Setting Initial Conditions for Inflation with Reaction-Diffusion Equation

Ajit M. Srivastava

Institute of Physics
Bhubaneswar, India

Collaborators: Partha Bagchi, Arpan Das,
Shreyansh S. Dave, Srikumar Sengupta

Outline:

1. Issue of initial conditions for different inflation models

2. Natural inflation: requirement for small field over several Hubble volumes. How natural?

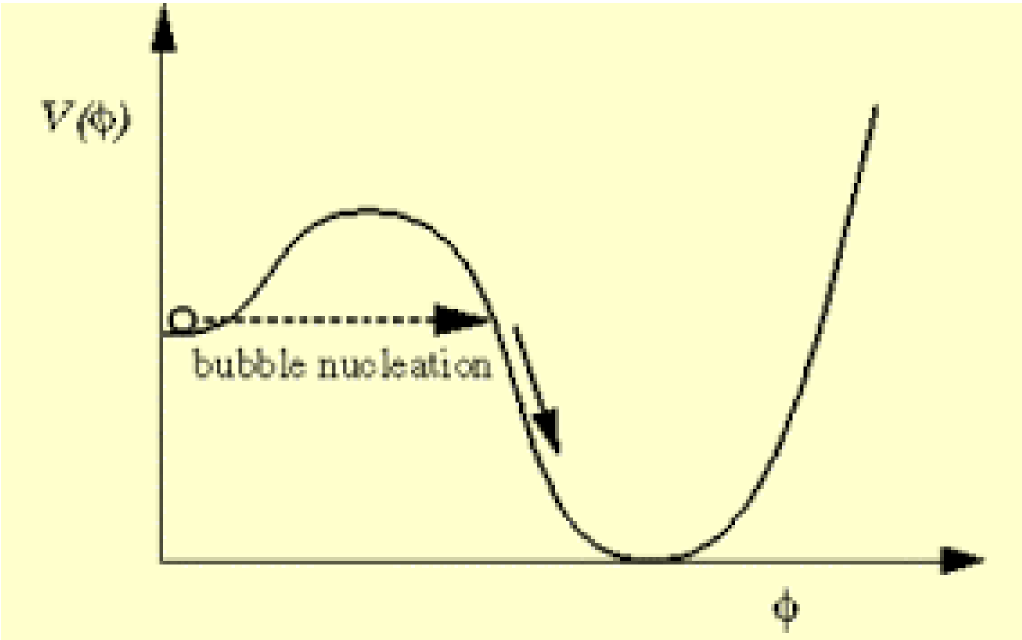
3. Review of propagating front solutions in reaction-diffusion equations.

4. Single domain with reaction-diffusion equation front leading to inflation: general picture

5. Results

6. Conclusions and future directions

Original: Old Inflation:



Inflation while in the metastable state

Tunnel by bubble nucleation
To end inflation

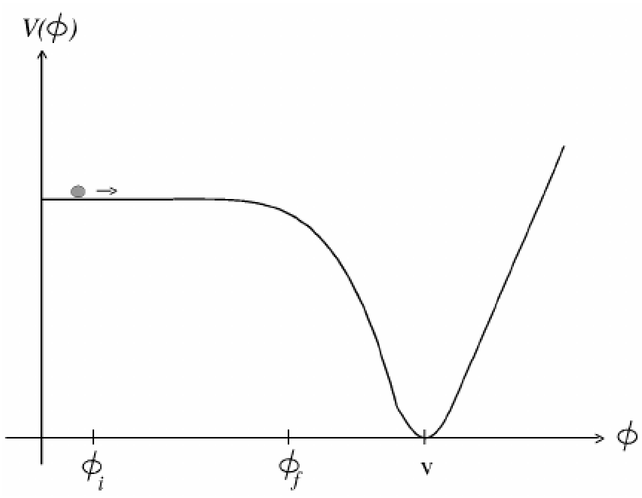
Problem: Contradictory requirements on nucleation rate

Sufficient inflation requires low nucleation rate

Reheating only possible by bubble collision:
requires large nucleation rate

No overlap for the two requirements

New Inflation



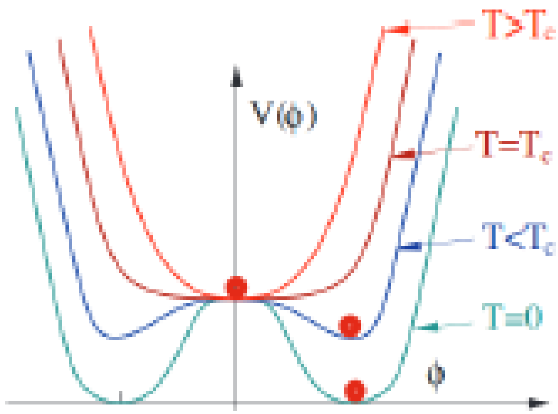
Field equation:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Inflate during slow roll down

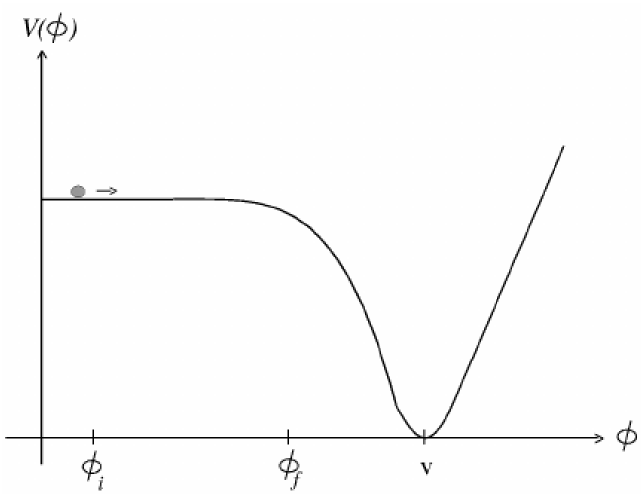
Reheat: roll down the slope
and oscillate

Question: Why should field start from very small values?
Note: this potential corresponds to a second order phase transition.



