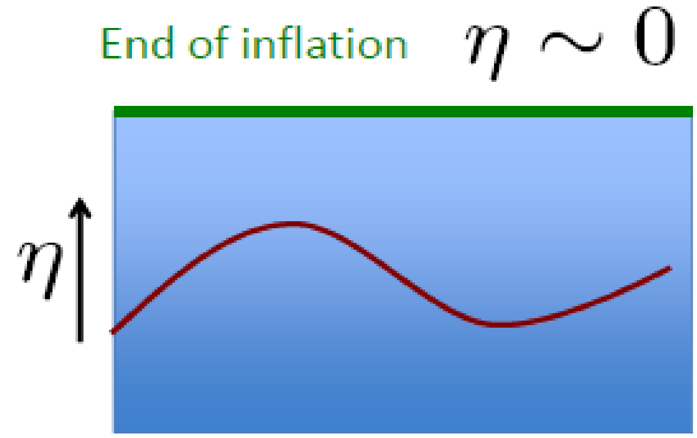

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$$k^3[\phi(\eta), \eta\partial_\eta\phi] = \eta^3 k^3 \quad \to 0 \;\; \text{as} \quad \eta k \to 0$$

This implies that fluctuation becomes classical when they exit the horizon.

- After inflation when reheating occurs one can write down a measure or more precisely a probability distribution function of fluctuation:

$$|\Psi(\phi(x))|^2 = \mu[\phi(x)]$$

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Testing Quantum Mechanics

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1. Many many successful predictions in Quantum Mechanics.
2. Fundamental deviation from the classical theory (probability distribution) via Bell's inequality violation.

But…..

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- If we view Alice and Bob as doing experiments after the end of inflation, then we will not be able to set up a Bell inequality for primordial perturbations.
- This is true if we make the realistic assumption that we cannot measure the conjugate momentum.

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- This is true if we make the realistic assumption that we cannot measure the conjugate momentum.

This is disappointing! But fortunately this is not the end of the story.

Any Hope.....?

- A measurement is a particular unitary evolution of the combined system +measuring apparatus, whose state can be viewed as classical.
- We need to produce all the elements of the Bell inequality discussion out of fluctuations.
- The initial entangled state would be a quantum fluctuation, the measurement apparatus would be another quantum fluctuation that has already become classical.

Any Hope.....?

- It should have shorter wavelength than the one corresponding to the entangled state. This shorter wavelength fluctuation should act both as the decider variable as well as measuring apparatus.
- The measurement should be some process which depends on the quantum state of one of the pieces of the entangled sate.
- The result of the measurement should be transmitted to us. Therefore the measurement should be some process which produces a large effect on the fluctuations so that we can see it today.

Any Hope.....?

- The state of the shorter wavelength fluctuations that acted as “decider” variables should also be preserved and transmitted to us.
- We know one mechanism for transmitting this information. Namely, through the inflationary evolution of massless (or nearly massless) scalar fields where small fluctuations are amplified and stretched to cosmic scales.
- Unfortunately, we have not been able to produce a suitable observable using the simplest single scalar field model.

Strategy

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- Choose a toy model of universe that will make the job easy.
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- Choose a toy model of universe that will make the job easy.
- Here we are not at claiming that this is the unique model of universe using which one can design the setup.
- Since we don’t have any direct observational evidence we also can’t claim that this toy model is our known universe. But may be in future this will be tested.
- In this toy model of universe we test the validity of Bell’s inequality with primordial fluctuations.

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- In this computation massive particles with spin “s” and additionally “isospin” quantum number plays important role.
- We provide the example for Stringy axion which has “isospin”.

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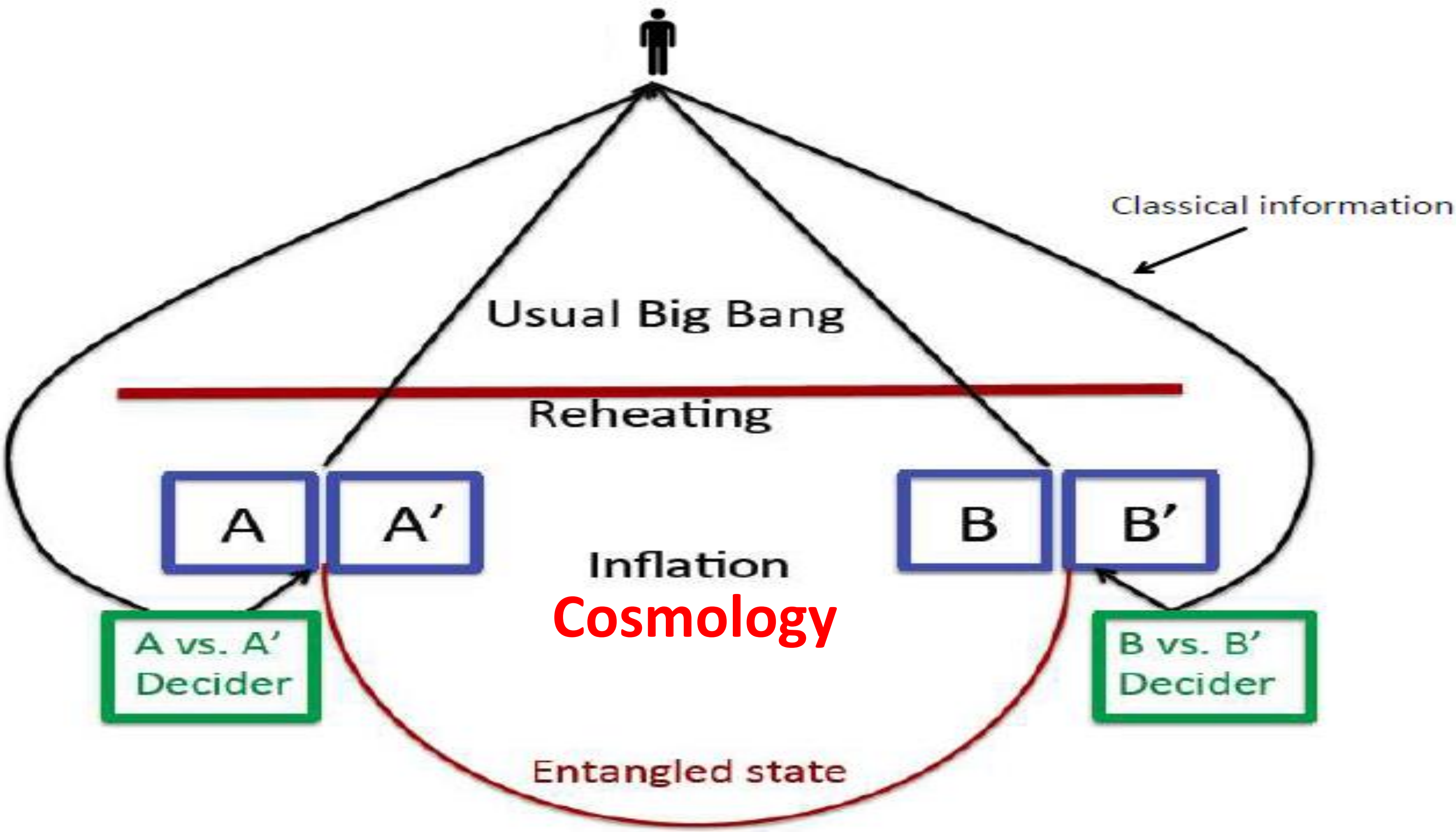
- In this computation massive particles with spin "s" and additionally "isospin" quantum number plays important role.
- We provide the example for Stringy axion which has "isospin".
- Time dependent mass profile with dependence in "isospin" makes the job easy here. Axions have such profile.
- Such time dependence in mass profile of the massive particles (ex. axion) produces classical perturbations on the inflaton. As a result hot spots produced in CMB by curvature fluctuations and all such massive particles are visible today.

A diagram of a successful set up where the whole process occurs during inflation. We generate an entangled state. Some time later we generate the variables that will decide whether we make an A or A' measurement and similarly for B and B'. These decider variables as well as the result of the measurement should remain as classical variables for the rest of the evolution and be visible to us.



