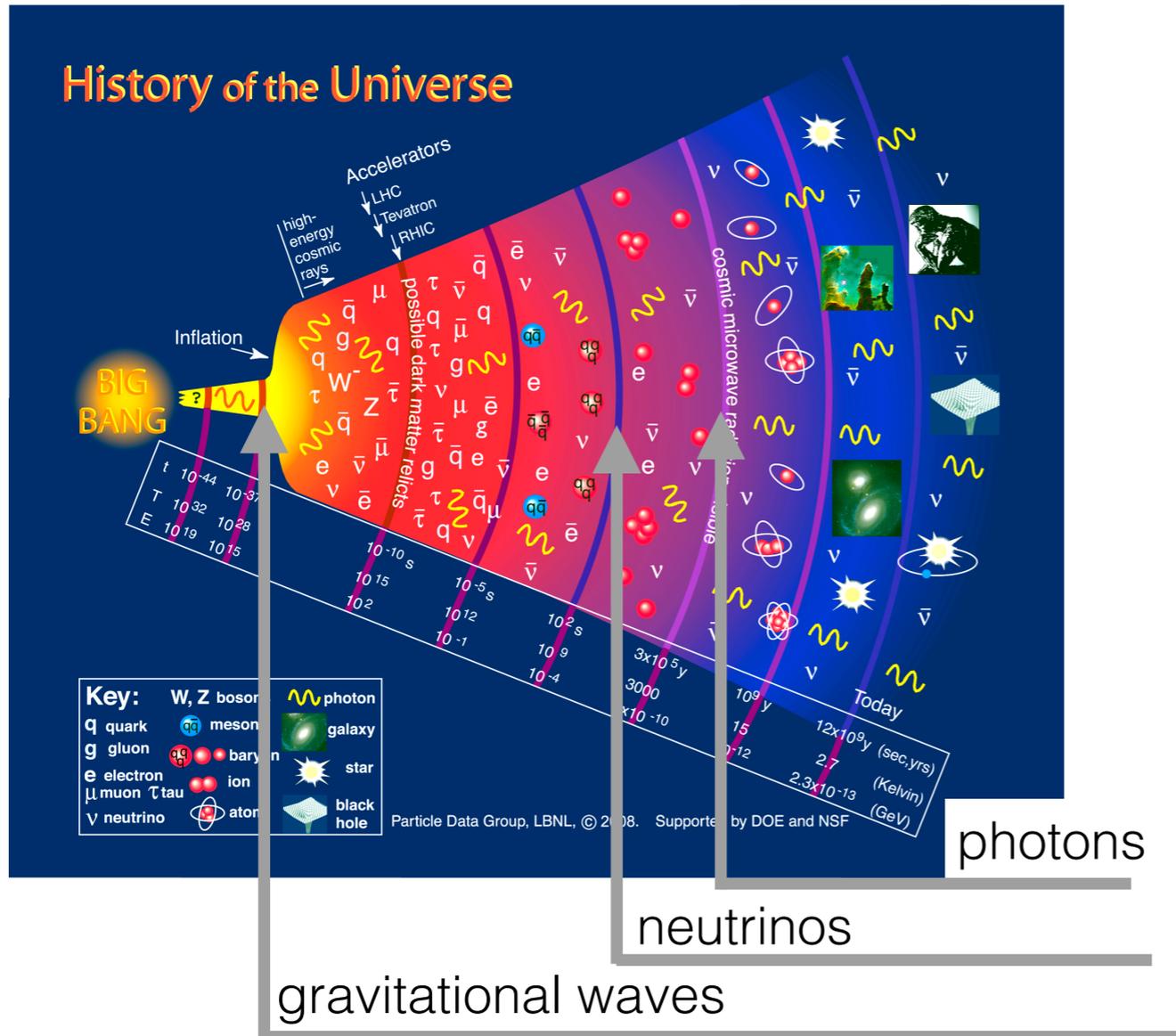


Inflation and Gravitational Waves

Probing the early Universe with gravitational waves

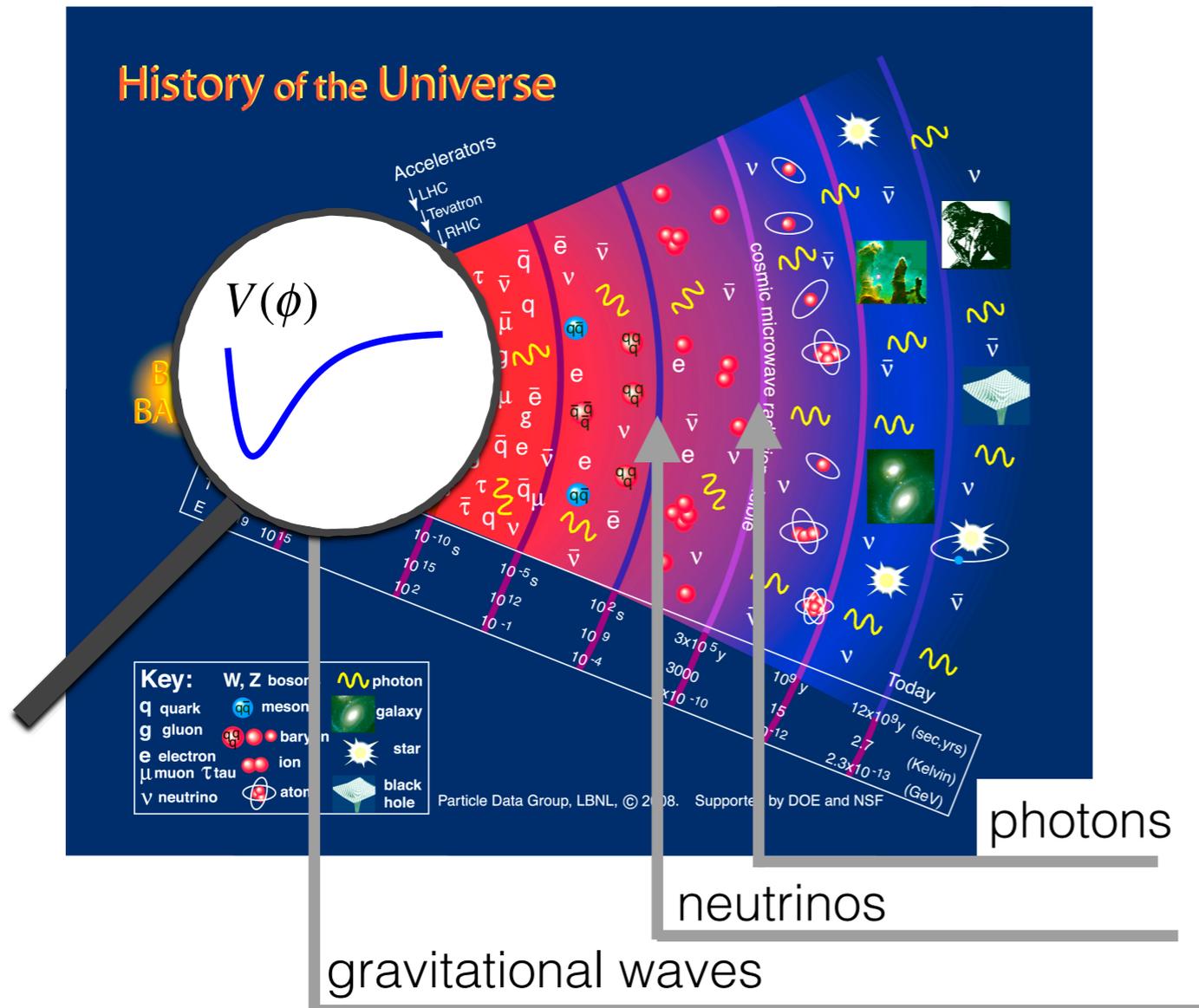


Valerie Domcke
APC, Paris

Saha Theory Workshop
Kolkata
16. - 20.1.2017

Inflation and Gravitational Waves

Probing the early Universe with gravitational waves



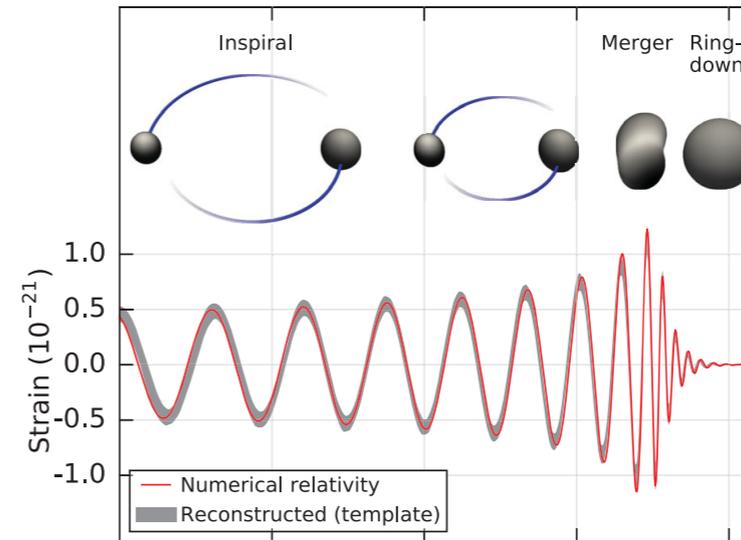
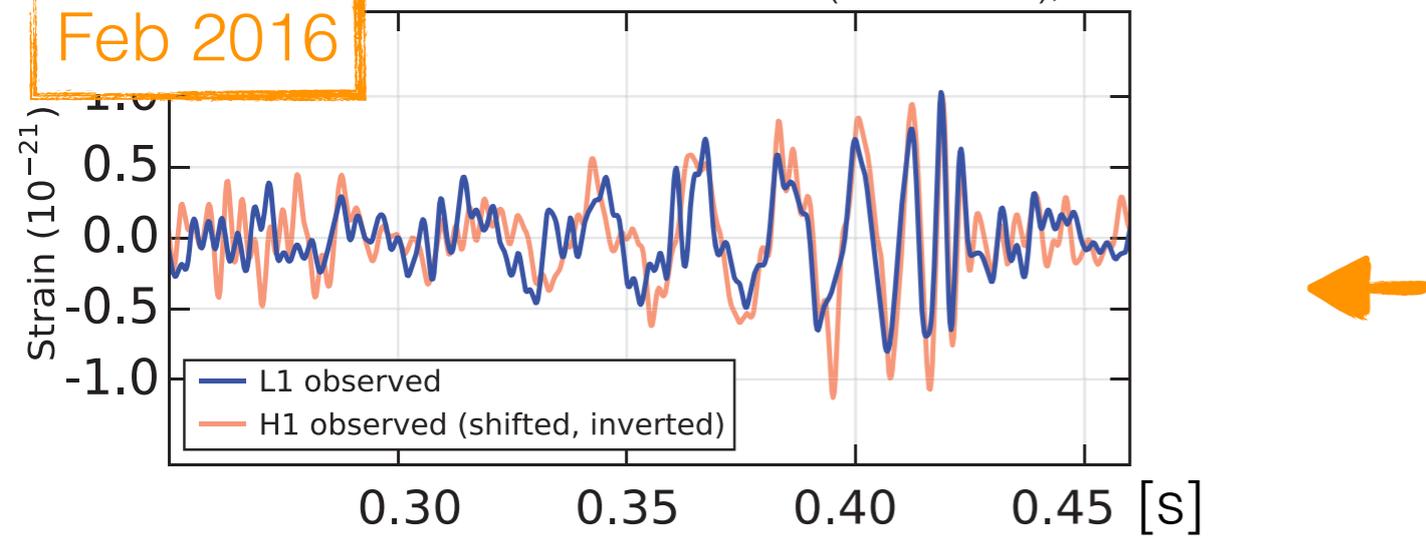
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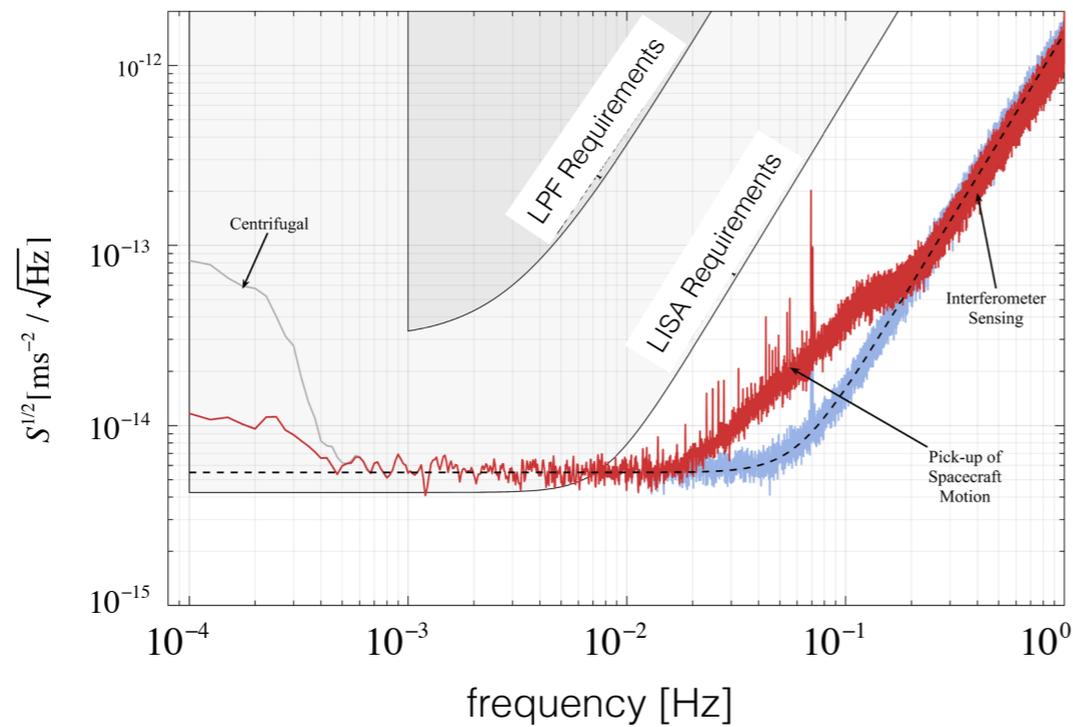
GWs in 2016

Feb 2016

PRL(116:061102), 2016



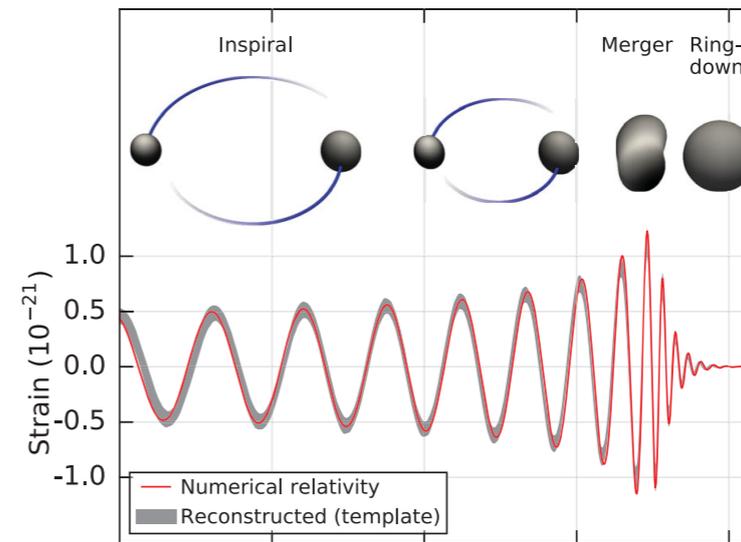
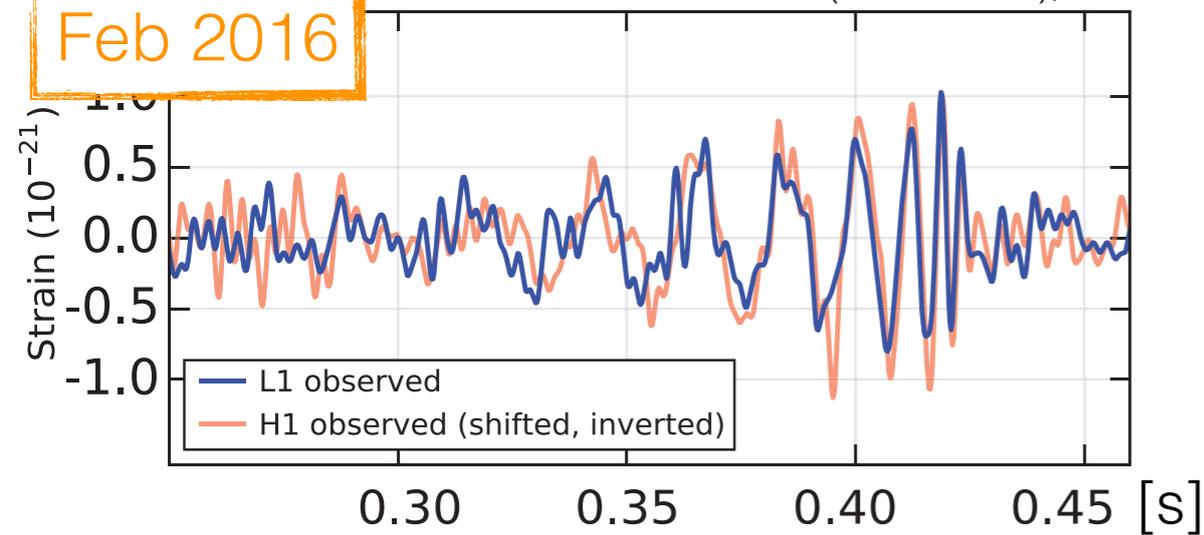
BH - BH
merger,
GW150914



