THE THEORY OF DYNAMICAL FRICTION

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with A. Kurkela and A. Soloviev, to appear soon

OUTLINE

- Motivation: why is this important? What do we already know?
- Description of the formalism
- Calculation of the drag force:
	- ideal gas: recover the known cases, generic expression for arbitrary distribution and arbitrary "bullet" velocity
	- recovery of the ideal liquid results
	- viscous liquids
	- generic results of interacting gases in the relaxation time approximation, interpolation between the gas and the liquid
- Comments on the boundary conditions effects
- Outlook

WHAT IS FRIR I

The problem is not new:

A massive particles propagates through a medium with a constant velocity. What is the gravitational drag force, induced by the wake? _

First analysis: Chandrasekhar,

1942

DYNAMICAL FRICTION AND MODERN HEP

- Traditionally: used to calculate motion of bodies in the galactic dynamics — with classical NR results being perfectly adequate
- More modern questions: effects of self-interacting DM (component) on the galactic dynamics — more complicated, selfinteractions effects should be taken into account
- Recently: calculate the the energy released by (primordial) black holes going through neutron stars and white dwarf — effects of matter in extreme conditions must be properly accounted for
- Exotic objects in the accretion discs of the black holes

WHAT DO WE KNOW ABOUT THE DYNAMICAL FRICTION? AB *^v*² log ⇤ *.* (24) (Translate to the relativistic language – ak) In this expression ⇢ is the energy density $\overline{\mathbf{R}}$ ˆ*R^E* $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{2}$ $\overline{2}$ $\overline{1}$ $\overline{2}$ $\overline{1}$ N *Ay* I C *A* I , F *R* I C T W 1 8⇡ ª **PPIONTO** . \vee . *^r*² (155)

tivistic particles using the results of $[1]$ and $[1]$ and $[1]$ and $[4]$ is:

Ideal non-relativistic gas $F = \frac{4\pi G^2 m_b^2 m_p}{r} \int_0^v dv' v'^2 f(v') \log \Lambda$

$$
F = \frac{4\pi G^2 m_b^2 m_p}{v^2} \int_0^v dv' v'^2 f(v') \log \Lambda
$$

of a gas of non-relativistic particles, namely ⇢*^v* = *mn*, where *n* is the number density of

Photons gas, relativistic bullet $\frac{1}{2}$ *B* $\frac{1}{2}$ *D* $\frac{1}{2}$

$$
F = \frac{64\pi}{3} G^2 m_b^2 \rho v \gamma^2 \log \Lambda \ .
$$

How do we interpolate between two these regimes? polate between two these regim

5

Ideal liquids: Supersonic: *<i><u>r deal*</u> induid

⇢ ´ ⇢¯

$$
F = \frac{4\pi m^2 G^2 \gamma^2 (1 + v^2)^2 (\rho_0 + P_0)}{v^2} \log \Lambda \; \theta(v - c_s) \qquad \qquad \frac{\sigma^2}{\sigma^2} \qquad \qquad
$$

10

Vanishes subsonically, though certain boundary effects can change this conclusion

Fig. 3.— Solid curves: Dynamical friction force in a gaseous medium as a function of Mach

CALCULATION OF DYNAMICAL FRICTION — HISTORICAL APPROACH

Chandrasekhar's approach: calculate the probability function of occurrence of force *F* acting on a star and an average time during which the force acts

In the case of the ideal fluid (Rephaeli & Salpeter, Ostriker) one calculates the drag due to the asymmetry in the flow associated with the front of the shock wave.