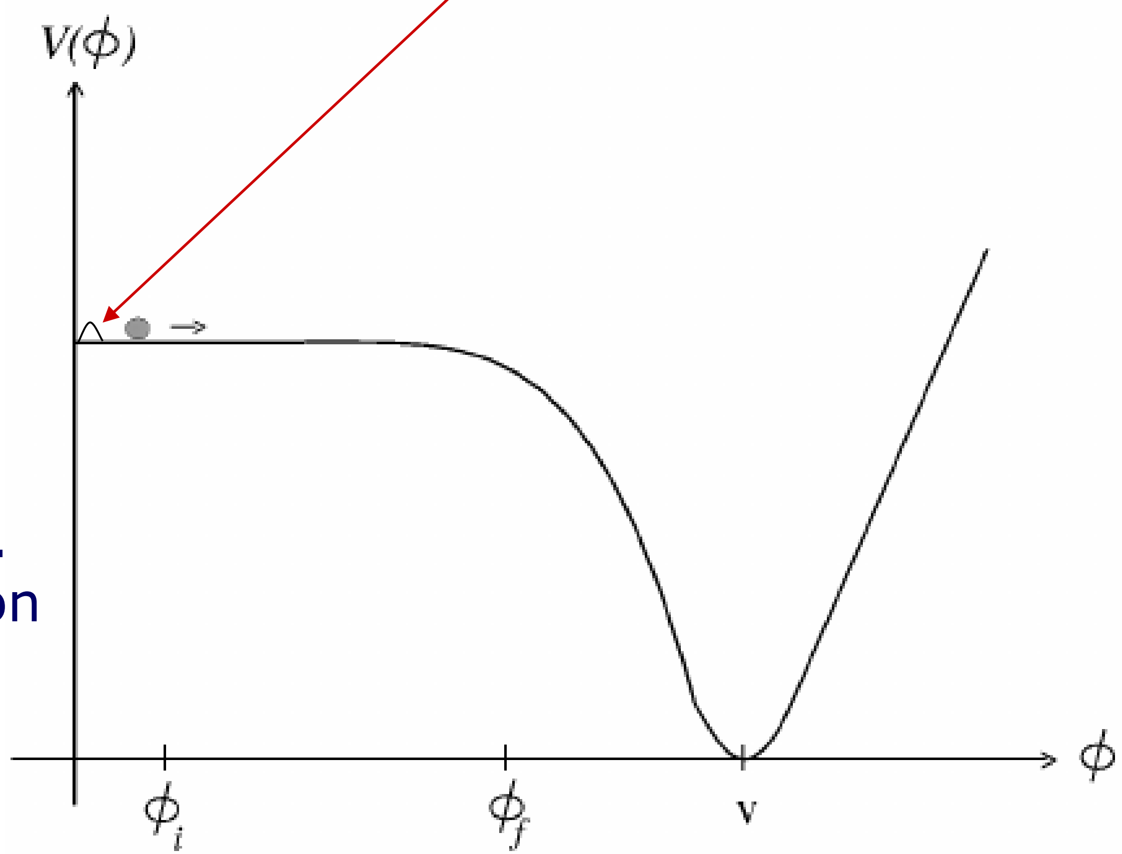
Note: the field always sits at the minimum: continuous change of vev for 2nd order transition.

New Inflation



Thus: Impossible to argue for small initial field amplitude for this (second order transition) effective potential.

Need: metastable vacuum with small barrier:



Field initially localized at 0 (after phase transition), tunnels through the barrier. Then rolls slowly for inflation

Important point about initial condition for these two models of inflation:

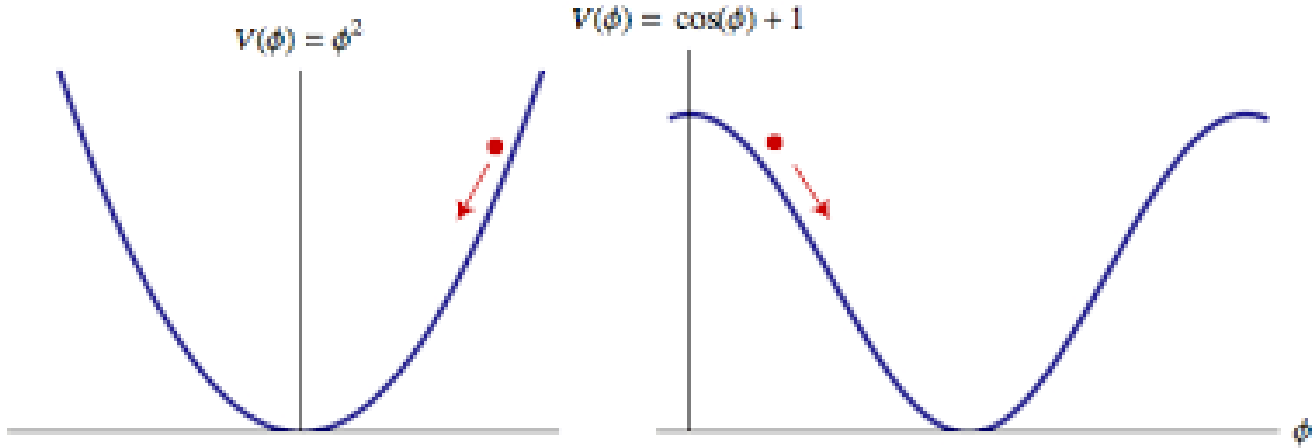
Required value of field (close to zero) naturally set by initially Thermal equilibrium stage with restoration of symmetry. After the transition, ϕ settles at $\phi = 0$ due to the presence of metastable vacuum.

So, $\phi = 0$ does not require any fine tuning.

For other models, chaotic, natural, initial value of the field is not set naturally in this manner. Rather, it is supposed to explore entire allowed (relevant) field values. Inflation occurs wherever the field has correct value.

Important: Correct initial value always refers to the value in the entire Hubble volume. For randomly varying field one says that it is the field value averaged over the Hubble volume which should satisfy required initial conditions.

This is an important statement: during initial stages of inflation This average value should change in the manner of “slow roll”.