GWs in 2016

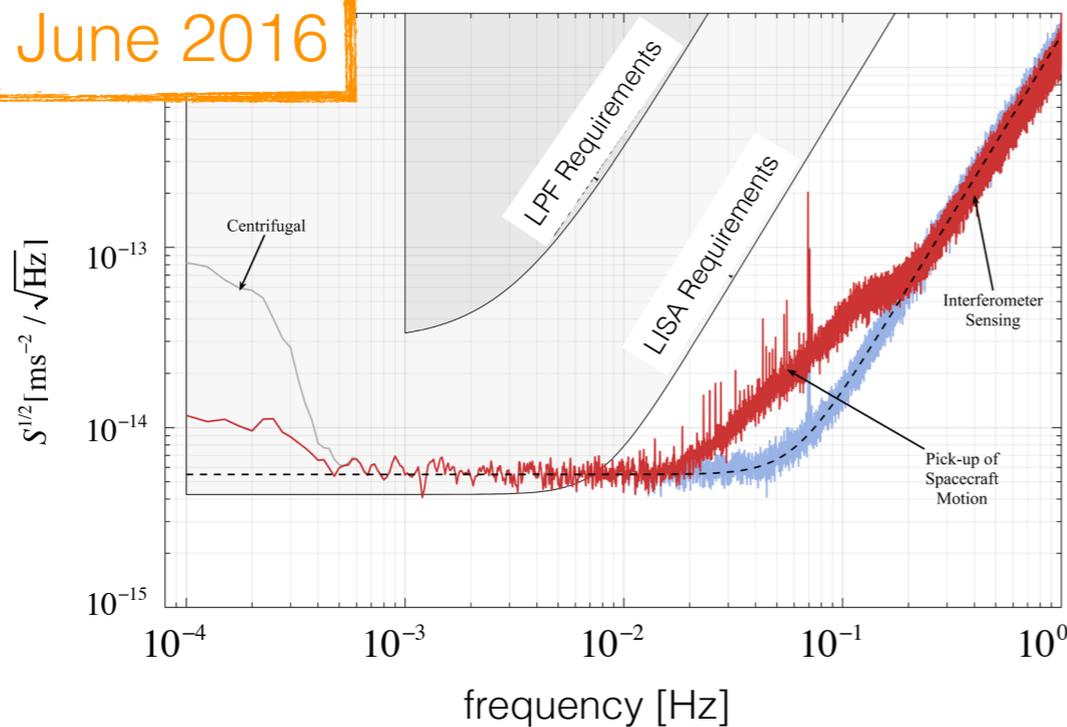
Feb 2016

PRL(116:061102), 2016



BH - BH merger,
GW150914

June 2016



PRL(116:231101), 2016

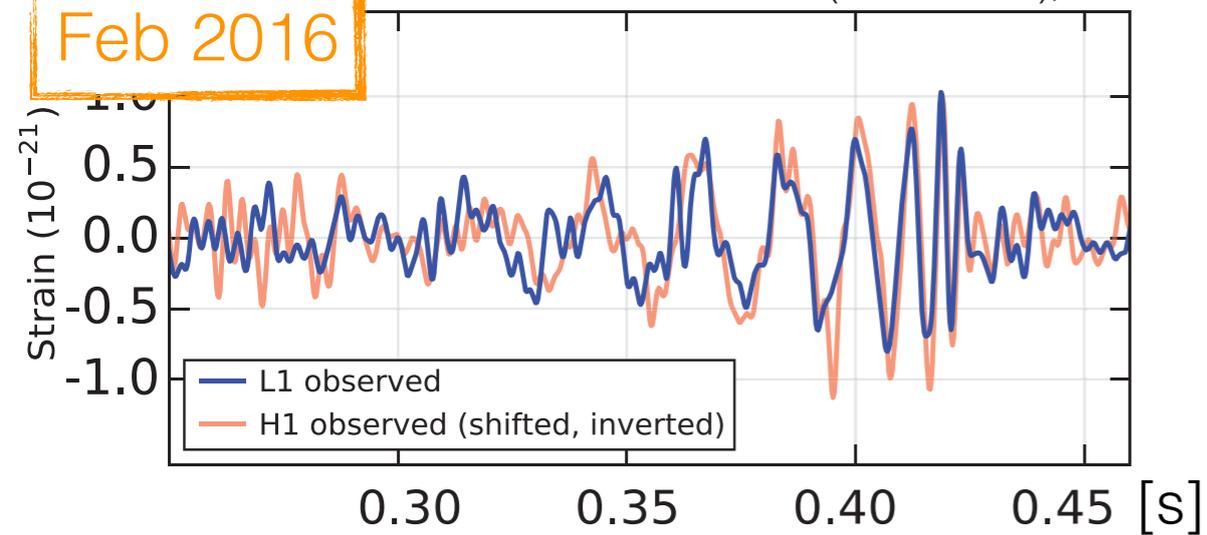
LISA Pathfinder, noise levels

- ESA mission, launch ~ 2034 (?)
- Interferometer(s) with free falling test masses in 3 satellites

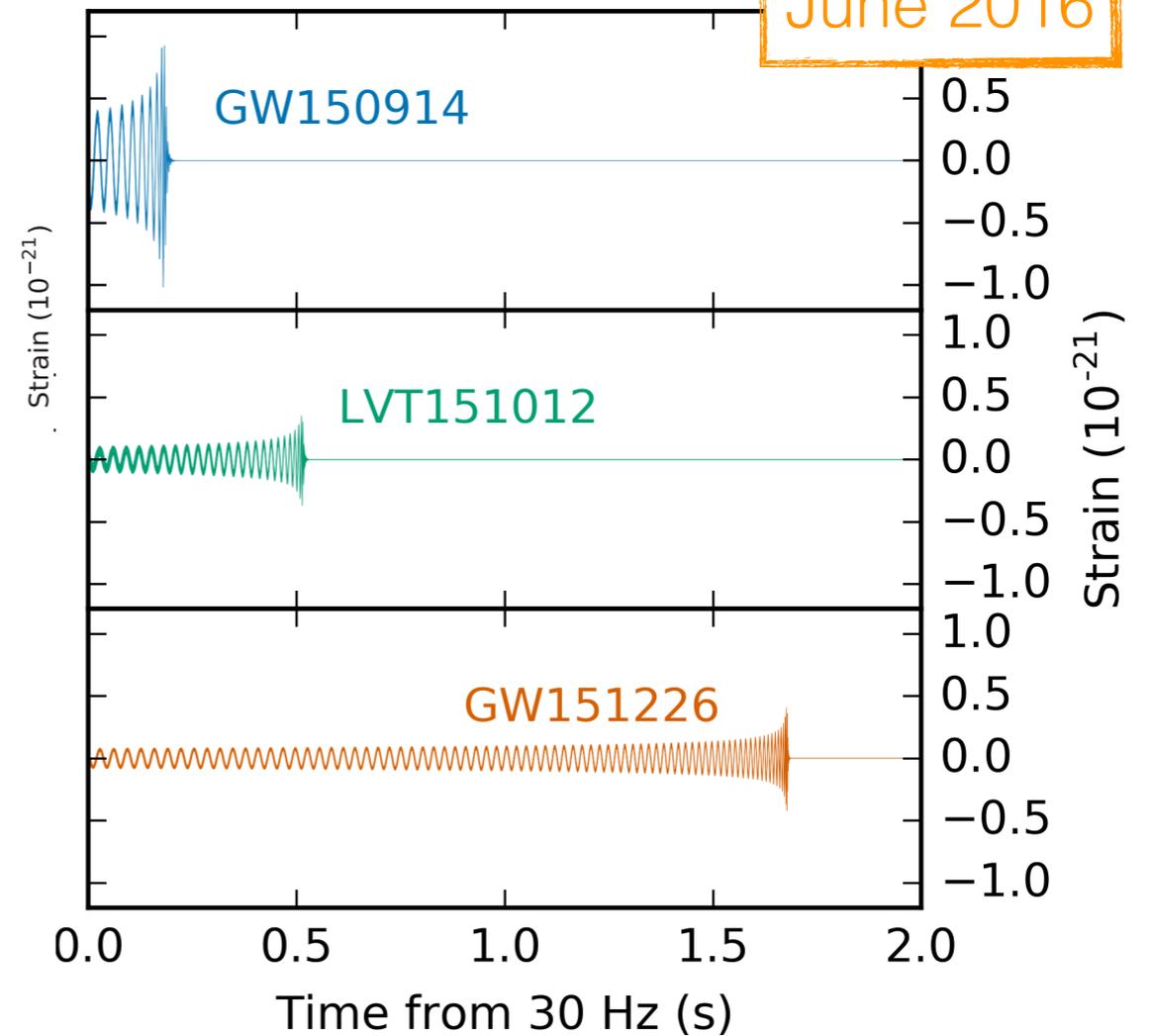
GWs in 2016

Feb 2016

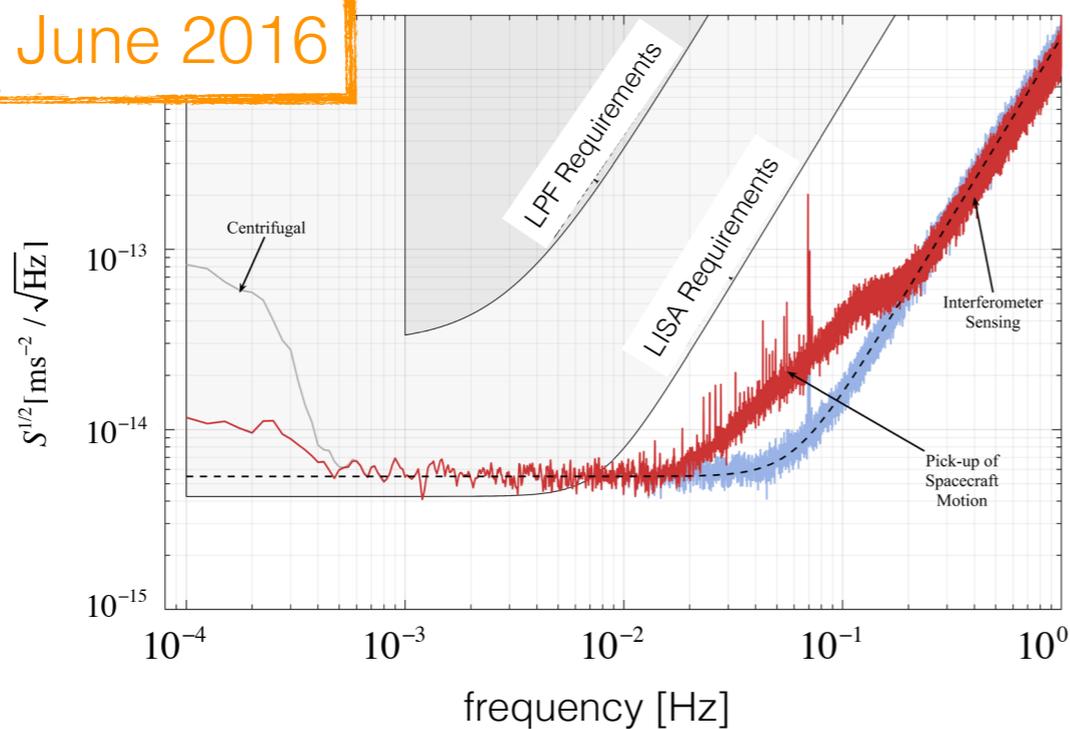
PRL(116:061102), 2016



June 2016



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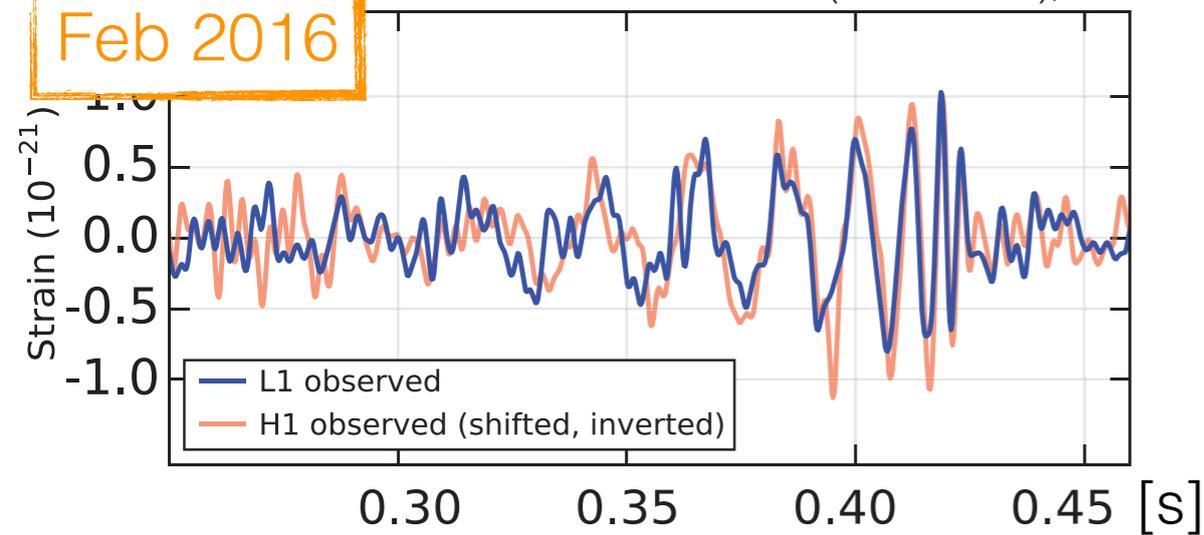


PRL(116:231101), 2016

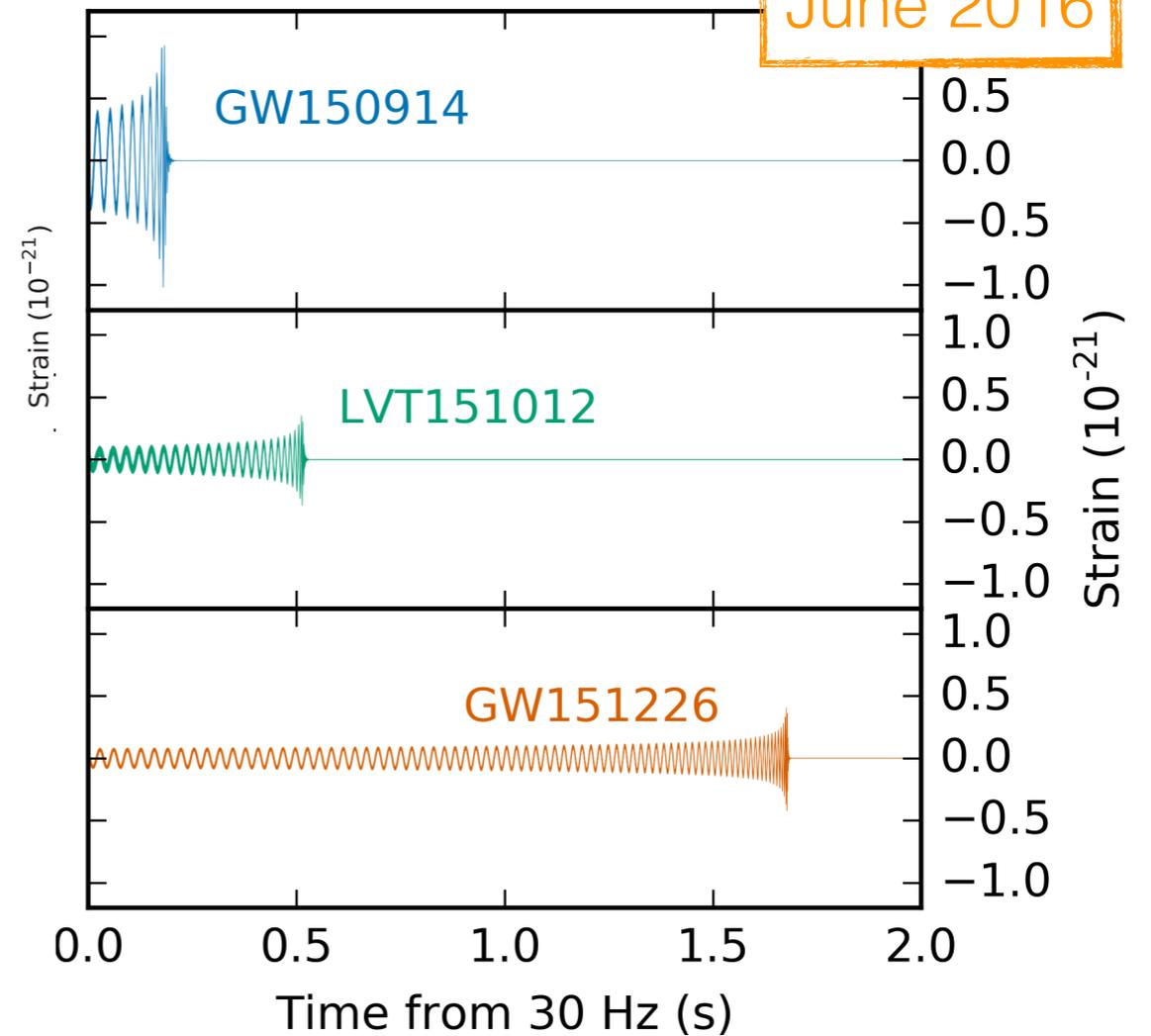
GWs in 2016

Feb 2016

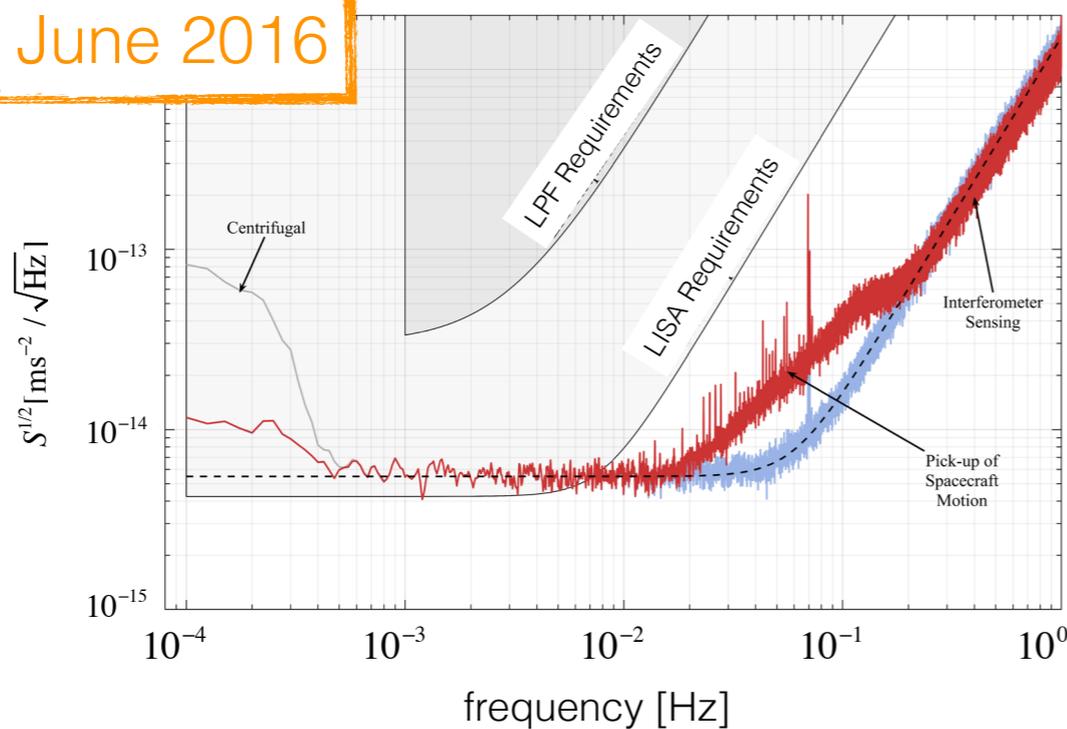
PRL(116:061102), 2016



June 2016



June 2016



PRL(116:231101), 2016

March 2016

LIGO India funded

October 2016

eLISA -> LISA (full 3 - arm mission)