A NEW APPROACH A NEW APPROACH *µ <i>x* ∞ *z* ∞ *z*

hwake

(Thermal) field theory approach + Gravity as an EFT a gas of particles. One can notice that in the linearized regime, the linearized regime, the force is exerted o
In the force is exerted on the force i he gravitational field the gravitational field of its own was can be written as

Definition of force induced

tion of force induced
by the wake:
$$
F^{\mu} = \frac{dp^{\mu}}{dt} = -m\gamma \Gamma^{\mu}{}_{\alpha\beta} (h^{wake}) v^{\alpha} v^{\beta}
$$

Our main objective is to calculate a force, exerted on a bullet that propagates through

µ⌫*,*↵(*x, x*⁰

)*T* ↵

wake(*x*⁰

)*.* (8)

Ggrav

That is, *hwake* represents the gravitational field that was created by a wake that was created

Finally, the wake itself will have itself will have itself will have its own gravitational field, which can again be computed with α

to field of the wake (Fourier space)[.] The graviton field of the wake (Fourier space):

$$
h_{\mu\nu}^{wake}(\omega, k) = G_{\mu\nu,\alpha\beta}^{dressed}(\omega, k) T_{bullet}^{\alpha\beta}(\omega, k)
$$

FORMAL EXPRESSION FOR THE FORCE with ⇢ the energy density and *P* the pressure. The gravitational perturbation *hbullet ^µ*⌫ caused \blacksquare the bullet is convolution with the Green function \blacksquare retarded function. As in all the cases we are interested, the cases we are interested, the retarded function is multiplied function. As in all the cases we are in all the cases we are in Γ in all the considering the considering the sign of the real part of the real part of the frequency turns turns to the frequency turns a retarded Green function into advanced, which is given by the complex conjugate of the complex conjugate of th retarded function. As in all the cases we are interested, the retarded function is multiplied **KMAL EXPRESSIC** Z d*k^z* 2⇡ *ikzGdressed ^µ*⌫*,*↵ = C *k* 2⇡ *ik^z* ² (*Gdressed ^µ*⌫*,*↵ (*Gdressed*

Let us first assume that we know the excitation of matter. *ikzGdressed ^µ*⌫*,*↵ = rst assun <u>*<u>unc</u>* chat we know the excitation of ma</u> $\ddot{\cdot}$ \overline{a} *kz*Im*Gdressed ^µ*⌫*,*↵ *.* (16)

Ggrav

)*T* ↵

The energy-momentum tensor of the bullet: y-mo *k*_{*a* $\frac{1}{2}$} The fact that we are interested merely in the imaginary part as well as the +*i*✏ prescription give us a convenient preserved in the *p* choice.

^µ⌫ (*x*) = ^Z

hbullet

$$
T_{bullet}^{\mu\nu} = \frac{m_{bullet}}{\gamma} u^{\mu} u^{\nu} \delta(z - vt) \delta^2(x_{\perp}) \implies \tilde{T}_{bullet}^{\mu\nu} = \frac{2\pi m_{bullet}}{\gamma} u^{\mu} u^{\nu} \delta(\omega - k^2 v)
$$

From here we can use the standard machinery: calculate the ³ We work with the convention *f*(*x*) = R *^d*4*^k* (2⇡)⁴ *eik·^xf*(*k*). Further, we will be interested in retarded Green dressed propagator, convolve with the energy momentum tensor of the bullet and further the Christoffel symbols. The fact that we are interested merely in the imaginary part as well as the +*i*✏ prescription give us a convenient present use the standard inactimery. Caredia
Research in the anaray momen nachi <u>.</u>
I the er energy mom *kz*Im⇥ dressed

dresseder

$$
F^{z} = -\frac{m^{2}}{2\gamma^{2}} \int \frac{\mathrm{d}^{2}k_{\perp}}{(2\pi)^{2}} \frac{\mathrm{d}k_{z}}{2\pi} k \left[\text{Im} \left[G^{dressed} \right] \right] G^{dr}
$$

n
*dependencies dependencies dependencies department*ing *department department department* the matter perfectly factorize the matter perfectly factorize Qualities of the perturber and

by a single power of *k^z*, we get for the integrand

Z d*k^z*

$$
\int G^{dressed} \equiv \left[G^{dressed}_{\mu\nu,\alpha\beta} \right] u^{\alpha} u^{\beta} u^{\mu} u^{\nu}
$$

