Calculating High Dimensional Integrals in QFT with MIXMAX

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in collaboration with

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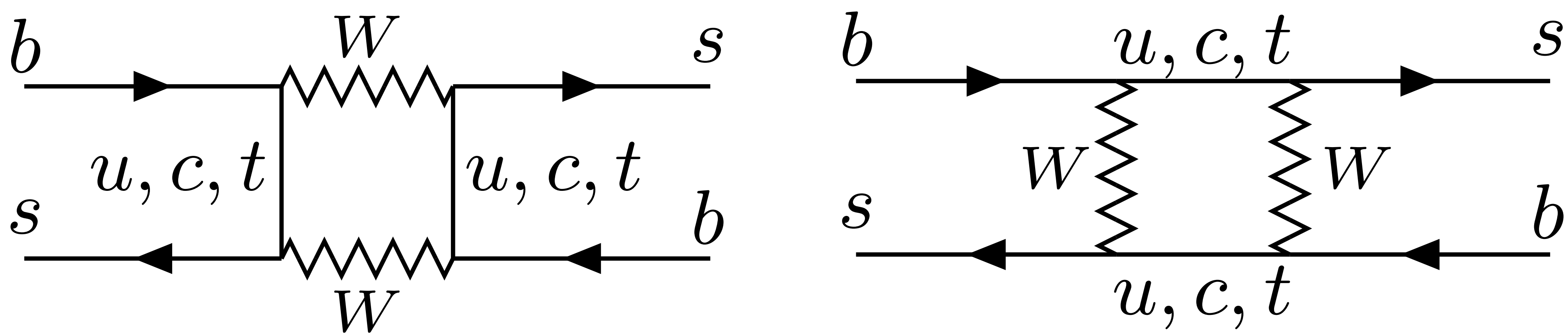
Outline

- We test on particular examples the application of the Monte Carlo method for integrals, which appear in multi-loop Feynman diagrams in QFT.
- We compare Monte Carlo Plain and MISER algorithms of integral calculation with Mersenne-Twister RNG against MixMax RNG.
- Summary

$B_s - \bar{B}_s$ mixing



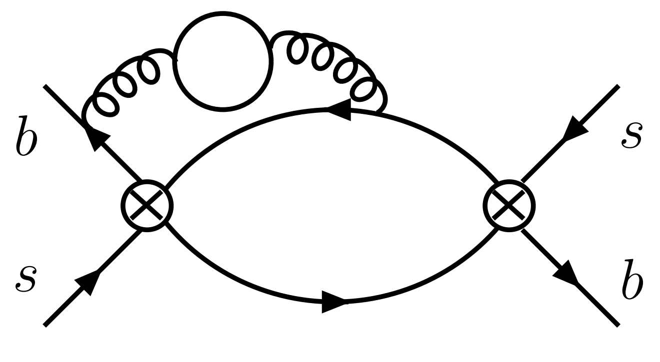
Leading order Feynman diagrams in strong coupling constant α_s.

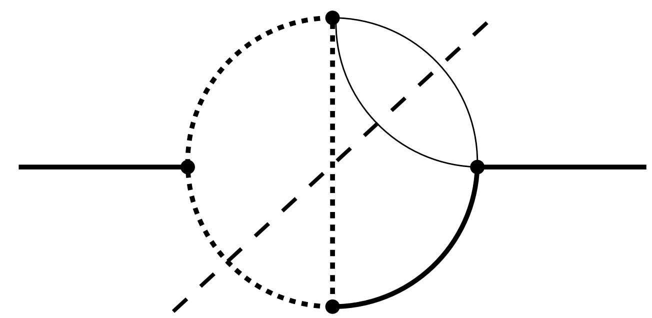
Lighter eigenstate (CP-even): $|B_L\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle$
Heavier eigenstate (CP-odd): $|B_H\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle$ with $|p|^2 + |q|^2 = 1.$

Decay rate difference: $\Delta\Gamma = \Gamma_L - \Gamma_H \simeq -\Delta\mathrm{m}\mathrm{R}e\left(\frac{\Gamma_{12}}{\mathrm{M}_{12}}\right) = 2|\Gamma_{12}|cos\phi$

$|\Gamma_{12}|$: absorptive part of box (tree-level decays), only internal u, c contribute

One of Feynman diagrams, which contributes at the three-loop calculation of $B_s - \bar{B}_s$ mixing





Cutkosky Rule

$$2i\mathrm{Im}(\mathcal{D})$$

$$= \mathcal{D}\left(\frac{i}{q_j^2 - m_j^2 - i\epsilon} \to 2\pi\delta^+(q_j^2 - m_j^2)\right)$$

Left: One of the Feynman diagrams, which contributes at $O(n_f\alpha_s^2)$ to the $B_s - \bar{B}_s$ process.

