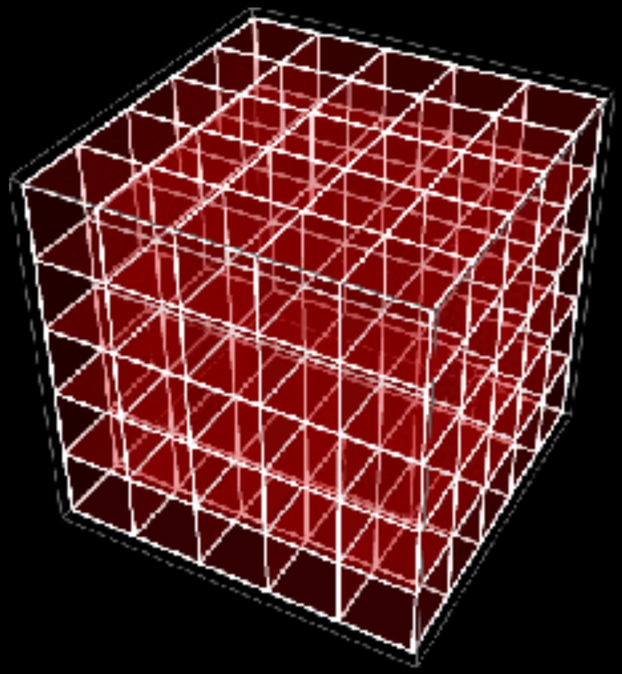
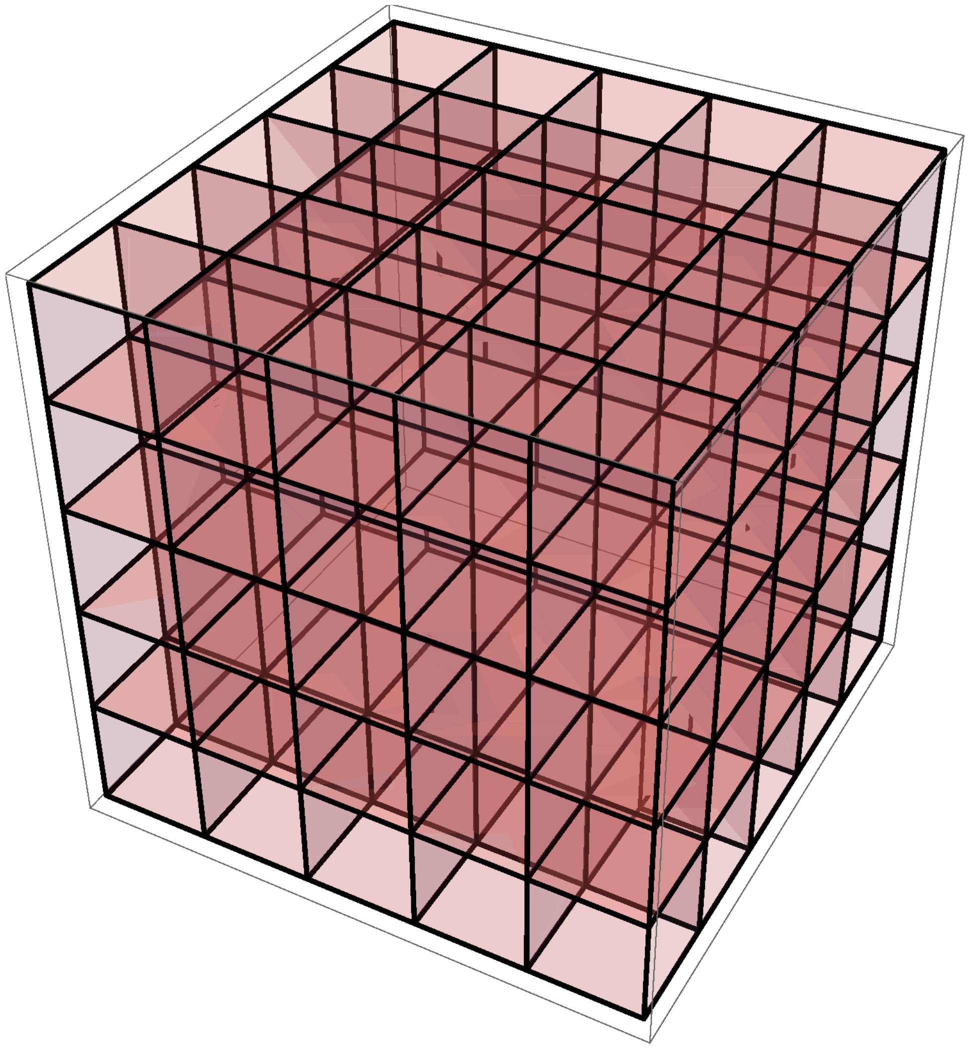
ICNFP2017
19/08/2017

Hadron Resonances from Lattice QCD



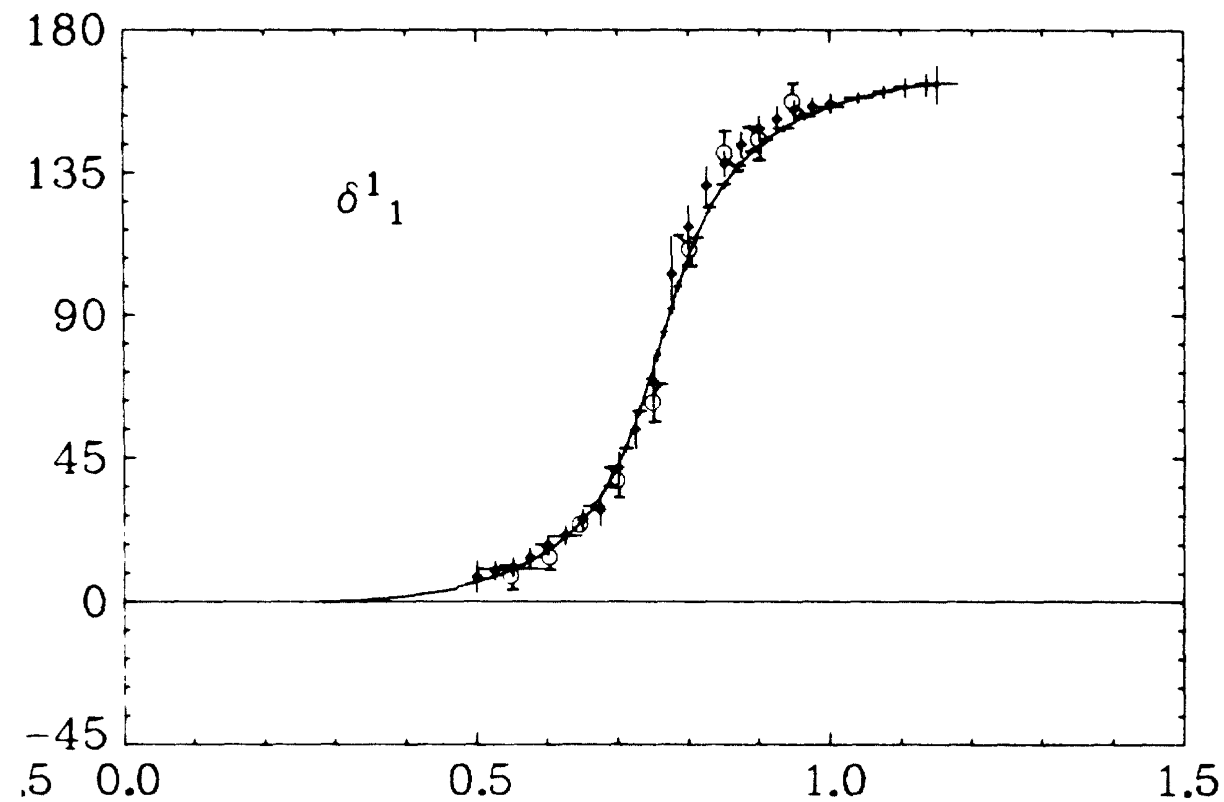
Christian B. Lang
University of Graz, Austria

Lattice QCD ➛ Analysis ➛ Phase shifts



Partial wave scattering amplitudes

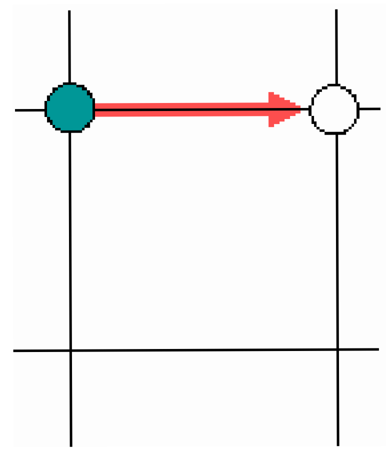
Phase shifts



The lattice approach

Regularization: Lattice QCD *(1974)*

Ken Wilson

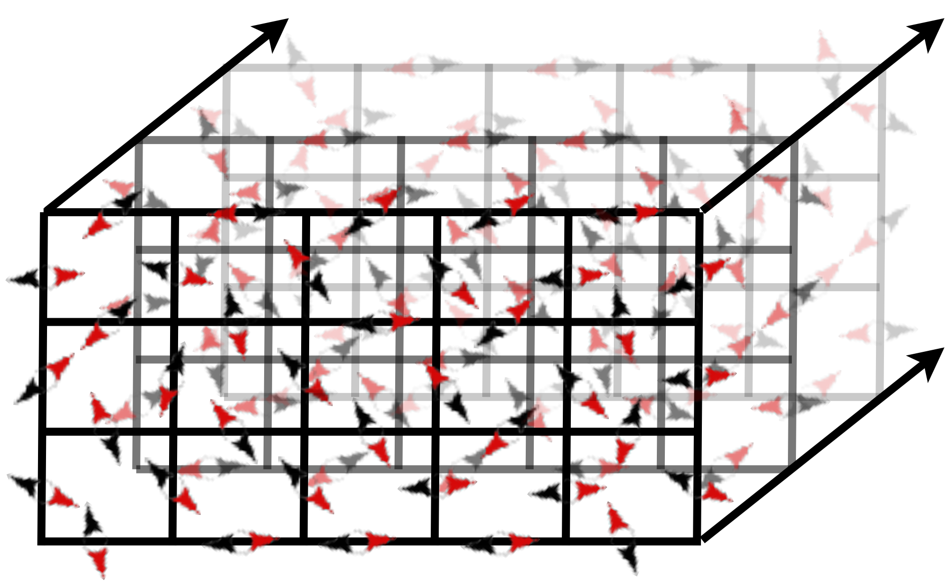


lattice spacing a

Gluon $U_\mu = e^{i\,A_\mu} \in \mathrm{SU}(3)$

Quark ψ

Antiquark $\overline{\psi}$



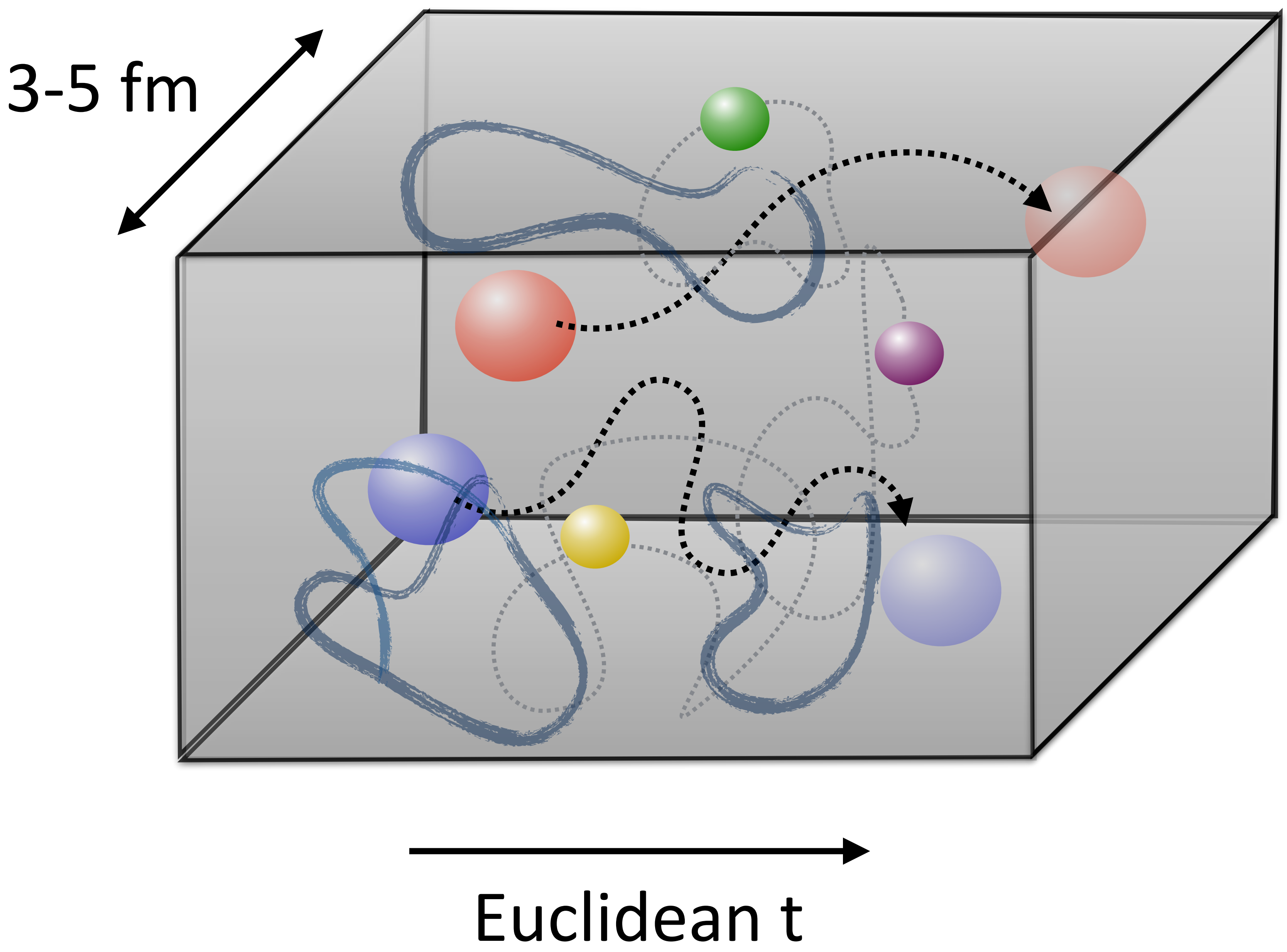
$U_\mu(x, y, z, \tau)$

Quantization:

$$\int [dU][d\psi][d\bar{\psi}] \to \sum_{\{U.\psi,\bar{\psi}\}}$$

The path integral becomes a well-defined (very large) sum over field configurations

Femto universe



Why the lattice?