⁸ ^{110*m*} optical the re perturber and **that** \blacktriangleright **a** Structure one could predict from optical theorem

RECIPE: HOW TO CALCULATE THE DRAG FORCE IN ANY REGIME

- Find an expression for the excitation of matter: particle retarded propagator in the case of the ideal gas, sound modes for the ideal fluid in ideal liquid…
- Use it to calculate the retarded propagator of the dressed graviton
- Contract with the four-velocity of the bullet
- Calculate the drag force as the Fourier transform of the imaginary part of the contracted dressed graviton

SCALES OF THE PROBLEM

Relevant scales:

- \bullet R_{IR} the largest relevant distance, the size of the cloud
- R_{UV} the smallest distance, the size of the bullet (Schwarzschild radius)
- The mean free path at this scale the liquid EFT starts breaking down, must be replaced by full kinetic theory

What is the k-dependence in various regimes?

General dimensional considerations: F ∝ m2 G2 10 *Comes from general expression Gravitational coupling2 from the dressed propagator*

STRUCTURE OF THE DRAG FORCE EXPRESSION 2 ª *dr* r 2 (155)
2 (155) *d* $\frac{1}{2}$ *v*12 q log ⇤ (156) *W* " \overline{T} p*E* [~] ¨ *^D* ` *B* [~] ¨ *^H* q*d*3 *x* (154) *e*2 ª *dr* ORCE EXPRESSION \mathbf{H} \blacksquare ⇢ $\overline{O}R$ \overline{O} $SLON$

1

*^F*⁹ *^m*²

*G*2

ª

x (154)

 \blacksquare

⇢⁰ log *k* (161)

Lowest frequencies – largest wavelengths: ideal liquid regime *v*² laigest wavelengths. Ideal *F* " *x* frequ *dv*¹ *v*12 $\mathbf{F} = \mathbf{P} \mathbf{F}$ igths: *^r*² (155)

0

~

*v*2

F "

 $T_{\mu\nu} = (\rho_0 + P_0)u_{\mu}u_{\nu} + P_0g_{\mu\nu} \implies F\circ(\rho_0 + P_0)G^2m^2$ $V = (\rho_0 + P_0)u_\mu u_\nu + P_0 g_{\mu\nu} \implies F \propto (\rho_0 + P_0)G^{-}m^{-}$ $P_0 + P_0$ ² m^2 ª *^v*

Already has dimensions of force. Namely the force is k-finite or ∝ *log k. Since gravity is a long-range force, we would expect a divergence log k.* **v**₂ *<u>we u</u>* $\mathop{\mathrm{odd}}$ expect a divergence log k. *v*2 log x defined with $\frac{1}{2}$ in the case of $\frac{1}{2}$ in the case o

Introduce first order corrections: viscous liquid.

$$
\delta T_{\mu\nu} = -2\eta \sigma_{\mu\nu} - \sigma_{\alpha} u^{\alpha} \Delta_{\mu\nu}
$$
\nBoth have one derivative –
\n
$$
v \text{iscosities are dim. 3}
$$
\nFrom dimensional analysis
\n
$$
\delta F \propto m^2 G^2 \eta_s k
$$

$\textbf{STRUCTURE OF THE DRAG}$ $FORCE$ EXPRESSION $MTID \cap T$ F OF THE DRAG *^Tµ*⌫ " p⇢⁰ ` *^P*0q*uµu*⌫ ` *^P*0*gµ*⌫ ùñ *^F*9p⇢⁰ ` *^P*0q*G*² *m*² (158) T *v*2 ª *^v* 0 *a*^c $\overline{\text{URL OF THE DRAG}}$ *F* " 4⇡*m*²*G*²²p1 ` *v*²q²p⇢⁰ ` *P*0q *p* ° *mpv p* † *mpv* (164) $\overline{1}$ $\overline{1}$ *hµ*⌫ L'ITTE DIATA