Role of new particles (Bell pairs)

- Massive particle pair creation is required to produce hot and cold spots in CMB.
- Massive particle mass should have time dependent profile.

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- Massive particle mass should have time dependent profile.
- Heavy fields become heavy at early and late times, but in an intermediate time scale where inflation occurs it behaves like a light field.
- Each massive particle pair is created in “isospin” singlet manner. This is required for measurement in the detectors.

- Eqn of motion for massive field:

$$h_k'' + \left\{c_S^2 k^2 + \left(\frac{m^2}{H^2} - 2\right)\frac{1}{\eta^2}\right\} h_k = 0 \qquad \text{for dS}$$

$$h_k'' + \left\{c_S^2 k^2 + \left(\frac{m^2}{H^2} - \left[\nu^2 - \frac{1}{4}\right]\right)\frac{1}{\eta^2}\right\} h_k = 0 \qquad \text{for qdS.}$$

$$\nu = \frac{3}{2} + \epsilon + \frac{\eta}{2} + \frac{s}{2}$$

$$\epsilon = -\frac{\dot{H}}{H^2},$$

$$\eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon},$$

$$s = \frac{\dot{c}_S}{Hc_S}.$$

$$h_k(\eta) = \begin{cases} \sqrt{-\eta}\left[C_1 H^{(1)}_{\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}}(-kc_S\eta) + C_2 H^{(2)}_{\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}}(-kc_S\eta)\right] & \text{for dS} \\ \sqrt{-\eta}\left[C_1 H^{(1)}_{\sqrt{\nu^2-\frac{m^2}{H^2}}}(-kc_S\eta) + C_2 H^{(2)}_{\sqrt{\nu^2-\frac{m^2}{H^2}}}(-kc_S\eta)\right] & \text{for qdS.} \end{cases}$$

Exact solution only possible if we assume that the time dependence in mass profile is very slow.

We choose Bunch Davies ($C_1 = \sqrt{\frac{\pi}{2}}$ and $C_2 = 0.$) and αδ vacuum ($C_1 = \cosh\alpha$ and $C_2 = e^{i\delta}\sinh\alpha$)

- Eqn of motion for massive field:

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WKB approximated solution for arbitrary time dependent mass profile.

$$h_k(\eta) = [D_1 u_k(\eta) + D_2 \bar{u}_k(\eta)]$$

$$u_k(\eta) = \frac{1}{\sqrt{2p(\eta)}} \exp\left[i \int^{\eta} d\eta' p(\eta')\right]$$

$$\bar{u}_k(\eta) = \frac{1}{\sqrt{2p(\eta)}} \exp\left[-i \int^{\eta} d\eta' p(\eta')\right]$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$$D_1 = \beta(k) = \frac{\mathcal{R}}{\mathcal{T}} = \int_{-\infty}^{0} d\eta \, \frac{\left(p'(\eta)\right)^2}{4p^3(\eta)} \exp\left[2i \int_{-\infty}^{\eta} d\eta' p(\eta')\right],$$

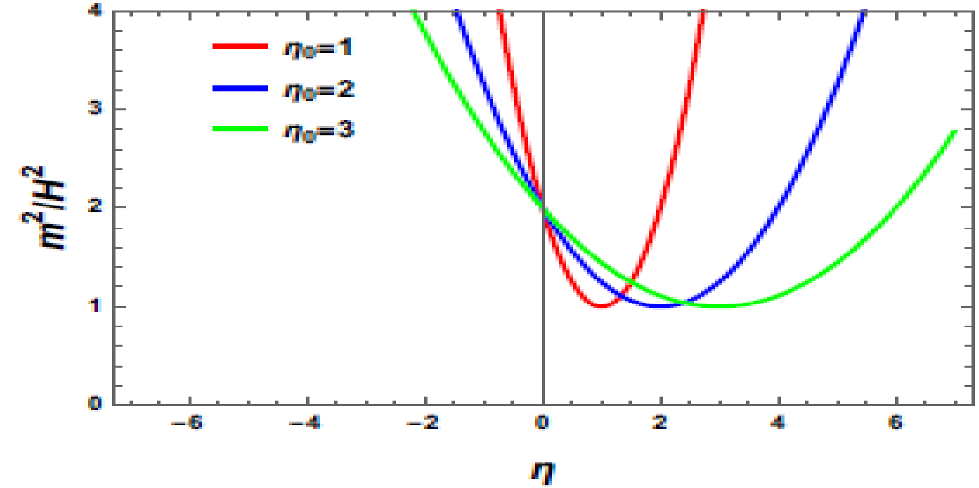
$$D_2 = \alpha(k) = \frac{1}{\mathcal{T}} = \sqrt{1 + |\beta(k)|^2} \, e^{i\phi},$$

$$p(\eta) = \begin{cases} \sqrt{\left\{c_S^2 k^2 + \left(\frac{m^2}{H^2} - 2\right)\frac{1}{\eta^2}\right\}} & \text{for dS} \\ \sqrt{\left\{c_S^2 k^2 + \left(\frac{m^2}{H^2} - \left[\nu^2 - \frac{1}{4}\right]\right)\frac{1}{\eta^2}\right\}} & \text{for qdS.} \end{cases}$$

Bogoluibov co-efficients

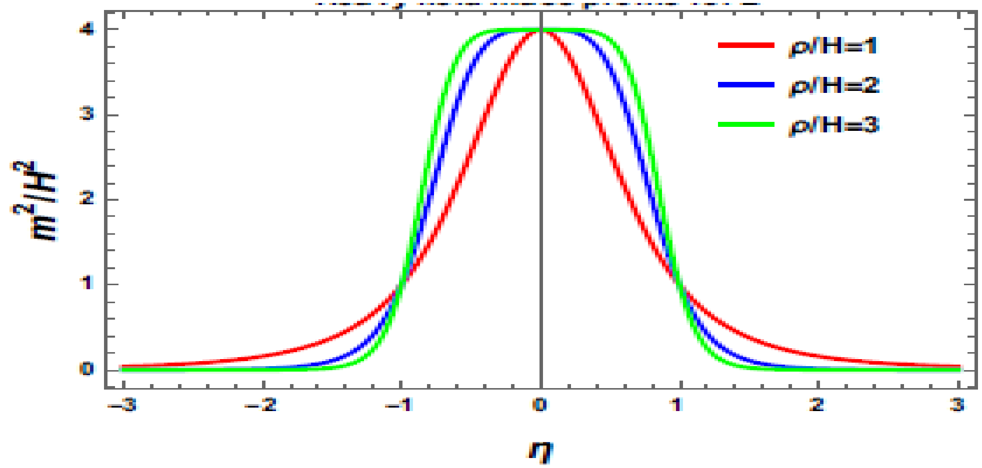
- <u>Toy model 1:</u>

$$m = \sqrt{\gamma\left(\frac{\eta}{\eta_0} - 1\right)^2 + \delta}\ H$$



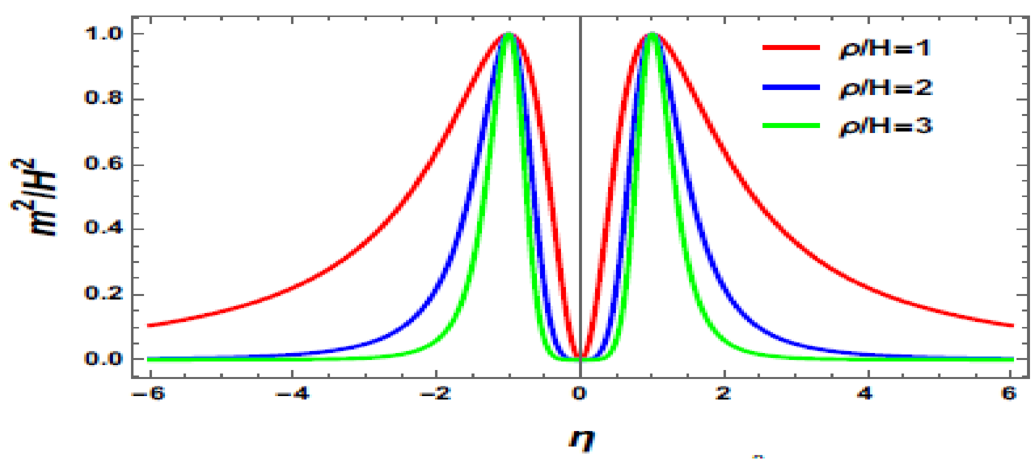
- <u>Toy model 2:</u>

$$m = \frac{m_0}{\sqrt{2}}\sqrt{\left[1 - \tanh\left(\frac{\rho}{H}\ln(-H\eta)\right)\right]}$$