- Continuum QFT is singular: regular formulation needed ✓
- Hadrons are non-perturbative compounds: non-pert. method needed ✓
- Gauge invariance is important: gauge inv. formulation preferred ✓
- Limit to "continuum QCD": should be controllable ✓

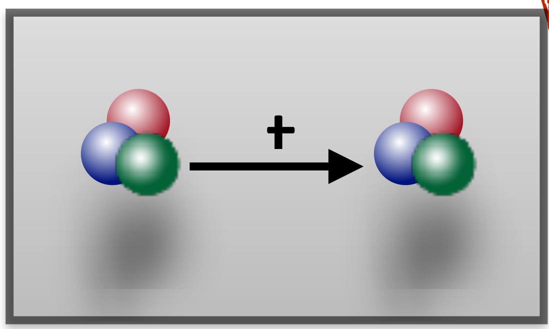
“Femto universe” - is it systematically controllable?

can be controlled/improved

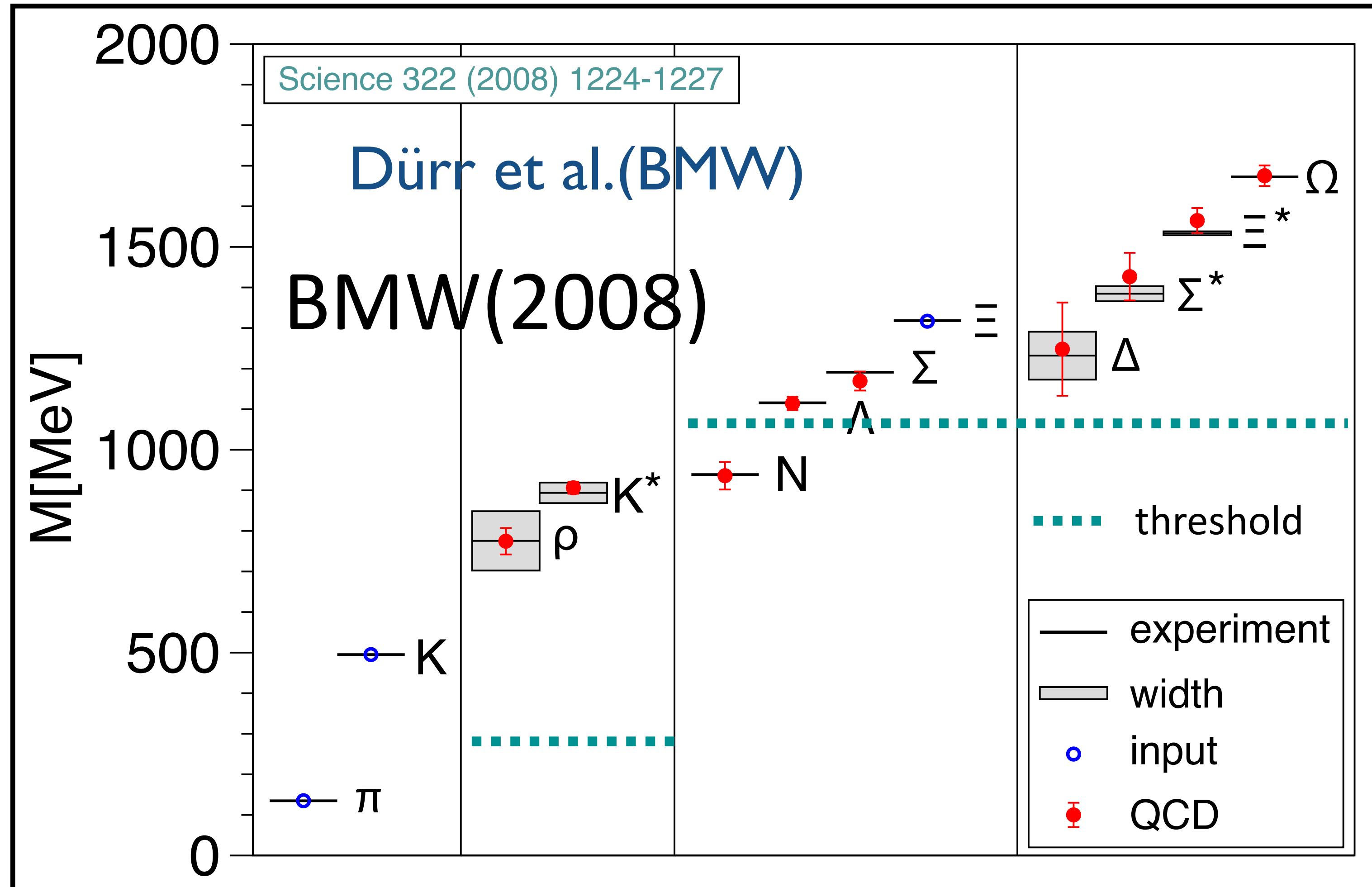
- dynamical quarks: u,d … s …c …b … ☑
- light quarks (pion mass ↘ 140 MeV) ☑
- lattice spacing (mostly a>0.05 fm) ☑
- lattice size (typically 2-6 fm) ☑
- limited statistics ☑
- analysis of correlators (operator set, asymptotic behavior) ?

Milestones

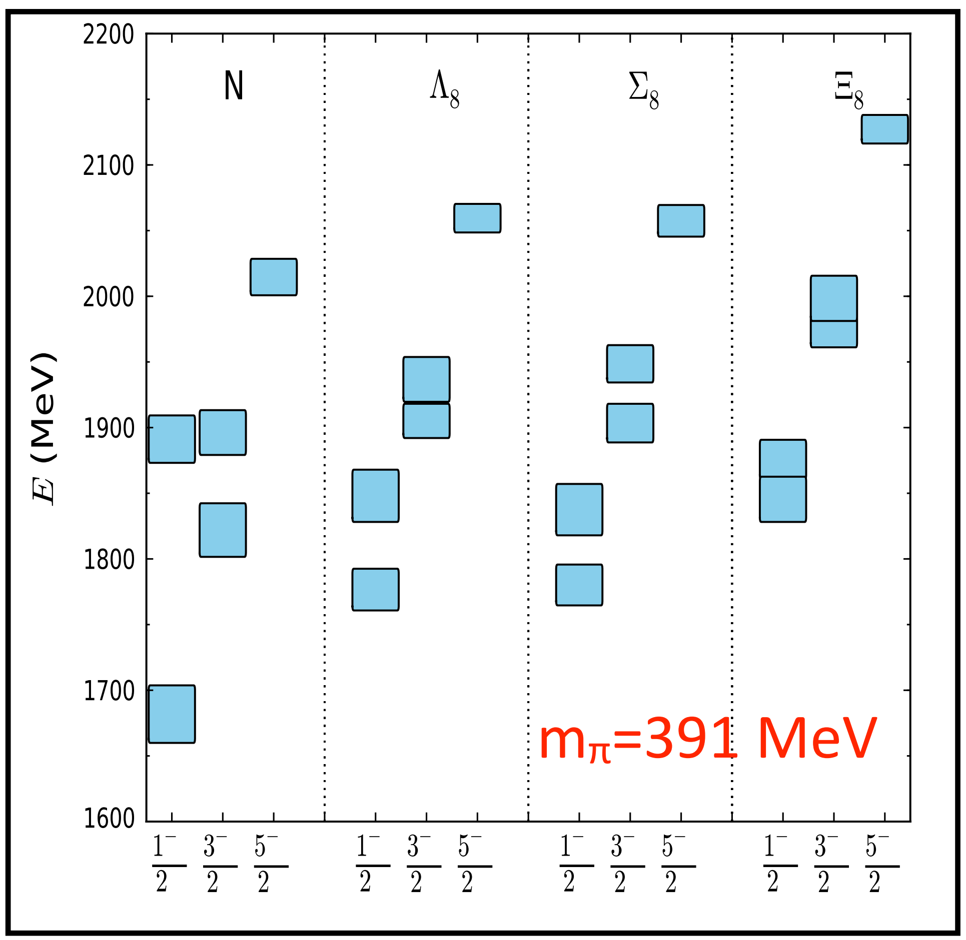
Simple hadron approximation

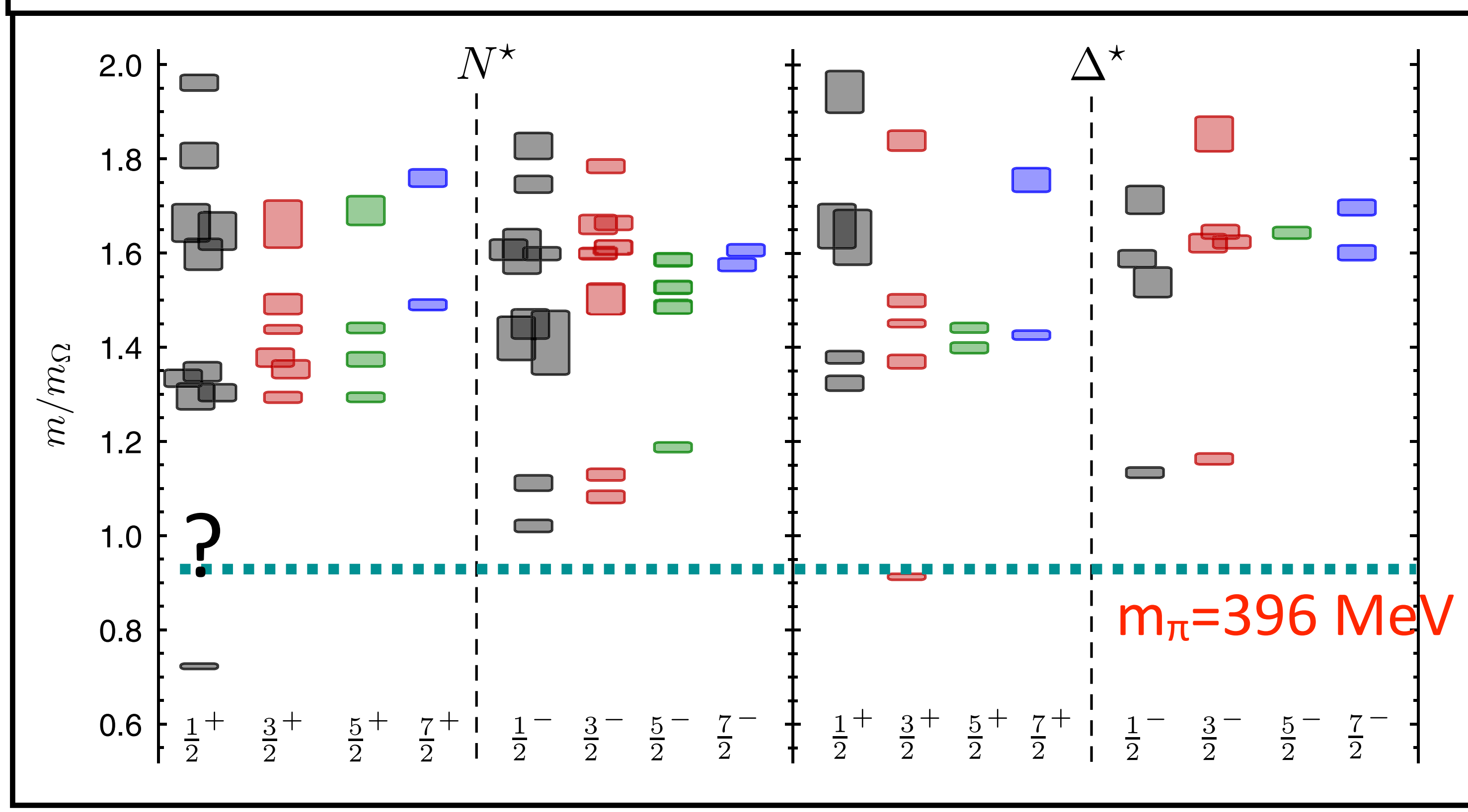


BMW(2008)



HSC(2011, 2013)





Edwards et al. (HSC) Phys.Rev. D87, 054506 (2013). Edwards et al. (HSC) PR D 84, 074508 (2011)