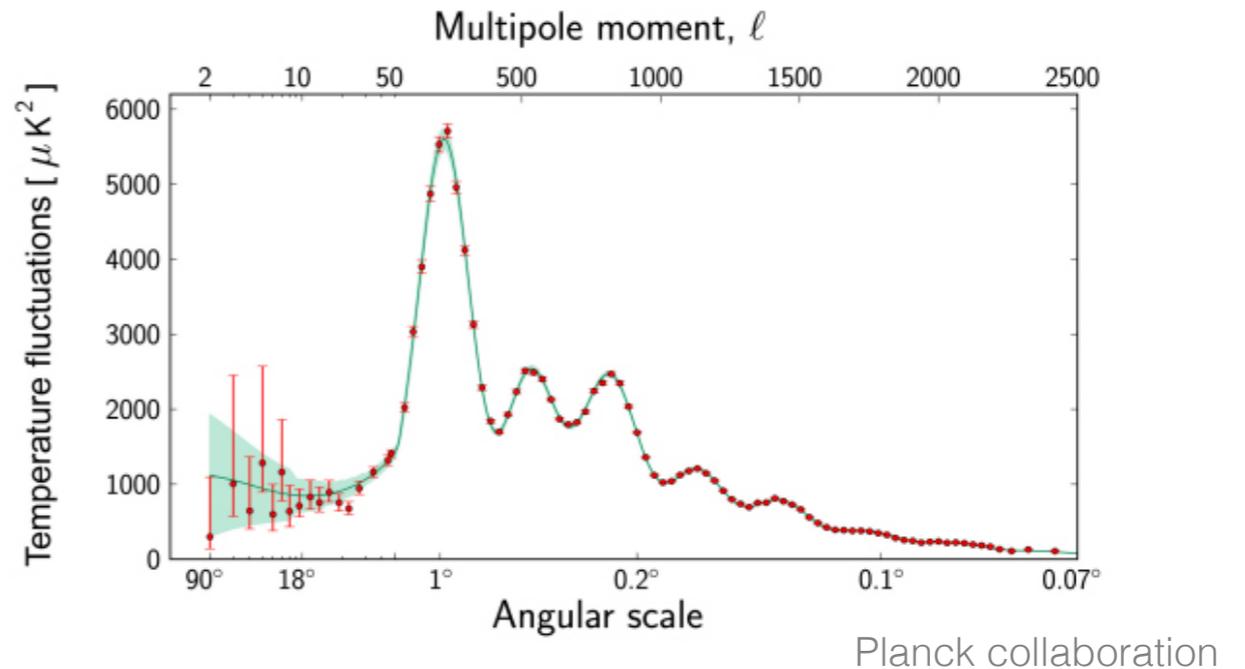
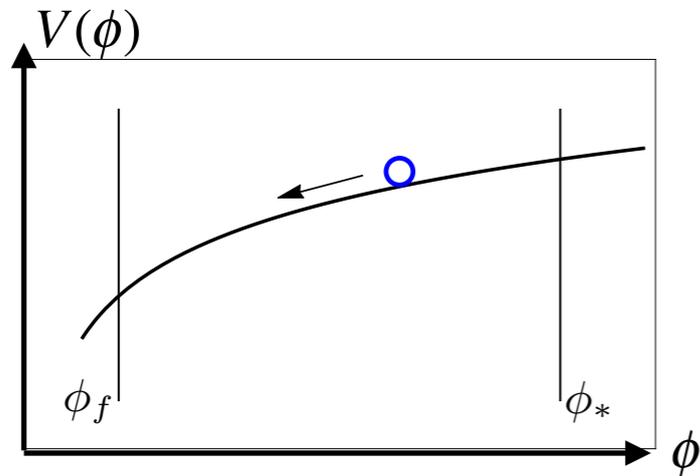
Outline

stochastic
GW backgrounds

- Primordial GW background
 -GWs from cosmic inflation
 -how to read to cosmic GW history book
 -searching for GWs: CMB versus direct detection

- An enhanced GW background ?
 -axion inflation
 -reduced symmetry during inflation
 -stiff equation of state after inflation

The paradigm of slow-roll inflation



large vacuum energy



exponential expansion



homogeneity of CMB

quantum fluctuations



become classical



tiny anisotropies in the CMB

The big question: $V(\phi) = ??$



$$\Delta_s^2 = \frac{V(\phi)}{24\pi^2 \epsilon(\phi)}, \quad \Delta_t^2 = \frac{2V(\phi)}{3\pi^2}; \quad \epsilon = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2$$

scalar spectrum, tensor spectrum

very successful paradigm, but very many possible realizations

Scales and horizons

co-moving perturbation modes
leave Hubble horizon during inflation,
re-enter after inflation

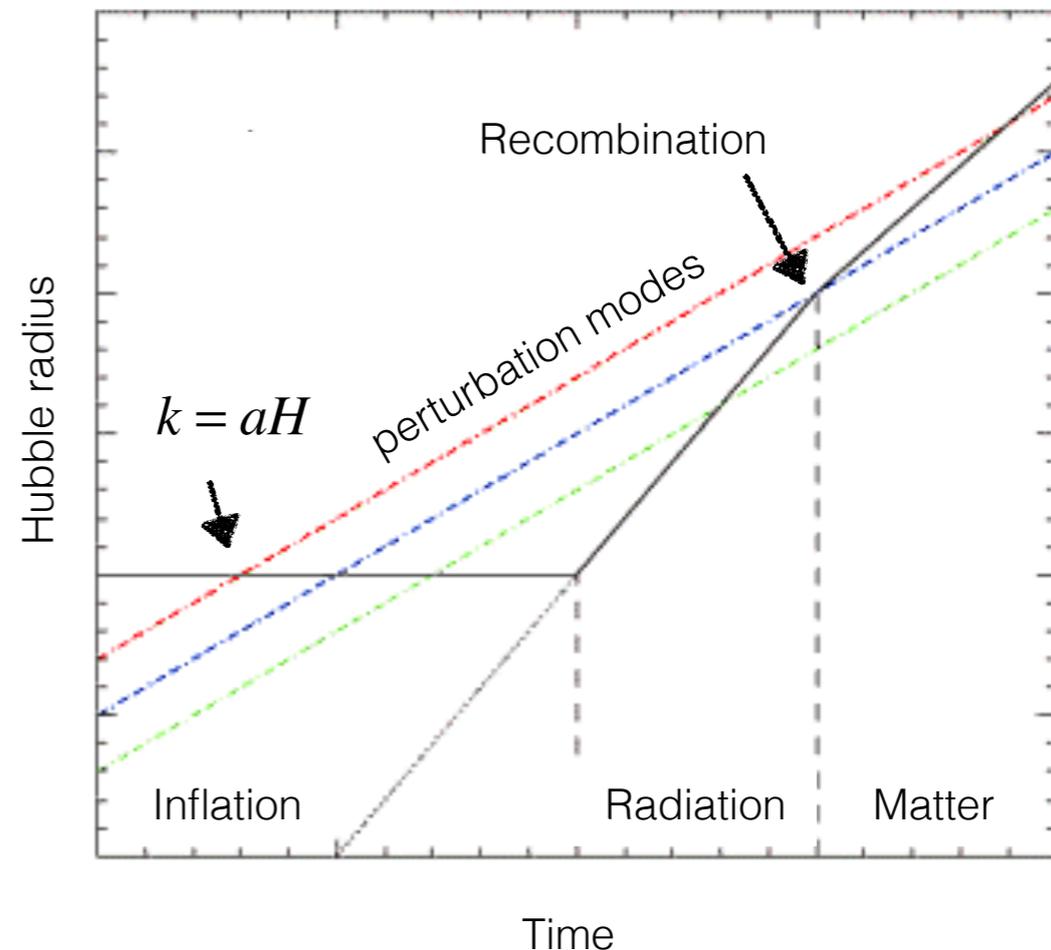


perturbation with given frequency today
corresponds to fixed time during inflation
and re-entry



1:1 relation: $f \rightarrow k \rightarrow H \rightarrow N_k \rightarrow V(\phi_k)$

$$N = N_{\text{CMB}} + \ln \frac{k_{\text{CMB}}}{0.002 \text{ Mpc}^{-1}} - 44.9 - \ln \frac{f}{10^2 \text{ Hz}}, \quad N = \int H dt$$



spectrum sensitive to primordial spectrum (scalar potential) and post-inflationary expansion

Some useful properties of GWs

perturbations of the background metric: $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x}, \tau))dx^\mu dx^\nu$

governed by linearized Einstein equation ($\tilde{h}_{ij} = ah_{ij}$, TT - gauge)

$$\tilde{h}_{ij}''(\mathbf{k}, \tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2 H^2} \tilde{h}_{ij}(\mathbf{k}, \tau) = \underbrace{16\pi G a \Pi_{ij}(\mathbf{k}, \tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic
(not spherical symmetric)
stress-energy tensor

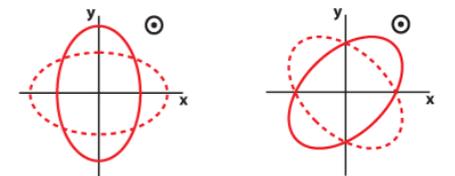
$$k \gg aH : h_{ij} \sim \cos(\omega\tau)/a, \quad k \ll aH : h_{ij} \sim \text{const.}$$

a useful plane wave expansion: $h_{ij}(\mathbf{x}, \tau) = \sum_{P=+, \times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2 \hat{\mathbf{k}} h_P(\mathbf{k}) \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)} e_{ij}^P(\hat{\mathbf{k}}) e^{-ik(\tau - \hat{\mathbf{k}}\mathbf{x})}$

transfer function, expansion coefficients, polarization tensor $P = +, \times$

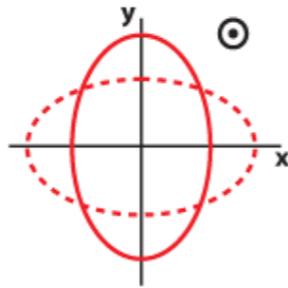
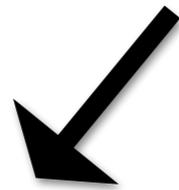
observational quantity in direct detection

$$\Omega_{\text{GW}} = \frac{1}{\rho_c} \frac{\partial \rho_{\text{GW}}(k, \tau)}{\partial \ln k}, \quad \rho_{\text{GW}}(\tau) = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle$$



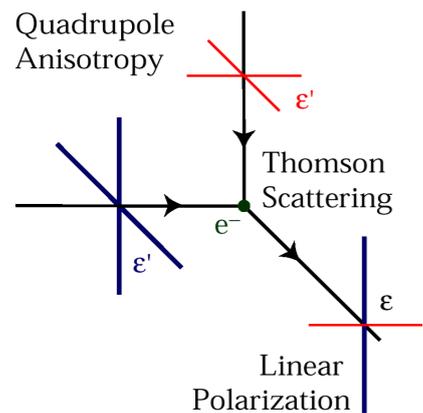
Hunting for primordial GWs

CMB

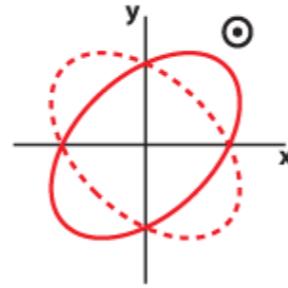


tensor anisotropies
on last scattering surface

polarization of CMB photons
through Thomson scattering



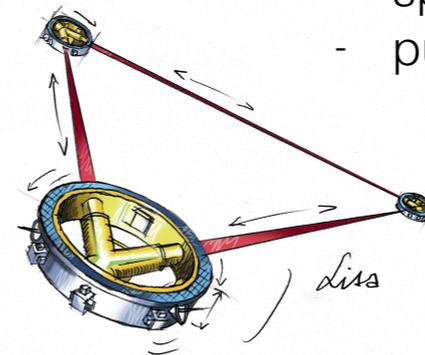
- Lensing: T \rightarrow E
- dust contaminates primordial signal
- B - modes most sensitive



direct

GW travels freely until today

distortion of space as GW
passes detector

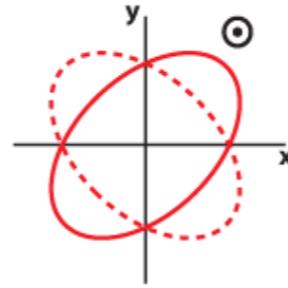
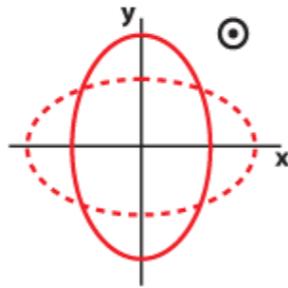
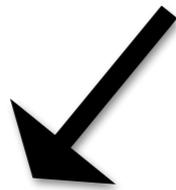


- ground-based interferometers
- space-based interferometers
- pulsar timing arrays