- <u>Toy model 3:</u>

$$m = m_0\ \mathrm{sech}\left(\frac{\rho}{H}\ln(-H\eta)\right)$$



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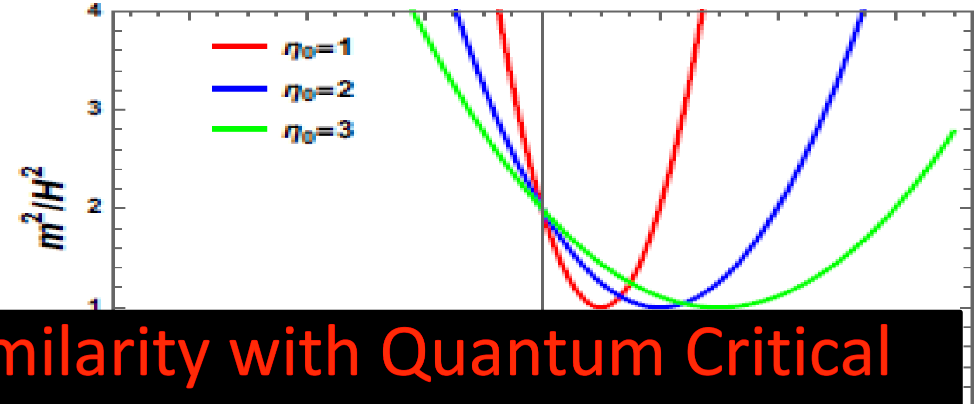

Similarity with Quantum Critical Quench and thermalization in CFT (Ref: John Cardy, Sumit Das, Gautam Mandal)
arXiv:1512.02187[hep-th],
JHEP 1007 (2010) 071
Phys.Rev.Lett. 96 (2006) 136801

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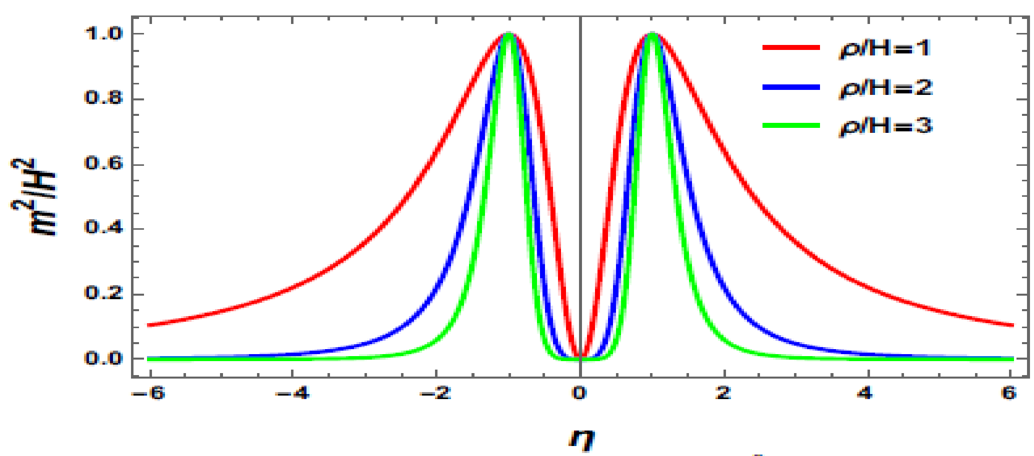

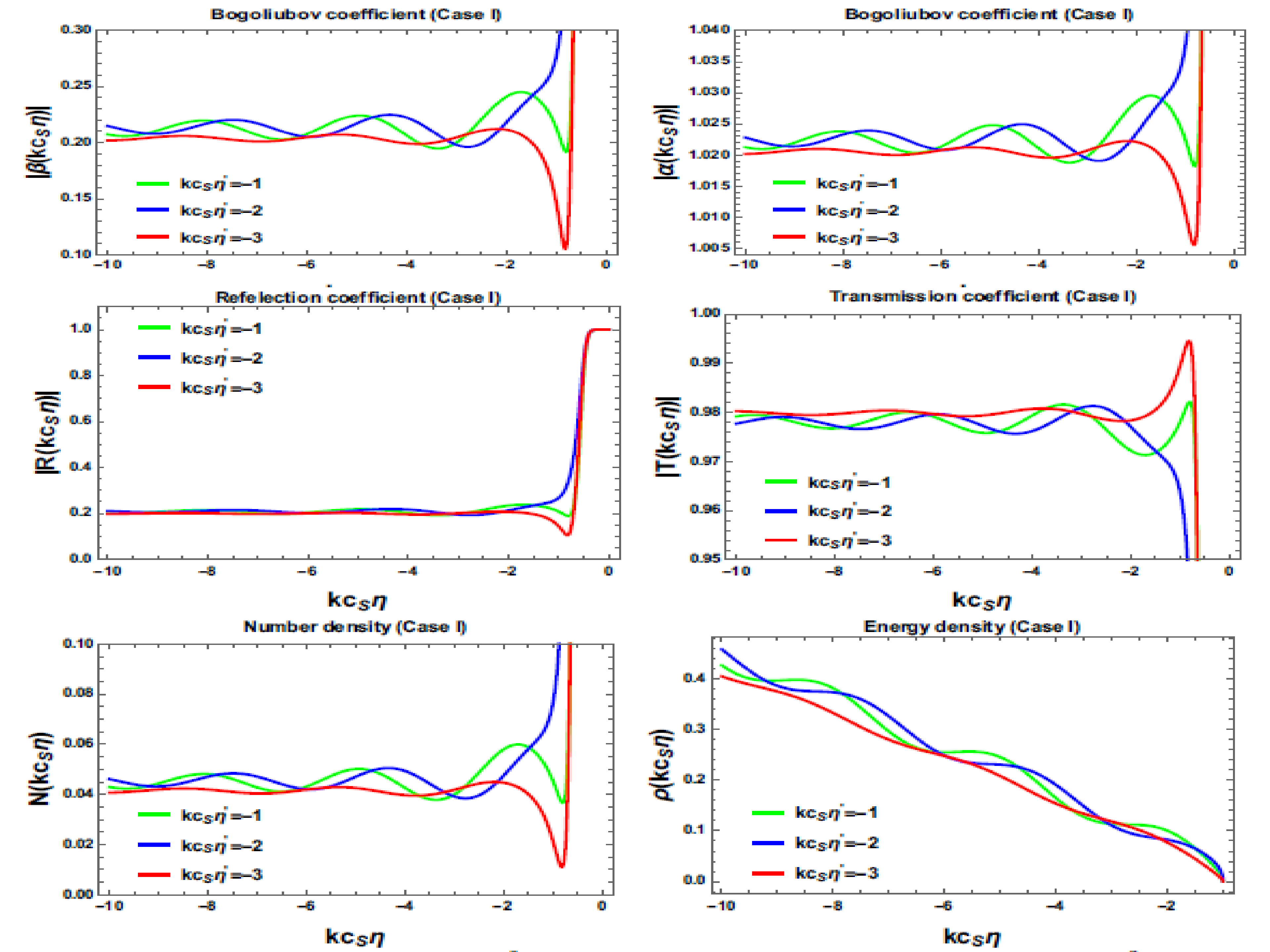

Bogoliubov coefficient (Case I)
|β(kcSη)|
kcSη=−1
kcSη=−2
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|α(kcSη)|
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Refelection coefficient (Case I)
|R(kcSη)|
kcSη=−1
kcSη=−2
kcSη=−3
kcSη
Transmission coefficient (Case I)
|T(kcSη)|
kcSη=−1
kcSη=−2
kcSη=−3
kcSη
Number density (Case I)
N(kcSη)
kcSη=−1
kcSη=−2
kcSη=−3
kcSη
Energy density (Case I)
ρ(kcSη)
kcSη=−1
kcSη=−2
kcSη=−3
kcSη