Spectroscopy

Ground state spectroscopy

Is correct only for stable particles.
Simple hadron approach qqq or qq
is valid only below scattering threshold
("bound states" or "artificial bound states")

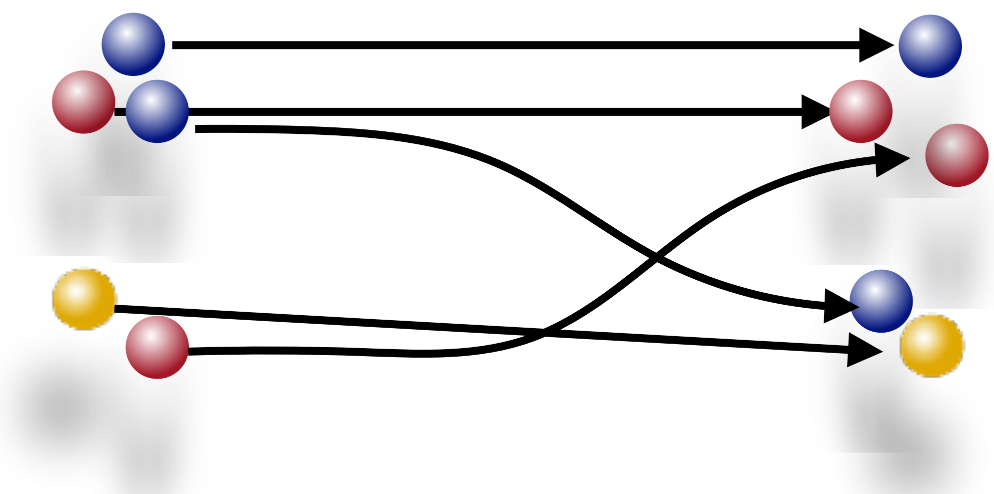
Resonances and bound states

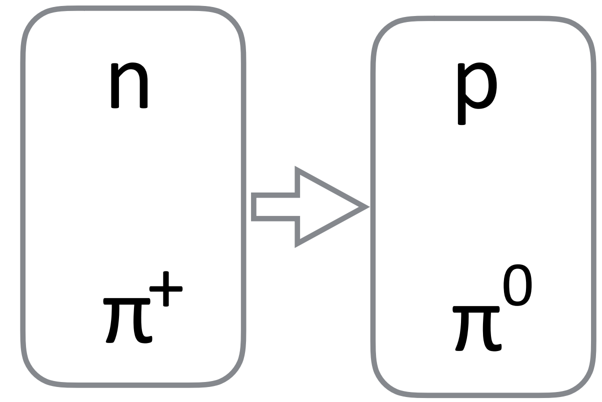
require inclusion of hadron-hadron channels in the calculation.
Multi-hadron approach: we need to extend the space of operators: (q$\underline{q}$)(q$\underline{q}$),(qqq)(q$\underline{q}$), (qqq)($\underline{qqq}$)…

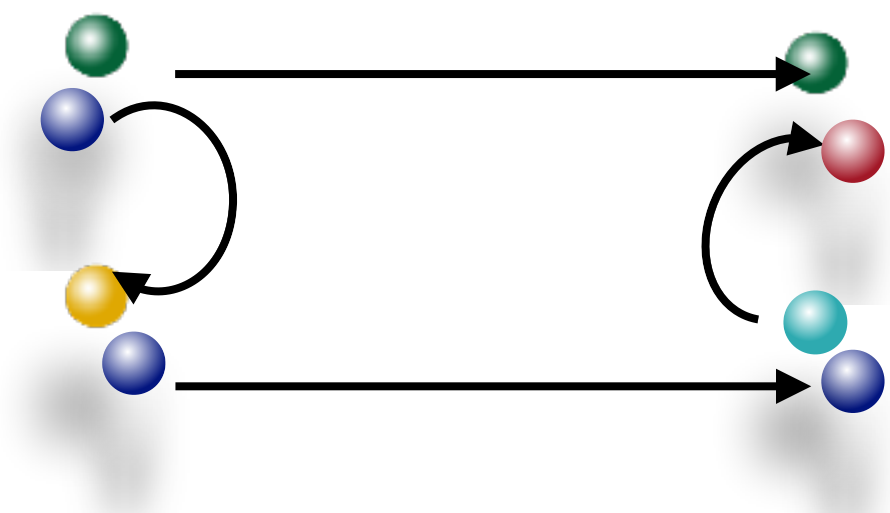
Excited states spectroscopy
and scattering amplitudes

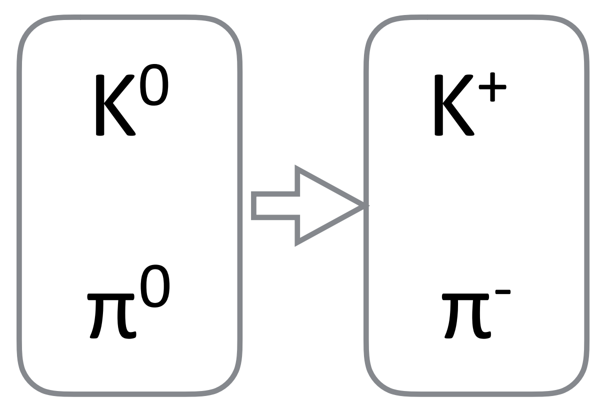
What is the challenge?

BM ➝ BM









MM ➝ MM

More terms

More quark propagators

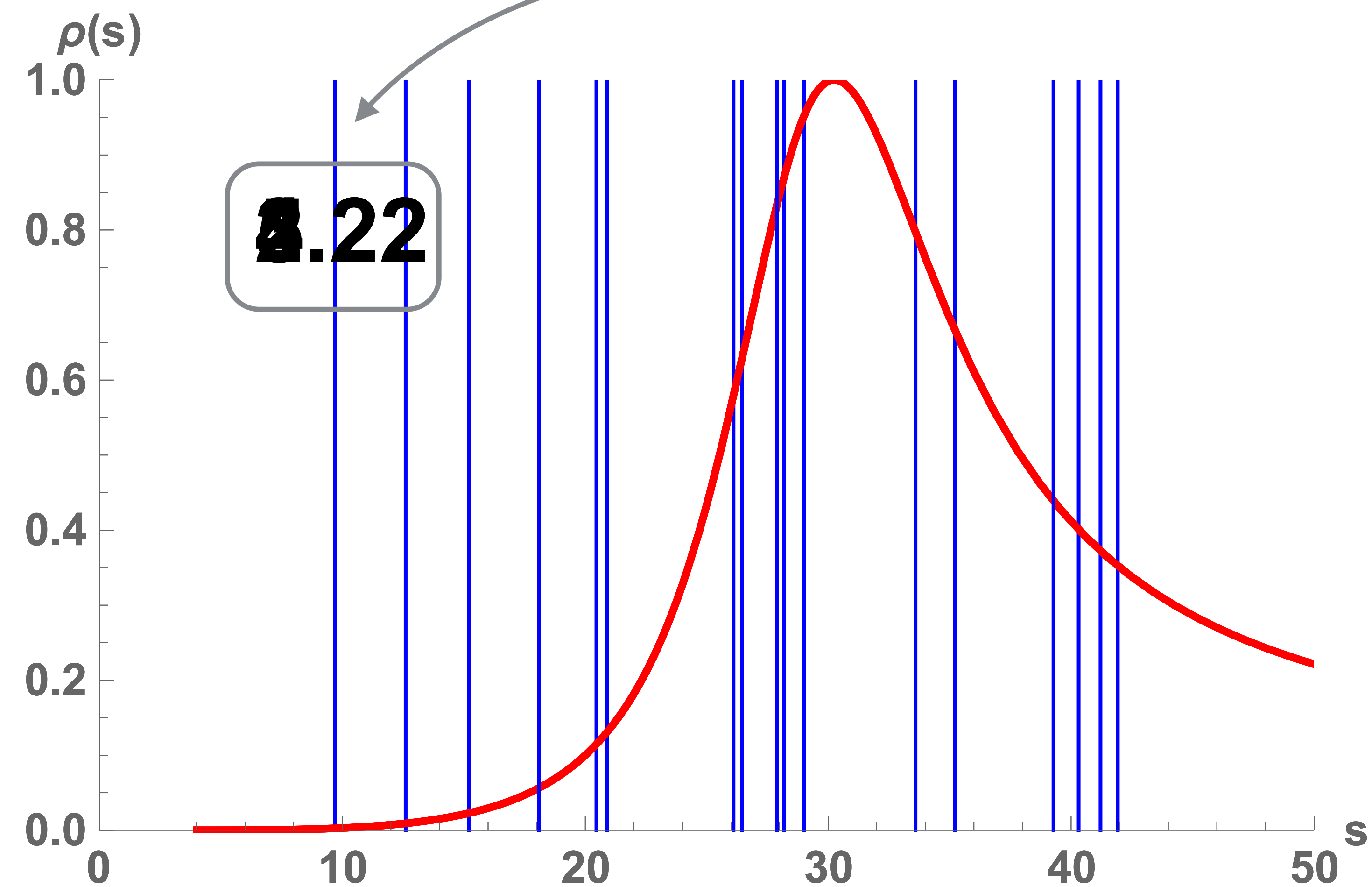
Backtracking loops are expensive! ⟶ Stochastic method and "Distillation method"
Peardon et al. (HSC), PR D 80, 054506 (2009).
Morningstar et al., PR D 83, 114505 (2011).

Continuum vs. lattice

Correlation functions have discrete energy levels!

$m_\pi L$=4 ~
L=5.6 fm

$m_\pi L$

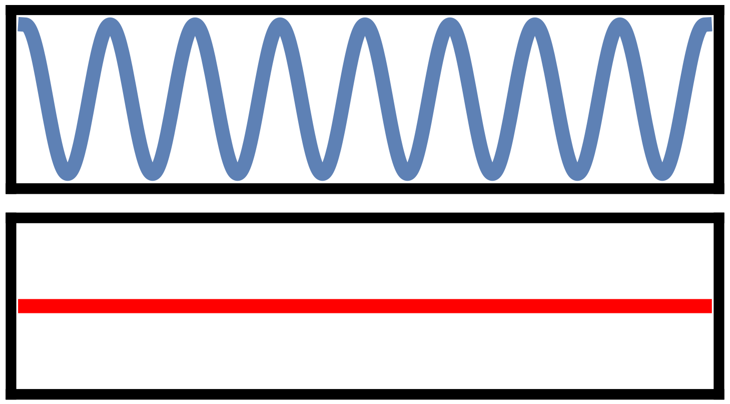


Example:

Spectral density of a simple resonance in continuum and the discrete energies for a lattice volume

One cannot arbitrarily fix the energies: they are eigenvalues depending on the control parameters (volume, couplings,...).

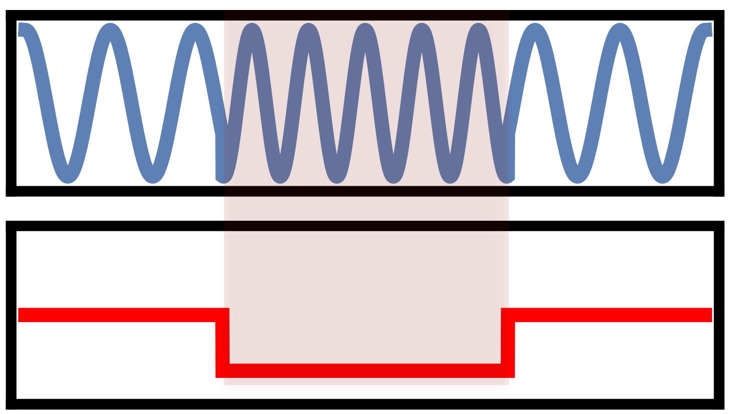
Energy levels ➤ Phase shift points (in the elastic region)



$V = const. \qquad \delta = 0$

$$\mathrm{e}^{\mathrm{i}pL} = 1 \qquad p_n = 2n\pi/L$$

periodic b.c.