Right: One of the master integrals, which originates from the diagram shown on the left side of this figure.

The leading term in the expansion over m_c/m_b of the imaginary part of a master integral:

$$J_2 = \int dxdy \left[\left(\frac{1}{(1-x)x(1-y)y} + \frac{1-y}{y^2}\right)\log\left((1-x)x(1-y)^2+y\right)\right.$$
$$\left. + \frac{\log((1-x)x)}{y} + \frac{1}{y} - \left(\frac{1}{(1-x)x(1-y)y} + \frac{1}{y^2}\right)\log((1-x)x(1-y)+y)\right],$$

where $x \in [0,1]$ and $y \in [0,1]$.

The exact value of this integral is $-1-2\zeta(2) \simeq -3.404114$, where $\zeta(x)$ is the Riemann zeta function: $\zeta(x) = \frac{1}{\Gamma(x)}\int_0^\infty \frac{t^x-1}{e^t-1}dx.$

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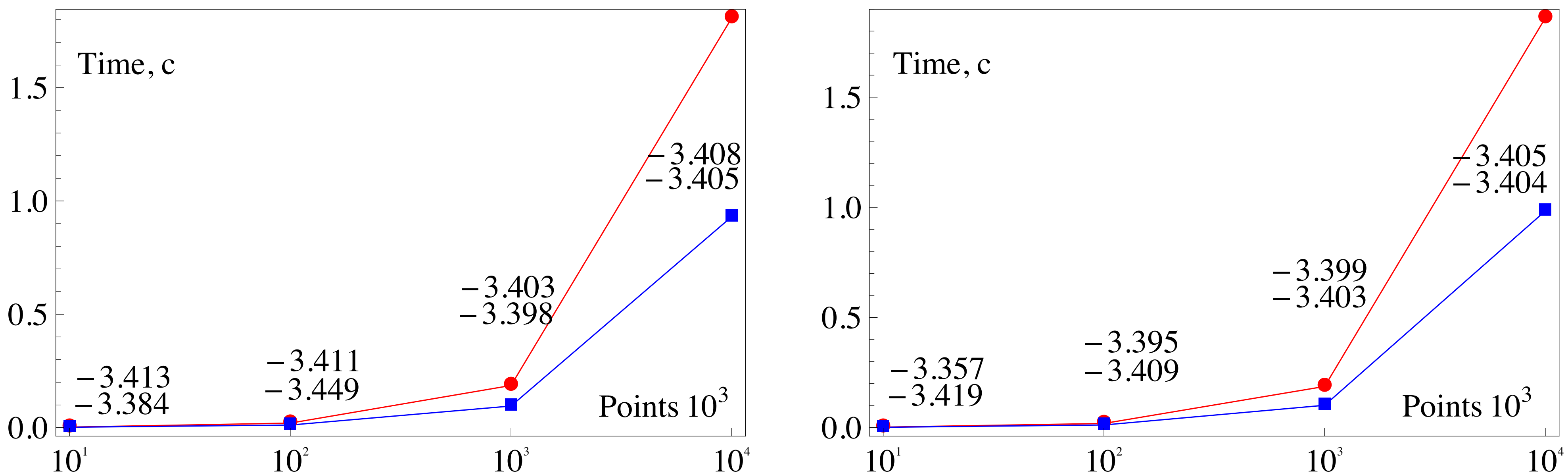