^F " *^m*² **T** $\frac{1}{2}$ **p** $\frac{1}{2}$ **p** $\frac{1}{2}$ *P* $\frac{1}{2}$ *p* $\frac{1}{2}$ *<i>P* $\frac{1}{2}$ *P* $\frac{1}{2}$ *<i>P* $\frac{1}{2}$ *P* $\frac{1}{2}$ *P \frac{* The viscous fluid EFT breaks down at the scale of MFP: $F \sim m^2 G^2 (\eta_s k + a_2 k^2 + a_3 k^3 + ...)$ **including the sum of the regular**

Tµ⌫

<u>r ne full</u> the *f* the full the *r* and *f* in *G*2 *S GPUS S COPUS* **GPU_S GPUS GPUS** " ´2⌘*µ*⌫ ´ ⇣r↵*u*↵4*µ*⌫ (159) $give a finite result$ *v* $\frac{1}{2}$ **c** $\frac{1}{$ The full theory should

Another limit: very high frequencies, ideal gas. The only parameter again is the energy density, we expect:

THE KINETIC THEORY it has been calculated for the massless gas in [13] and the viscid hydrodynamical case was analyzed in $[12]$. However, the generic case is not known and will be analyzed here (though here (though here (though

particularly numerically and the second control of the second second second second second second second second
Entre de la control de la We will closely follow the prescription of [13] and will also work in the time relaxation α and α in the B in the distribution for the distribution for the distribution function α To capture the effects of the interactions between the gas particles, use relaxation time approximation. τ is the time scale of the interactions

Boltzmann equation:

 $p^{\mu}\partial_{\mu}f - \Gamma^{\alpha}{}_{\beta\gamma}p^{\gamma}p^{\gamma}\nabla^{(p)}_{\alpha}f$ $p^{\alpha}u_{\alpha}$ τ $(f - f_{eq})$

1

 $\frac{d}{d\theta}$ $\frac{d}{d\theta}$

limiting regimes

quantified

graviton propagator (10). Note, that this function is known in certain limiting cases, *e.g.*

 $\frac{1}{\text{interactions time scale}}$ interaction kernel, with τ is the *interactions time scale*

- $m = \text{diam}(k\tau)$ and λ is the relaxation time. The relaxation of λ is the stage will not assume any specific form λ is the specific form of λ is the specific form of λ is the specific form of λ is the spec \bullet Ideal liquid regime: (k τ) << 1
- for the distribution function function α , β *req.* • viscosities: ∽τ
- Interacting gas the "transitional regime": $(k\tau) \sim O(1)$

13

MATTER EXCITATIONS AND KINETIC THEORY *v b x excita pp*r(*p*) ↵ *f*⁰ 8⇡ 2 ª *dr FOM2* \overline{C} <u>1</u> \overline{C} *v*2 *dv*¹ *v*12 *f*p*v*¹ **q** log ∈ lo

k + 1/2 μ *+ 1/2 \mu* Note that since the unperturbed distribution *f*⁰ depends only on *p*⁰, one can calculate the end (technical) are calculated as a function of the metric perturbation *hµ*⌫. We *F* Machinery: how do we solve the kinetic theory $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

 $s = \frac{1}{2} \log \frac{$ *Pasic assumptions:* f_{eq} *is isotropic, v pic,* $\left| \right|$ due to $\left| \right|$ α equation can be four ρ° *Tµ*⌫ " ´2⌘*µ*⌫ ´ ⇣r↵*u*↵4*µ*⌫ (159) depends only on *p0*

> Formal solution: Formal solution: $\delta f =$

are calculated as a function of the metric perturbation *hµ*⌫. We

feq/⌧ + ↵

When we know the perturbation *f* we can finally calculate the correlator

assume a locally thermal distribution *feq* = *f*⁰

i! + *i*~

the spatial derivatives. These two approaches are fully equivalent. Note that for *feq* we

^v^p · [~]