$V = localized \qquad \delta \neq 0$

$$\mathrm{e}^{\mathrm{i}pL+2\mathrm{i}\delta(p)} = 1 \qquad 2\delta(p_n) = 2n\pi - p_n L$$

$$\cot\delta(p_n) = -\cot(p_n L/2)$$

Solving the Helmholtz eq. for d=3

$$\boxed{p_n \cot\delta(p_n) = c\mathcal{Z}_{00}\left(1; \left(\frac{p_n L}{2\pi}\right)^2\right)}$$

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

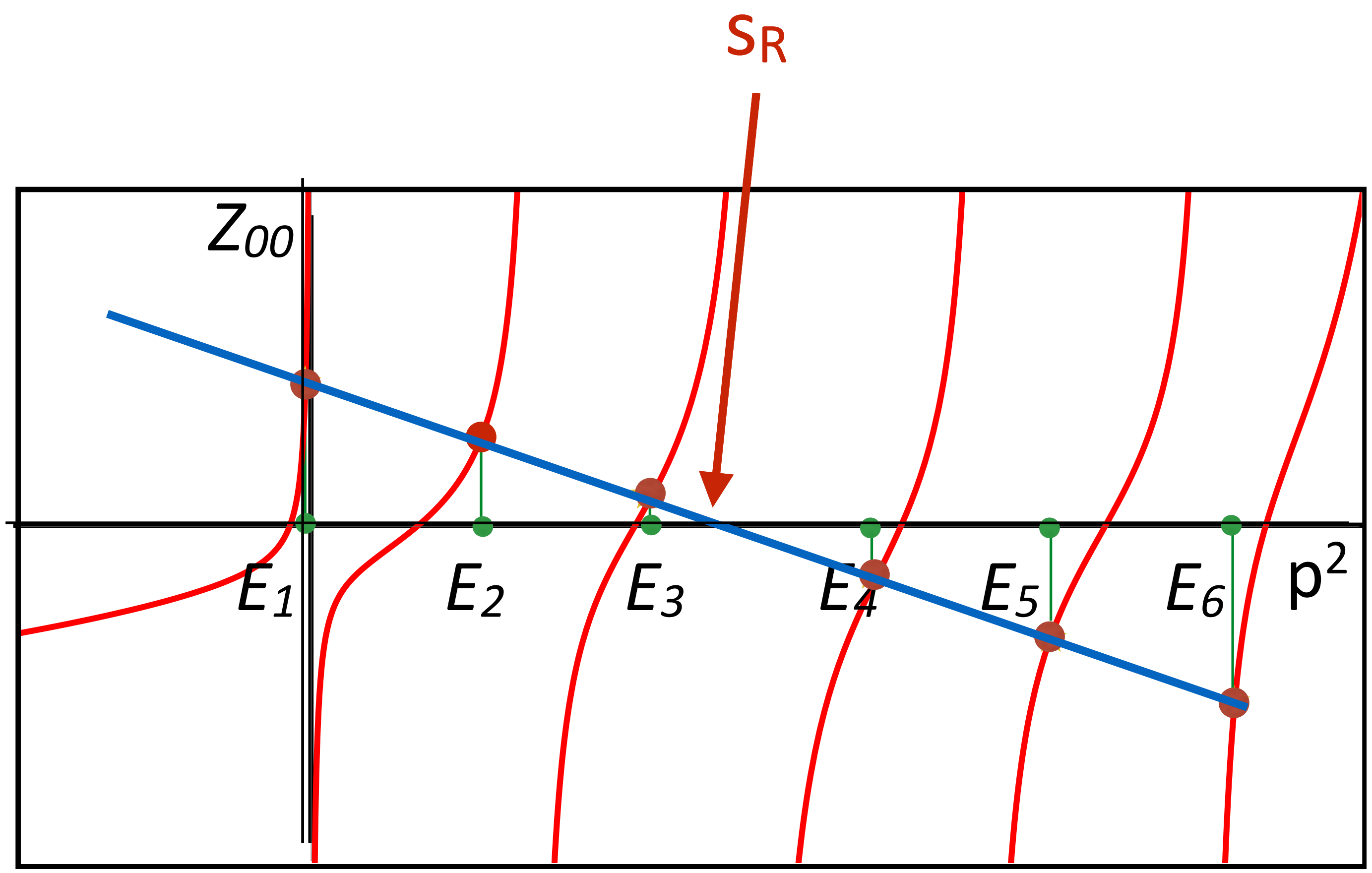
Elastic scattering

Partial wave amplitude t_ℓ

$$t^{-1} = p\,\cot\delta(p) - \mathrm{i}\,p$$

$$\equiv k^{-1} - \mathrm{i}p$$

e.g. $k^{-1} \propto (s_R - s)$



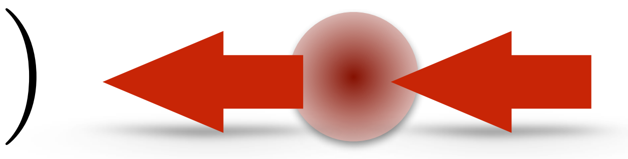
"Lüscher curves"

$$\boxed{p_n \cot\delta(p_n) = c\,\mathcal{Z}_{00}\left(1;\left(\frac{p_n L}{2\pi}\right)^2\right)}$$

(lattice loop term)

Energy levels E_n → p cot δ → $\delta(E_n)$

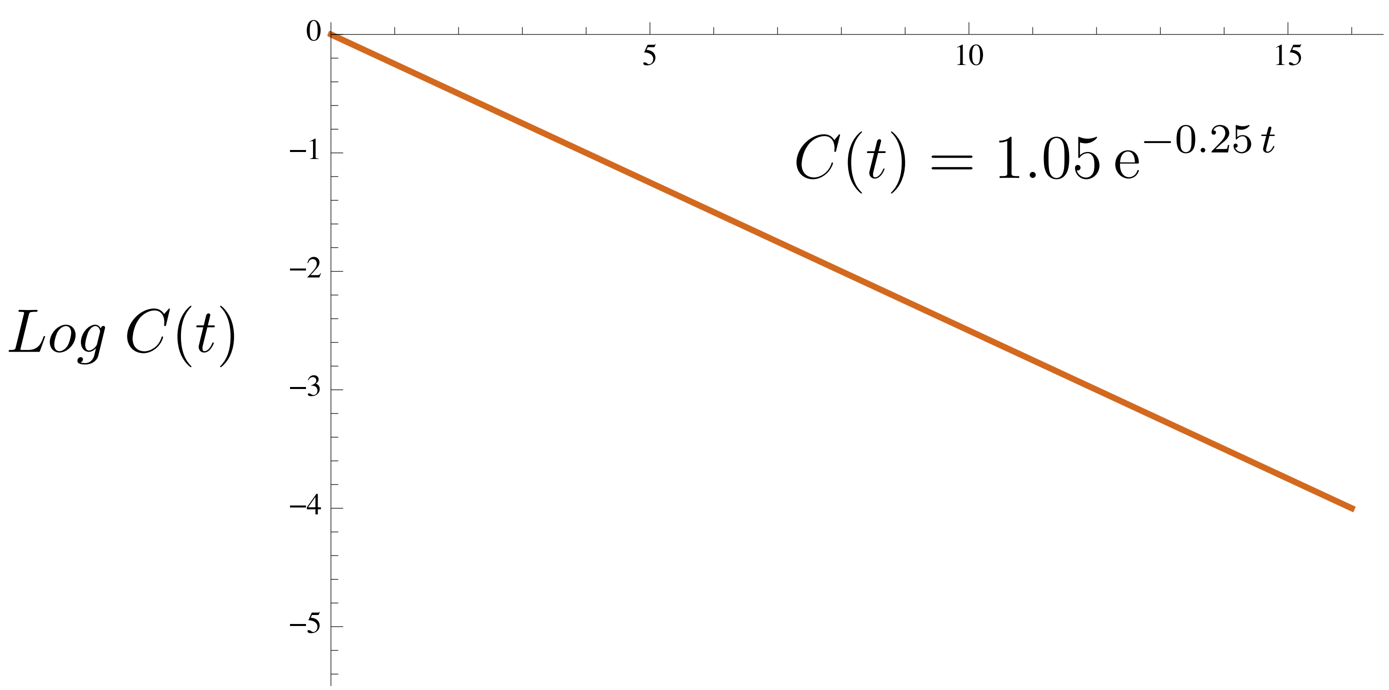
Lattice tools: correlation functions

$X_i(t)$  $\overline{X}_j(0)$

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\, \langle n|\overline{X}_j\rangle$$

X_i lattice operator

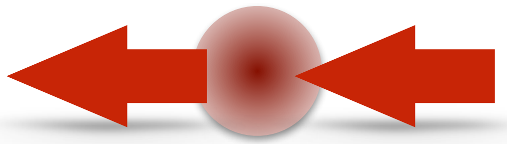
n “physical” eigenstate



slope=
energy of
ground state

Lattice tools: correlation functions

$X_i(t)$

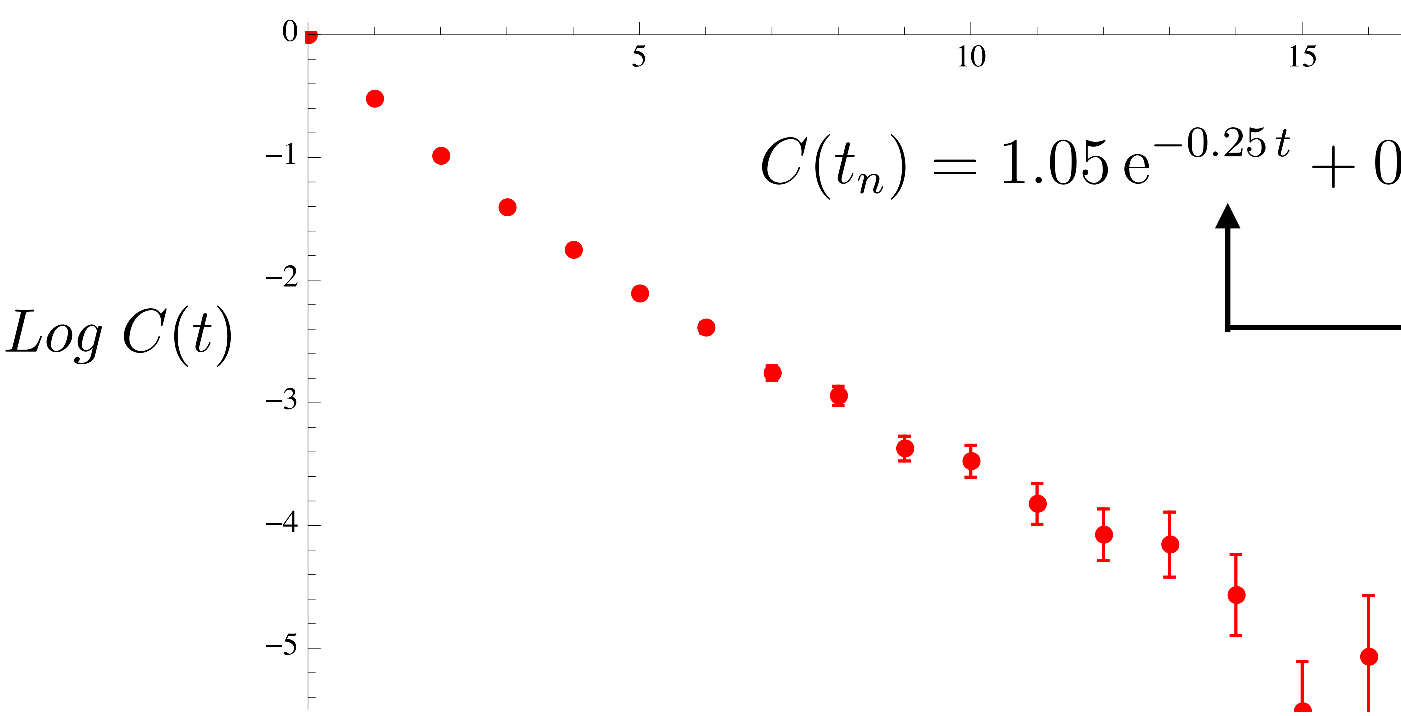


$\overline{X}_j(0)$

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\, \langle n|\overline{X}_j\rangle$$

X_i lattice operator

n “physical” eigenstate



$$C(t_n) = 1.05\,\mathrm{e}^{-0.25\,t} + 0.78\,\mathrm{e}^{-0.55\,t} + 0.54\,\mathrm{e}^{-0.85\,t} + \ldots + \text{noise}$$

energy levels

slope=
energy of
ground state

How to get the energy levels?

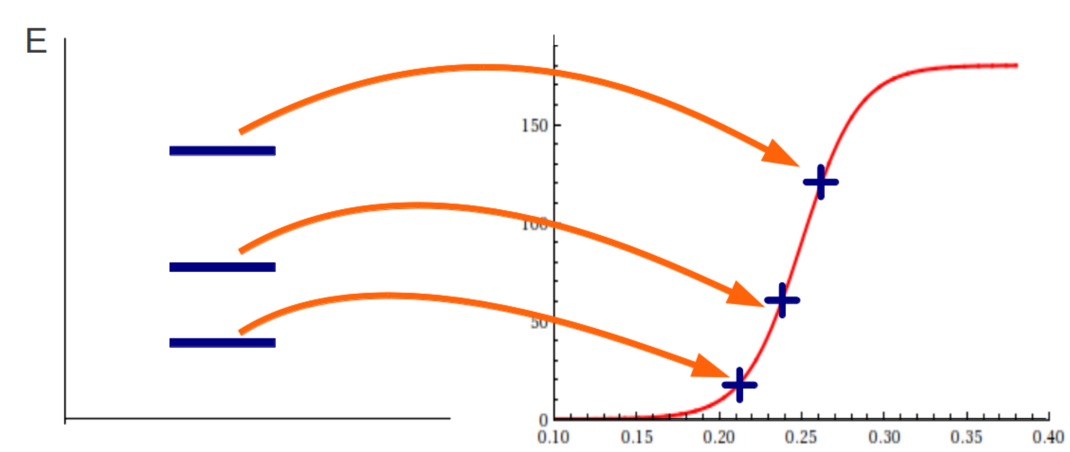
Lüscher, Wolff: NPB339(90)222
Michael, NPB259(85)58
See also Blossier et al., JHEP0904(09)094

- Compute all cross-correlations for several lattice operators with the same quantum numbers:

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle$$

- Solve the eigenvalue problem. The eigenvalues give the energy levels:

$$\lambda^{(n)}(t) \propto e^{-t\boxed{E_n}}\left(1+\mathcal{O}(e^{-t\Delta E_n})\right)$$