Hunting for primordial GWs

$$r = \Delta t^2 / \Delta s^2$$

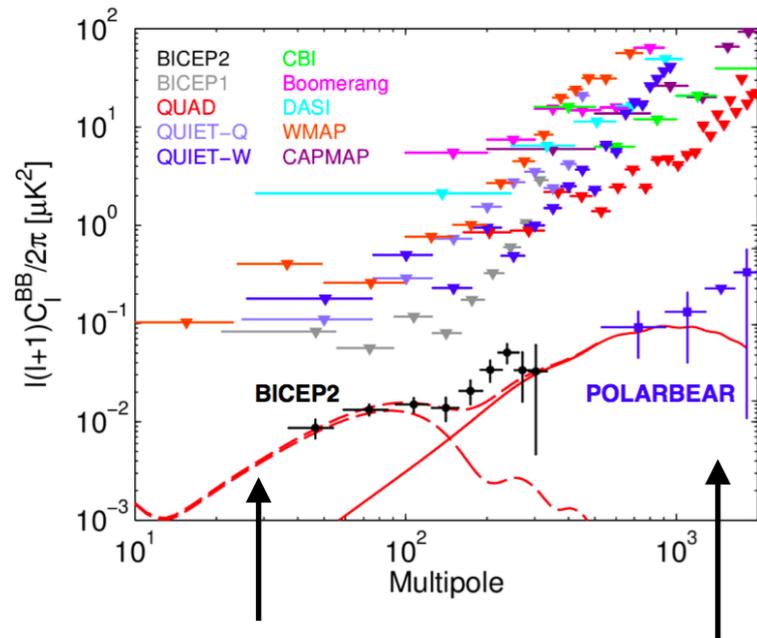
CMB



$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2$$

direct

BICEP2 '14



hypothetical primordial contribution with $r \sim 0.17$

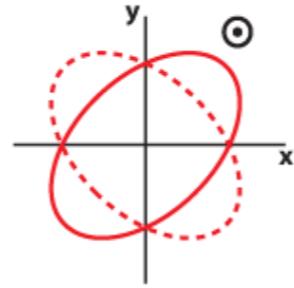
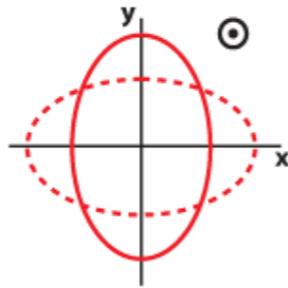
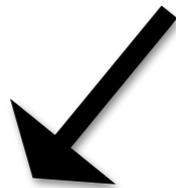
Lensing

sensitive to CMB scales

Hunting for primordial GWs

$$r = \Delta t^2 / \Delta s^2$$

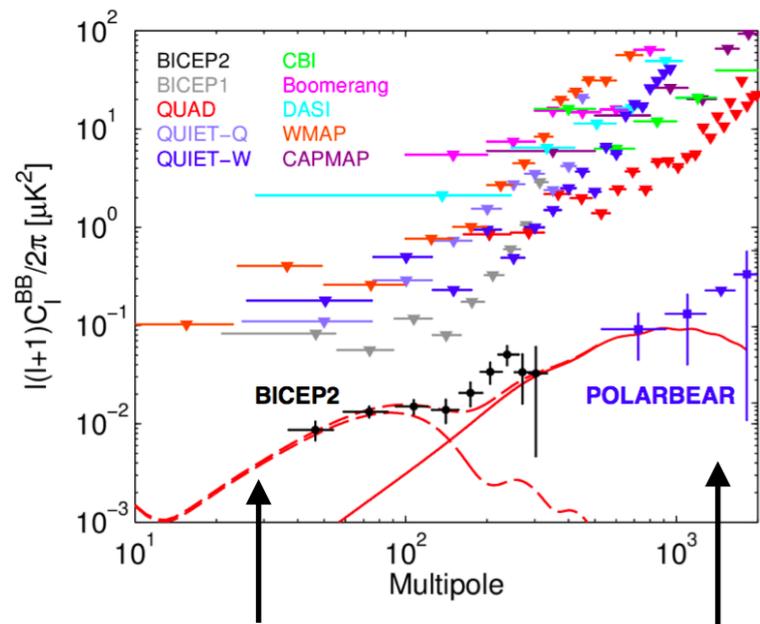
CMB



$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12 a_0^2 H_0^2} T_k^2$$

direct

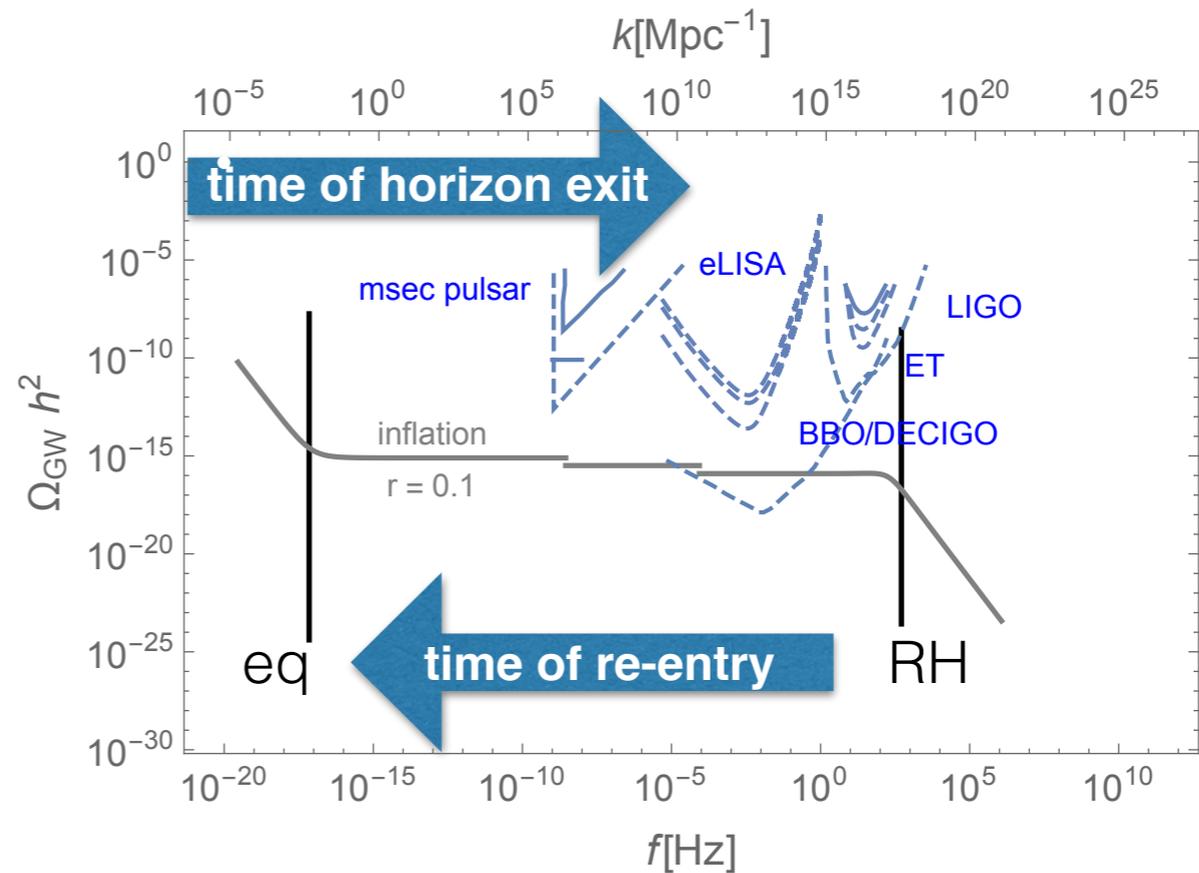
BICEP2 '14



hypothetical primordial contribution with $r \sim 0.17$

Lensing

sensitive to CMB scales



with suitable detectors, probe 30 orders of magnitude

Any hope of detection?

consistency relation of single-field slow-roll inflation:

$$n_T = -r / 8 < 0$$

Is this the end of the story ?

Any hope of detection?

consistency relation of single-field slow-roll inflation:

$$n_T = -r / 8 < 0$$

violated in slow-roll inflation if:

- extra dofs/sources during inflation
→ additional classical source of GWs

Cook, Sorbo '12

particle production during inflation

Fujita et al '15

spectator fields with $c_s < 1$

- broken spatial homogeneity during inflation

Endlich et al '13,
Dimastro-
giovanni et al '16

eg solid inflation,
chromo-natural inflation

- second order GW production from large scalar perturbations

Assadulahi, Wands '09

evaded in non-standard cosmology:

- non-standard equation of state during reheating

kination domination ($w = 1$)

Spookily '93; Joyce '96;
Giovannini '99; Sa, Henriques '10

- alternatives to inflation

bouncing cosmology

Gasperini, Veneziano '02

-

See also: Figueroa, Ricciardone et al '16

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Axion inflation

a generic coupling for a pseudoscalar inflaton:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Turner, Widrow '88,
Garretson, Field, Carroll '92,
Anber, Sorbo '06./'10/'12,
Barnaby, Namba, Peloso '11,
Barnaby, Pajer, Peloso '12 ,
.....

resulting background equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle.$$

$$\frac{d^2 A_\pm^a(\tau, k)}{d\tau^2} + \left[k^2 \pm 2k \frac{\xi}{\tau} \right] A_\pm^a(\tau, k) = 0, \quad \xi = \frac{\alpha\dot{\phi}}{2\Lambda H}$$

- tachyonic instability for the gauge field, controlled by $\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)$
 - exponential growth of gauge field modes towards end of inflation
 - backreaction on inflaton eom, new friction term: $\langle\vec{E}\vec{B}\rangle \simeq \mathcal{N} \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi}$
- + additional source for scalar and tensor fluctuations

power spectrum of scalar and tensor perturbations affected

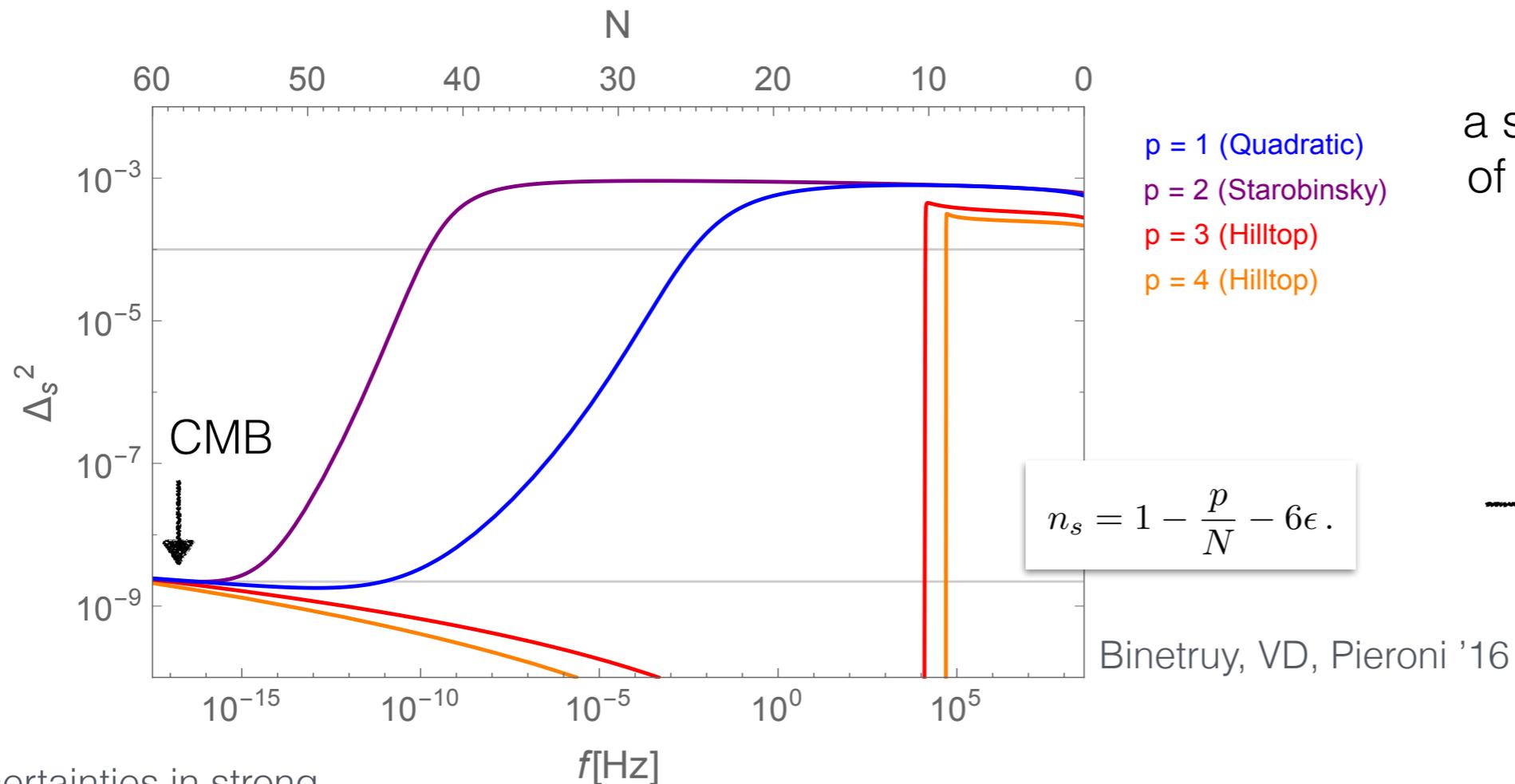
Scalar power spectrum

vacuum + sourced contribution:

$$\Delta_s^2(k) = \Delta_s^2(k)_{\text{vac}} + \Delta_s^2(k)_{\text{gauge}} = \left(\frac{H^2}{2\pi |\dot{\phi}|} \right)^2 + \left(\frac{\alpha \langle \vec{E}\vec{B} \rangle}{3bH\dot{\phi}} \right)^2$$

$$b = 1 - 2\pi\xi \frac{\alpha \langle \vec{E}\vec{B} \rangle}{3\Lambda H \dot{\phi}},$$

$$\langle \vec{E}\vec{B} \rangle \simeq \mathcal{N} \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi}$$



a simple parametrization of the scalar potential:

$$\epsilon_V \simeq \frac{\beta}{N^p} \quad \text{Mukhanov '13}$$

→ 3 parameters:
 α, β, p

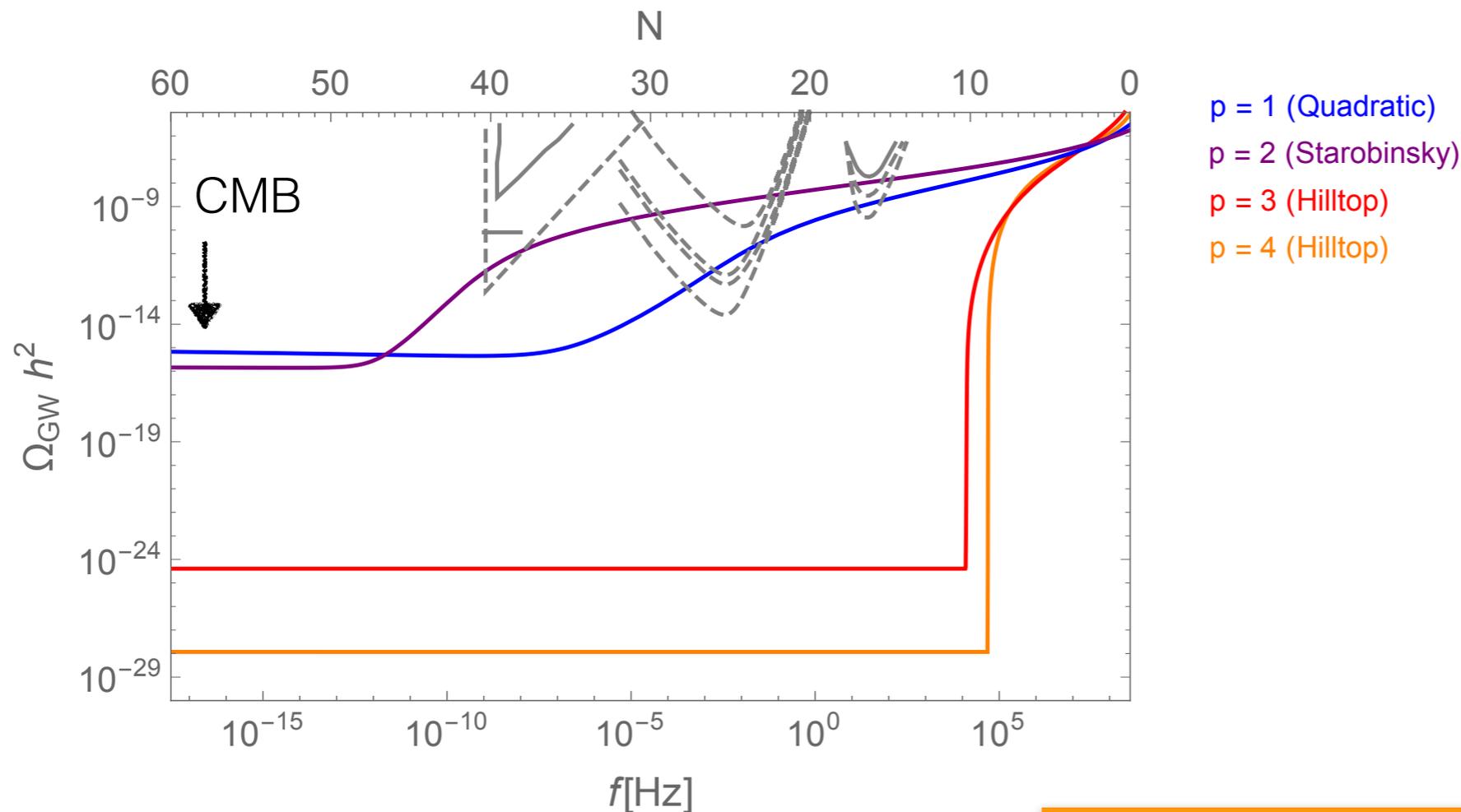
uncertainties in strong back reaction regime, Sloth '15, Peloso '16

strong, quasi-universal enhancement at small scales

Tensor power spectrum

vacuum + sourced contribution:

$$\Omega_{\text{GW}} = \frac{1}{12} \left(\frac{H}{\pi M_P} \right)^2 \left(1 + 4.3 \times 10^{(-7)} \frac{H^2}{M_P^2 \xi^6} e^{4\pi\xi} \right)$$



