The linear covariant gauge beyond perturbation theory

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Talk at ICNFP2017, 19/08/2017

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Overview

Gribov copies and how to deal with them in the path integral

(Softly broken) standard BRST

BRST symmetric formulation of Gribov-Zwanziger theory in Landau gauge

Comparison with gauge fixed lattice theory

Generalization to the linear covariant gauge

Application of BRST symmetry: Nielsen identities in the Gribov-Zwanziger theory

LKF transformations

QCD

Our goal?

- We wish to start from “elementary” knowledge: the basic ingredients of YM: gluon & ghost (& quark) propagators and the interactions (vertices) between them
- We shall work in a specific gauge, and try to describe/understand nonperturbative effects
- These basic ingredients then enter in studies of spectrum, phase transition, etc.

QCD

Our goal?

- QCD is a gauge theory, so physics should be gauge invariant (gauge independent).
- QCD is also strongly coupled $\rightarrow$ nonperturbative machinery indispensable
- Continuum methods require partial modelling of nonperturbative interactions $\rightarrow$ how well is gauge covariance under control?
- This is our main motivation, since most continuum methods rely on Landau (or to lesser extent Coulomb) gauge. Nowadays, lattice QCD is also starting to offer insights into other gauges.

Faddeev-Popov quantization of QCD

Classical level

- Classical $SU(N)$ Yang-Mills action in $d=4$ Euclidean space time

$$S_{YM} = \frac{1}{4}\int \mathrm{d}^4 x F^a_{\mu\nu} F^a_{\mu\nu}$$

- S_{YM} possesses enormous local invariance,

$$A_\mu \quad \rightarrow \quad A^S_\mu \;=\; S^+\partial_\mu S + S^+ A_\mu S \qquad S \in SU(N)$$

or in infinitesimal form

$$\begin{aligned} A^a_\mu &\rightarrow A^a_\mu + D^{ab}_\mu \omega^b \\ D^{ab}_\mu &\equiv \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu \end{aligned}$$

- Freedom to choose gauge

Faddeev-Popov quantization of QCD

- Faddeev and Popov implemented a linear covariant gauge choice as follows

$$
\begin{aligned}
Z_{FP} &= \int [dA]\delta(\partial A - B) \det \mathcal{M}^{ab} e^{-S_{YM}} \\
\mathcal{M}^{ab} &= -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c) \quad = \text{Faddeev-Popov operator}
\end{aligned}
$$

+ Gaussian sampling over B via $\int [dB] e^{-\frac{B^2}{2\alpha}}$.

- The FP trick is based on the functional version of

$$
\int dx \delta[f(x) - y] g(x) = \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x)=y}
$$

- Notice that this *assumes* that $f(x) = y$ only has a single solution! Otherwise one needs

$$
\int dx \delta[f(x) - y] g(x) = \sum_i \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x_i)=y}
$$

Faddeev-Popov quantization of QCD

The Faddeev-Popov action

- Faddeev and Popov implemented a gauge choice as follows

$$Z_{FP} = \int [dA] \underbrace{\delta(\partial A - B) \det \mathcal{M}^{ab}}_{\to \text{ unity}} e^{-S_{YM}}$$

- $\delta(\partial A) \Rightarrow \partial A = 0 \equiv$ Landau gauge if $\alpha = 0$
- Faddeev-Popov determinant $\det \mathcal{M}^{ab}$ is corresponding Jacobian
- This form is not suitable to work/compute with (we want Feynman rules from local action)

Faddeev-Popov quantization of QCD

The Faddeev-Popov action in the Landau gauge

- We shall work with linear covariant gauge $\partial_\mu A_\mu^a = \alpha b^a$.
- Very popular gauge, as it has many nice (quantum) properties.
- The eventual gauge fixed action reads

$$S_{YM} + S_{gf} \quad = \quad \int \mathrm{d}^4 x \left(\frac{1}{4} F_{\mu\nu}^2 + b^a \partial_\mu A_\mu^a - \frac{\alpha}{2} b^a b^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right)$$