Results

Results

- First steps: Light quark elastic resonance region results

  ππ p-wave (ρ) and s-wave (σ)
  πK p-wave (K*) and s-wave (κ)
  $\rho\pi(a_1)$, $\omega\pi$ (b_1)…
  phase shifts, scattering lengths

- Coupled channel formalism (beyond the elastic region)

  (ππ, KK), (πK, ηK)

- Heavy-heavy and heavy-light quark mesons near thresholds

  ψ(3770) ,X(3872), Z(3900)
  D_s and DK, D*K
  B_s and BK, B*K

- Meson-baryon resonances

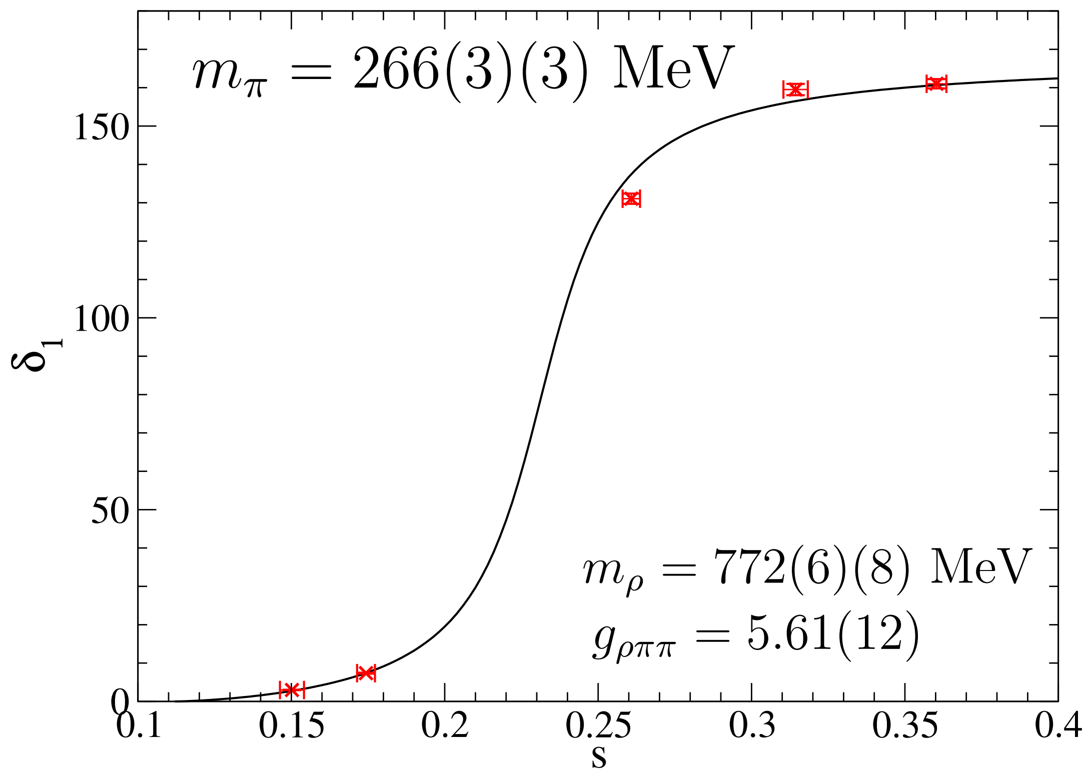
  Nπ

Recent review: Briceno, Dudek, and Young: *arXiv*:1706.06223

Mesons

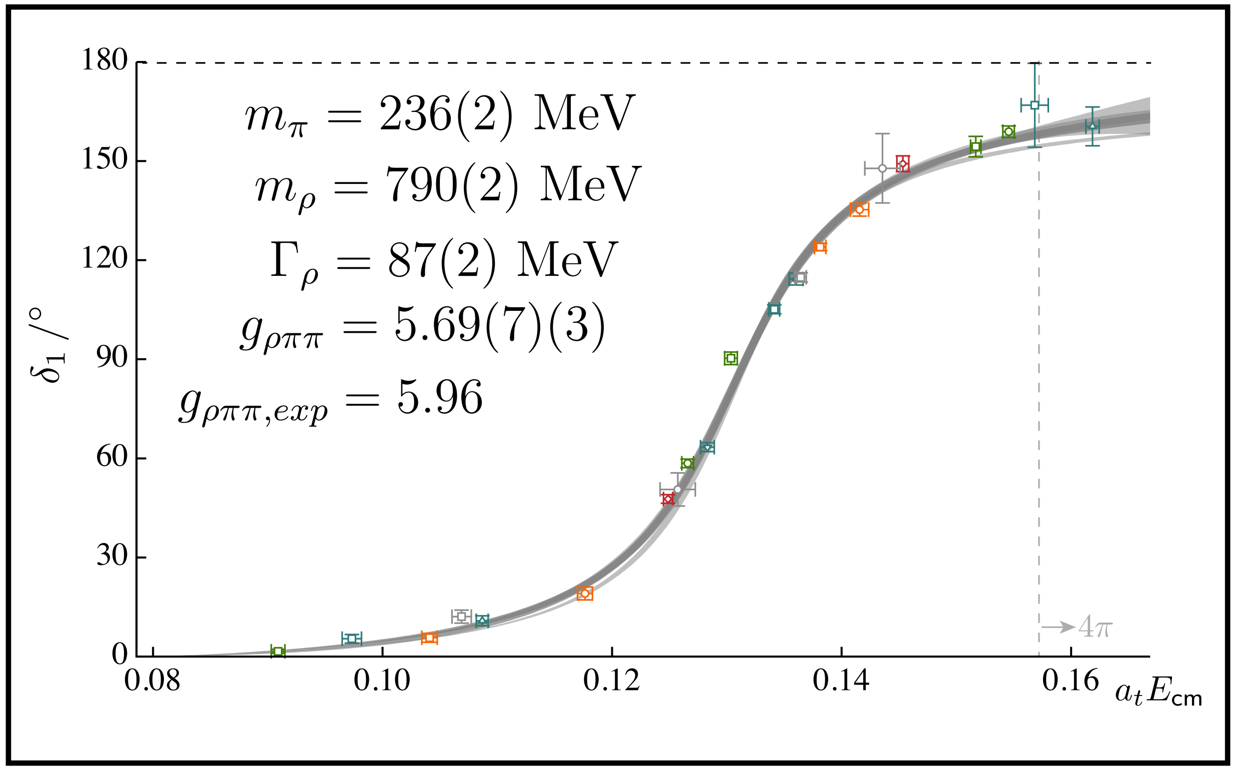
Example: ρ

2011: ππ ➛ ρ ➛ ππ



CBL, Mohler, Prelovsek, Vidmar;
PR D **84**, 054503 (2011)
Erratum PR D 89 (2014) 059903(E)

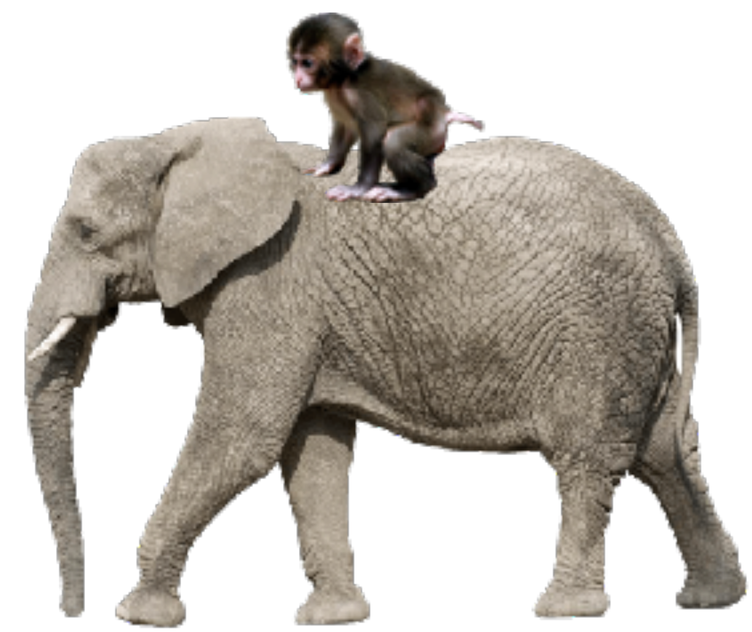
2015: (ππ,KK̲) ➛ ρ ➛ (ππ,KK̲)



D. J. Wilson et al., (HSC), PRD 92, 094502 (2015)

Heavy quark results (with meson-meson operators)

Heavy-Heavy $c\bar{c}$



Heavy-Light $c\bar{s}$, $b\bar{s}$

What is the effect of nearby thresholds?

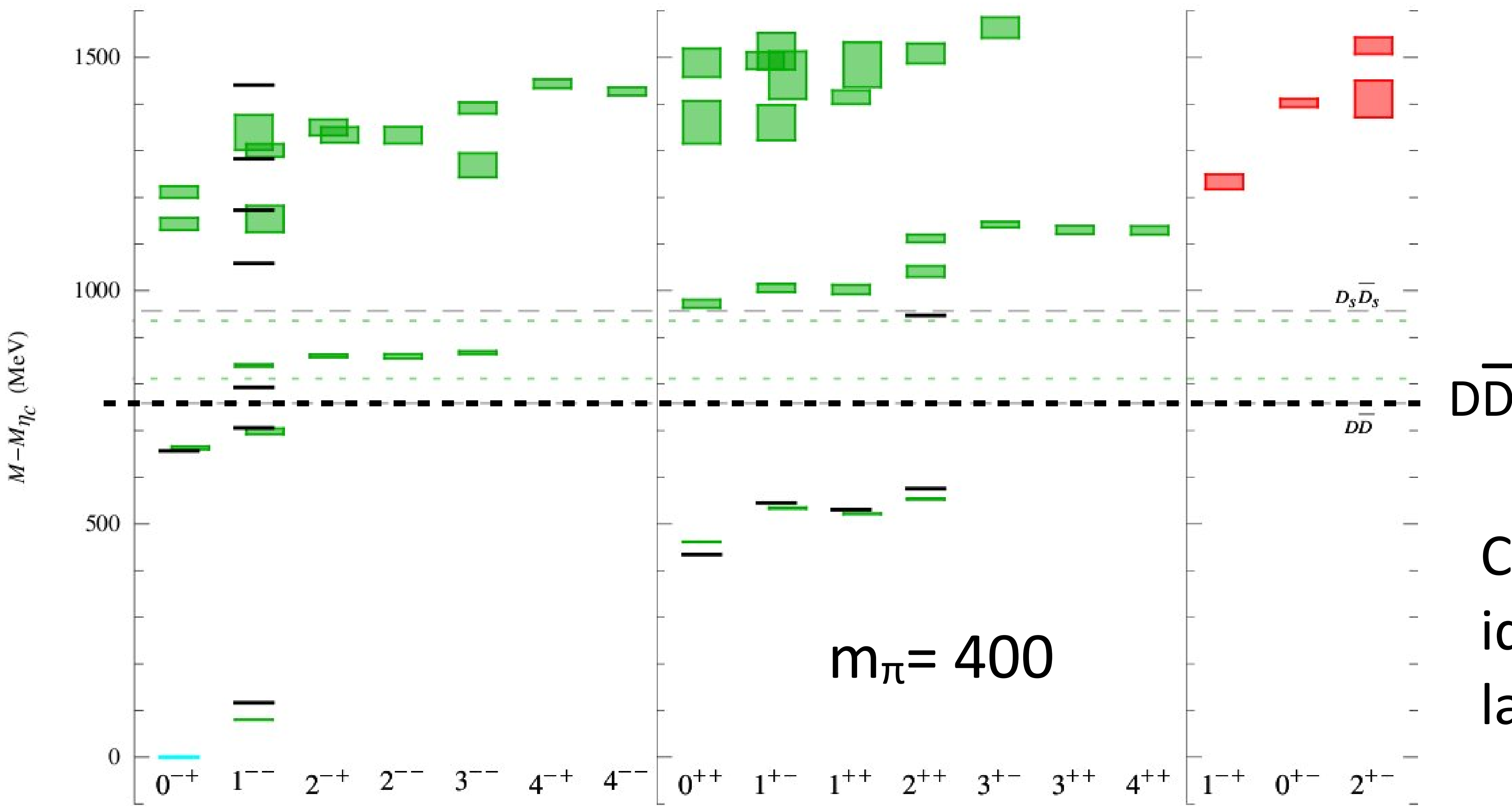
Examples: Charmonium
$\bar{D}D$ threshold and $\psi(3770)$
$\bar{D}K$ and D^*K in $D_{sn}^{(*)}$

Are there “new” states?