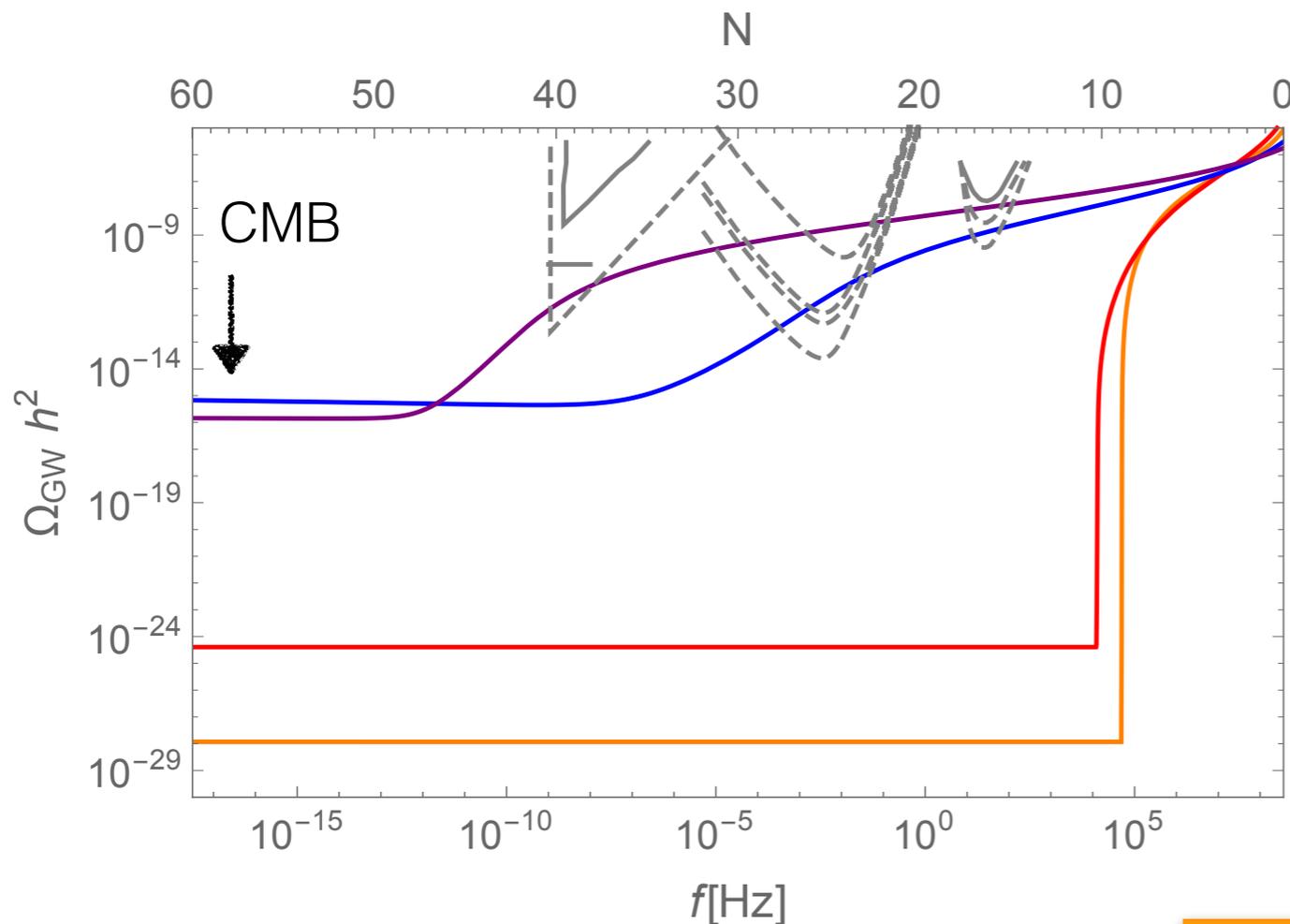
Binetruy, VD, Pieroni '16

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- p = 1 (Quadratic)
- p = 2 (Starobinsky)
- p = 3 (Hilltop)
- p = 4 (Hilltop)

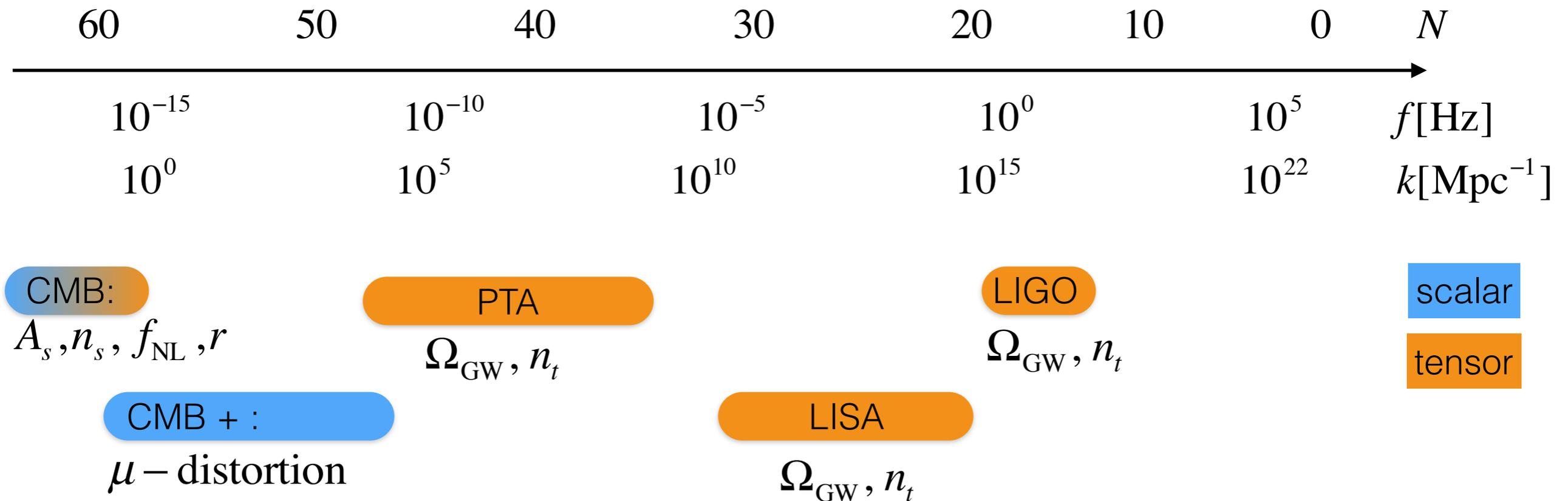
polarized
non-gaussian

$$\langle h(k_1)h(k_2)h(k_3) \rangle_{\text{equil}} \propto \Omega_{\text{GW}}(k)^{3/2}$$

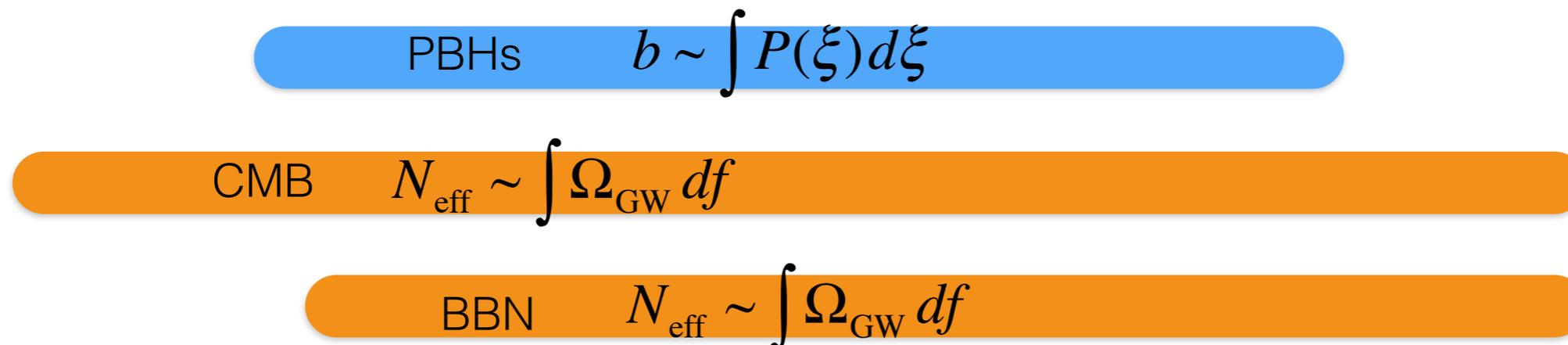
Binetruy, VD, Pieroni '16

strong enhancement on small scales

Observables & scales



constraints dominated by small scales (to be taken with grain of salt):

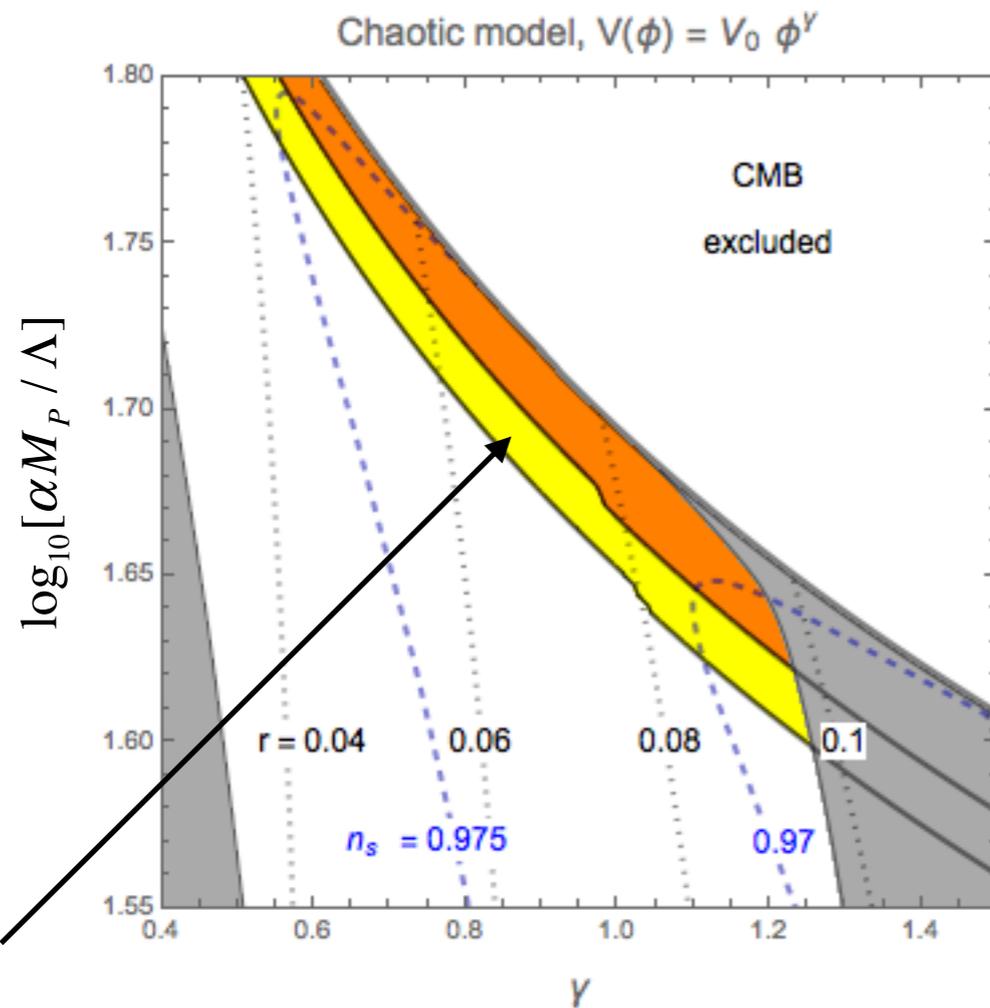


Chaotic model

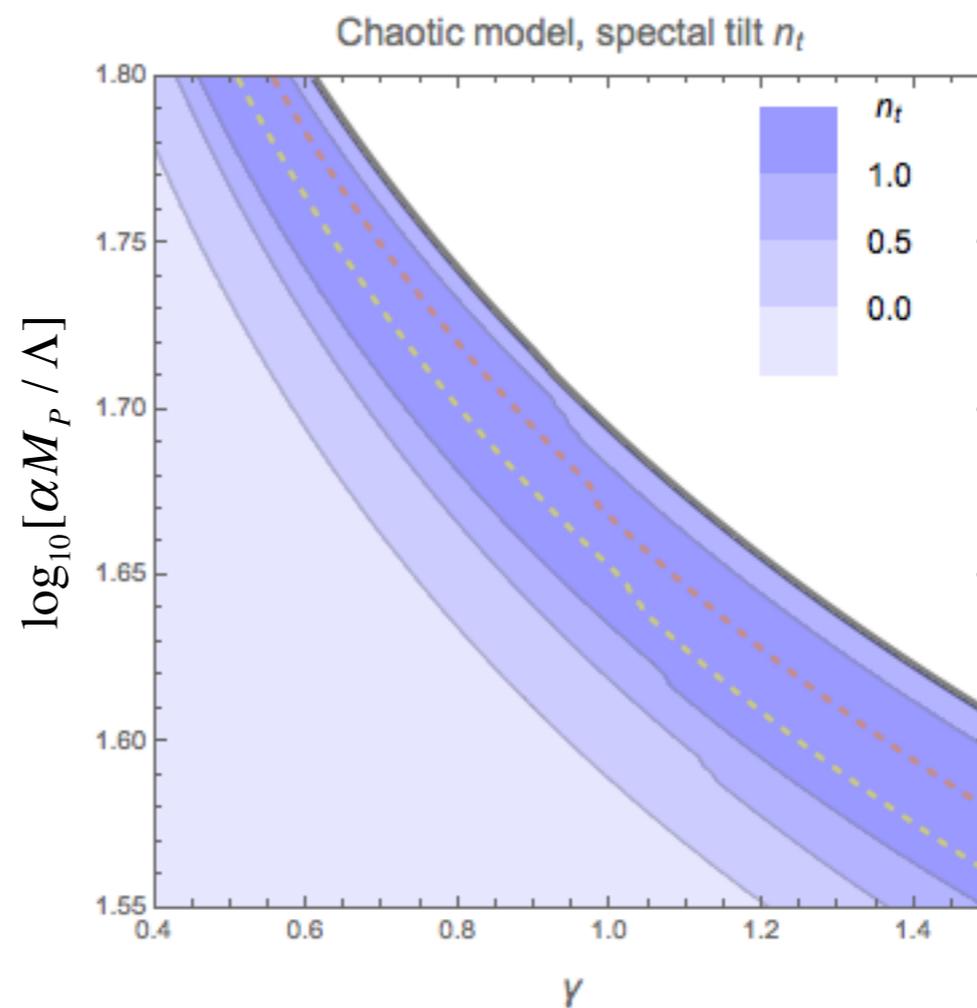
3 parameters:
 $\alpha / \Lambda, \beta, p$

$$V(\phi) = V_0 \phi^\gamma$$

Binetruy, VD, Pieroni '16; Figueroa, Ricciardone et al '16
 $\rightarrow p = 1$, vary $\beta = \gamma / 4$ and α



LISA sensitivity



Remarkable complementarity between CMB and direct GW searches

Outline

stochastic
GW backgrounds

- Primordial GW background
 -GWs from cosmic inflation
 -how to read to cosmic GW history book
 -searching for GWs: CMB versus direct detection
- An enhanced GW background ?
 -axion inflation
 -reduced symmetry during inflation
 -stiff equation of state after inflation

Broken spatial reparametrization invariance

standard picture: inflaton is time-dependent, but homogeneous

what if this space-reparametrization invariance is broken?

tensor modes may acquire mass m_h

consistency relation $n_T = -r / 8 < 0$ evaded

explicit examples: chromo-natural inflation, solid inflation

general effective field theory description:
(preserving isotropy and homogeneity)

Adshead, Wyman '12

Maleknejad, Sheikh-Jabbari '13

Endlich, Horn, Nicolis, Wang '14

Cannone, Tasinato, Wands '15

$$S_{(2)} = \frac{M_P^2}{8} \int dt d^3x a^3(t) n(t) \left[\dot{h}_{ij} - \frac{c_T^2(t)}{a^2} (\partial_l h_{ij})^2 - m_h^2(t) h_{ij}^2 \right]$$

renormalization of Planck mass

tensor sound speed

mass for tensor modes

simplified setup: $n(t) \rightarrow 1$

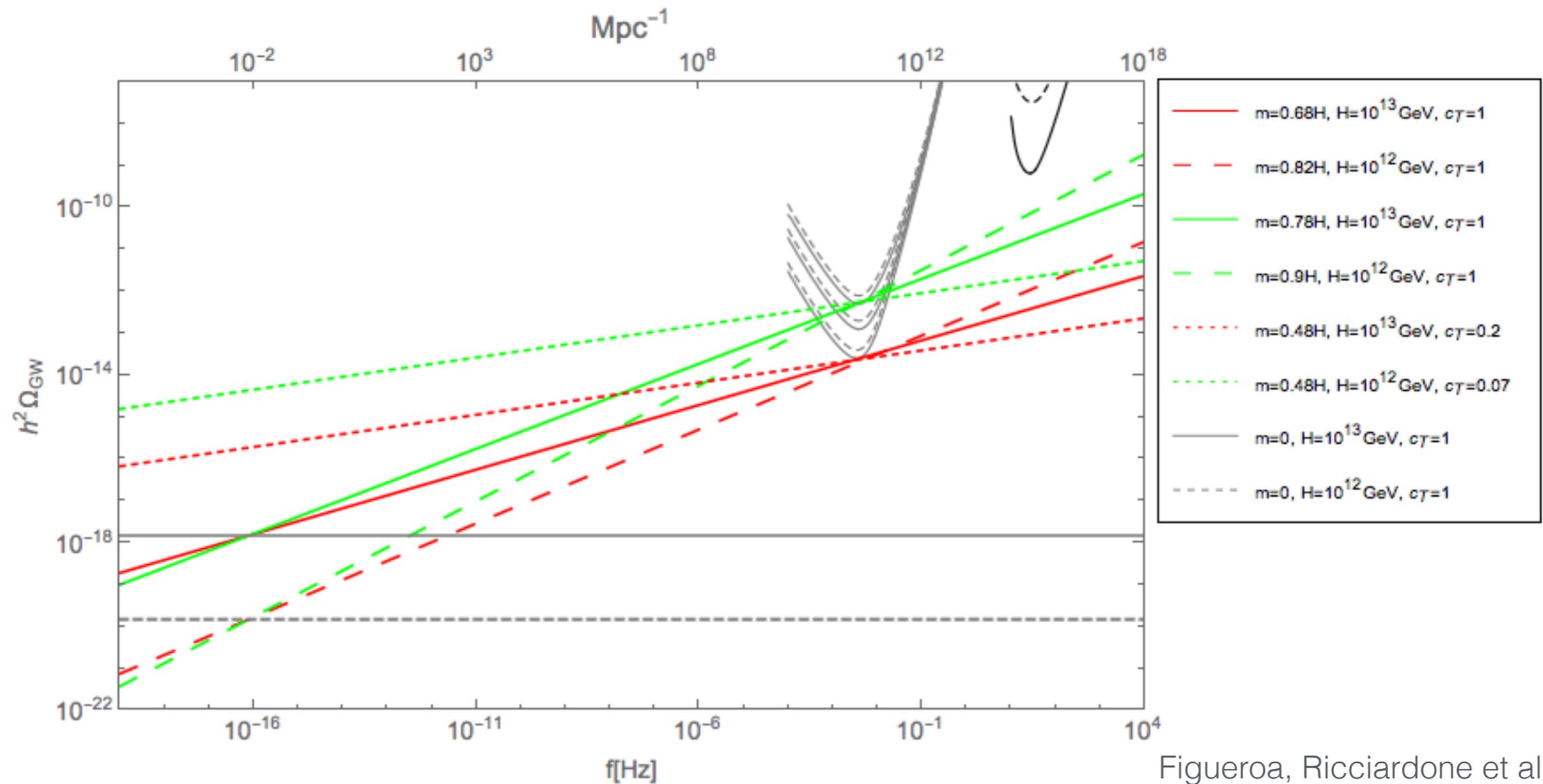
$c_T(t) \rightarrow c_T$

$m_h(t) \rightarrow m_h$

Broken spatial reparametrization invariance

effective field theory description with time-independent m_h and c_T :