Chaotic inflation and Natural inflation: No fine tuning of Shape: Require large field amplitudes for slow roll.
Very large roll down time (very flat potential shape), so large distance to travel (large field amplitude), with low gradient

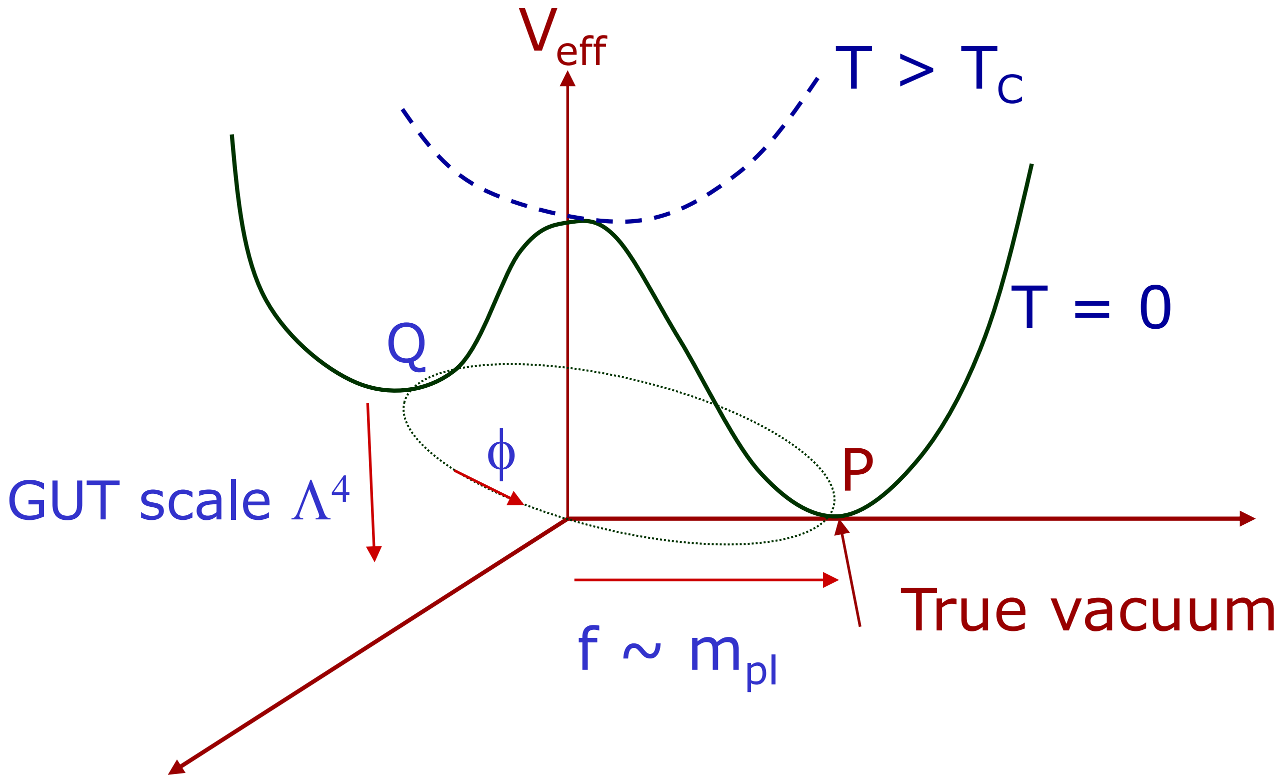
We consider natural Inflation as example: We will not discuss the Issue of the viability of the models. Any model having inflaton field has to address the issue of initial conditions for the field.
Also: our model can be extended to other models of inflation

Potential for natural inflation: $V(\phi) = \Lambda^4 [1 + \cos(\phi / f)]$

Minimum at $\phi = \pi f$:

$f \sim m_{pl}$, $\Lambda \sim 10^{15}$ GeV

Note: This potential is exactly of the same shape as axion potential for QCD (and also for chiral sigma model in Low energy effective theory of QCD): Two very different energy scales arise naturally.



Note: Potential always tilted: Tilt is negligible at energy scales much larger than Λ. Successful inflation requires field very close to saddle point Q, most importantly: over several Hubble volumes.

Can one assume that the entire required value of the field spans the Hubble volume?

May be more natural near the Planck scale as all relevant length Scales (including the Hubble scale) are of same order.

BUT: No reason to expect this when scale of inflation is below the Planck scale. Then all relevant correlation lengths are much smaller than the Hubble scale. So field must have large variations Within the entire Hubble volume.

One can say that the relevant thing is the average value of the field over the Hubble volume.

Suppose average value of the field in the Hubble volume is 0.1.

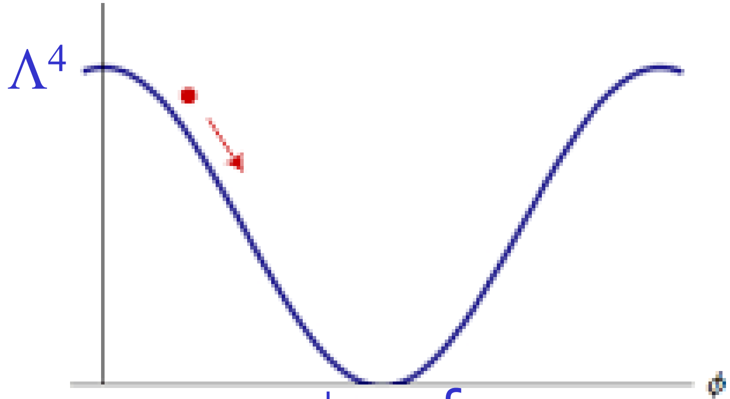
With ϕ varying randomly over many correlation volumes (domains), this means that many such "domains" will have ϕ larger than 0.1.

ϕ will roll down much faster in those domains. This means that the average value of ϕ will increase faster than the roll down time calculated for the averaged ϕ itself. So calculation not consistent.

Let us then assume that ϕ has roughly the required initial value
In the entire Hubble volume...

$f \sim m_{pl}$
$\Lambda \sim 10^{15}$ GeV

$$V(\phi) = \Lambda^4[1 + \cos(\phi / f)]$$



$\phi = \pi f$

Natural setting: Thermal initial conditions with $T > \Lambda$ (but T much less than m_{pl}). Non-zero probability for $\phi_{initial} \sim 0$ (recall, this is a saddle point).

Take conservatively, $\phi_{initial} \sim 0.1\,(2\pi f)$, probability ~ 0.1

We need this to hold for at least several Hubble volumes. Then only FRW equations are relevant leading to vacuum Energy dominance for inflation.

Again, conservatively, take only one Hubble volume.

For radiation dominated era down to $T \sim \Lambda$, we have:

$H \sim 1/t$, and $T \sim t^{-1/2}$

So: $H^{-1}_{GUT} = H^{-1}_{pl}\, T^2_{pl}/T^2_{GUT}$

Take thermal correlation length $\xi \sim T^{-1}$, then

$H^{-1}_{GUT}/\,\xi_{GUT} = (H^{-1}_{pl}/\,\xi_{pl})T_{pl}/T_{GUT}$

With all Planck scale quantities of order m_{pl}, and $T_{GUT} \sim 10^{15}$ GeV,

$H^{-1}_{GUT}/\,\xi_{GUT} = 10^4$

So, each Hubble volume has about 10^{12} correlation volumes.