Examples: “level hunting”: X(3872), Z(3900), X(5568)
$B\bar{K}$ and $B^*\bar{K}$ in $B_{sn}^{(*)}$

Heavy quarks: Charmonium

L. Liu et al. (HSC), JHEP 1207, 126 (2012), [1204.5425]



Challenge: Spin identification (from lattice irreps)

See also: Mohler et al., Phys.Rev. D 87, 034501 (2013) [arXiv:1208.4059]
Galloway et al. (HPQCD) PoS (LATTICE2014) 092 [arXiv:1411.1318]
(extrapolation to physical point)

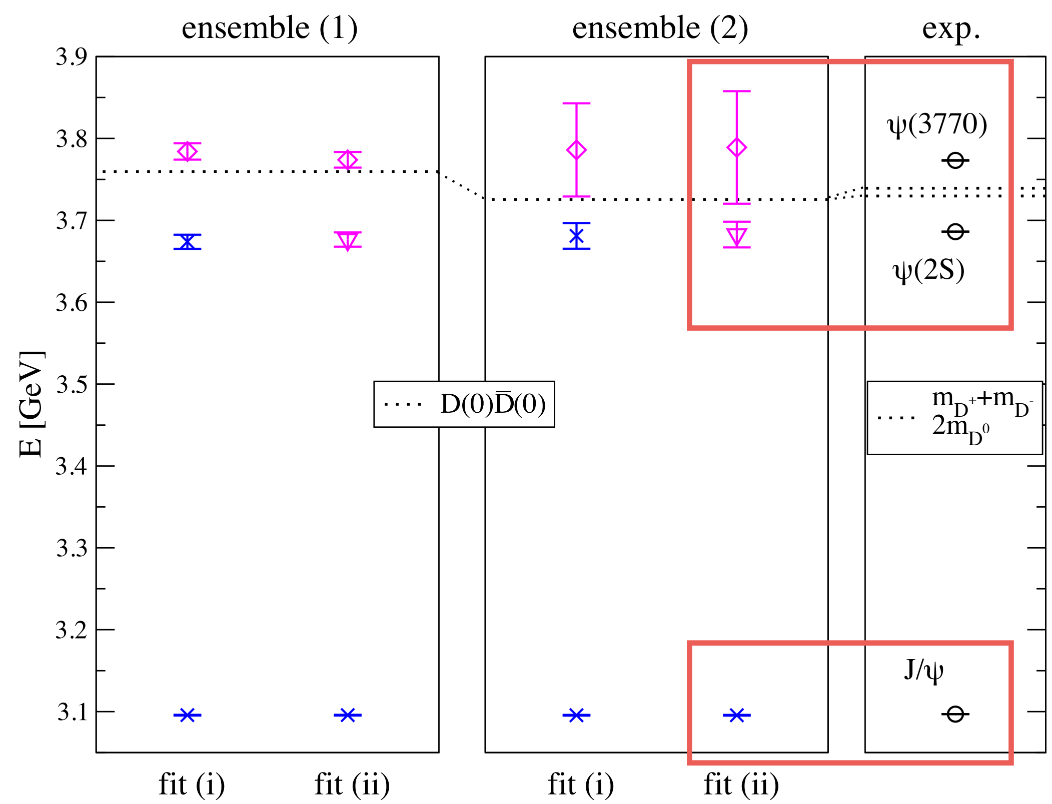
CBL et al., JHEP (2015) [arXiv:1503.05363]

Charmonium

ψ(3770): resonance close to $D\underline{D}$ threshold

Lattice study: $D\underline{D}$ scattering on two volumes and m_π=266 and 157 MeV

15 interpolators of $c\underline{c}$ type
2 operators of type $D\underline{D}$



[same paper:
$\eta_{c0}(2P)$ or X(3915): 0^{++}
controversial signal]

	ψ(3770), m_R	g(no unit)	ψ(2S), m_R
m_π=266 MeV	3774(6)(10)	9.7(1.4)	3676(6)(9)
m_π=157 MeV	3789(68)(10)	28(21)	3682(13)(9)
Exp.	3773.15(33)	18.7(1.4)	3686.11(1)

Charmonium: “Level hunting”

Search for $1^+(1^{+-})$ ($Z_c^+(3900)$ $c\bar{c}u\bar{d}$)

Prelovsek et al. , Phys. Rev. D 91 (2015) 014504;
arXiv:1405.7623v2

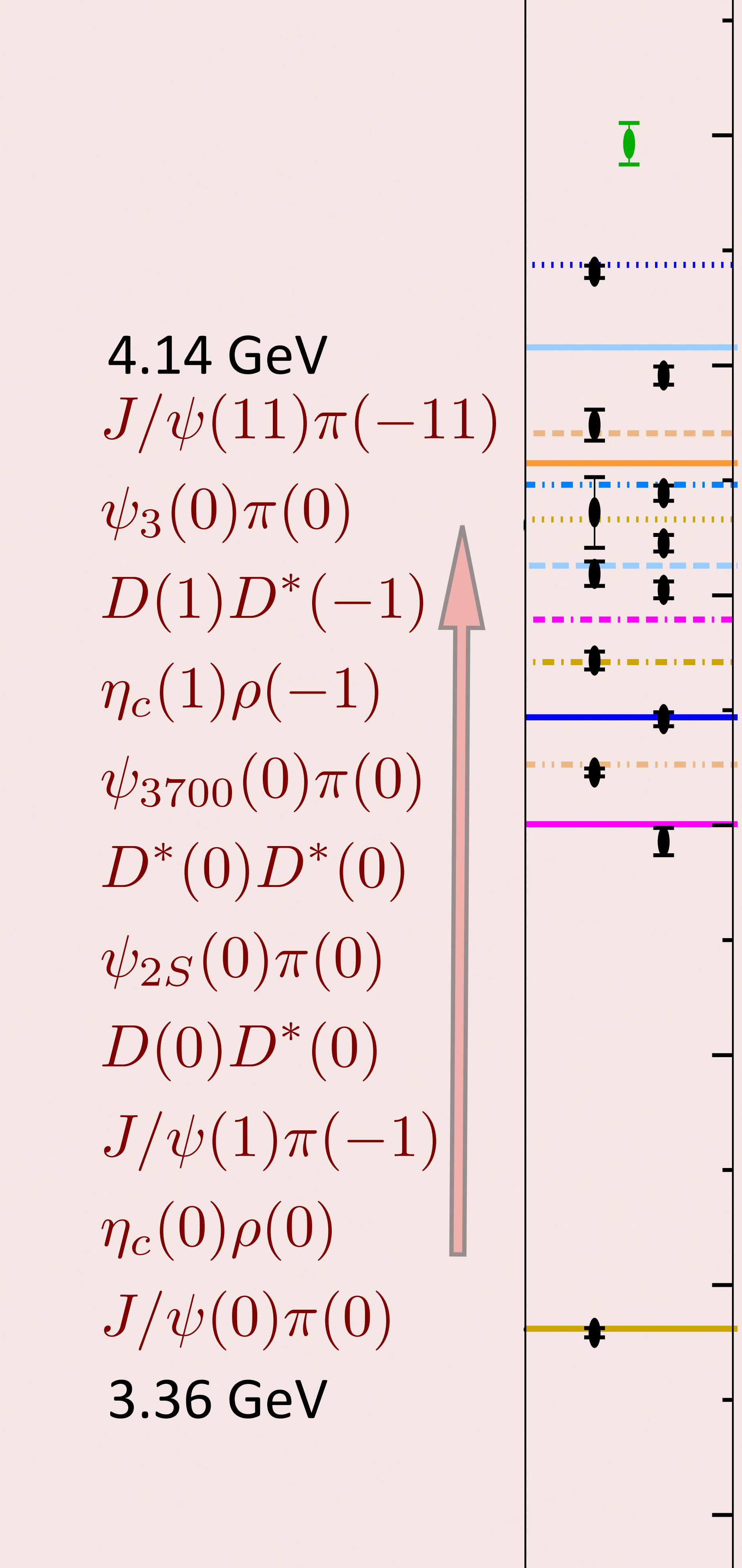
18 interpolators of meson-meson type
covering all imaginable states up to 4.1 GeV
(small volume bonus)
4 tetraquark operators

$\boxed{\rightarrow \text{No signal for } Z_c^+(3900)}$

Lee et al. (FNAL/MILC), [arXiv:1411.1389]

Chen et al.,(CLQCD) Phys. Rev. D 89, 094506 (2014)

DD* weakly repulsive!



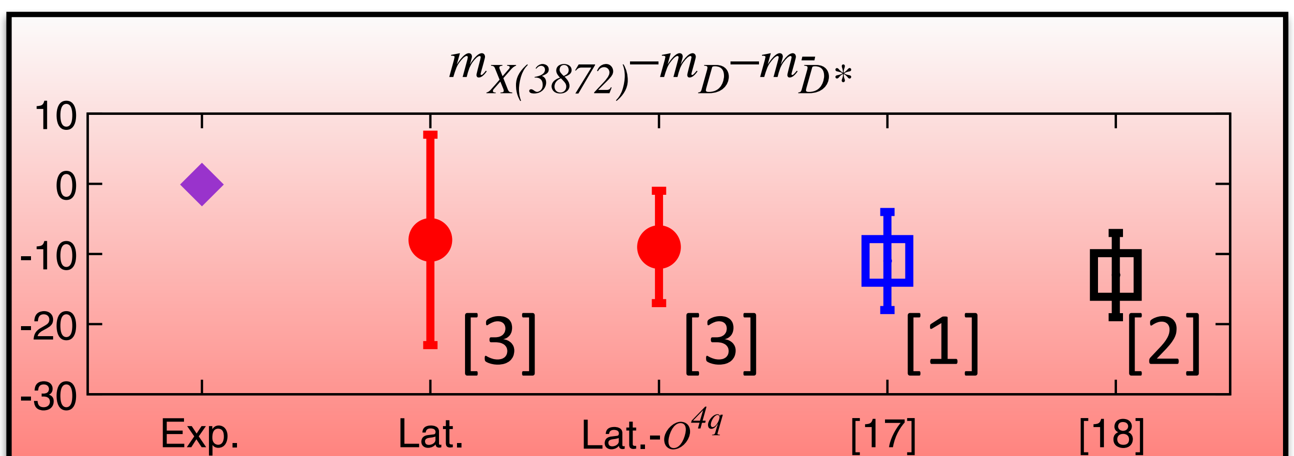
X(3872)

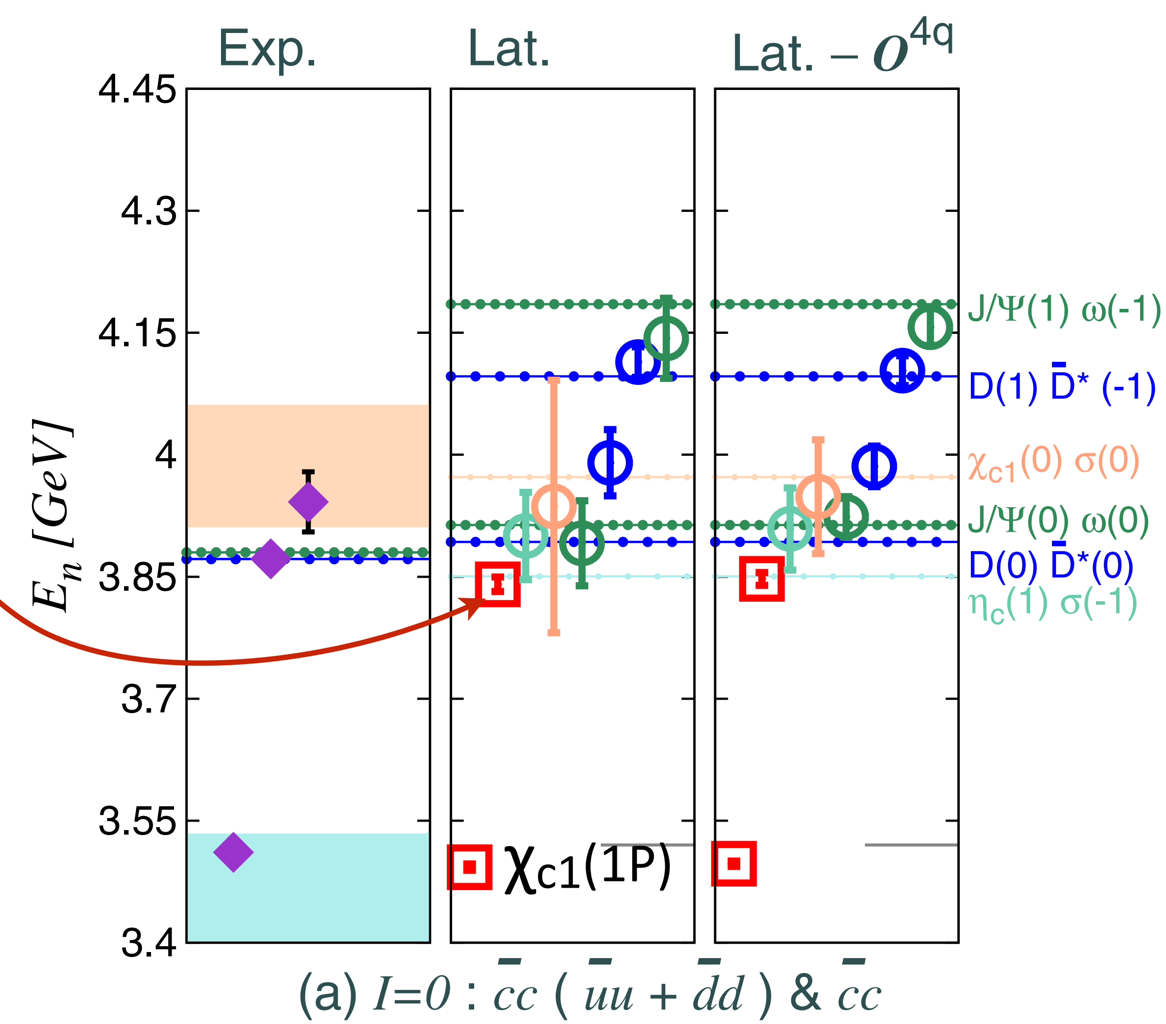
X(3872) $0^+(1^{++})$

[1] Prelovsek/Leskovec, Phys. Rev. Lett. 111, 192001 (2013)
. [2] Lee et al. (FNAL/MILC), [arXiv:1411.1389]
[3] Padmanath et al,Phys. Rev. D 92 (2015) 034501 [arXiv:1503.03257]

at m_π=266 MeV
22 $c\bar{c}$ and $c\bar{c}u\bar{u}$,$c\bar{c}u\bar{d}$,..interpolators
for I=0 and 1
($D\bar{D}^*$, $J/\psi\ \rho$, $J/\psi\ \omega$, $\eta_c\ \sigma$, $\chi_{c0}\ \pi$, $\chi_{c2}\ \pi$, 4q)

we find X(3872) closely below $D\bar{D}^*$
(with strong $c\bar{c}$ component)





(large scatt.length 1.1 fm)

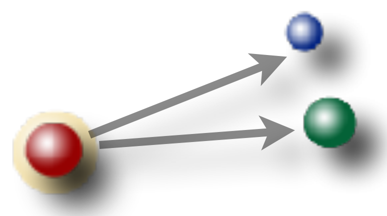
Heavy quark sector: D_s (0^+, 1^+, 2^+)

$c\bar{s} \leftrightarrow c\bar{u}+u\bar{s}$

$D_s \leftrightarrow D+K$

Quark model and LQCD
in single hadron approximation:
unclear picture,
threshold important?