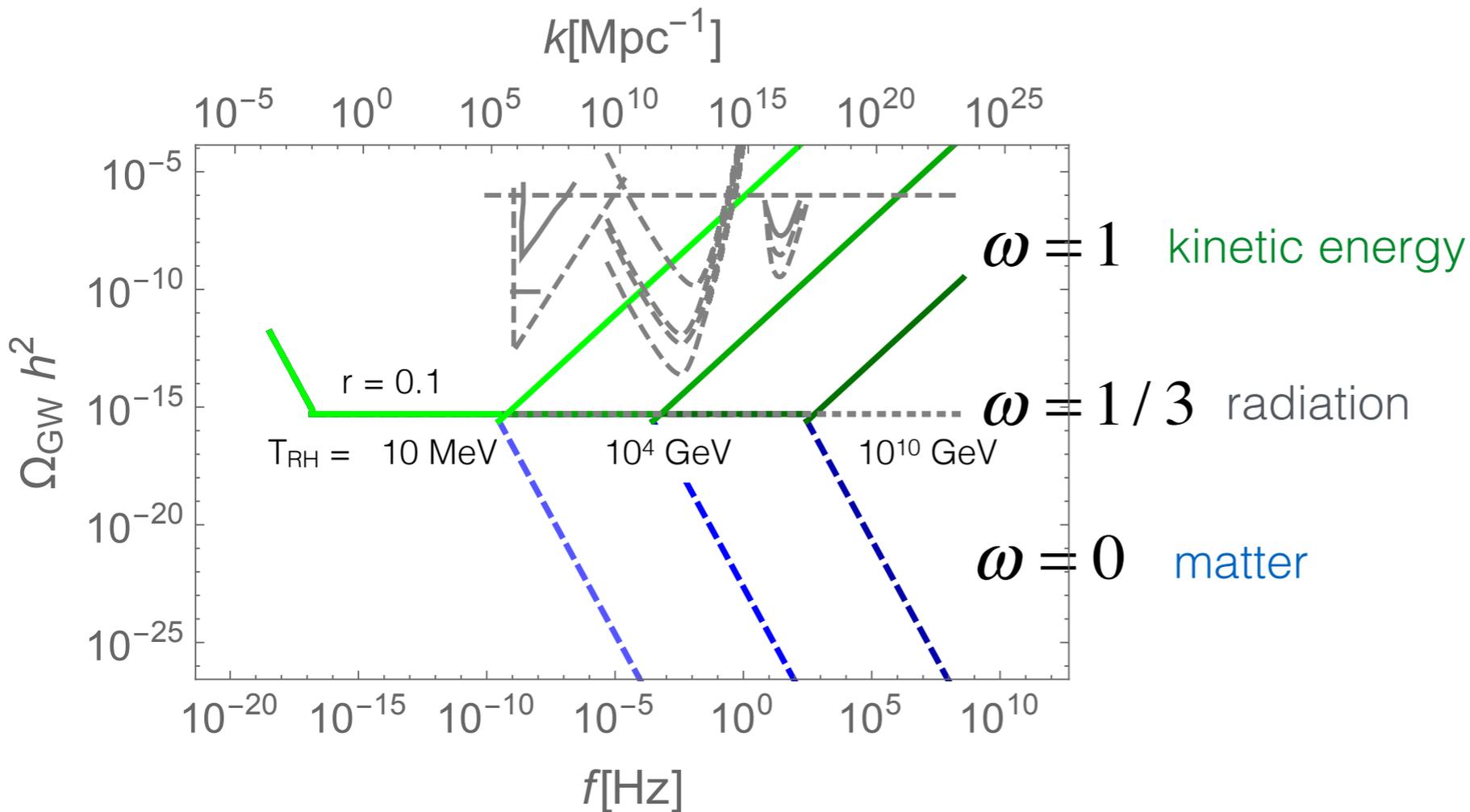
3 parameters:
 H, m_h, c_T



Figueroa, Ricciardone et al '16

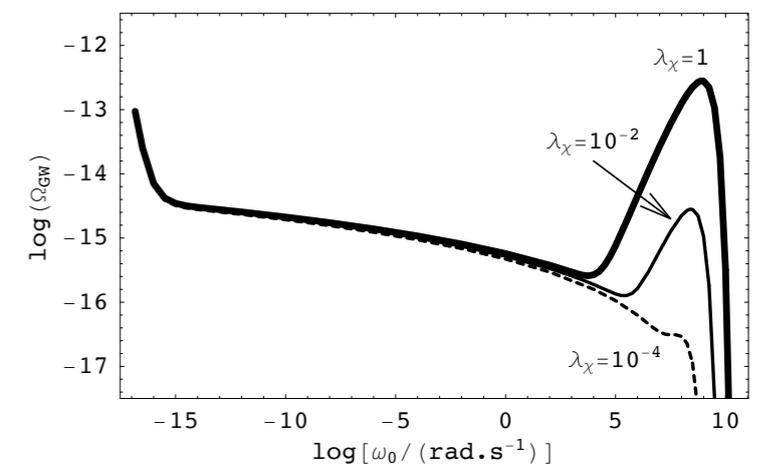
Non-standard equation of state after inflation

$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2, \quad T_k(t) = \frac{a(t_i)}{a(t)} = \left(\frac{t_i}{t}\right)^{\frac{2}{3(1+\omega)}} \rightarrow \Omega(f) = \Omega(f_0) \left(\frac{f}{f_0}\right)^{\frac{2(3\omega-1)}{1+3\omega}}$$



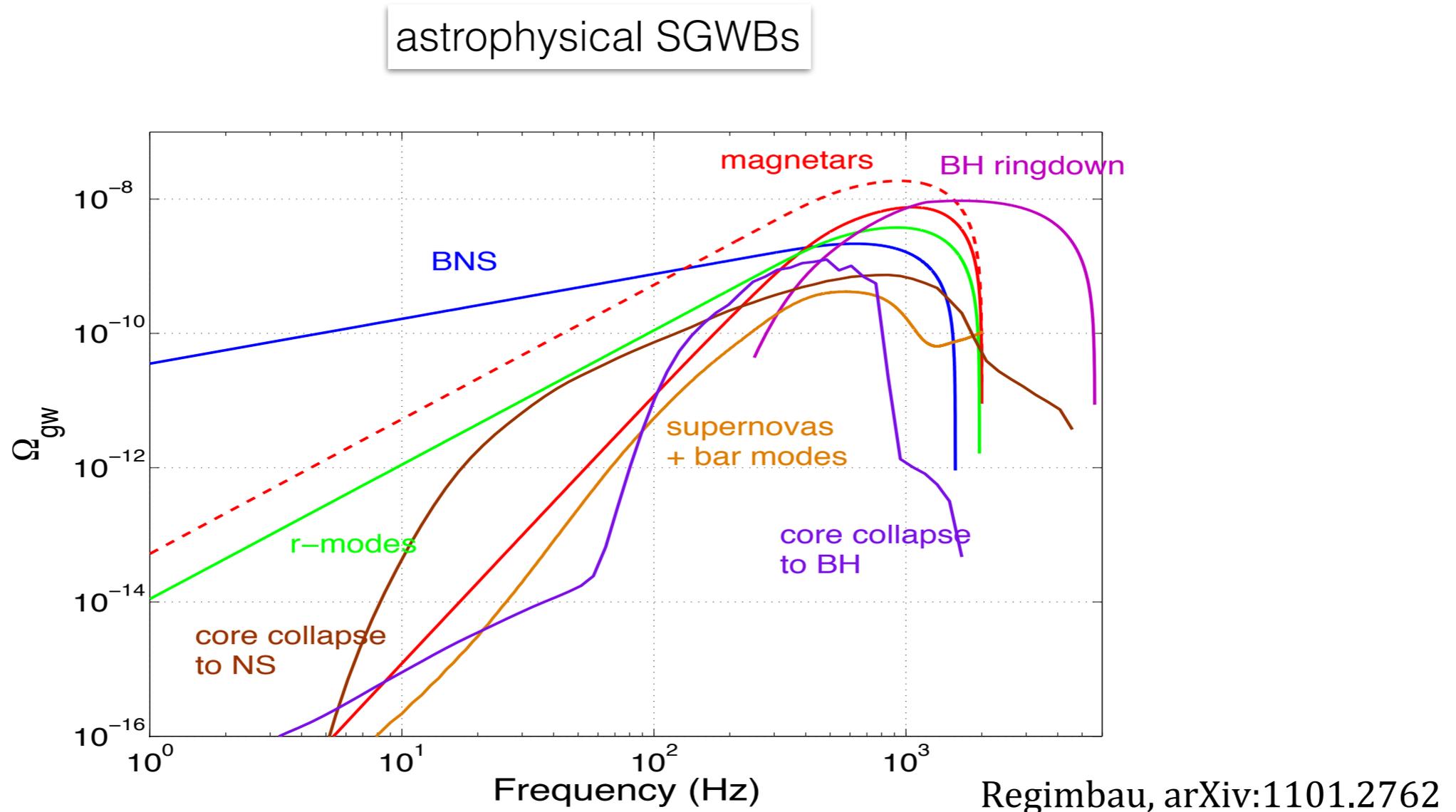
kination phase after inflation:
Spookily '93; Joyce '96

GW production in
(hybrid) quintessential models:
Giovannini '99; Sa, Henriques '10



stiff equation of state during reheating can enhance primordial GW signal

A note of caution...



LIGO events suggest large astrophysical backgrounds, possibly substractable due to 'popcorn' nature

Conclusion and Outlook

- The stochastic background of cosmic inflation is an extremely powerful tool: It would shed light on the microphysics of inflation, as well as the entire subsequent cosmological history
- The standard primordial tensor vacuum fluctuations are too small to be detectable within the near future, but an enhancement by many orders of magnitude is possible in well-motivated models
- If the inflaton is a pseudoscalar, both scalar and tensor spectrum are strongly enhanced towards the end of inflation - at the small frequencies of LISA and LIGO
- The complementarity of CMB and direct GW measurements provides a powerful probe of the physics of cosmic inflation.

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Thank you!

backup slides

pseudoscalar inflation background dynamics

a useful classification of inflation models:

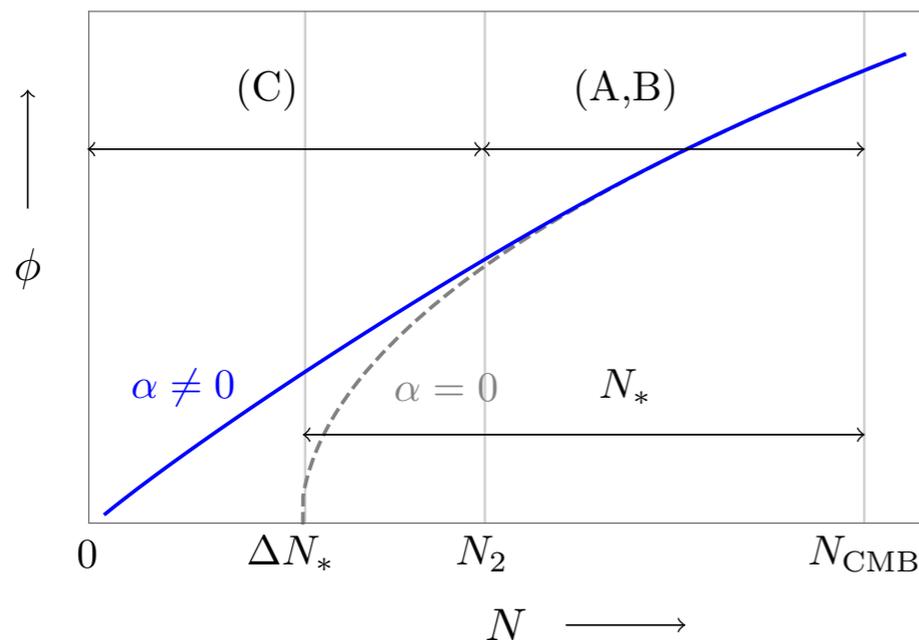
$$\epsilon_V = \frac{\beta}{N^p} + \mathcal{O}(1/N^{p+1})$$

Mukhanov '13

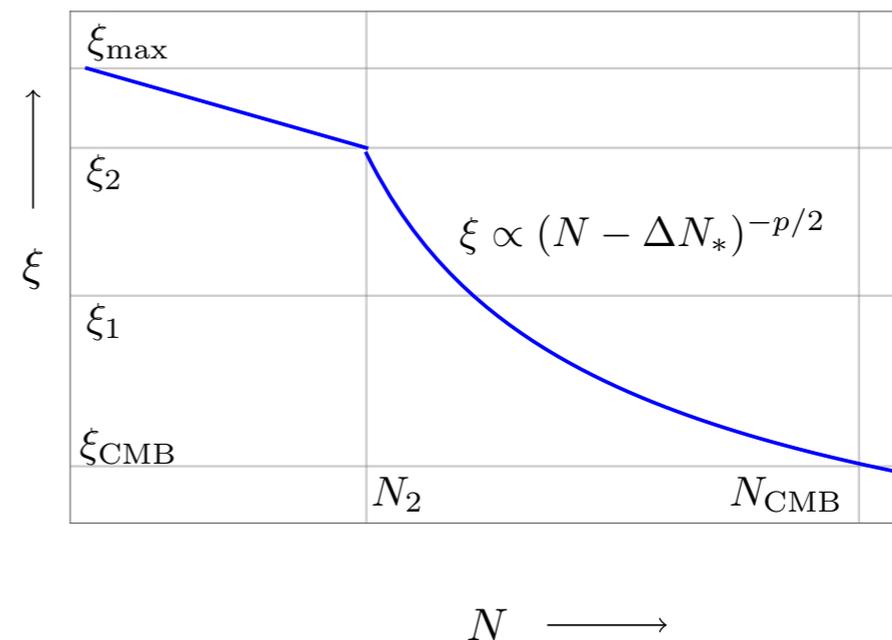


3 parameters:
 α, β, p

evolution of the inflation field



evolution of $\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)$



- additional friction, CMB observables evaluated at 'later' point on $V(\phi)$

$$n_s \simeq 1 - \frac{\mathcal{O}(1)}{N_*}, \quad r = \frac{16\beta}{N_*}, \quad N_* < N_{\text{CMB}} \simeq 60$$

n_s reduced, r increased

- rapid increase of ξ for large values of p , enters eom exponentially

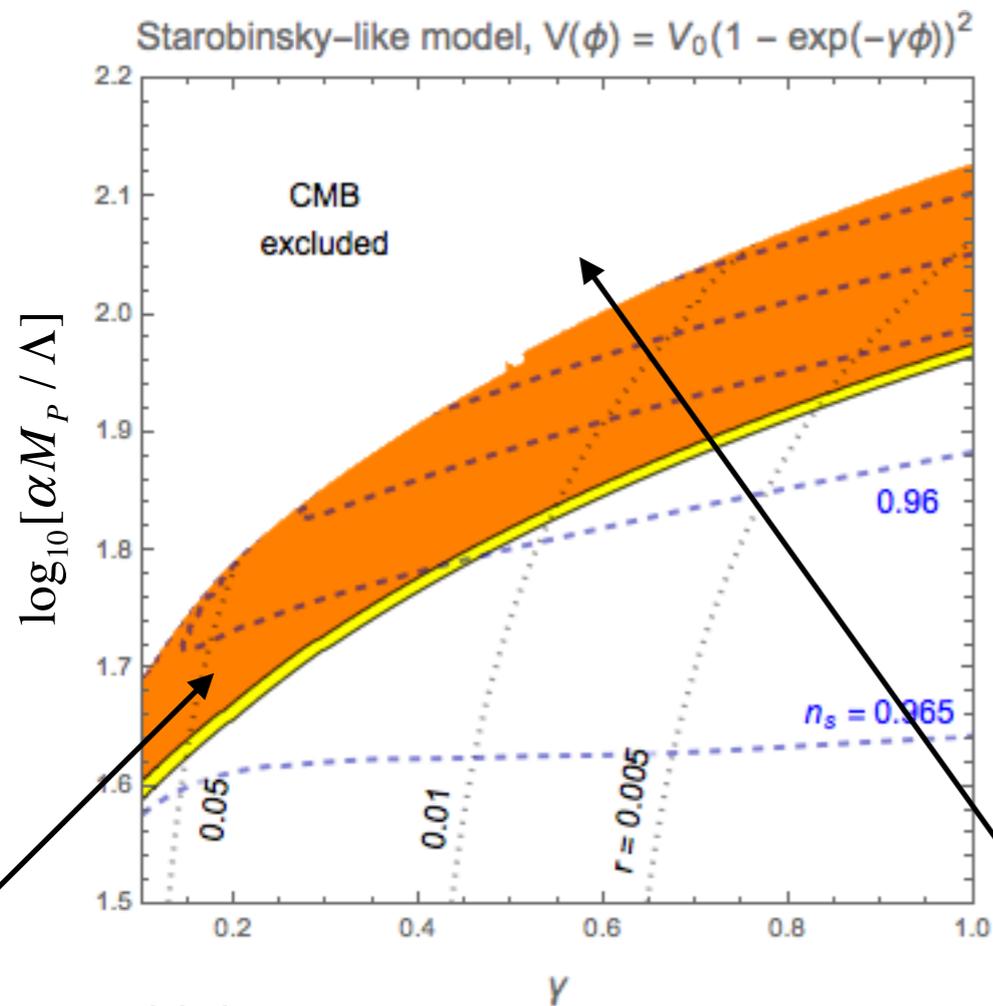
large gauge field effects at the end of inflation, in particular for large p (small r !)