- Curvature fluctuations in presence of massive particles:

$$S = S_1 + S_2$$

$$S_1 = \frac{1}{2}\int d\eta d^3x \frac{2\epsilon M_p^2}{\tilde{c}_S^2 H^2}\left[\frac{(\partial_\eta\zeta)^2 - c_S^2(\partial_i\zeta)^2}{\eta^2} - \frac{m_{inf}^2}{H^2\eta^2}\right],$$

$$S_2 = -\int \frac{d\eta}{\tilde{c}_S H} m(\eta)\partial_\eta\zeta(\eta, \mathbf{x} = 0).$$

Mass parameter

$$\Lambda = \begin{cases} \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} & \mathbf{for} \quad \mathbf{dS} \\ \sqrt{\nu^2 - \frac{m^2}{H^2}} & \mathbf{for} \quad \mathbf{qdS}. \end{cases}$$

- Scalar fluctuation:

$$\zeta(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \zeta_{\mathbf{k}}(\eta) \exp(i\mathbf{k}.\mathbf{x})$$

$$\zeta_{\mathbf{k}}(\eta) = \frac{h_{\mathbf{k}}(\eta)}{zM_p} = \frac{h(\eta, \mathbf{k})\, a(\mathbf{k}) + h^*(\eta, -\mathbf{k})\, a^\dagger(-\mathbf{k})}{zM_p}$$

Mukhanov Sasaki Variable: $z = \frac{a\sqrt{2\epsilon}}{\bar{c}_S}$

$$\left[a(\mathbf{k}), a^\dagger(-\mathbf{k}')\right] = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}'),$$
$$\left[a(\mathbf{k}), a(\mathbf{k}')\right] = 0,$$
$$\left[a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}')\right] = 0.$$

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$$\left[a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}^{'})\right] = 0.$$

In-In formalism:

$$\langle\zeta_k(\eta=0)\rangle = -i\int_{-\infty}^{0} d\eta\ a(\eta)\ \langle 0|\left[\zeta_{\mathbf{k}}(0), H_{int}(\eta)\right]|0\rangle,$$

One point function

Interaction part due to massive field:

$$H_{int}(\eta) = -\frac{m}{\tilde{c}_S H}\partial_\eta\zeta(\eta,\mathbf{x}=0)$$

- Curvature fluctuations in presence of massive particles:

$$\langle\zeta(\mathbf{x},\eta=0)\rangle_{|kc_S\eta|\to-\infty} \approx -\frac{2H}{M_p^2\epsilon\pi}\left[|C_2|^2 O_1 - |C_1|^2 O_2 - i\left(C_1^* C_2 e^{i\pi\left(\Lambda+\frac{1}{2}\right)} + C_1 C_2^* e^{-i\pi\left(\Lambda+\frac{1}{2}\right)}\right) O_3\right],$$

$$\langle\zeta(\mathbf{x},\eta=\xi\to 0)\rangle_{|kc_S\eta|\to 0} = \frac{H\sqrt{\xi}c_S}{2M_p^2\epsilon\pi^2}\left(C_1^* C_2 + C_1 C_2^* - |C_1|^2 - |C_2|^2\right) O_4^{\xi\theta},$$

$$\langle\zeta(\mathbf{x},\eta=\xi\to 0)\rangle_{|kc_S\eta|\approx 1} = \frac{H\sqrt{\xi}c_S}{2M_p^2\epsilon\pi^2}\left(C_1^* C_2 + C_1 C_2^* - |C_1|^2 - |C_2|^2\right) O_5^{\xi\theta}$$

One point function

$$O_{1,2} = \mp\frac{i}{2\pi^2 c_S^2} m(\eta=-|\mathbf{x}|/c_S), \qquad O_3 = \frac{1}{2\pi^2 c_S^2} m(\eta=-|\mathbf{x}|/c_S),$$

$$O_4^{\xi\theta} = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2\, e^{ikx}\int_{-\infty}^{\xi} d\eta\, \frac{c_S\left(\Lambda-\frac{1}{2}\right)m(\eta)}{a(\eta)\epsilon\sqrt{-\eta}}\left[\left(-\frac{kc_S\eta}{2}\right)^{-\Lambda}\left(-\frac{kc_S\xi}{2}\right)^{-\Lambda} + \left(\frac{kc_S\eta}{2}\right)^{-\Lambda}\left(\frac{kc_S\xi}{2}\right)^{-\Lambda}\right],$$

$$O_5^{\xi\theta} = \frac{1}{2\pi^2}\int_0^\infty dk\, e^{ikx}\, k^{2-\Lambda}\int_{-\infty}^{\xi} d\eta\, \frac{c_S\left(\Lambda-\frac{1}{2}\right)\left(\frac{1}{2}\right)^{-\Lambda} m(\eta)}{a(\eta)\epsilon\sqrt{-\eta}}\left[\left(-\frac{c_S\xi}{2}\right)^{-\Lambda} + (-1)^{-\Lambda}\left(\frac{c_S\xi}{2}\right)^{-\Lambda}\right].$$

- Curvature fluctuations in presence of massive particles:

$$\langle\zeta(\mathbf{x},\eta)\zeta(\mathbf{y},\eta)\rangle_{|kc_S\eta|\to-\infty} \approx \frac{1}{4\pi^4}\frac{H^2}{2\epsilon M_p^2}\frac{\eta^2\tilde{c}_S^2}{c_S}\left[\left(|C_2|^2+|C_1|^2\right)J_1+\left(C_1^*C_2e^{i\pi\left(\Lambda+\frac{1}{2}\right)}J_2+C_1C_2^*e^{-i\pi\left(\Lambda+\frac{1}{2}\right)}J_3\right)\right],$$

$$\langle\zeta(\mathbf{x},\eta)\zeta(\mathbf{y},\eta)\rangle_{|kc_S\eta|\to 0} \approx \frac{H^2}{2\epsilon M_p^2}\frac{(-\eta c_S)^{3-2\Lambda}}{2^{2(2-\Lambda)}\pi^4}\frac{\tilde{c}_S^2}{c_S^3}\left|\frac{\Gamma(\Lambda)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[\left(|C_2|^2+|C_1|^2\right)-\left(C_1^*C_2+C_1C_2^*\right)\right]K_I,$$

$$\langle\zeta(\mathbf{x},\eta=0)\zeta(\mathbf{y},\eta=0)\rangle_{|kc_S\eta|\approx 1} \approx \frac{H^2}{2\epsilon M_p^2}\frac{1}{2^{2(2-\Lambda)}\pi^4}\frac{\tilde{c}_S^2}{c_S^3}\left|\frac{\Gamma(\Lambda)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[\left(|C_2|^2+|C_1|^2\right)-\left(C_1^*C_2+C_1C_2^*\right)\right]Z_I$$

Two point function

$$J_1 = -\frac{4\pi}{|\mathbf{x}-\mathbf{y}|^2},\quad J_{2,3} = -\frac{4\pi}{(|\mathbf{x}-\mathbf{y}|\pm 2c_S\eta)^2},$$

$$K_I = 4\pi\left(\frac{i}{|\mathbf{x}-\mathbf{y}|}\right)^{3-2\Lambda}\Gamma(3-2\Lambda),\quad Z_I = 4\pi\left\{\ln\left(\frac{L_{IR}}{|\mathbf{x}-\mathbf{y}|}\right)-\gamma_E\right\}$$