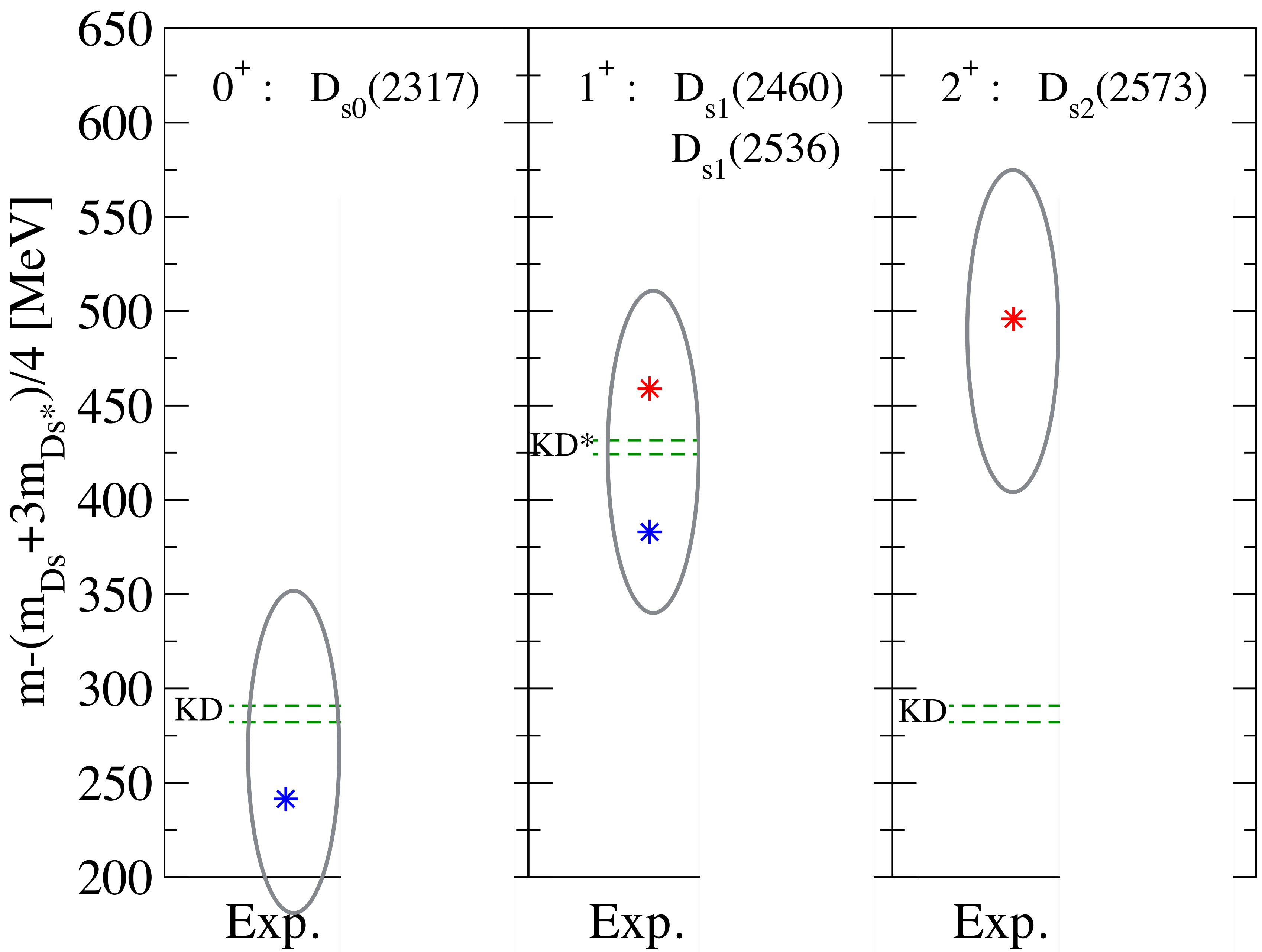
Mohler et al.,PRL. 111, 222001; (2013)
CBL et al., Phys. Rev. D 90, 034510 (2014)



Include meson meson interpolators!

(PACS-CS lattices.
m_π=157 MeV)

See also Martinez Torres et al., JHEP 1505 (2015) 153

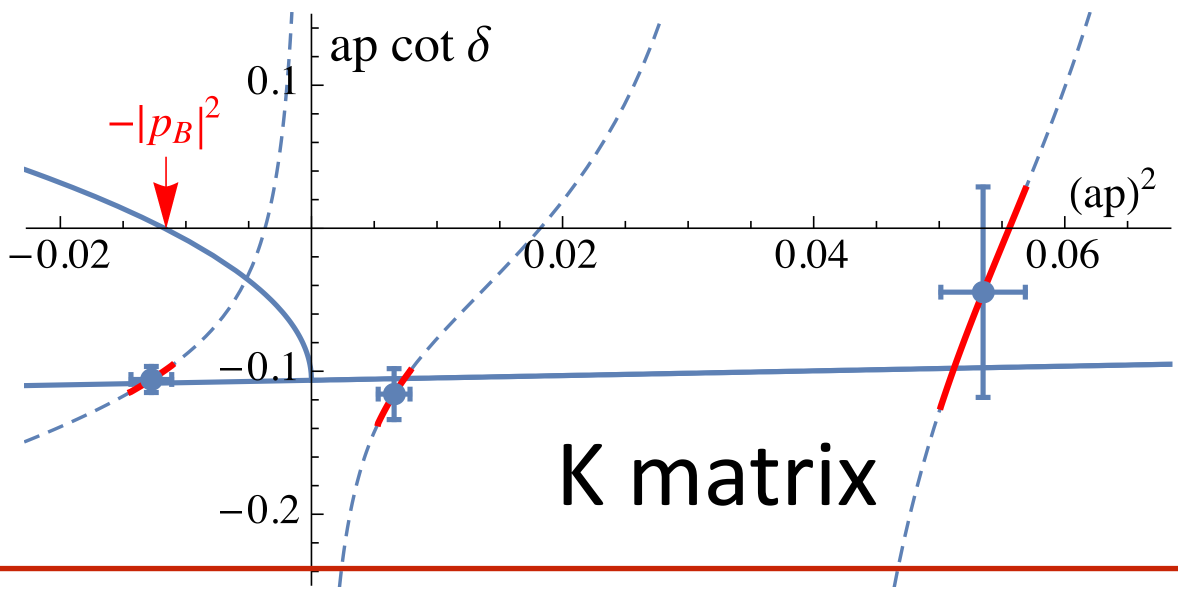


Heavy quark sector: B_s (0^+, 1^+, 2^+)

CBL et al., Phys. Lett. B 750 (2015) 17 [arXiv: 1501.01646]

BK, B*K scattering (PACS-CS lattices, m_π=157 MeV)

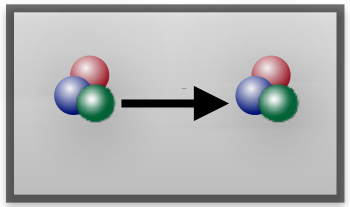
0^+: Bound state B_{s0} with $m(B_{s0})$ = 5.711(13)(19) GeV (prediction)



1^+: Bound state B_{s1} with $m(B_{s1})$= 5.750(17)(19) GeV (prediction)

Close to threshold weakly coupled state B_{s1}' at m=5.831(9)(6) GeV (Exp: B_{s1}(5830) at 5.8287(4) GeV)

Baryons

Baryons in  approximation

Zillion LQCD papers,
mainly ground states

Namekawa et al. (PACS-CS), PRD 87, 094512 (2013)

Alexandrou et al.(ETMC), PRD 90, 074501 (2014).

Brown et al. PRD 90, 094507 (2014).

Rubio et al. (RQCD), PRD 92, 034504 (2015)

Hadron Spectrum Collaboration et al.:
Excited levels

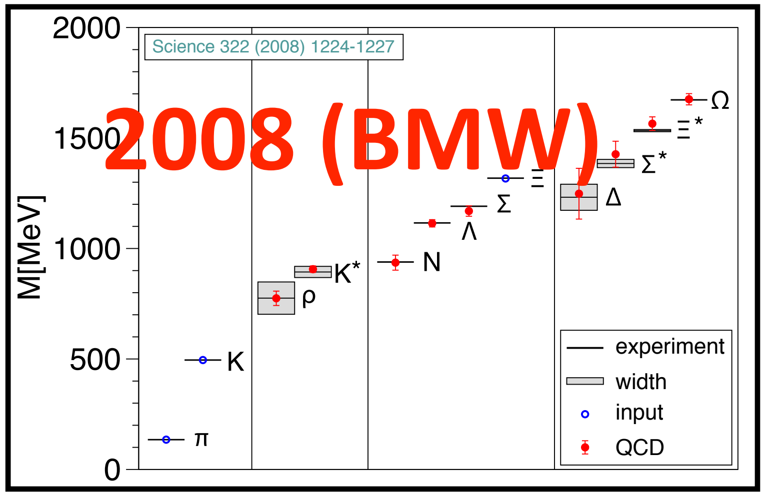
ccx Baryons

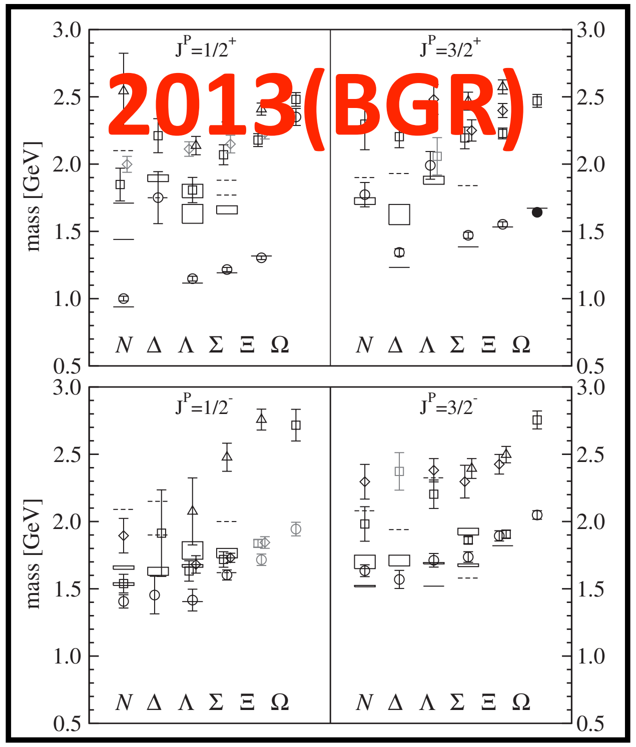
Padmanath et al., Phys.Rev. D91 (2015) 094502