Starobinsky-type model

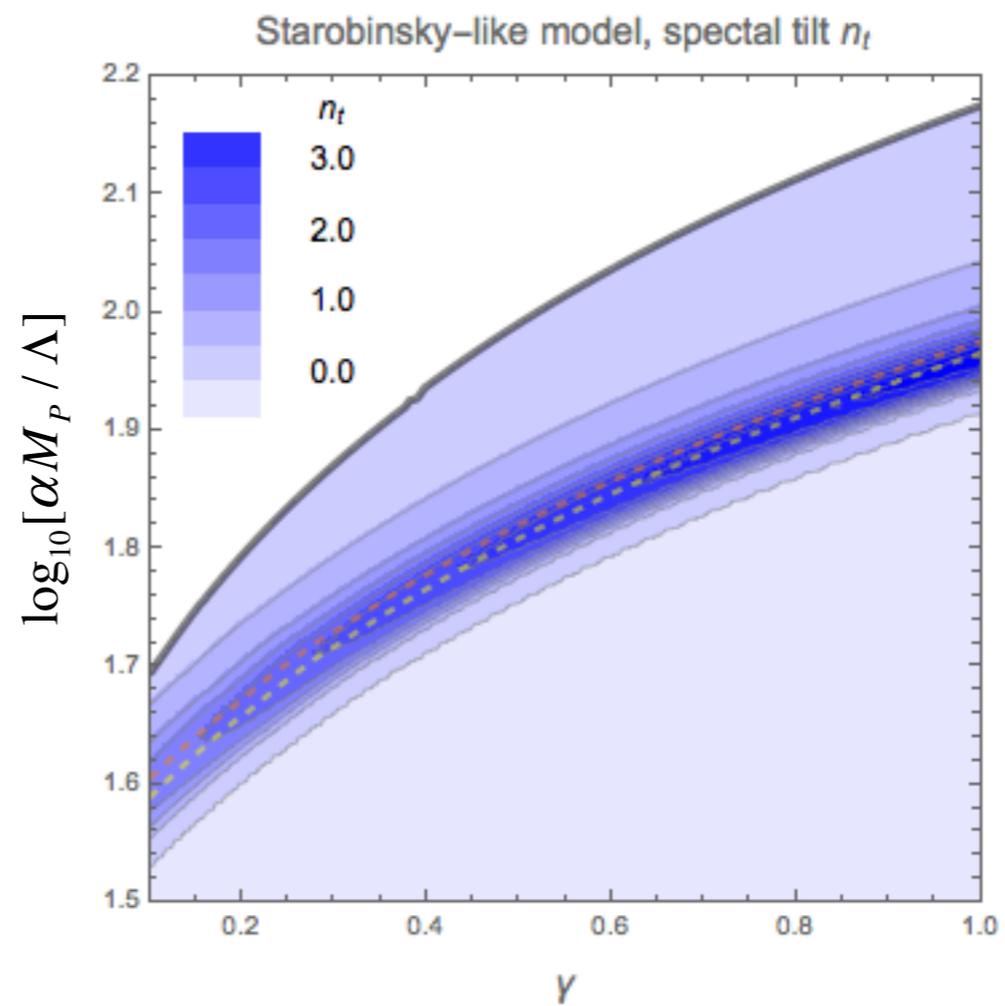
3 parameters:
 $\alpha / \Lambda, \beta, p$

$$V(\phi) = V_0(1 - e^{-\gamma\phi})^2$$

Binetruy, VD, Pieroni '16; Figueroa, Ricciardone et al '16
 $\rightarrow p = 2$, vary $\beta = 1/(2\gamma^2)$ and α



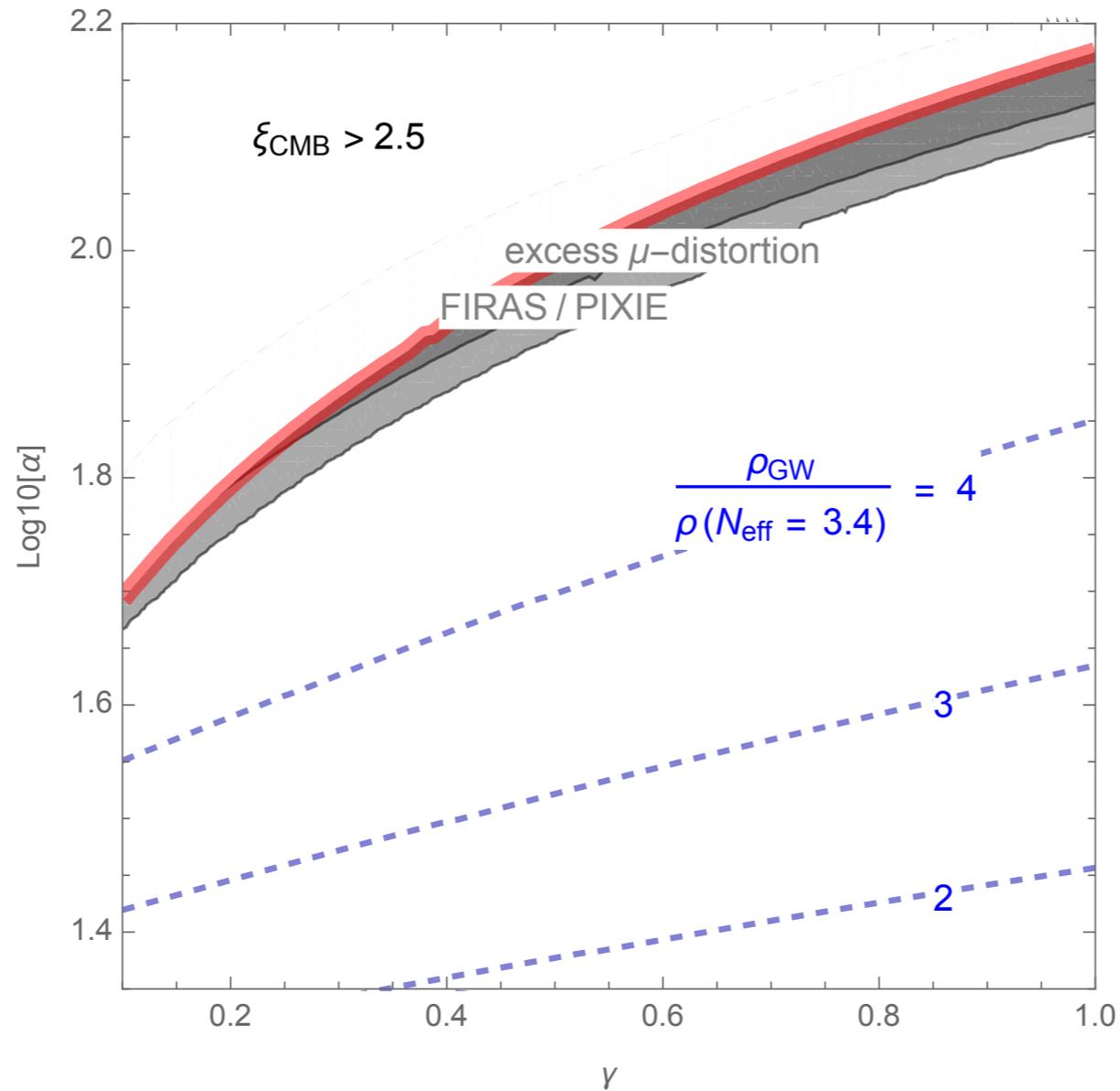
LISA sensitivity



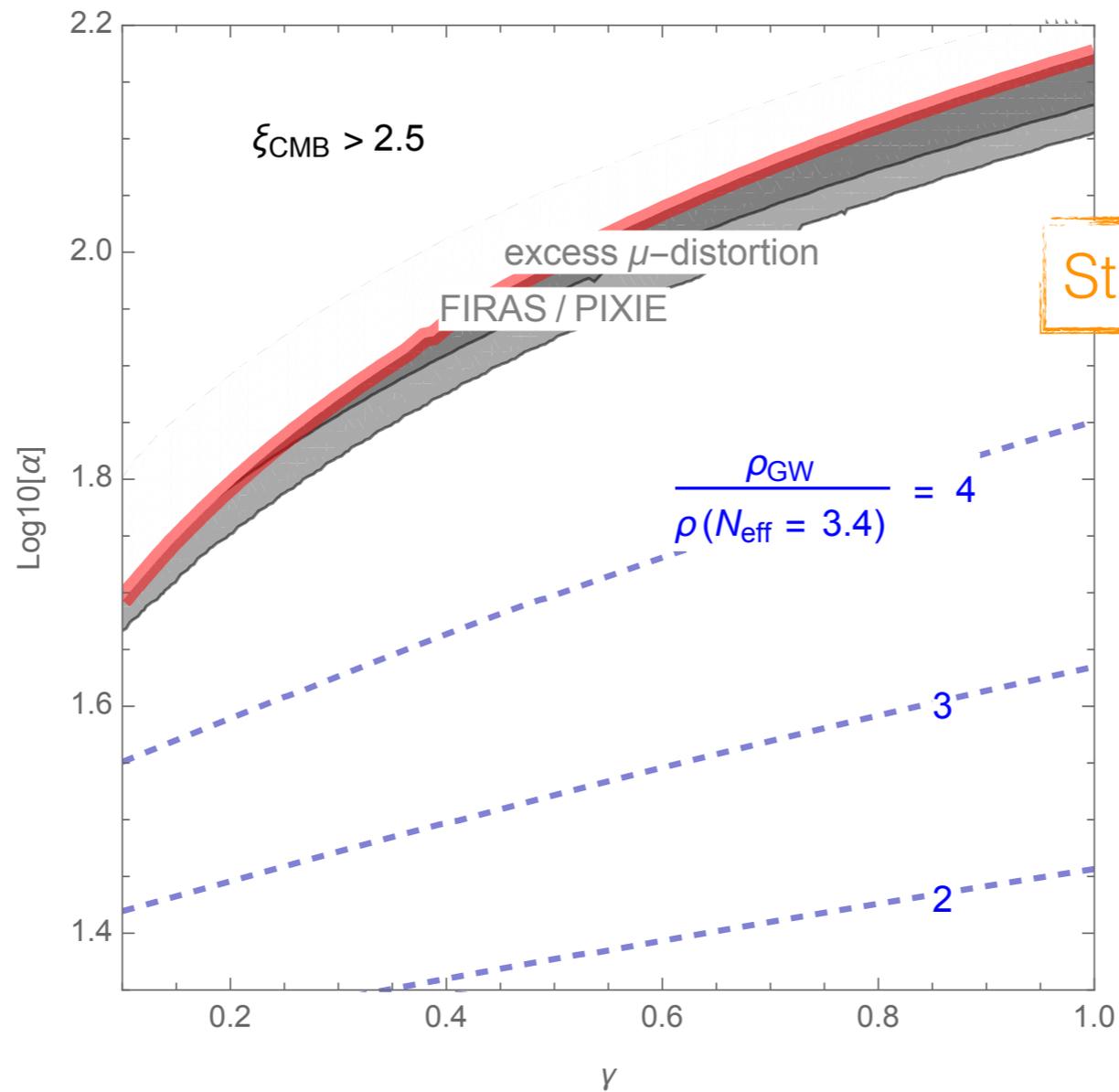
non-gaussianities, mu-distortions (+ LIGO)

complementarily between CMB and direct GW observations

pseudoscalar inflation mu - distortion and N_eff



pseudoscalar inflation mu - distortion and N_eff



model independent parametrization:

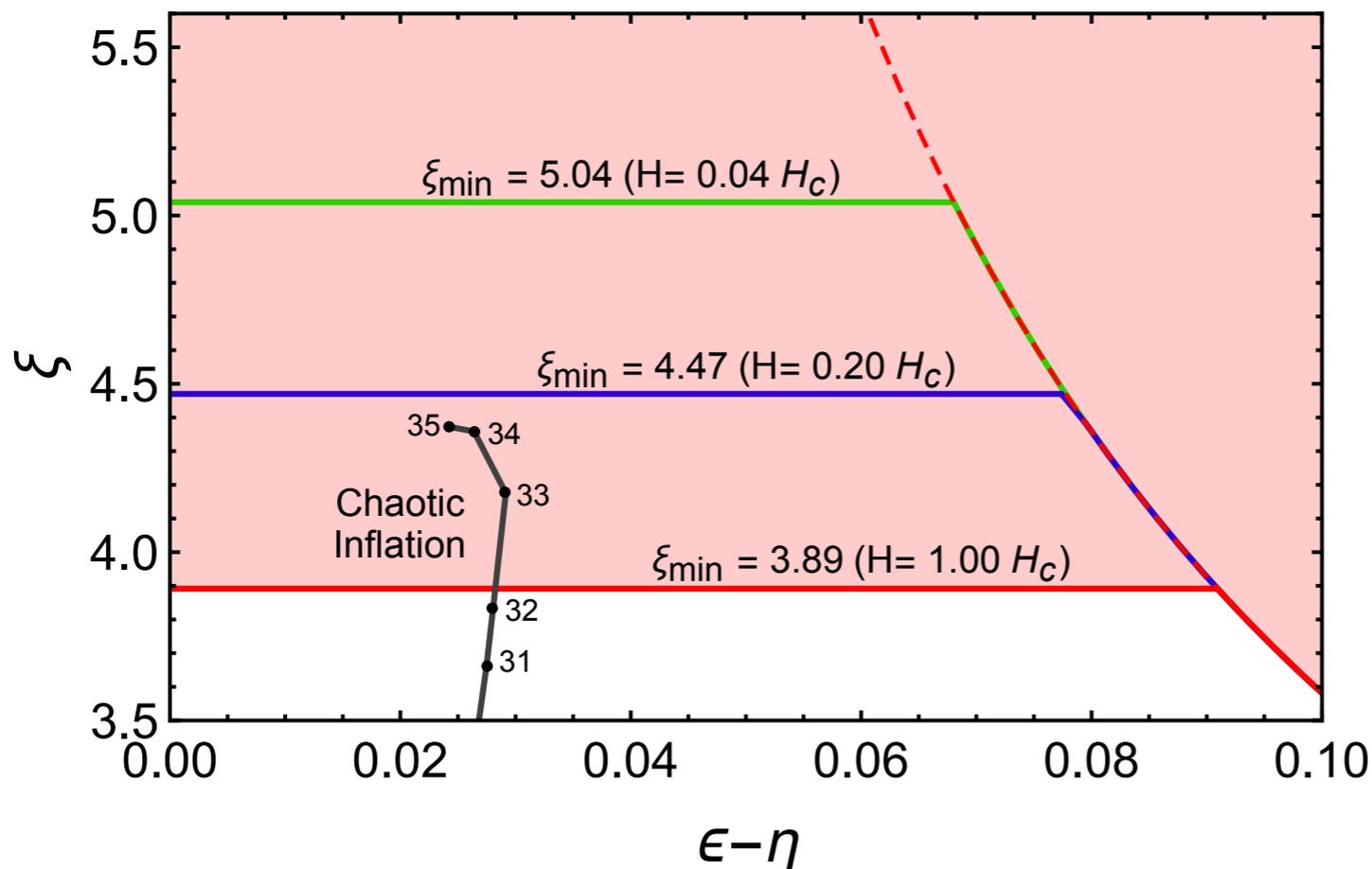
Figueroa, Ricciardone et al '16

local parametrization: $\Omega_{\text{GW}} h^2 \propto \left(\frac{f}{f_*} \right)_t^n$, $\epsilon_H = -\frac{\dot{H}}{H^2}$, $\eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$:

$$\rightarrow \Omega_{\text{GW}} h^2 \simeq 1.5 \cdot 10^{-13} \frac{H^4}{M_P^4} \frac{e^{4\pi\xi}}{\xi^6}, \quad n_t \simeq (4\pi\xi - 6)(\epsilon_H - \eta)$$

3 parameters:
 $H, \xi, \epsilon_H - \eta$

A5M5 (Best Config.)