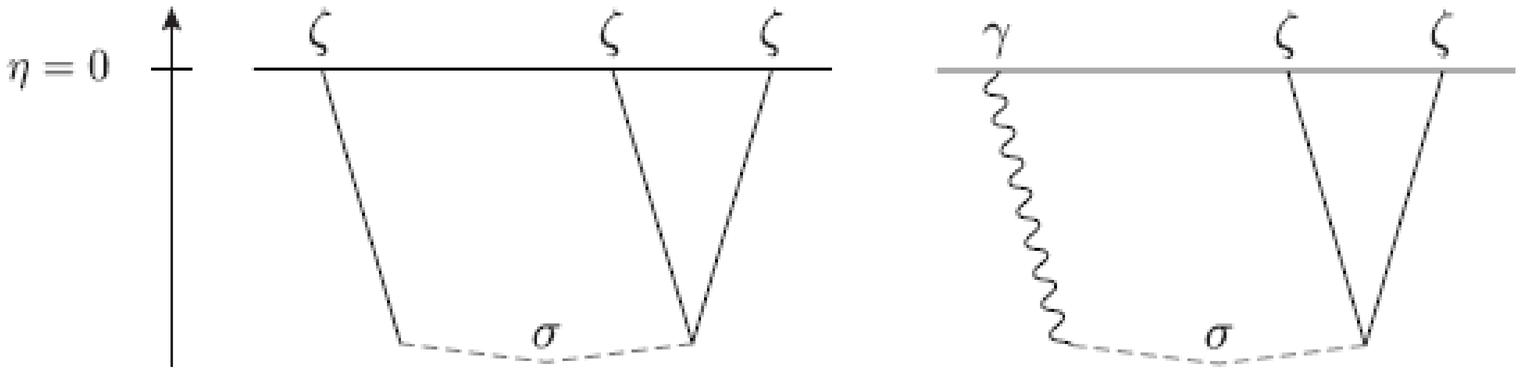
- Curvature fluctuations in presence of massive particles:

Bunch Davies Vacuum+Canonical model

One point Function:	$\langle\zeta(\vec{x})\rangle = \frac{1}{4\pi} m(\eta = -\lvert\vec{x}\rvert)\left(\frac{H}{2\epsilon M_{pl}^2}\right)$
Two point Function:	$\langle\zeta(\vec{x})\zeta(0)\rangle = \frac{1}{(2\pi)^2}\left(\frac{H^2}{2\epsilon M_{pl}^2}\right)\log(L/\lvert\vec{x}\rvert)$

Important Note

- Scale of inflation and the associated new physics can be predicted if the amount of Bell violation is known.
- Here we need to measure the mass of the particles participating in evolution of universe and to serve this purpose we need to measure the one point function scalar fluctuations.
- One can derive the expression for the three point function in terms of the time dependent mass of new particles (Bell pairs). Enhancement in NG in slow roll.



Analogy with axion fluctuations from String Theory

- <u>Axion model:</u>

$$S_{axion} = \int d\eta\, d^3x \left[\frac{f_a^2(\eta)}{2H^2} \frac{\left[(\partial_\eta a)^2 - (\partial_i a)^2\right]}{\eta^2} - \frac{U(a)}{H^4\eta^4} \right]$$

$$U(a) = V(af_a) = \mu^3 a f_a + \Lambda_C^4 \cos a$$
$$= \mu^3 f_a \left[a + b\cos a\right].$$

$$b = \frac{\Lambda_C^4}{\mu^3 f_a}.$$

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$$\Lambda_C = \sqrt{m_{SUSY} M_p}\, e^{-cS_{inst}}$$

$$M_p = \frac{L^3}{\sqrt{\alpha'}\, g_s}$$

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$$b = \frac{\Lambda_C^4}{\mu^3 f_a}.$$

$$U(a) \simeq \frac{1}{2} m_{axion}^2 (a - a_0)^2$$

Scale of the effective potential:

$$V_0 = \mu^3 f_a = \frac{1}{\alpha'^2 g_s} e^{4A_0} + \frac{R^2}{\alpha' L^4} m_{SUSY}^4 e^{2A_0}$$

- Axion model:

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Warp factor:

$$e^{A_0} = \left(\frac{\Lambda_C}{m_{SUSY}}\right)^2 \frac{L}{R}\sqrt{g_s \alpha'} = \frac{L^4}{m_{SUSY} R}\sqrt{\frac{\alpha'}{g_s}}\, e^{-2cS_{inst}}$$

String Scale:

$$M_s = \frac{e^{A_0}}{\sqrt{\alpha'}} = \left(\frac{\Lambda_C}{m_{SUSY}}\right)^2 \frac{L}{R}\sqrt{g_s} = \frac{L^4}{m_{SUSY} R\sqrt{g_s}}\, e^{-2cS_{inst}}$$

- Axion have time dependent decay constant. It serves the purpose of time dependent mass for heavy particles as mentioned earlier.
- Time dependent decay constant is large at early and late times. It becomes small during some time, a few e-foldings after creation of massive axion pairs. Then it becomes large.
- Fluctuations are produced at the characteristic scale of axions.

- Axion fluctuation with time dependent decay constant:

$$\partial_\eta^2 \vartheta_{\mathbf{k}} + \left(k^2 - \frac{\partial_\eta^2 \left(\frac{f_a^2}{H^2\eta^2}\right)}{\left(\frac{f_a^2}{H^2\eta^2}\right)} + \frac{m_{axion}^2}{f_a^2 H^2 \eta^2}\right)\vartheta_{\mathbf{k}} = 0$$

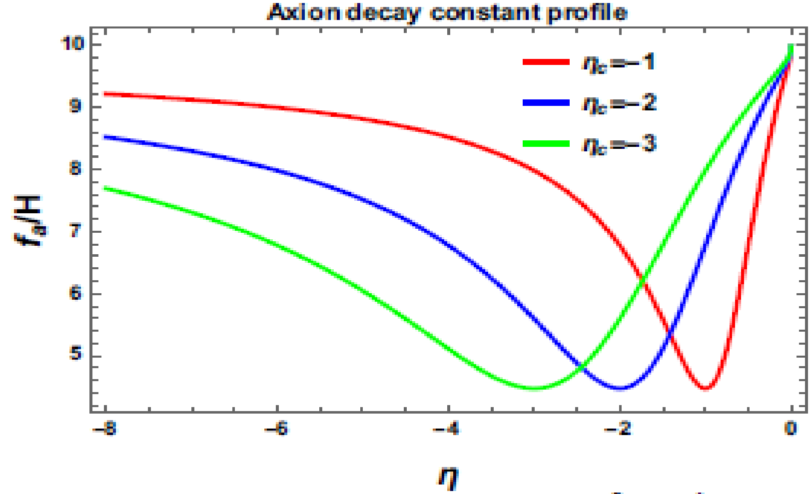
$$\vartheta_{\mathbf{k}} = \frac{f_a^2}{H^2\eta^2 M_p^2}\bar{a}_{\mathbf{k}}.$$

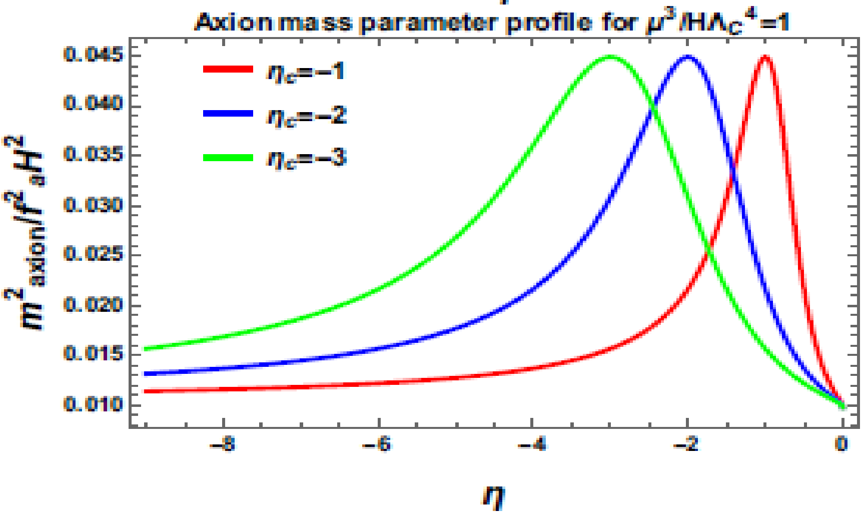
Toy model for Axion decay constant profile

$$f_a = \sqrt{100 - \frac{80}{1+\left(\ln\frac{\eta}{\eta_c}\right)^2}}\, H$$

$$\frac{m_{axion}^2}{f_a^2 H^2} = -\frac{\Lambda_C^4}{\left[100 - \frac{80}{1+\left(\ln\frac{\eta}{\eta_c}\right)^2}\right]^2 H^4} \times \Sigma_C$$

$$\Sigma_C = \begin{cases} \cos\left(\sin^{-1}\left(\frac{\mu^3 H}{\Lambda_C^4}\sqrt{100 - \frac{80}{1+\left(\ln\frac{\eta}{\eta_c}\right)^2}}\right)\right) & \text{for total } U(a) \\ (-1)^m & \text{for osc. } U(a). \end{cases}$$