Field not expected to be correlated beyond correlation length.

So, each correlation volume should have independently varying field magnitude, especially with axionic almost flat potential.

For each correlation volume, required value of $\phi \sim 0.1$ had Probability 0.1

So, for the Hubble volume with 10^{12} correlation domains, the Probability that $\phi \sim 0.1$ in the entire Hubble volume is $(0.1)^{10^{12}}$ which is completely negligible.

This is a serious problem in assuming a reasonable initial Condition for inflation.

One needs to specify the specific conditions under which the Required initial conditions can arise in any inflation model.

We argue that this can be achieved "naturally" by precisely invoking (rather than neglecting) the correlation domain structure of variation of field inside the Hubble volume.

We will not require initial small value of the field over any extended region, not even the entire correlation volume. It will be very small in a very tiny region inside the correlation domain, and will be assumed to smoothly change to large value (vev) over the correlation length.

We use specific features of the reaction-diffusion equation to address this issue

Reaction-Diffusion equations : Quick review

Diffusion equation: $$\frac{\partial u}{\partial t} = D\nabla^2 u$$

In 1-d it has solution of the form: $$u(x,t) = \frac{u_0}{(4\pi Dt)^{1/2}} e^{-x^2/4Dt}$$

Note: Diffusion equation has no traveling wave solution of the form u(x-vt).

Modify the equation by introducing "reaction term" f(u). (used e.g. in the context of biological systems, where the reaction Term represents interaction of species).

$$\frac{\partial u}{\partial t} = \nabla.(D\nabla u) + f(u)$$

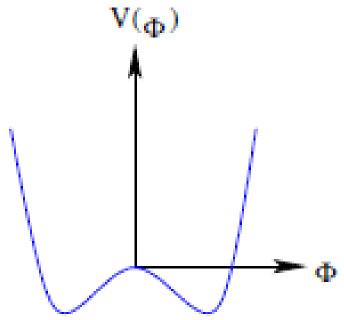
This is the "Reaction-Diffuion equation. It has traveling wave (and static) solutions, with appropriate boundary conditions.

Compare with the field equations: $$\ddot{\phi} - \nabla^2\phi + \eta\dot{\phi} = -V'(\phi)$$

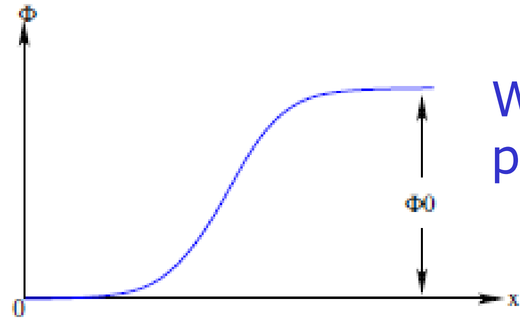
In the high dissipation limit, this is same as the reaction-diffusion Eqn

Nontrivial field evolution with reaction-diffusion equation
For specific boundary conditions:

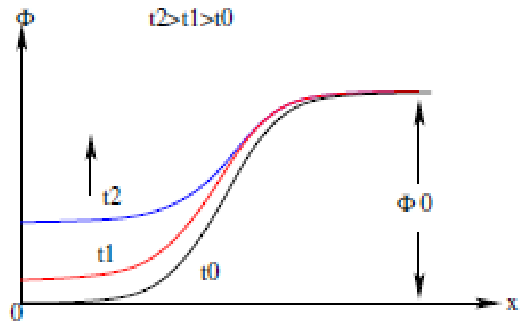
Take this potential



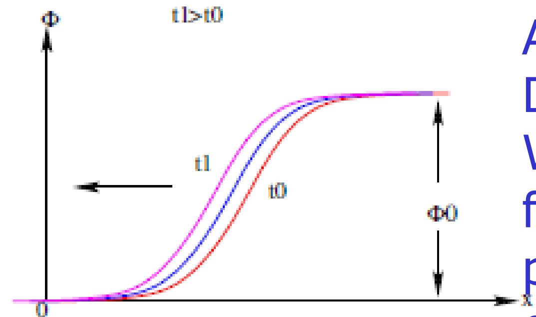
With this initial profile for ϕ

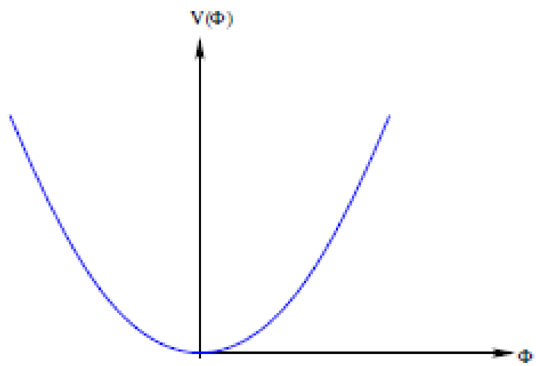


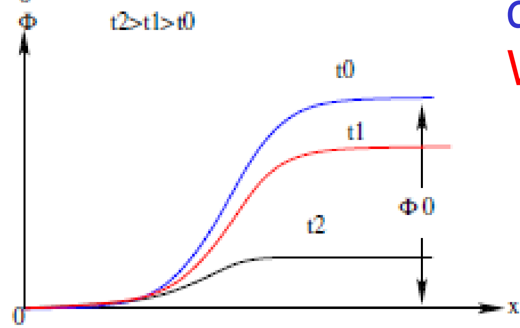
Expected Evolution: ϕ Rolls down to vev = ϕ_0



Actual evolution Development of Well formed front which propagates with definite velocity Which can be 0