Formal solution:
$$
\delta f = \frac{1}{p^0} \frac{\delta f_{eq}/\tau + \Gamma^{\alpha}{}_{\beta\gamma} p^{\beta} p^{\gamma} \nabla_{\alpha}^{(p)} f_0}{-i\omega + i\vec{v}_p \cdot \vec{k} + 1/\tau}
$$

p*E*

0

` *B*

*m*² (158)

⇢⁰ log *k* (161)

Energy momentum tensor international momentum tensor and Matter excited as a Matter excited as a momentum tensor

 μ the uncer and the unitation of *Matter excitation (retarded correlator of* energy momentum tensor): *^F*⁹ *^m*² *dF* energy momentum tensor): *Matter excitation (retarded correlator of*

When we know the perturbation *f* we can finally calculate the correlator

$$
\delta T^{\mu\nu} = \int \frac{d^3 p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{p^0} \delta f
$$

$$
\delta T^{\mu\nu} = \int \frac{d^3 p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{p^0} \delta f \qquad \langle [T^{\alpha\beta}, T^{\mu\nu}] \rangle = \frac{\delta T^{\alpha\beta}}{\delta h_{\mu\nu}} \underbrace{ \text{contact terms}}_{\text{analytic in } k, \omega}
$$

Gkin is not fully equivalent to the object that we have defined in the Eq. (7). Nonetheless, 14

WARM UP: IDEAL GAS expression that smoothly interpolates between two these limits. r(*p*) *ⁱ ^f*⁰ ⁼ *^pⁱ J.D.EAL GAS*

The rest of the calculation is pretty straightforward. First, we are interested in the calculation in the calculation is \mathcal{L}_max

dressed graviton propagator. In our formalism the integral expression for this object reads the integral expression for this object reads to the integral expression for the integral expression for the integral expression f

The formal solution is much simpler than in the generic case: $p^0 - i\omega + i\vec{v}_p$ (2⇡)³ (! + *i*✏)² + *k*² $\delta f = \frac{1}{\phi} \frac{\rho \rho \rho \rho}{\rho}$

Notice that with our assumptions *f*0(*p*⁰*/T*) , we have

$$
\delta f = \frac{1}{p^0} \frac{\Gamma^i_{\ \beta\gamma} p^\beta p^\gamma \nabla_i^{(p)} f_0}{-i\omega + i\vec{v}_p \cdot \vec{k}}
$$

cases to the best of our knowledge does not appear in the literature and we present it here

 $N_{\rm eff}$ this case is relatively simple compared to the full case, since the expression for expression for expression for α

$$
\sqrt[3]{k_zG^{dressed}}dk_z = \sqrt[3]{k_zdk_z \int dp \frac{f'(p^0)p\gamma^4}{240\pi^2(k_1^2 + k_z^2)^4(p^0)^2(k_1^2 - k_z^2(-1 + p^2))^2} \times \left[\frac{R(k_z, k_\perp; p, p^0; v)}{\left(R(k_z, k_\perp; p, p^0; v) + \frac{S(k_z, k_\perp; p, p^0; v)}{k_z p^0 v} - \sqrt{k_z^2 + k_\perp^2 p}\right)}\right]}
$$
\n
$$
completely analytic \ncontributes to the branch
$$

functions of kz
and and along the real axis. However, since $\frac{1}{2}$ two

 K_Z are ω rest of ω in the meal are interested in the meal are interested in the mean ω *y anchoris by* κ *z cut along the real axis*

dressed graviton propagator. In our formalism the integral expression for the integral expression for this object re

pµp⌫*p*!*p*⇠

~² + *m*². One can analytically perform the angular integration in the

Gdressed = For sufficiently big $p > ym_p v$ the log is always negative and the trom the jump ove *I,µ*⌫*I*↵*,*⇢ entire imaginary part comes from the jump over the branch cut