Characteristics	New particle	Axion
Mass parameter	$m(\eta)$	$\frac{m_{axion}}{f_a} = \begin{cases} \sqrt{-\frac{\Lambda_C^4}{f_a^2}\cos\left(\sin^{-1}\left(\frac{\mu^3 f_a}{\Lambda_C^4}\right)\right)} & \text{for total } U(a) \\ \sqrt{\frac{\Lambda_C^4}{f_a^2}(-1)^{m+1}} & \text{for osc. } U(a). \end{cases}$
Rescaled mode function	$h_{\mathbf{k}} = zM_p\zeta_{\mathbf{k}}$	$\vartheta_{\mathbf{k}} = \frac{f_a^2}{H^2\eta^2 M_p^2}\bar{a}_{\mathbf{k}}$
Mukhanov-Sasaki variable	$z = \frac{a\sqrt{2\epsilon}}{c_S} = \frac{\sqrt{2\epsilon}}{H\eta c_S}$	$\frac{f_a^2}{H^2\eta^2 M_p^2}$
Scalar mode equation	$h''_{\mathbf{k}} + \left(c_S^2 k^2 + \frac{\left(\frac{m^2}{H^2} - \delta\right)}{\eta^2}\right) h_{\mathbf{k}} = 0$ where $\delta = \frac{z''}{z} = \begin{cases} 2 & \text{for dS} \\ \left(\nu^2 - \frac{1}{4}\right) & \text{for qdS.} \end{cases}$	$\partial_\eta^2 \vartheta_{\mathbf{k}} + \left(k^2 - \frac{\partial_\eta^2\left(\frac{f_a^2}{H^2\eta^2}\right)}{\left(\frac{f_a^2}{H^2\eta^2}\right)} + \frac{m_{axion}^2}{f_a^2 H^2 \eta^2}\right)\vartheta_{\mathbf{k}} = 0$ where $\frac{\partial_\eta^2\left(\frac{f_a^2}{H^2\eta^2}\right)}{\left(\frac{f_a^2}{H^2\eta^2}\right)} \approx \begin{cases} \frac{6}{\eta^2} & \text{for } \eta \sim \eta_c \\ \frac{6}{\eta^2} & \text{for early \& late } \eta \\ \frac{6+\Delta_c}{\eta^2} & \text{for } \eta < \eta_c. \end{cases}$
Parametrization	$\frac{m^2}{H^2} = \begin{cases} \gamma\left(\frac{\eta}{\eta_0} - 1\right)^2 + \delta & \text{Case I} \\ \frac{m_0^2}{2H^2}\left[1 - \tanh\left(\rho\frac{\ln(-H\eta)}{H}\right)\right] & \text{Case II} \\ \frac{m_0^2}{H^2}\text{sech}^2\left(\rho\frac{\ln(-H\eta)}{H}\right) & \text{Case III.} \end{cases}$	$\frac{m_{axion}^2}{f_a^2 H^2} = \frac{m_{axion}^2}{\left[100 - \frac{80}{1+\left(\ln\frac{\eta}{\eta_c}\right)^2}\right]H^4}$

Role of isospin breaking interaction

- Isospin breaking interaction:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\mathcal{H})^\dagger(\partial_\nu\mathcal{H}) + \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \mathcal{H}^\dagger\left(\sum_{n=0}^{\infty}\mathcal{M}_n^2(\phi)(\sigma.\mathbf{n})^n\right)\mathcal{H} + \cdots\right]$$

$$(\sigma.\mathbf{n})^n = (\sigma_x\cos m\theta + \sigma_y\sin m\theta)^n = \begin{cases} I, & \textbf{for even } n \\ (\sigma.\mathbf{n}), & \textbf{for odd } n. \end{cases}$$

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$$(\sigma.\mathbf{n})^n=(\sigma_x\cos m\theta+\sigma_y\sin m\theta)^n=\begin{cases}I, & \textbf{for even } n\\ (\sigma.\mathbf{n}), & \textbf{for odd } n.\end{cases}$$

$$=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\mathcal{H}_1)^*(\partial_\nu\mathcal{H}_1)+\frac{1}{2}g^{\mu\nu}(\partial_\mu\mathcal{H}_2)^*(\partial_\nu\mathcal{H}_2)+\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)\right.$$
$$\left.-\left[|\mathcal{H}_1|^2+|\mathcal{H}_2|^2\right]\mathcal{M}^2_{\mathbf{even}}(\phi)-\left[\exp(-im\theta)\mathcal{H}_1^*\mathcal{H}_2+\exp(im\theta)\mathcal{H}_2^*H_1\right]\mathcal{M}^2_{\mathbf{odd}}(\phi)+\cdots\right]$$

$$\mathcal{M}^2_{\mathbf{even}}(\phi)=\sum_{n=0,2,4}^{\infty}\mathcal{M}_n^2(\phi),\qquad \mathcal{M}^2_{\mathbf{odd}}(\phi)=\sum_{n=1,3,5}^{\infty}\mathcal{M}_n^2(\phi)$$

$$\mathcal{M}^2(\phi)=\begin{pmatrix}\mathcal{M}^2_{\mathbf{even}}(\phi) & \exp(-im\theta)\mathcal{M}^2_{\mathbf{odd}}(\phi)\\ \exp(im\theta)\mathcal{M}^2_{\mathbf{odd}}(\phi) & \mathcal{M}^2_{\mathbf{even}}(\phi)\end{pmatrix}$$

Mass eigen value

$$\lambda_\pm(\phi)=\sqrt{\mathcal{M}^2_{\mathbf{even}}(\phi)\pm\mathcal{M}^2_{\mathbf{odd}}(\phi)}$$

Toy model for mass parameter

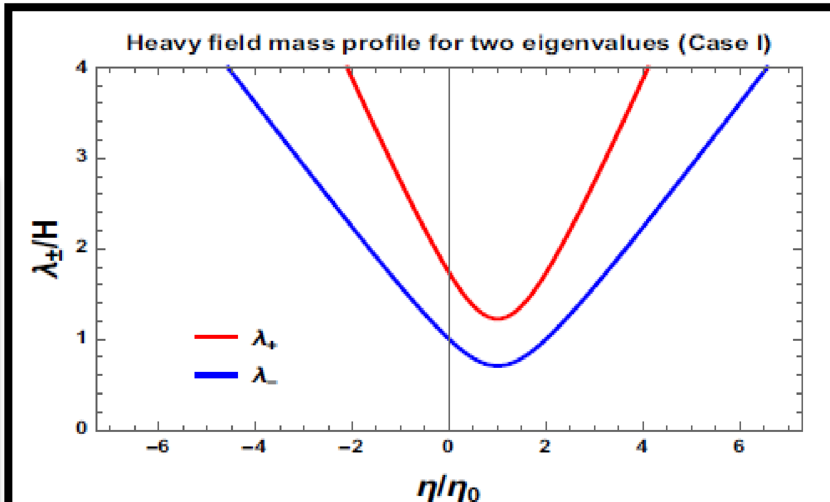
$$\frac{\lambda_\pm(\eta)}{H} = \sqrt{\left(\frac{M_{\rm even}(\eta)}{H}\right)^2 \pm \left(\frac{M^2_{\rm odd}(\eta)}{H}\right)^2}$$