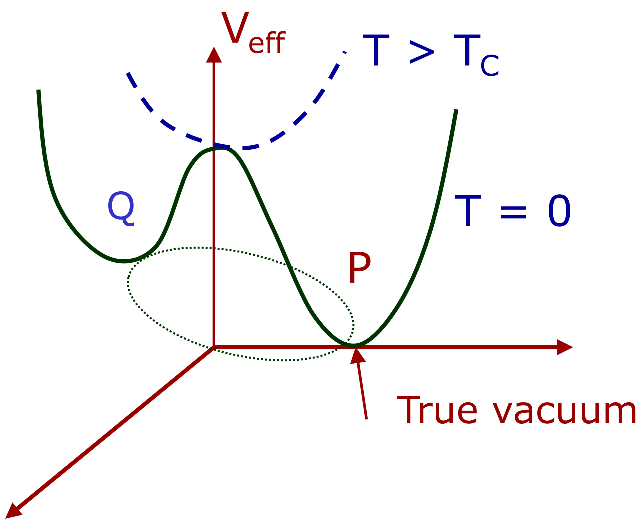






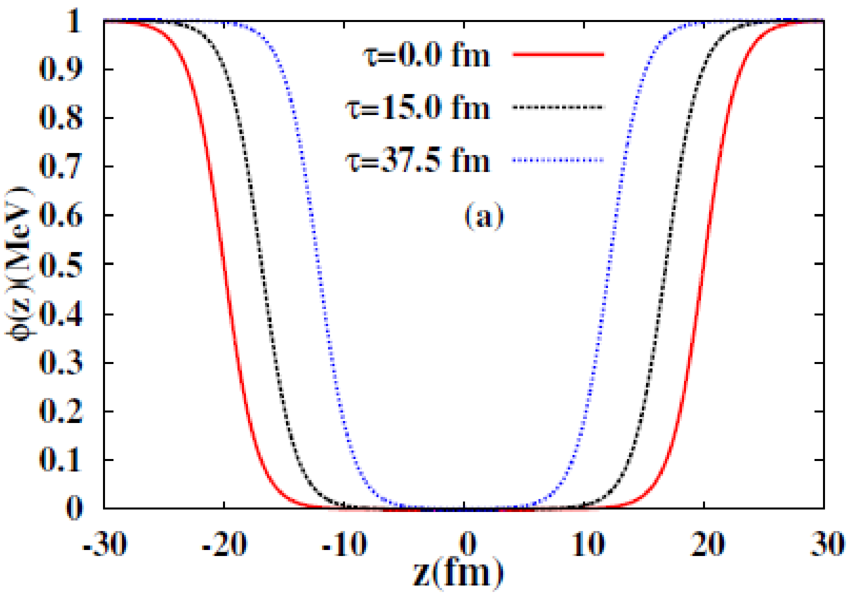
Note: Non-zero vev (and a maximum) necessary for the propagating (or static) front. For the above potential field rolls down quickly

Recall: potential for QCD axion case, or for QCD chiral sigma model:
As mentioned earlier, Natural inflation has the same form of potential



For QCD chiral sigma model, we had Studied evolution of a field profile Interpolating between points Q and P with dissipative field equations. These Equations became exactly the same as specific reaction-diffusion Equation known as The Newell-Whitehead equation.

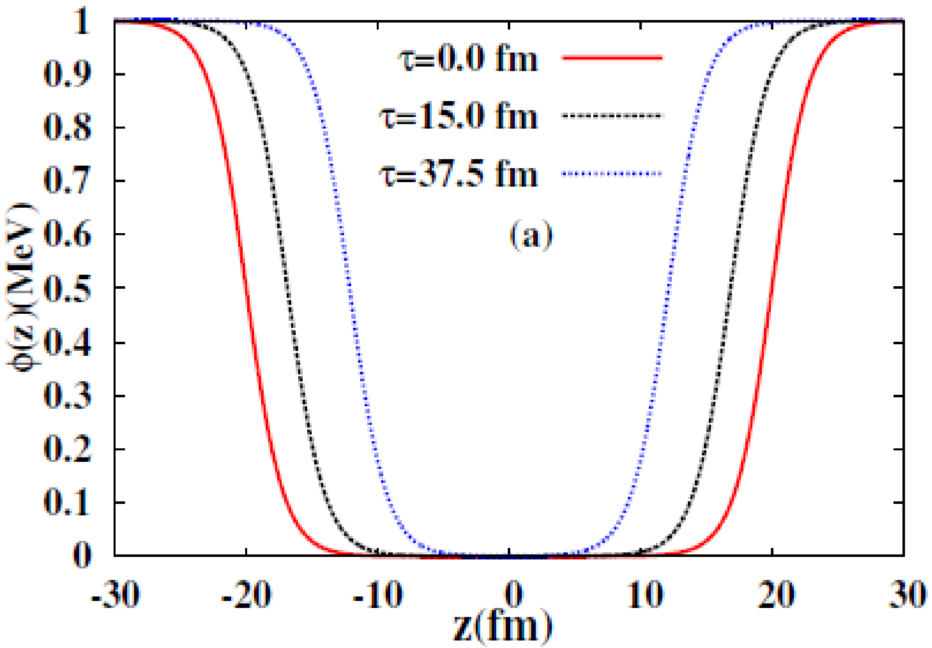
One would expect field at point Q to Quickly roll down to P. However:



well defined front forms and moves inwards, just as the interface moves for a first order phase transition.

Note: no metastable vacuum here, so no first Order transition interface

Lessons to learn from this:



Note: the velocity of the front Depends on the specific profile (different solutions have different velocities). There are Also static fronts.

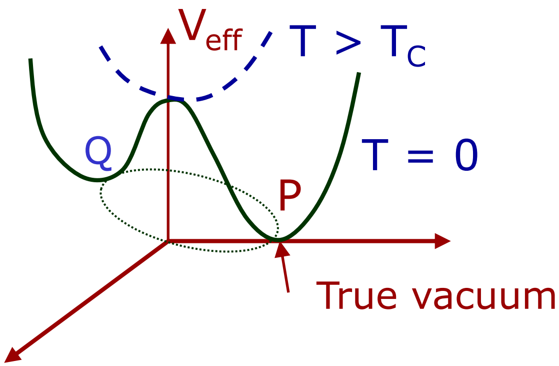
So, in an expanding system (expanding plasma as for Heavy-ion collisions, or Expanding universe), expansion may dominate over Slow shrinking of domain.

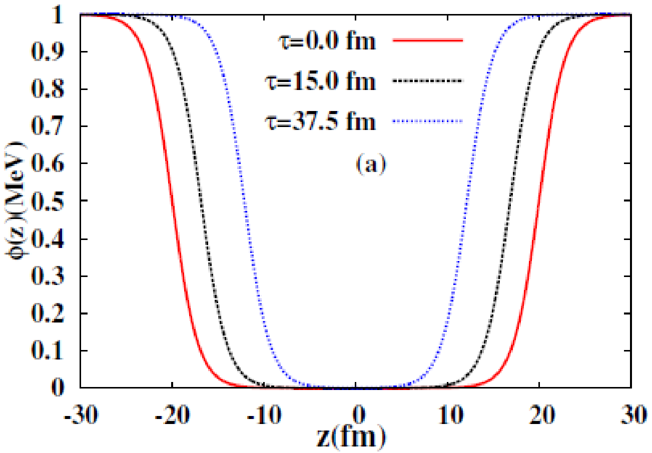
In such a case, domain will actually expand, being stretched by the expansion, even though energetically it should have shrunk.