$\text{IDEAL GAS: ANALYTIC STRUCTURE}$ $OF THE DRESSED PROPAGATOR$ \blacksquare *^F* " *^m*² *G*² *^s k* (160) *G^µ*⌫↵ *kin* ⁼ *^T ^µ*⌫ \cup with *T ^µ*⌫ = Z *d*³*p* \mathbf{L} N DDADACAT∩D *Gkin is not fully equivalent* to the object that we have defined in the Eq. (7). Nonetheless, \mathbf{I} \mathbf{I} \mathbf{O} \mathbf{I} \mathbf{I} \mathbf{D} \mathbf{I} P **ATOR** \overline{C} \overline{C} \overline{C} \overline{D}

^Tµ⌫ " p⇢⁰ ` *^P*0q*uµu*⌫ ` *^P*0*gµ*⌫ ùñ *^F*9p⇢⁰ ` *^P*0q*G*²

*^v*² log ⇤ ✓p*^v* ´ *^cs*^q (157)

When we know the perturbation *f* we can finally calculate the correlator

^y À

*F*9*m*²

one can show that they merely di↵er by contact terms, i.e. terms that do not contain new

^RS! (147)

^d log *^k* log *^k* (162)

*G*² ⌘*^s k* (160)

IDEAL GAS: GENERIC SOLUTION

Because of the analytic structure of the k_z integral, the generic solution breaks down into two integrals over p, one of them running from *p=0* to $p = \gamma m_p v$, and another running from $p = \gamma m_p v$ to infinity. The integrands are continuous at the stitching point.

The known limits (Chandrasekhar+photons gas) are easily reproduced from the general expression

INVISCID FLUID ⇢⁰ log *k* (161) ⇢⁰ log *k* (161) multiplies a log, which contains a log, which contains a cut on the real axis. Which contains a cut on the real
The real axis. Which contains a cut of the real axis, which contains a cut of the real axis. Which contains a There are two distinct structures that the integral over *k^z* can have depending on the

parameter *p*. For suciently large *p*, namely

*G*² ⌘*^s k* (160)

*G*² ⌘*^s k* (160)

^d log *^k* log *^k* (162)

Here the energy momentum tensor and the propagators are well known. Dressed propagator: $\frac{1}{2}$ $\frac{1}{2}$ wall known Dreagad pro α are the energy momentum tensor and the propagators are

$$
G^{dressed}(\omega = vk^{z} + i\epsilon) = \left(\frac{\kappa\gamma^{2}}{(1 - v^{2})k_{z}^{2} + k_{\perp}^{2}}\right)^{2} \frac{f_{\text{sound}}^{\text{ideal}}(\vec{k})}{(c_{s}^{2} - v^{2})k_{z}^{2} + c_{s}^{2}\vec{k}_{\perp}^{2} - i\epsilon v k_{z}}
$$

all the $\frac{1}{2}$ **ravita** *imaginary axis velocities, real* $\frac{1}{2}$ *v*² al pole:
_Wavie ⇣ all the gravitational poles are on

*F*9*m*²

*F*9*m*²

*^F*⁹ *^m*²

*^F*⁹ *^m*²

dF

^r*^T* ↵*, T ^µ*⌫^s

*G*2

are on velocities, real otherwise. The virtual phonons sound poles: on the imaginary axis for subsonic become real *p* $\frac{1}{2}$ *m*^p *n*^p*p*^{*x*} (*n m*^p (*n m*^p) ational poles are on welcotties real otherwise. The virtual phonons 42
422 - Pierre Barnett, amerikansk matematik
422 - Pierre Barnett, amerikansk matematik *df*⁰

 $\frac{k_1}{(1 - v^2)}$ **b** $\frac{k_z}{(1 - v^2)}$ **d** $\frac{k_z}{(1 - v^$ shell and the propagator is the contribution from the gravity **Example 12** As in the lue at gas case, without dissipation the gravity p ² the idea
re is no c *mgv d*
p de n $\frac{1}{2}$ without dissipation \overline{a} (27 + 29*v*²) ⁶*p*²