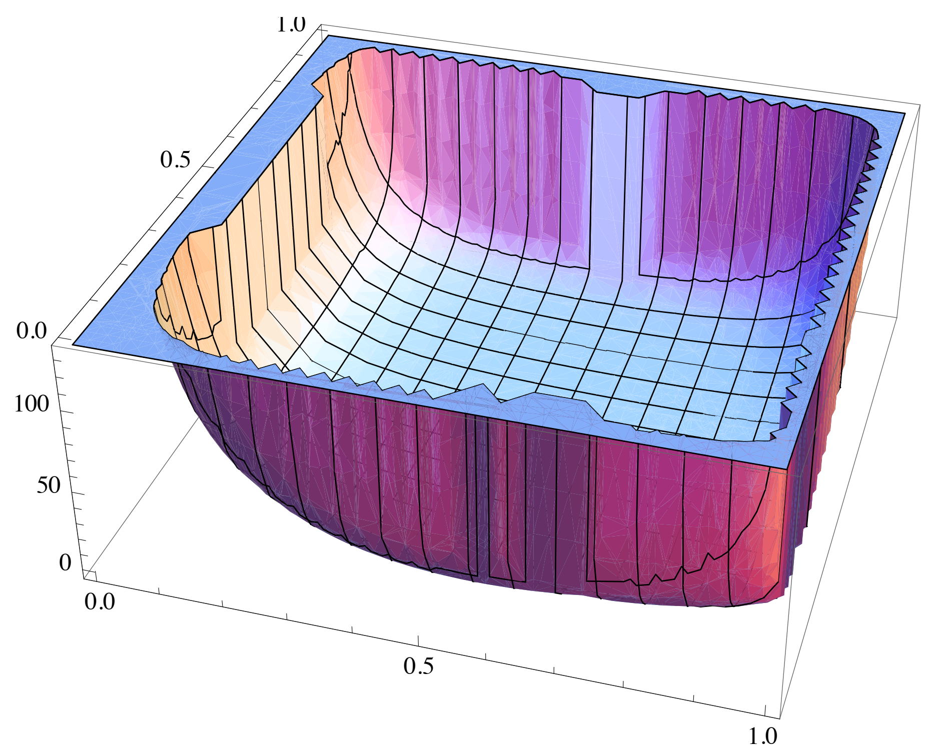
Left: The red dots represent the results of integrations by MC Plain of the integral with Mersenne-Twister RNG and the blue dots with - MixMax RNG. Right: The same for MISER algorithm.

	N	10^4	10^5	10^6	10^7	10^8	10^9
MT,	Time, c	0.002055	0.01976	0.1858	1.808	18.11	180.8
Plain	Result	-3.413520	-3.411583	-3.403035	-3.407948	-3.403809	-3.403920
MX,	Time, c	0.001576	0.01123	0.0952	0.931	8.68	87.1
Plain	Result	-3.384531	-3.449768	-3.398291	-3.404808	-3.403767	-3.404495
MT,	Time, c	0.002035	0.01872	0.1868	1.859	18.65	186.9
MISER	Result	-3.357564	-3.394729	-3.398947	-3.404698	-3.403930	-3.404185
MX,	Time, c	0.001388	0.01183	0.1013	0.984	9.97	100.5
MISER	Result	-3.418790	-3.409007	-3.403363	-3.403912	-3.404168	-3.404087

$\bar{B} \to s\gamma\gamma$ process

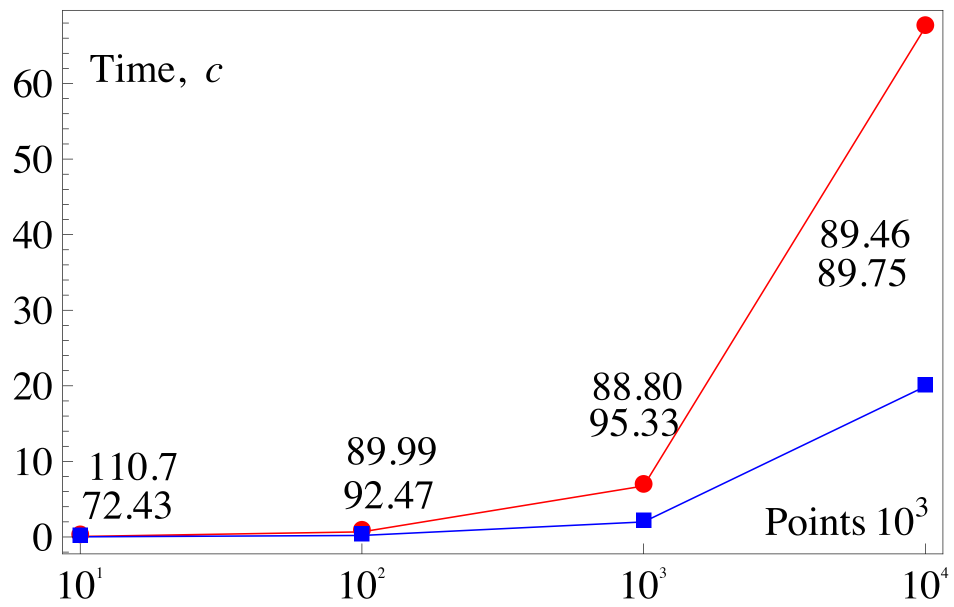
A master integral, which contributes at $O(\alpha_s)$ to the $\bar{B} \to s\gamma\gamma$ process:

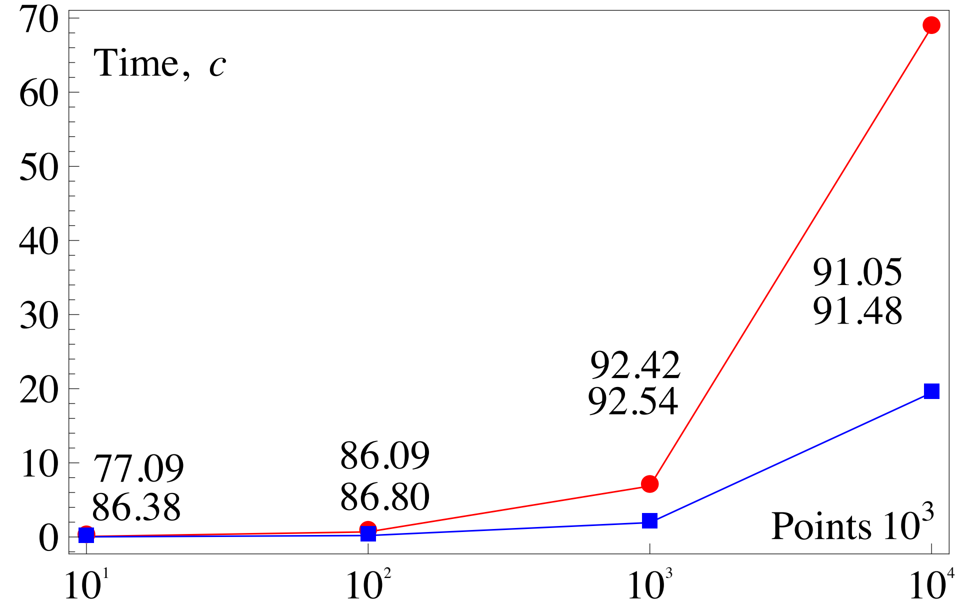
$$F_2 = \frac{1}{\sqrt{x(1-x)}\sqrt{y(1-y)}\,(1+x^2+y)} \left[x + y^2 + x^3 y \log(1-x)\log(1-y)\right.$$
$$+x^2y^2\log^2(1-x) + 3x^2\log^2 y + xy^4\log^2(1-y) + xy^3\log(1-y) + \left(x+y^3\right)\log x$$
$$\left. + \left(5x+y^2\right)\log y + \left(xy^2+2\right)\log x \log y + 2y\log^2 x + xy\log(1-x)\right].$$



The dominant contribution of this integral comes from the points close to the boundaries of the integration region.

The numerical integration on Wolfram Mathematica for this integral gives 91.63316.





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Right: The same for MISER algorithm.

	N	10^4	10^5	10^6	10^7	10^8	10^9
MT,	Time, c	0.007305	0.06825	0.6755	6.746	67.44	676.7
Plain	Result	110.7305	89.99293	88.79852	89.45729	90.23272	91.04224
MX,	Time, c	0.002308	0.02036	0.2012	1.992	19.92	199.2
Plain	Result	72.42657	92.46804	95.32528	89.74804	91.86384	91.33837
MT,	Time, c	0.006843	0.07110	0.6859	6.866	68.77	688.6
MISER	Result	77.09064	86.09166	92.42170	91.05356	91.60778	91.63024
MX,	Time, c	0.002130	0.01946	0.1938	1.937	19.45	194.1
MISER	Result	86.37929	86.79966	92.54222	91.47741	91.62262	91.62835

Conclusion

- On particular examples in QFT we studied the application of the Monte Carlo method for integrals, which appear in multi-loop Feynman diagrams.
- The comparison of Mersenne-Twister and MixMax RNG-s shows that integral calculation with MixMax RNG is faster than with Mersenne-Twister RNG.