For QCD case, we showed that this leads to formation of large DCC domains (disoriented chiral condensate domains).

Its implications for expanding universe are then obvious:

For expanding universe: think of this QCD potential as that for Natural inflation. Think of the chiral field domain as domain for Inflaton (axion) field, with zero of the field being at point Q (large vacuum energy).





As the domain shrinks slowly (by inward motion of the wall), or it remains static, it gets stretched by the universe expansion. If expansion dominates, the domain will become larger, and eventually will dominate the energy in the Hubble volume.

When vacuum energy starts dominating, universe will inflate. After the domain exits the horizon, inflation will be established.

Small value away from point Q will decide eventual roll down Of the field and end of inflationary stage.

Natural inflation with reaction-diffusion equation:

Potential: $V(\phi) = \Lambda^4[1 + \cos(\phi / f)]$

We take f = m_{pl} , $\Lambda = 10^{15}$ GeV

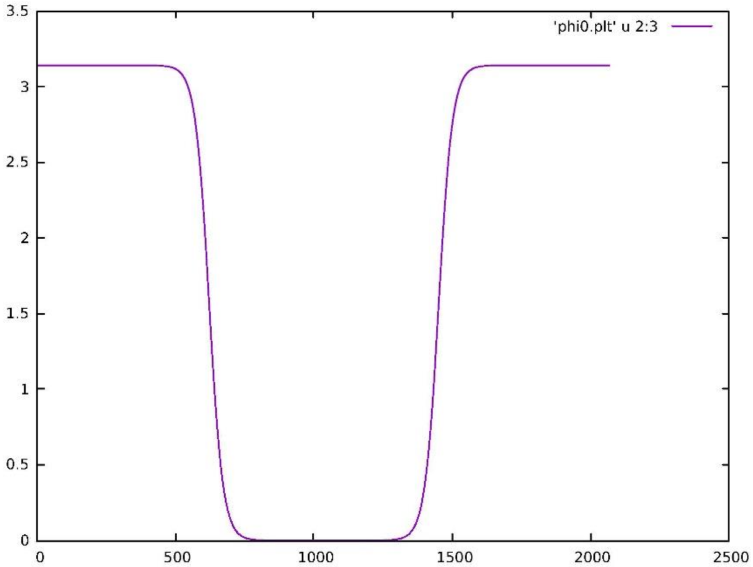
Field equations for the inflaton: $\ddot{\phi} - \frac{\nabla^2 \phi}{a^2} + 3H\dot{\phi} + V'(\phi) = 0$

Scale factor evolution: $H = \frac{\dot{a}}{a} = [\frac{8\pi G}{3}(\rho_\phi + \rho_{radiation})]^{1/2}$

Field energy: $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{(\nabla\phi)^2}{2a^2} + V(\phi)$

Radiation energy:
(starting at GUT scale) $\rho_\phi \sim a^{-4}$

Field profile taken as Tanh (for planar 1-d solution as well as for spherical 3-d solution) interpolating between the minimum of V at $\phi = \pi$ m_{pl} and a point close to the Maximum of V with $\phi = \varepsilon$.
Our results very insensitive to the initial profile. Even linear segments evolve into smooth profile and lead to inflation.



Results:

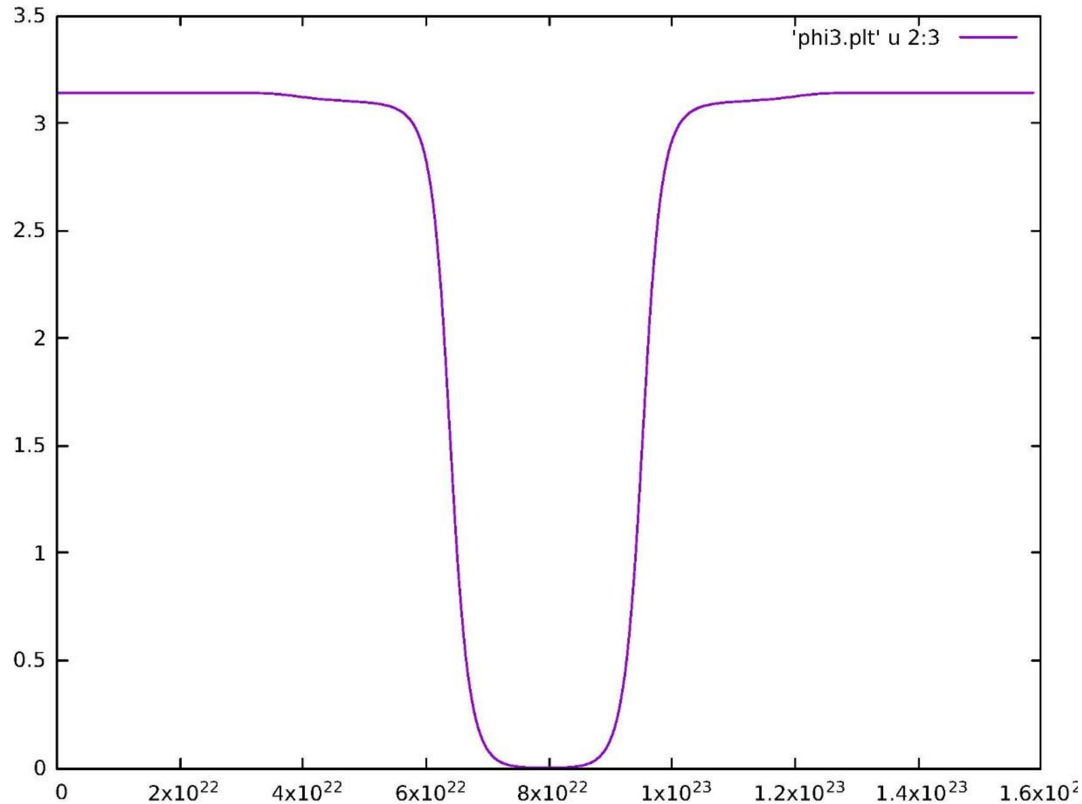
Initial profile of field in a domain (lengths in the units of Λ^{-1}, ϕ in units of f).