 $c_s k_{\perp}$ bubbonic build $f(x)$ in form in the original confirming the $\sqrt{c_s^2 - v^2}$ result of Rephaeli-Slapeter Subsonic builet — no poles on the real axis,
 $\frac{2}{s^2 - v^2}$ integral is perfectly real, confirming the result of Rephaeli-Slapeter Subsonic bullet — no poles on the real axis,

) + *p*²

in the hydrodynamic medium that cannot propagate far. There is no phase space available

MORE REALISTIC PICTURE: ADD THE VISCOSITY T T T T T *T* ↵ *hµ*⌫ Γ contact terms Γ

In the system with dissipations the calculation is more involved and we cannot disregard the analytic structure outside the real axis. *p* ° *mpv p* † *mpv* (164) *F* and *n* ^p⌘*s^k* ` *^a*2*k*² ` *^a*3*k*³ ` *...*^q (165)

$$
G^{dressed}(\omega + i\epsilon) \neq G^{*dressed}(\omega - i\epsilon)
$$

The viscosity contributions blow up at the speed of sound. The results are also linearly sensitive to the cutoff of the EFT

^d log *^k* log *^k* (162)

Perturber's velocity v

SOLVE THE FULL KINETIC $THEORY$ have a notation of the set of the *v^p* ⌘ *p/p* ~ ⁰. Assuming a small correction *f* = *f f*⁰ due to S. THE TUATIONS IN THE VIOLET IS Now we have generic regimes of the regime Γ in the regime Γ *feq* = *f*0(*p*⁰ *p*0 *T* ✓ ~ *<u><i>v*</u> */* \overline{P} U $\mathbf I$ *T T* \blacksquare IVECTER THEORETICAL MACHINERY THEORETICAL MACHINERY THAT WE HAVE A LICENSE IN SECTION AND ASSUMING THAT WE HAVE **p**

feq = *f*0(*p*⁰

interpolates between two these regimes, yielding them as the limiting cases.

)
)
)

tuations in the equilibrium distribution functions in the equilibrium distribution function $\mathcal{L}_\mathbf{p}$

 $\frac{1}{2}$

Z *p*²*dp*

2⇡² *T*

Eq. (51) as an equation on *T* and

uⁱ

=

Formal solution:
$$
\delta f = \frac{1}{p^{\rho}} \underbrace{\oint f_{eq}}_{-i\omega + i\vec{v}_p} \frac{\tau + \Gamma^{\alpha}{}_{\beta\gamma} p^{\beta} p^{\gamma} \nabla_{\alpha}^{(p)} f_0}{-i\omega + i\vec{v}_p \cdot \vec{k} + 1/\tau}
$$

u +

the spatial derivatives. These two approaches are fully equivalent. Note that for *feq* we

^v^p · [~]

the metric perturbation the metric perturbation that the Fourier space of the Fourier space of the Fourier space

^v^p · [~]

~

$$
\delta f_{eq} = f_0(p^0) \frac{p^0}{T} \left(\vec{v}_p \cdot \vec{\delta u} + \frac{\delta T}{T} \right)
$$
\nHere closely following Romatschke, 1512.02641

we get the following expression for the fluctuating expression for the fluctuating energy-momentum tensor in th

however the se results looks vastly divided by the set of the smoothly divided by the smoothly that smoothly that smoothly divided by the smoothly divided by the smoothly divided by the smoothly divided by the smoothly div

Here closely following Romatschke, $^{1512.02641}$ *1512.02641* ϵ *ere closely following* Zomatschke,

 \mathcal{L}^{max}

#

↵*p*↵*p*r(*p*)