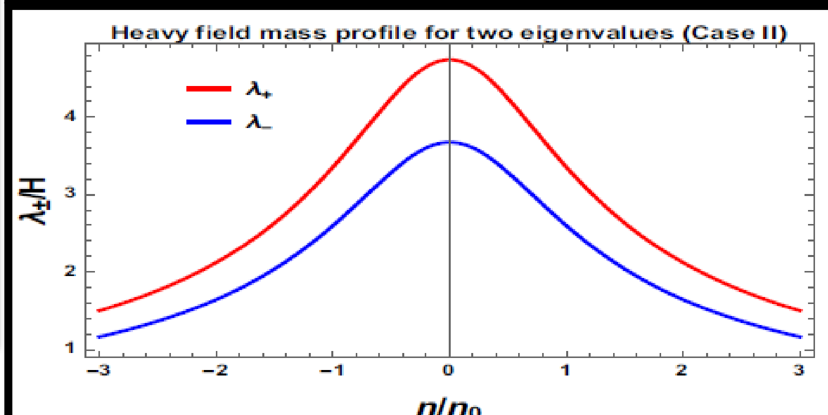
$$= \begin{cases} \sqrt{\gamma_\pm\left(\frac{\eta}{\eta_0}-1\right)^2+\delta_\pm} & \textbf{Case I} \\ \frac{m_{0\pm}}{\sqrt{2}H}\sqrt{\left[1-\tanh\left(\rho\frac{\ln(-H\eta)}{H}\right)\right]} & \textbf{Case II} \\ \frac{m_{0\pm}}{H}\text{sech}\left(\rho\frac{\ln(-H\eta)}{H}\right) & \textbf{Case III.} \end{cases}$$

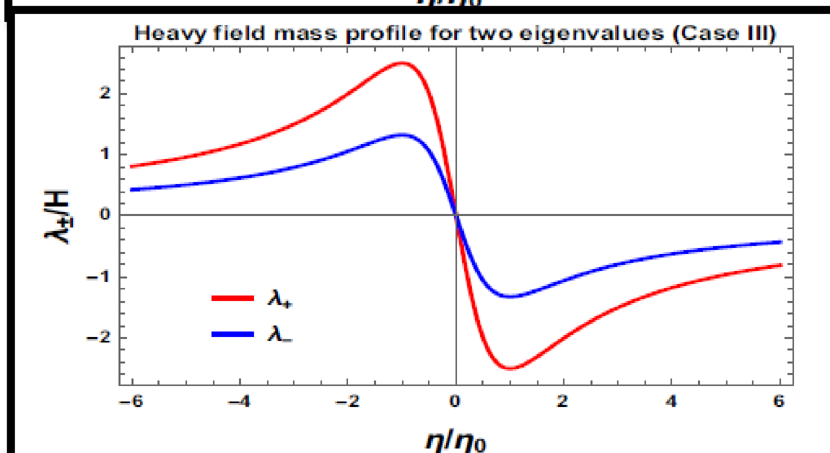
$$\gamma_\pm = \gamma_{\rm even} \pm \gamma_{\rm odd} = \gamma_{\rm even} + \textbf{sign}(\sigma.\text{n})\gamma_{\rm odd},$$

$$\delta_\pm = \delta_{\rm even} \pm \delta_{\rm odd} = \delta_{\rm even} + \textbf{sign}(\sigma.\text{n})\delta_{\rm odd},$$

$$m_{0\pm} = \sqrt{m^2_{\rm even} \pm m^2_{\rm even}} = \sqrt{m^2_{\rm even} + \textbf{sign}(\sigma.\text{n})m^2_{\rm even}}.$$







How we interpret?

- To avoid any instability, eigen values of the mass matrix are always positive definite.
- At late times, so that the two eigenvalues of the mass differ by a distinguishable amount.
- The field *H* is a complex field and when we produce a particle pair one member of the pair will be a particle and the other an antiparticle.

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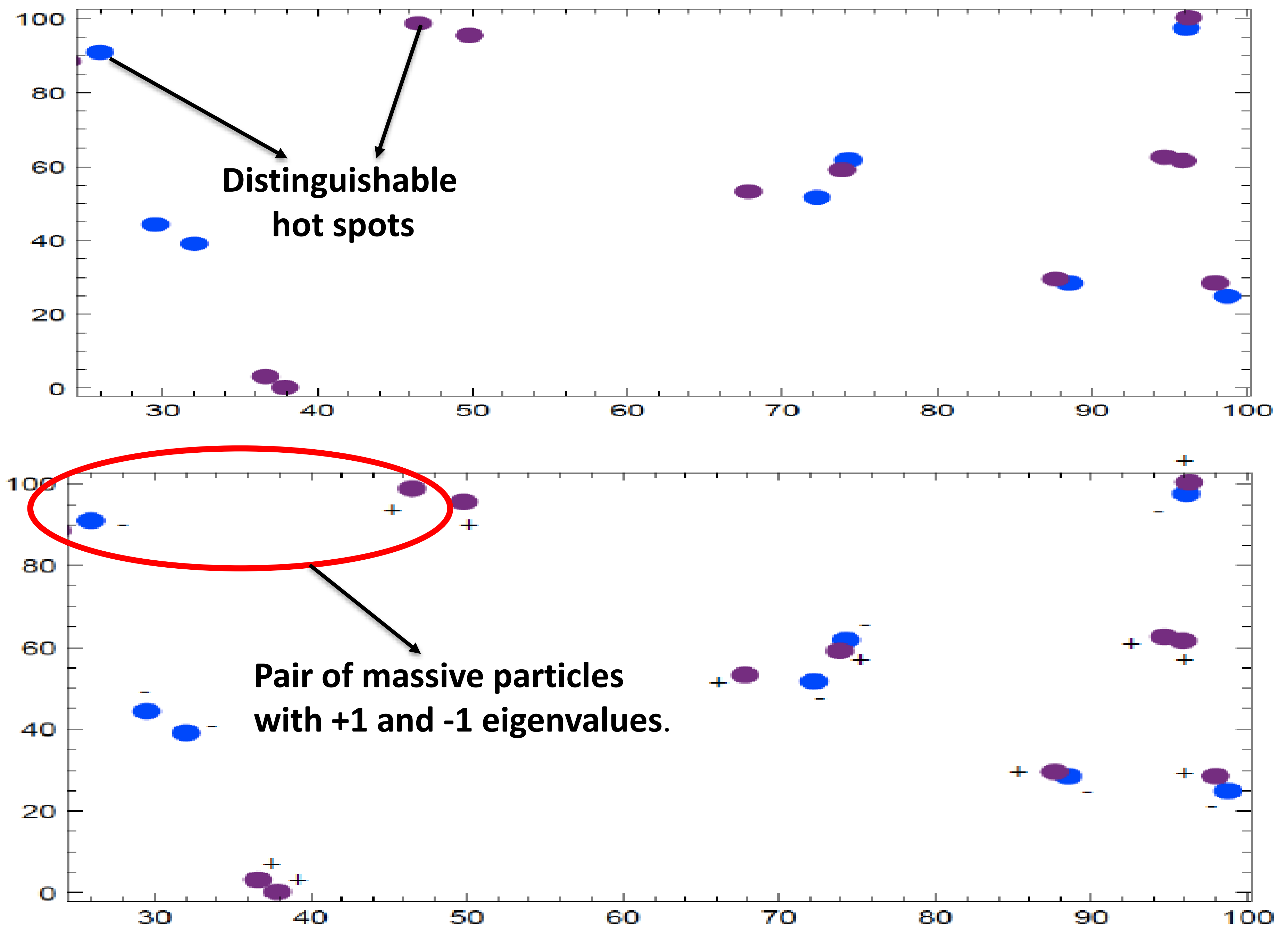
- At late times, so that the two eigenvalues of the mass differ by a distinguishable amount.
- The field *H* is a complex field and when we produce a particle pair one member of the pair will be a particle and the other an antiparticle.
- The complex conjugate field is, of course, also a SU(2) doublet.
- In our prescription, at late times, one can measure both the primordial scalar fluctuations as well as the primordial axion fluctuations.
- If we want to make the model consistent with present day data one can imagine that the axion corresponds to a sub-leading component of the dark matter.