Region of size $H^{-1}/2$.

Value of ϕ at domain center is $\varepsilon \sim 10^{-7}$

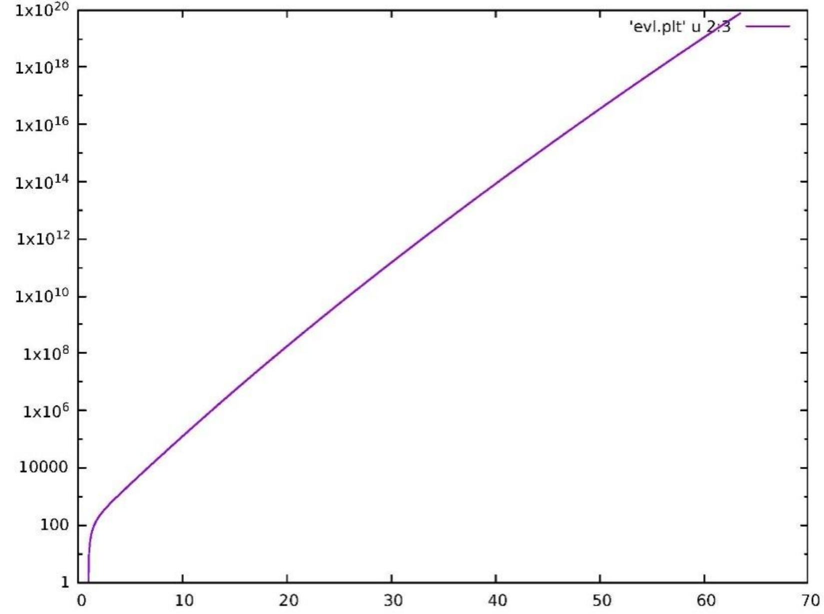
Field profile after 60 e-fold evolution.

Very little change in the field

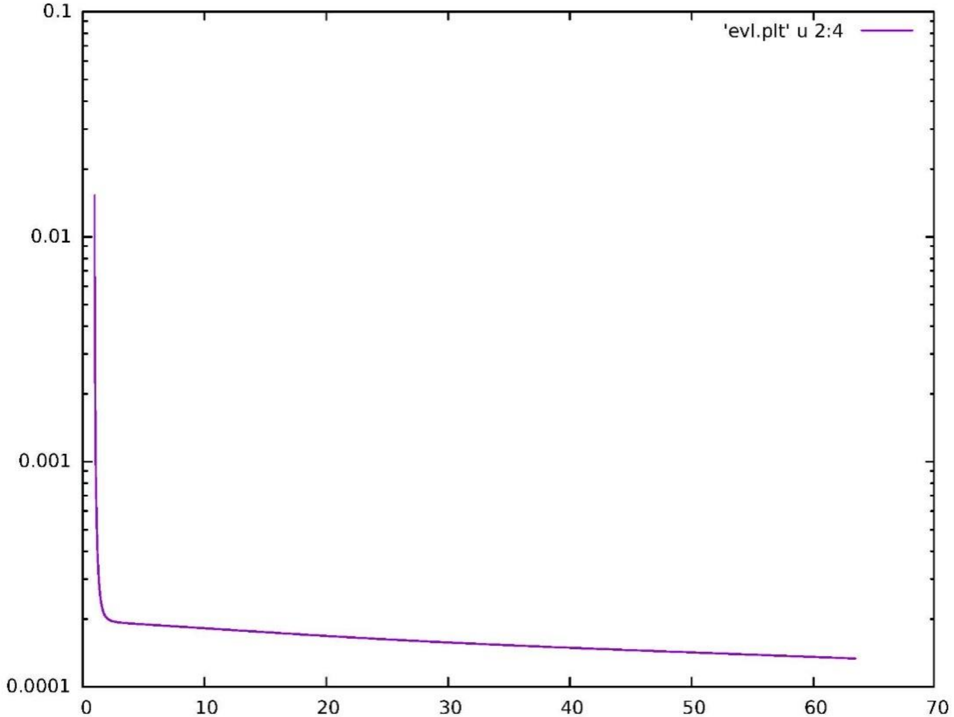


Evolution of scale factor:

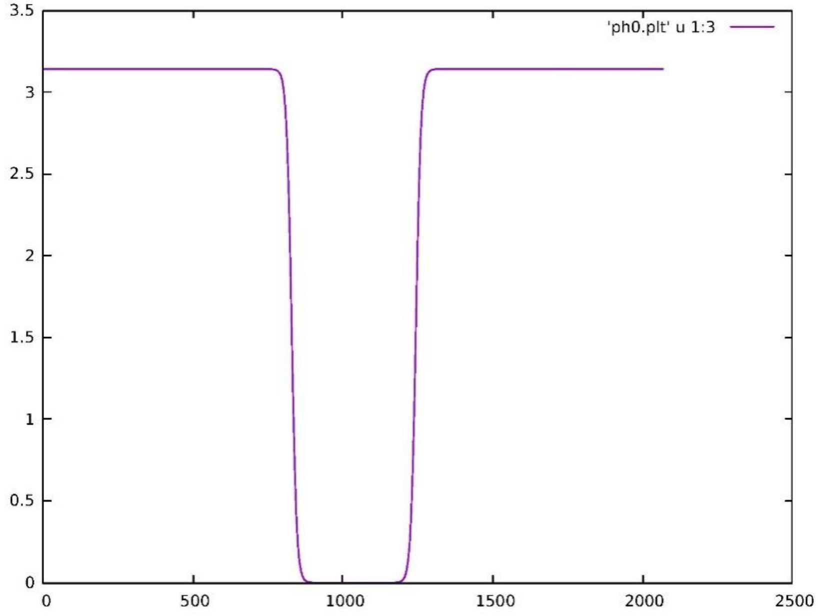
Time in units of H^{-1}_{iniial}

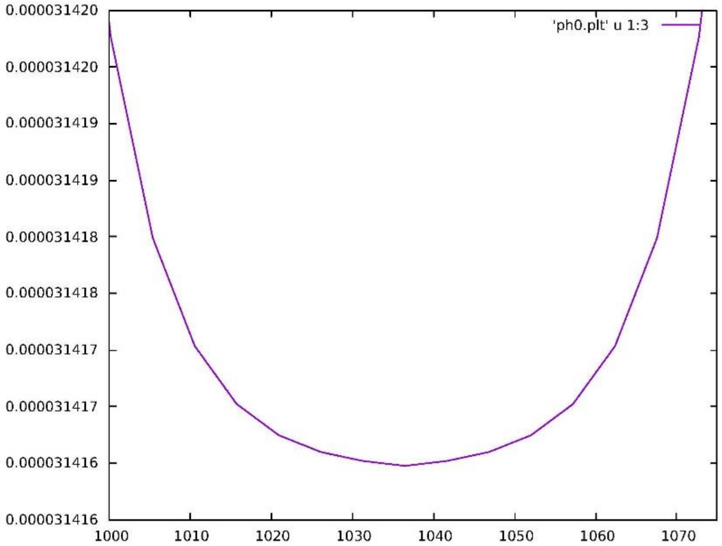


Evolution of H:



Ending inflation: Take larger value of ϕ at domain center : $\varepsilon \sim 10^{-5}$



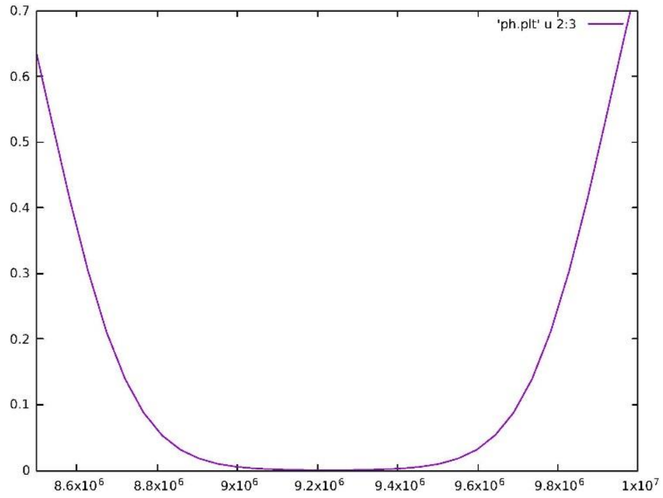


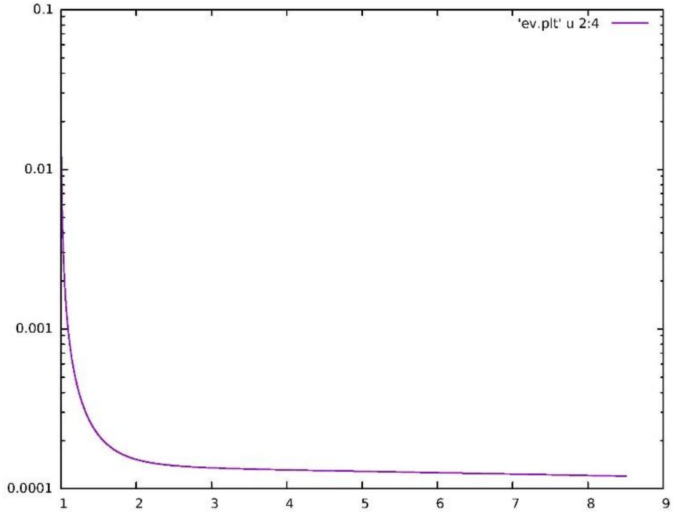
Profile near center: note it has smooth variation

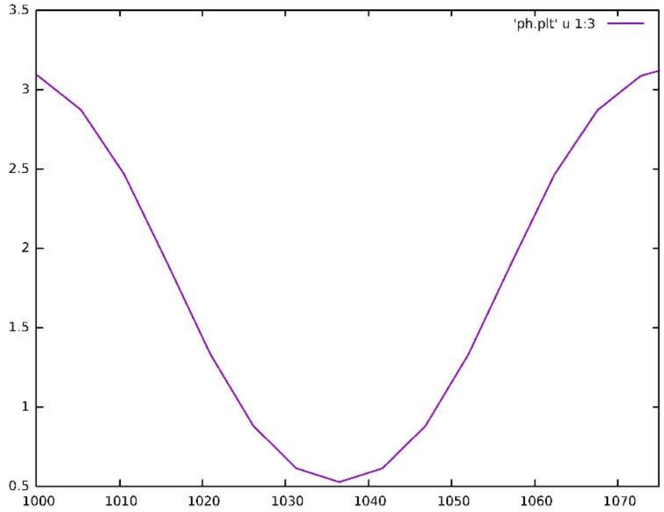
Domain stretches beyond Hubble size after t ~7 H^{-1}. This is the stage of entering full inflation.

Field roll down at the center is more rapid here so after 60 e-fold evolution field significantly rolls down.

Profile after t ~ 7 H^{-1}. Evolution of H shows domain is superhorizon at this stage







Due to larger ε, field rolls
Down significantly after 60
e-fold evolution ending inflation.
(comoving coordinates)

Main points about the results:

We show that one does not need to assume finely tuned value
Of the field in the entire Hubble volume for inflation.

A single domain, much smaller than the Hubble volume, with field varying smoothly across the domain, can lead to Inflation. This occurs because of special reaction-diffusion equation solutions of dissipation dominated field equations.

They lead to slowly moving (or static) fronts, instead of rapidly rolling field.

Such smooth field profiles expected with general picture of Correlation domains. Value close to the top of the potential (saddle point) taken only at one point, then it smoothly changes to the vacuum Value across the domain.

Any Hubble volume with even such domain will have Inflation.

Future directions:

Existence of slowly moving propagating front (instead of rapidly rolling field) is a general consequence of reaction-diffusion equations.

Also, smoothly varying field profile across a correlation domain needs to be assumed for the beginning of inflation. This can be applied to other models of inflation (which have potentials of correct types).

Warm inflation has extra dissipation, this should lead to closer correspondence with the Reaction-diffusion equation.

Effect of fluctuations etc. has to be studied, especially on the Propagating front profile and its velocity.

For Reaction-diffusion equations: For different reaction terms One gets different specific equations with well defined solutions. (We have shown that, with chiral sigma model potential one gets the Newell-Whithead equation, while for Polyakov loop potential, one gets the Fitzhugh-Nagumo equation used in population genetics).

What will be the properties of the reaction-diffusion equation with $\cos(\phi)$ interaction term (as for the Natural inflation case)?

THANKS TO THE ORGANIZERS

FOR A GREAT WORKSHOP