The BRST symmetry

an important (crucial) symmetry

- Quantum action enjoys nilpotent BRST symmetry, $s(S_{YM}+S_{gf}) = 0$

$$sA_\mu^a = -D_\mu^{ab}c^b\,, \qquad sc^a = \frac{g}{2}f^{abc}c^b c^c$$
$$s\bar{c}^a = b^a\,, \qquad sb^a = 0\,, \qquad s^2 = 0$$

- Quantum replacement for classical gauge invariance
- Used for proofs of
  - perturbative renormalizability via *Slavnov-Taylor identity*
  - perturbative unitarity: ghost, antighost, longitudinal and timelike gluon polarizations cancel, only 2 transverse gluon degrees of freedom survive!
  - BRST symmetry is an important concept

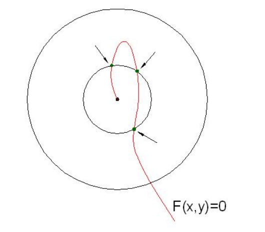
Potential flaw in FP quantization

The Gribov problem

- Take A_μ in linear covariant gauge $\Leftrightarrow \partial_\mu A_\mu = \alpha b$
- Consider (infinitesimal) gauge transform: $A'_\mu = A_\mu + D_\mu \omega$
- $\partial_\mu A'_\mu = \alpha b$ if $\partial_\mu D_\mu \omega = 0$
  GAUGE COPY if FP operator has (normalizable) zero modes!
- Was investigated (and drawn attention to) by Gribov in late seventies for Landau ($\alpha = 0$) and Coulomb gauge ($\partial_i A_i = 0$).

Potential flaw in FP quantization

The Gribov problem



- There is still some overcounting when using FP action (which is mathematically seen “wrong”)
- Solution: use the more correct functional version of δ-function where the argument has multiple zeros.
- Unfortunately: cannot be put in useful partition function

Treating the copy problem

From Faddeev-Popov to Gribov-Zwanziger: first Landau gauge $\partial A = 0$

- A class of copies was related to zero modes of Faddeev-Popov operator $M = -\partial D$
- Let us restrict path integral to region Ω where $\partial A = 0$ and $M > 0$. Only sensible if $\partial A = 0$ as only then $M = -\partial D$ is Hermitian!
- Ω corresponds to local minima of the functional $\int d^4 x A_\mu^2$!
- $\Rightarrow$ This is already an improvement of Faddeev-Popov!
- Compare with lattice where one seeks for (in theory) global minima of $\int d^4 x A_\mu^2$
- How to implement restriction to Ω in continuum???

The Gribov-Zwanziger action

Gribov-Zwanziger

- Gribov and later on Zwanziger worked out this problem and proved many properties of region Ω (with Dell'Antonio)
- Example: every gauge orbit passes through Ω, it is convex and bounded in every direction.

Gribov-Zwanziger

- Restricts the integration to the Gribov region to all orders (work of Zwanziger)
- The Gribov-Zwanziger action is given by

$$S_{GZ} = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4 x\, h(x)$$

with the horizon function

$$h(x) = g^2 f^{abc} A^b_\mu \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A^e_\mu$$

- $$\text{horizon condition (= gap equation)}\ \langle h(x) \rangle = d(N^2-1)$$
- For $\gamma = 0$, everything reduces to Faddeev-Popov.

Gribov-Zwanziger

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM} + S_{gf} + S_h$$

with now

$$S_h = \int d^4x \left(\overline{\varphi}_\mu^{ac}\partial_\nu\left(\partial_\nu\varphi_\mu^{ac} + gf^{abm}A_\nu^b\varphi_\mu^{mc}\right) - \overline{\omega}_\mu^{ac}\partial_\nu\left(\partial_\nu\omega_\mu^{ac} + gf^{abm}A_\nu^b\omega_\mu^{mc}\right) - g\left(\partial_\nu\overline{\omega}_\mu^{ac}\right)f^{abm}\left(D_\nu c\right)^b\varphi_\mu^{mc} - \gamma^2 g\left(f^{abc}A_\mu^a\varphi_\mu^{bc} + f^{abc}A_\mu^a\overline{\varphi}_\mu^{bc} + \frac{4}{g}\left(N^2-1\right)\gamma^2\right)\right)$$

- horizon condition (= gap equation)

$$\frac{\partial\Gamma}{\partial\gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle gf^{abc}A_\mu^a(\varphi+\overline{\varphi})_\mu^{bc}\right\rangle = 2d(N^2-1)\gamma^2}_{d=2\,\text{condensate}}$$

- $\gamma \propto \Lambda_{QCD}$: source of dimensional transmutation.