How we interpret?

- This dark matter density depends on the value of θ left over from the end of inflation, since this determines the deviation from the minimum of the axion potential.
- This is the source of isocurvature fluctuation (f_a small).
- In this prescription, the scalar fluctuations are given by the usual gaussian random field + some characteristic hot.

How we interpret?

- This dark matter density depends on the value of θ left over from the end of inflation, since this determines the deviation from the minimum of the axion potential.
- This is the source of isocurvature fluctuation (f_a small).
- In this prescription, the scalar fluctuations are given by the usual gaussian random field + some characteristic hot.
- There are two types of hot spots that differ by their overall amplitude. Let us call them the superhot and the very hot spots (± sign in the eigen values).
- Each hot spot can be assigned a ±1 (superhot or very hot spot).
- Hotspot =individual massive particle.
- The final result of this procedure is a set of (Bell) pairs.


Distinguishable hot spots
Pair of massive particles with +1 and -1 eigenvalues.

Final Note:

- Here for each θ we have a collection of ± measurements. Let us identify the pair as, $(\pm 1_{\theta_A}, \pm 1_{\theta_B})$. These outcomes are similar to Bell experiment in QM.
- Here one can define the observable,

$$C_{AB} = \frac{1}{2}[C(\theta_A, \theta_{A'}, \theta_B, \theta_{B'}) + C(\theta_B, \theta_{B'}, \theta_A, \theta_{A'})]$$

where θ_A, $\theta_{A'}$ correspond to the two choices of detector at Alice's location and , θ_B, $\theta_{B'}$ similarly at Bob's location.

- Here one can define Alice's location to be the location of the particle and Bob's location that of the antiparticle.
- Orientation of the detector in such setup is proportional to $\cos(\theta_A - \theta_B)$.

Final Note:

- Next we set , $\theta = \frac{\pi}{4}$, in A, A′ ,B,B′:

$$A = \sigma_x \ , \quad A' = \sigma_y$$

$$B = \sin\theta\sigma_x + \cos\theta\sigma_y \ , \rightarrow \frac{\sigma_x + \sigma_y}{\sqrt{2}}$$

$$B' = \cos\theta\sigma_x - \sin\theta\sigma_y \rightarrow \frac{\sigma_x - \sigma_y}{\sqrt{2}}$$

$[A, A']=[\sigma_x, \sigma_y]=2i\sigma_z$ $\quad$ $<|C_{AB}|>=\sqrt{4+[A,A'][B,B']}=\sqrt{4+4\sigma_z^2}=2\sqrt{2}$

$[B, B']=[\frac{\sigma_x+\sigma_y}{\sqrt{2}}, \frac{\sigma_x-\sigma_y}{\sqrt{2}}]=-2i\sigma_z$

Then by observing C_{AB} we would observe maximal violation of Bell's inequality in primordial cosmology.

Role of spin

- Eqn of motion for heavy field with spin S:

$$h_k'' + \left\{c_S^2 k^2 + \left(\frac{m_S^2}{H^2} - \left[\nu_S^2 - \frac{1}{4}\right]\right)\frac{1}{\eta^2}\right\} h_k = 0.$$

$$\nu_S = \begin{cases} \left(\mathcal{S} - \frac{1}{2}\right) & \text{for dS} \\ \left(\mathcal{S} - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2} & \text{for qdS.} \end{cases}$$

$$h_k(\eta) = \sqrt{-\eta}\left[C_1 H^{(1)}_{\sqrt{\nu_S^2 - \frac{m_S^2}{H^2}}}(-k c_S \eta) + C_2 H^{(2)}_{\sqrt{\nu_S^2 - \frac{m_S^2}{H^2}}}(-k c_S \eta)\right]$$

$$\frac{m_S^2}{H^2} = \begin{cases} \frac{m^2}{H^2} - 2 & \text{for dS} \\ \frac{m^2}{H^2} - \left[\nu^2 - \frac{1}{4}\right] & \text{for qdS.} \end{cases}$$

For dS :

$$\sqrt{\nu_S^2 - \frac{m_S^2}{H^2}} \approx \begin{cases} \sqrt{\left(\mathcal{S} - \frac{1}{2}\right)^2 - 1} & \text{for } m_S \approx H \\ \left(\mathcal{S} - \frac{1}{2}\right) & \text{for } m_S << H \\ i\sqrt{\Upsilon_S^2 - \left(\mathcal{S} - \frac{1}{2}\right)^2} & \text{for } m_S >> H. \end{cases}$$

For qdS :

$$\sqrt{\nu_S^2 - \frac{m_S^2}{H^2}} \approx \begin{cases} \sqrt{\left(\left(\mathcal{S} - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2}\right)^2 - 1} & \text{for } m_S \approx H \\ \left(\mathcal{S} - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2} & \text{for } m_S << H \\ i\sqrt{\Upsilon_S^2 - \left(\left(\mathcal{S} - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2}\right)^2} & \text{for } m_S >> H. \end{cases}$$

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$$h_k'' + \left\{c_S^2 k^2 + \left(\frac{m_S^2}{H^2} - \left[\nu_S^2 - \frac{1}{4}\right]\right)\frac{1}{\eta^2}\right\} h_k = 0.$$

$$\nu_S = \begin{cases} \left(S - \frac{1}{2}\right) & \text{for dS} \\ \left(S - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2} & \text{for qdS.} \end{cases}$$

WKB approximated solution

$$h_k(\eta) = [D_1 u_k(\eta) + D_2 \bar{u}_k(\eta)]$$

$$u_k(\eta) = \frac{1}{\sqrt{2p(\eta)}} \exp\left[i\int^{\eta} d\eta' p(\eta')\right]$$

$$\bar{u}_k(\eta) = \frac{1}{\sqrt{2p(\eta)}} \exp\left[-i\int^{\eta} d\eta' p(\eta')\right]$$

$$\left|\frac{m_S}{H}\right|_{|kc_S\eta|<<1, |kc_S\eta|>>1} \geq \sqrt{\nu_S^2 - \frac{1}{4}} = \begin{cases} \sqrt{\left(S - \frac{1}{2}\right)^2 - \frac{1}{4}} & \text{for dS} \\ \sqrt{\left(\left(S - \frac{1}{2}\right) + \epsilon + \frac{\eta}{2} + \frac{s}{2}\right)^2 - \frac{1}{4}} & \text{for qdS.} \end{cases}$$

Future prospects and Bottom lines

Bottom lines

- We provide a toy model for Bell’s inequality violation in cosmology.
- Model consist of inflaton and additional massive field with time dependent behaviour.
- For each model massive particle creation in “isospin” singlet state plays crucial role.

Bottom lines

- We provide a toy model for Bell's inequality violation in cosmology.
- Model consist of inflaton and additional massive field with time dependent behaviour.
- For each model massive particle creation in "isospin" singlet state plays crucial role.
- Prescribed methodology is consistent with axion fluctuations appearing in the context of String Theory.
- Signature of the Bell violation is visualized from non-zero one point (+two point) function of curvature fluctuation.

Future Prospect

- Effect on three point function (PNG) for curvature fluctuations in presence of Bell's inequality violation. Role of high spin of new particles (Bell pairs) in PNG.
- There are other signatures of quantum mechanics: e.g. Looking at phase oscillations in the 3 or 4 point function produced by massive particles. This is an interference effect.

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- Connection with thermalization, quantum critical quench etc (where post quench dynamics is described by a CFT with initial state of the generalized Calabrese-Cardy form and pre quench states are ground/squeezed state).
- Role of open quantum systems, dissipation etc in dS and its connection with Bell pairs.

Thanks for your time……