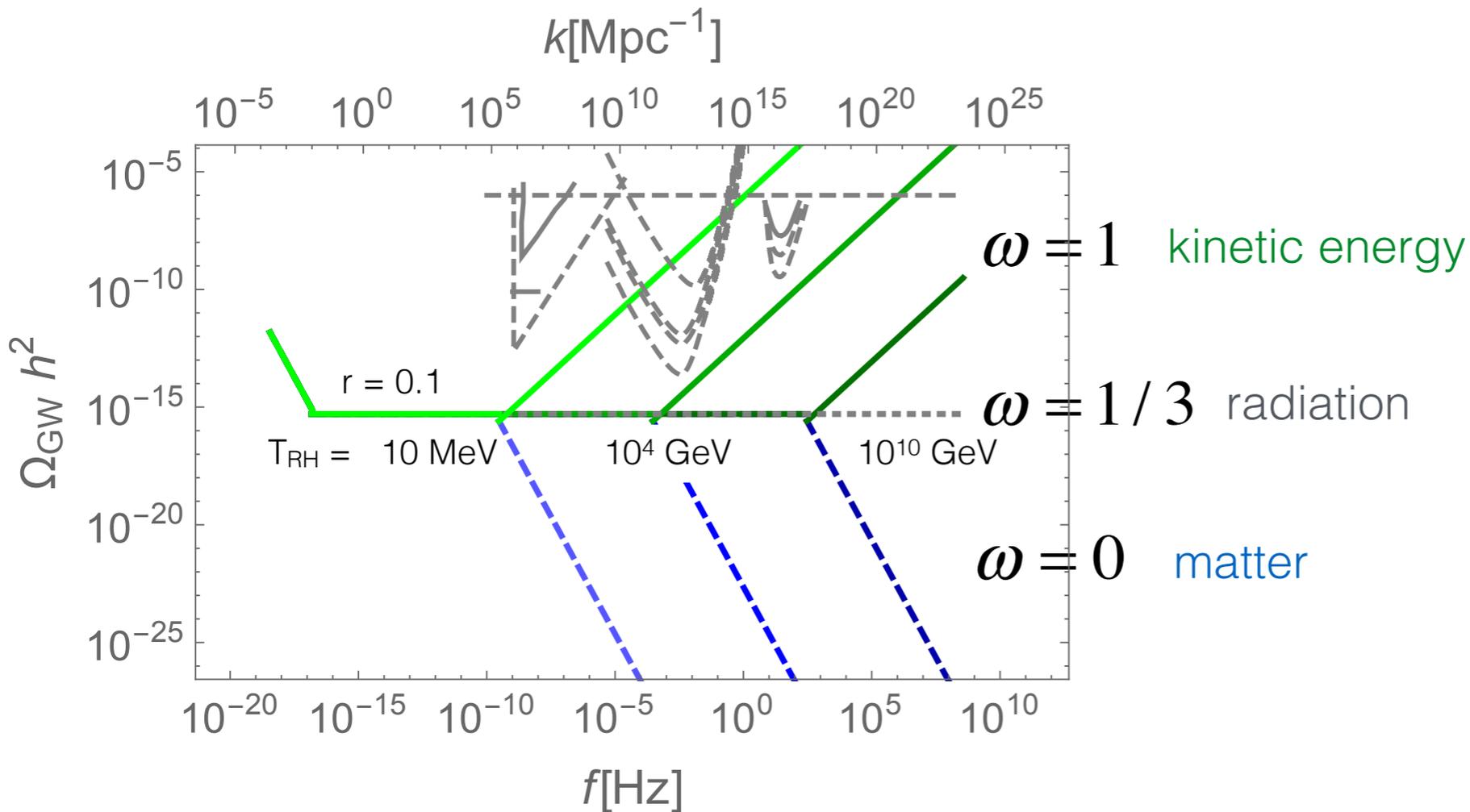
Significant part of parameter space can be probed

Amplitude and spectral tilt could determine 2 out of 3 parameters

Tensor non-gaussianities: consistency relation

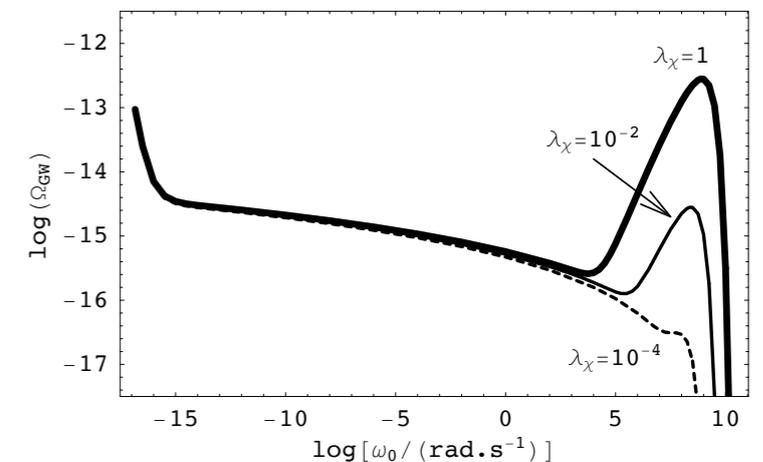
non-standard equation of state after inflation

$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2, \quad T_k(t) = \frac{a(t_i)}{a(t)} = \left(\frac{t_i}{t}\right)^{\frac{2}{3(1+\omega)}} \rightarrow \Omega(f) = \Omega(f_0) \left(\frac{f}{f_0}\right)^{\frac{2(3\omega-1)}{1+3\omega}}$$



kination phase after inflation:
Spookily '93; Joyce '96

GW production in
(hybrid) quintessential models:
Giovannini '99; Sa, Henriques '10



stiff equation of state during reheating can enhance primordial GW signal

second order GW production

Large scalar perturbations re-entering the horizon after inflation



grow in a matter-dominated reheating phase



source second order tensor perturbations

max. amplitude: $\Omega_{\text{GW}}^{\text{max}} \approx \Delta_s^4 \Omega_r \left(\frac{k_{\text{inf}}}{k_{\text{RH}}} \right)^2$

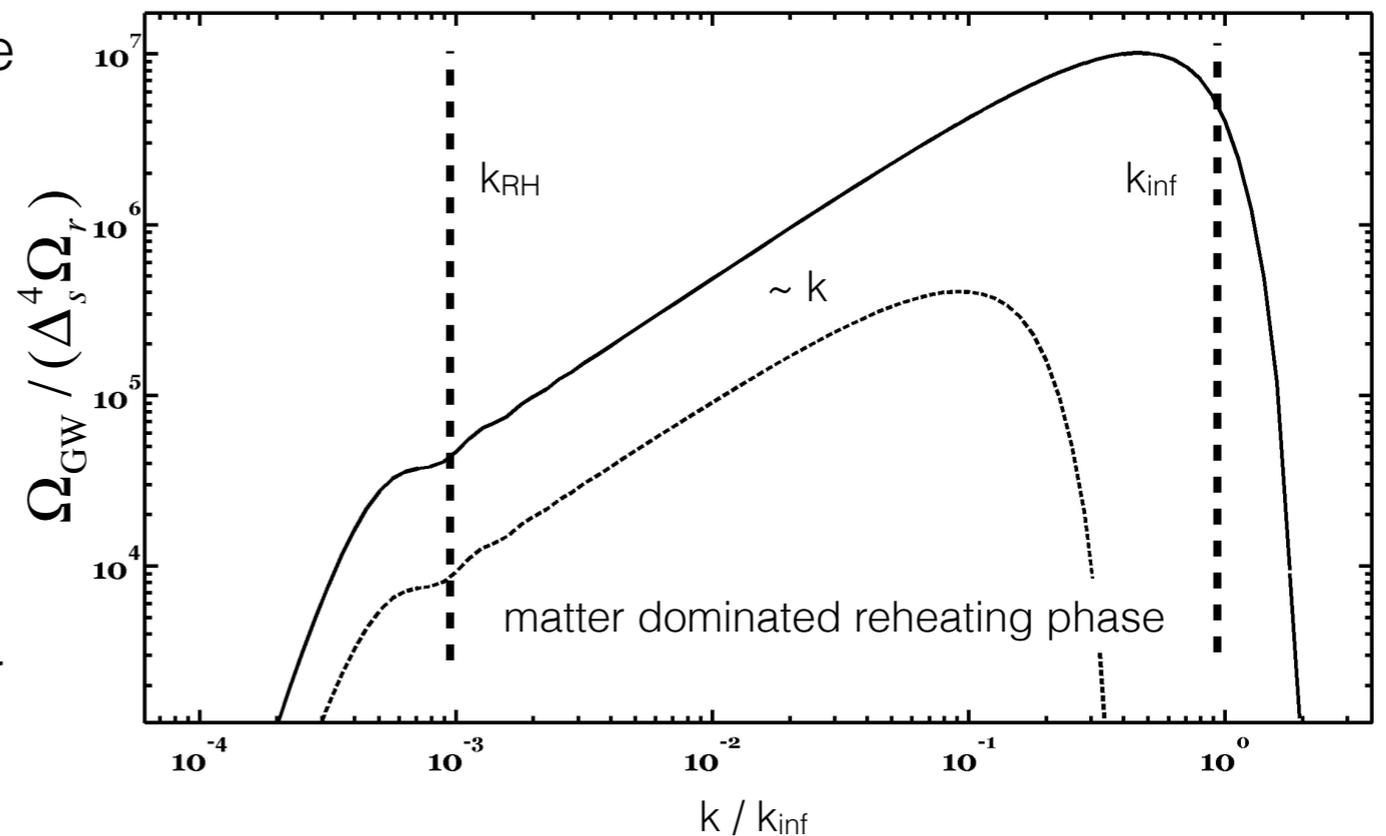
detectable signal for eLISA/LIGO/VIRGO for relatively small reheating temperatures and

$$(\Delta_s^2)_{\text{small scales}} \gg (\Delta_s^2)_{\text{CMB}}$$

note: very large Δ_s^2 on small scales leads to the formation of primordial black holes, which in turn can produce GWs in merger processes.

Tomita '67,

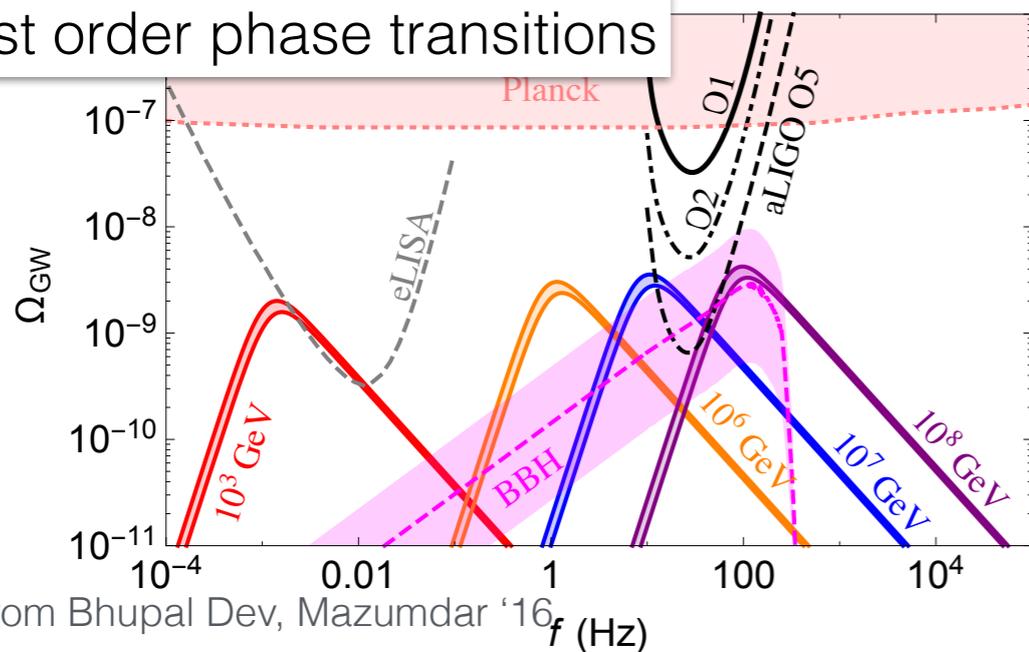
Assadulahi, Wands '09



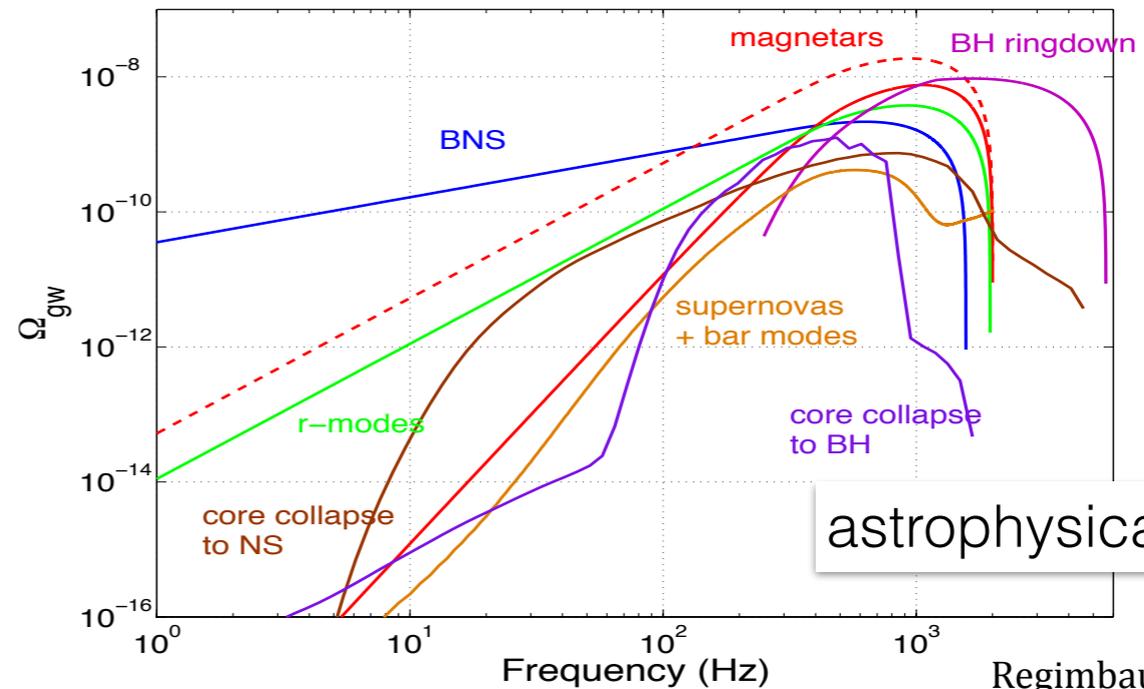
primordial scalar fluctuations can source gravitational waves after inflation

Other (cosmological) sources

first order phase transitions

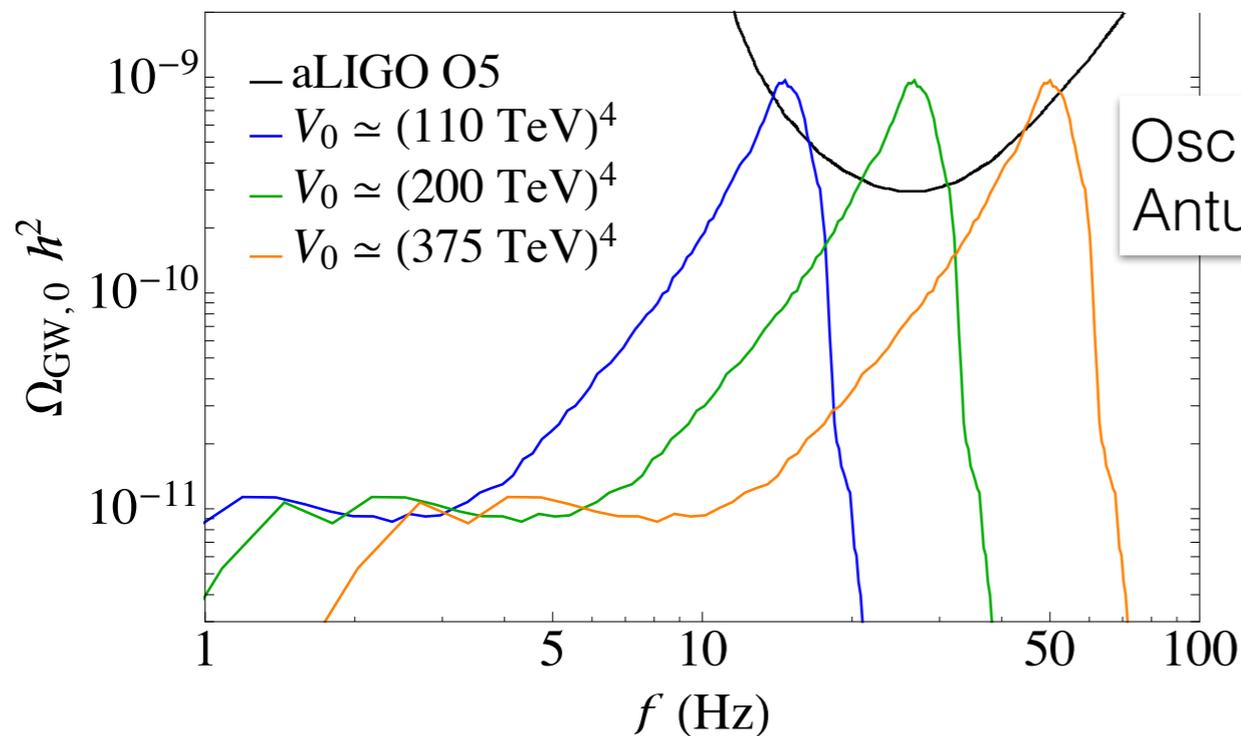


plot from Bhupal Dev, Mazumdar '16



astrophysical SGWBs

Regimbau, arXiv:1101.2762



Oscillons
Antusch, Orani, Cefala '16

Cosmic strings,
preheating,
....