(*p*) (53)

assum wighter of and there com *T* $nonents of$ </u> α ingthic hack into the expression for energy strain the free strength \mathbb{R}^n $\frac{1}{\sqrt{1-\frac{1}{n}}}$ When we know the perturbation finally calculate the perturbation \mathcal{C} we can finally calculate the correlator \mathcal{C} *p d d d d different equations to momentum tensor we have 4 different equations to* 2⇡² *T* δ u. Plugging this back into the expression for energy *^v^p ·* [~] 1 VC. We have 4 unknown variables: $\delta \rm{T},$ and three components of momentum tensor we have 4 different equations to solve: we nave 4 annown vanaores. *o*r, and three components or י
ח nsor we have 4 different equation again, containing the derivation of \mathbb{R}^n and the upper round of ve nave 4 different equa io $\overline{\text{CNOTV}}$

$$
\delta \rho = \delta T^{00} \qquad \delta u^{i} = \frac{\delta T^{0i}}{\rho + P}
$$
 Use the unperturbed f to
calculate the energy density

⇢ ⁺ *^P* with ⇢ ⁼

4⇡

*T*⁰*ⁱ*

culate the ene: *<u>rbed</u>* \int *¹* dens *p^µp*⌫ *^p*⁰ *^f* (23) Use the unperturbed *f* to (*p*) (53)

Z *d*³*p*

(2⇡)³ *^f*0(*p*)*p*⁰

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RESULTS FROM THE FULL KINETIC THEORY

Smooth interpolation:

MAPPING THE HYDRO EFT ONTO THE FULL THEORY

The viscosities can be extracted from Kubo relations:

Viscosities for the Boltzmann distribution. In the conformal limit $\eta \rightarrow T\tau/5$.

Bulk viscosity mapped onto m and τ maximized at m~ T

$\mathbf{COMMENTS ON "OSTRKIER'S}$ **²** *^F* " *^m*² \mathcal{I}^{max} is a community of \mathcal{I}^{max} . Computed the community of \mathcal{I}^{max} In the previous section we considered a stationary situation where the medium extends

While these are exactly the assumptions of the classical dynamical friction problem, some

In contrast with our results and the scope of the scope of this our work, we will show the can attack non-trivial boundary results of Rephaeli-Salpeter the interesting to see the inter force does not vanish identically in the subsonic regime \overline{h} s not vanish iden 1101 v \mathbf{f} $\text{vec does not vanish identically in}$ \forall the subsonic regime

$$
F = \frac{4\pi G^2 m_b^2 \rho_0}{v^2} \log \left(\frac{1+\mathcal{M}}{1-\mathcal{M}}\right)
$$

Seconding the energy-momentum with particle velocity of sound Finite (but might be numerically important), boundary effect in time

Way to reproduce in our formalism:

$$
T_{bullet}^{\mu\nu}(\vec{x},t) = \gamma m \delta(z - vt) \delta^2(\vec{x}_{\perp}) v^{\mu} v^{\nu} \times \theta(t)
$$

$$
T_{bullet}^{\mu\nu}(\vec{k},\omega) = \gamma m \frac{-iv^{\mu}v^{\nu}}{-\omega + k^z v - i\epsilon}
$$

Z *d*⁴*k*

(2⇡)⁴ *^eit*(!*ikzv*)

 $(\vec{x}_\perp)v^\mu v^\nu \times \theta(t)$ with a similar **calculation** And proceed with a similar

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 $F = \frac{1}{2}$ and the Speed of Sound to large

^µ⌫*,*↵ (*k*) *iv^µv*⌫

CONCLUSIONS

- Proposed a completely generic method to calculate the classical dynamical friction in any regime
- Showed a completely generic analytic expression for the ideal gas in any regime of \bullet temperatures/ velocities
- Developed the expression for viscous fluids within the EFT that make important contributions in the subsonic regime
- Showed the smooth interpolations between the low and high-k regimes within the full kinetic theory
- These improved results can now be used in plethora of physical problems: interacting DM, \bullet propagating of the BHs through the compact objects, physics of the accretion discs etc.
- Extension of the existing results to include the boundary effects, post-Newtonian corrections is straightforward (though laborious) using our formalism