Spin identification up to 7/2

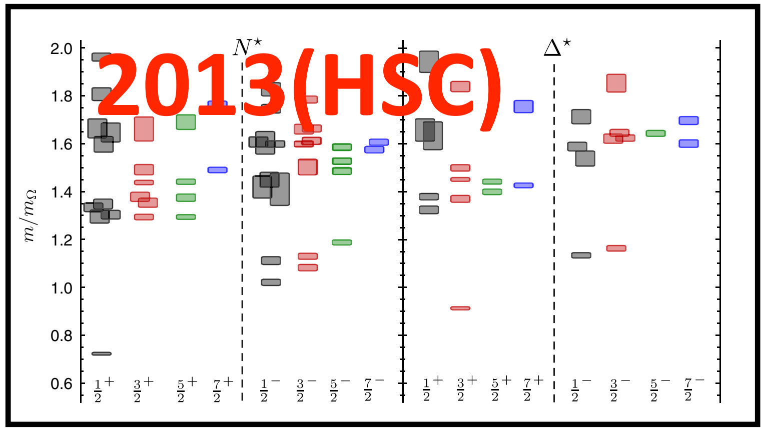
ccc Baryons

Padmanath et al., Phys.Rev. D90 (2014) 074504





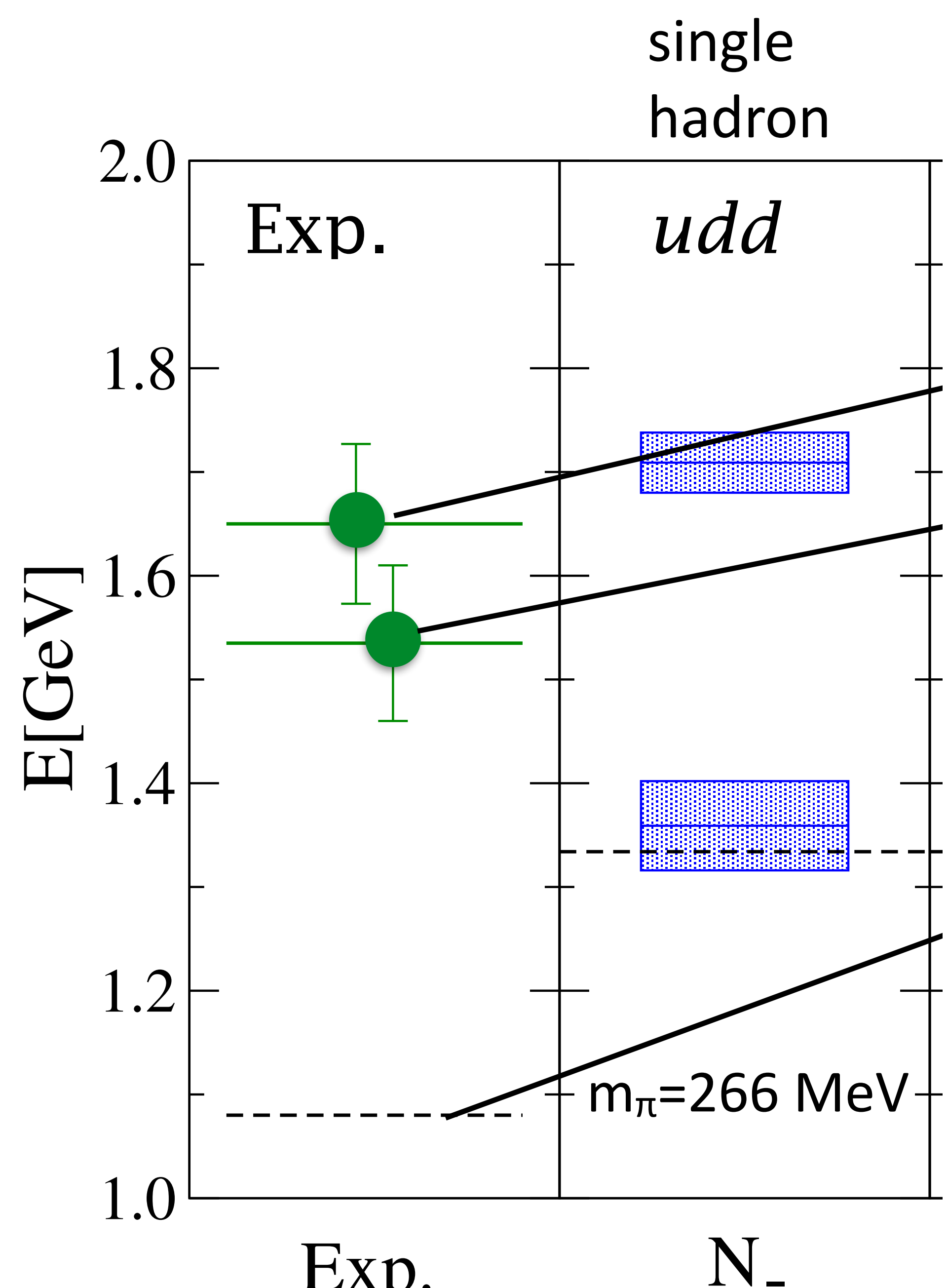
Engel et al., PRD 87, 074504 (2013)



πN (1/2⁻): Effect of open 2-hadron channel?

N*(1535),N*(1650)

Nπ negative parity

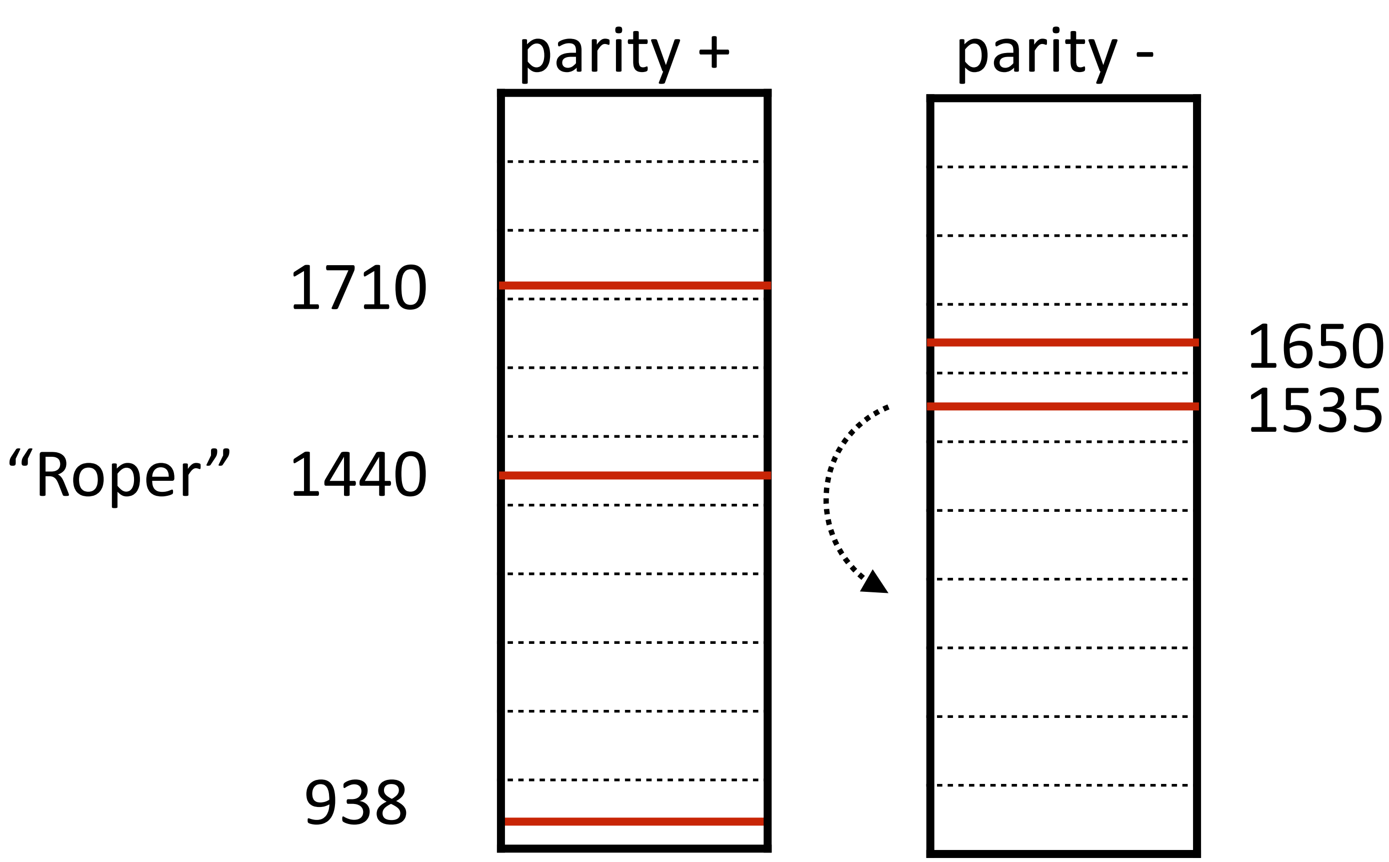


CBL&Verduci, PRD87 (2013) 054502 [arXiv:1212.5055]

πN ($1/2^+$): And what about the Roper?

$N\pi$ in p-wave

baryon	*width*	*decay*	*%*
p(938)	-	-	-
N(1440)	250-450	$N\pi$	75–52
		$N\pi\pi$	30-40
		through $\Delta\pi$	20-30
		through $N(\pi\pi)_s$	10-20
N(1710)	50-250	$N\pi$,$N\omega$,$N\eta$,ΛK, $N\pi\pi(\Delta\pi,n\rho,..)$..	



Parity puzzle:
why not + - + - ?

Comparison with HEFT model

Operator basis ($1/2^+$):

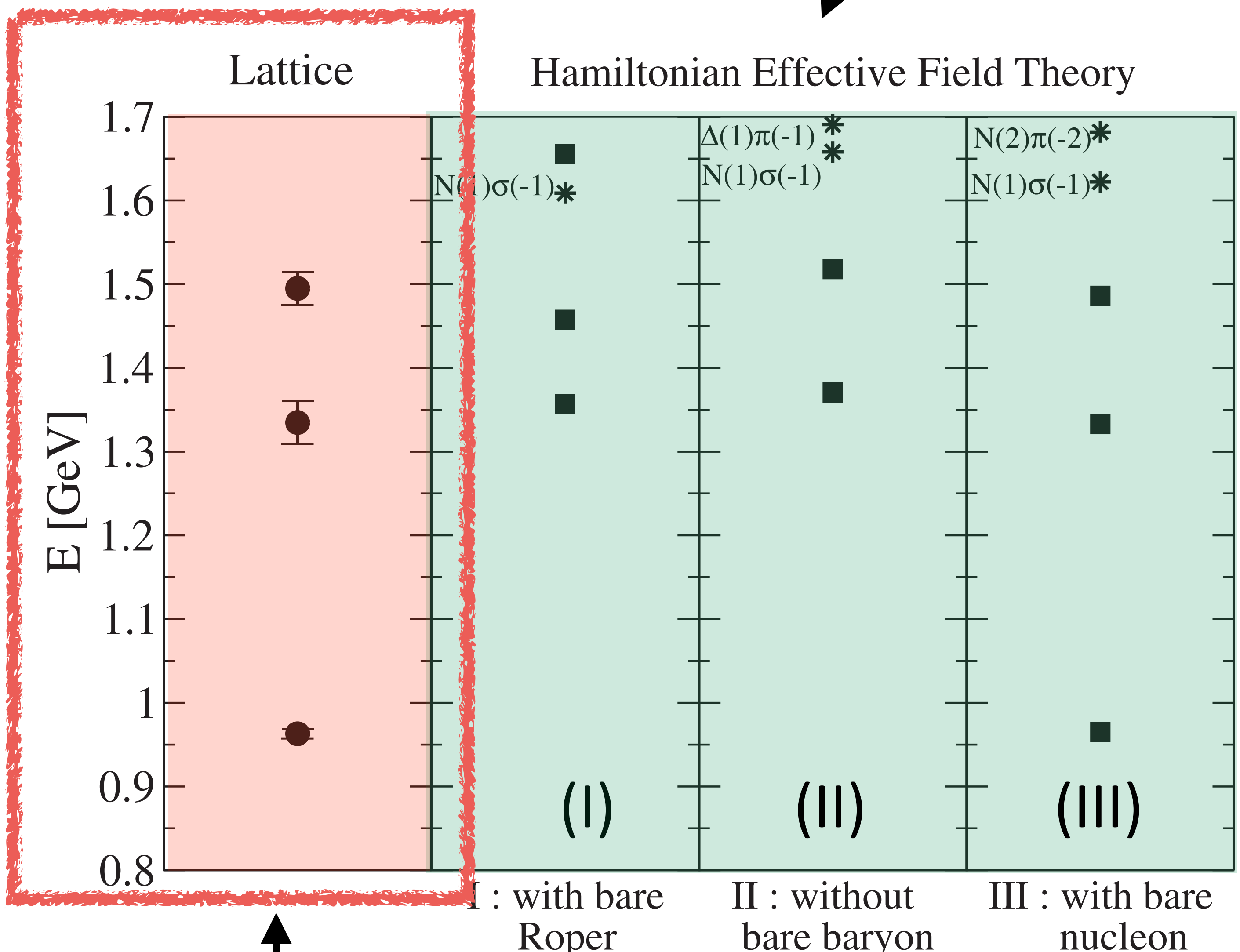
N, Nπ (p wave), Nσ (s wave)

m_π=157 MeV

Z.-W. Liu et al.(CSSM), PRD 95 034034 (2017); arXiv: 1607.04536.

Hamiltonian EFT (parameters fitted to exp. phase shift)

Coupled channels study



best agreement

CBL et al. PRD 95 (2017) 014510 ; arXiv:1610.01422

(I) Nπ,Δπ,Nσ+Roper

(II) Nπ,Δπ,Nσ

(III) Nπ,Δπ,Nσ+N

(Δ and σ assumed stable)

Possible reasons

Why LQCD studies did not (yet) see the Roper?

- Coupled channel phenomenon; some channels missing ($N\pi\pi$, $\Delta\pi$, $N\eta$)? ***
- Spatial size of operators and lattice? **
- Pentaquark operator needed? *
- Chiral symmetry? *

Summary

- Simple hadron propagators give excited level - however, their positions are shifted: a decay channel study is needed.
- The Lüscher formalism (+extensions) becomes standard
- Light quark hadrons: threshold parameters and resonance phase shifts become available for 2→2 processes, approaching physical masses
- 2→3 in progress
- Heavy quarks: work nicely in simple cases but no multi-channel study yet; solves quark model puzzles.
- Baryons: some results for Nπ, but not always conclusive; in the near future I see no hope for trustworthy results > 1.7-2.0 GeV
- Model calculations help understand the LQCD results (and vice versa)


The End