Gribov-Zwanziger

Gribov-Zwanziger quantization

The GZ formalism is a geometrically inspired path-integral construction (cf. boundary condition to stay within the Gribov region) with good quantum properties that improves upon the standard FP quantization.

Gribov-Zwanziger quantization

Nice property: closely related to lattice formulation, as in both cases minimization of $\int A^2$ along the gauge orbit is used to define the (a) nonperturbative Landau gauge.

There is the technical issue of having multiple local minima, unknown how to be dealt with at the quantitative level.

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The Gribov-Zwanziger action

What about the BRST symmetry?

- The (naturally extended) BRST symmetry

$$s\overline{\omega}_\mu^{ab} = \overline{\varphi}_\mu^{ab} , \qquad s\overline{\varphi}_\mu^{ab} = 0 , \qquad s\varphi_\mu^{ab} = \omega_\mu^{ab} , \qquad s\omega_\mu^{ab} = 0 ,$$

  is softly broken

$$sS_{GZ} = g\gamma^2 \int d^4x \left(f^{abc} A_\mu^a \omega_\mu^{bc} - (D_\mu^{am} c^m)(\overline{\varphi}_\mu^{bc} + \varphi_\mu^{bc}) \right)$$

- Apparently: treating Gribov copy leads to soft breaking of BRST.
- What about gauge parameter independence of correlation functions of BRST invariant operators if we were to generalize GZ to other gauges?

  Let us first find a BRST!

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Preliminaries

- Consider A^2-functional

$$f_A[u] \equiv \mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu = \mathrm{Tr}\int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u\right)^2$$

and set $v = he^{ig\omega}$.

- Working up to 2nd order to identify minima:

$$f_A[v] = f_A[h] + 2\mathrm{Tr}\int d^4x\, \left(\omega\partial_\mu A^h_\mu\right) - \mathrm{Tr}\int d^4x\, \omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3)$$

$\Rightarrow$ $\partial_\mu A^h_\mu = 0 \quad \& \quad -\partial_\mu D_\mu[A^h] > 0$

We recognize the Landau gauge and defining condition of the Gribov region (positive FP operator).

Preliminaries

- The “minimum configuration” can be solved for

$$A_\mu^h = A_\mu - \frac{1}{\partial^2}\partial_\mu \partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu \frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3)$$

  It is transverse and gauge invariant order by order.

- Observation: if $\partial A = 0$, $A = A^h$. More precisely

$$A = A^h + \text{non-local power series in } (A, \partial A)$$

Rewriting GZ action

- $$A = A^h + \text{non-local power series in } (A, \partial A)$$
- Consider GZ action with non-local horizon action
$H(A) = g^2 \int d^4x d^4y\ f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)$

$$\begin{aligned} S_{GZ} &= S_{YM} + \int d^4x \,(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A) \\ &= S_{YM} + \int d^4x \,(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\ &= S_{YM} + \int d^4x \left(b^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) \end{aligned}$$

with a new field b^h

$$b^h = b - \gamma^4 R(A)$$

Rewriting GZ action

- Introduce auxiliary fields to obtain

$$S_{GZ} = S_{YM} + \int d^4x \left(b^h \partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \int d^4x \, \left(\bar{\phi}\mathcal{M}(A^h)\phi - \bar{\omega}\mathcal{M}(A^h)\omega + \gamma^2 A^h(\bar{\phi}+\phi)\right)$$

and rename $b^h \to b$ again.

- This new (equivalent) GZ action in the Landau gauge enjoys a nilpotent BRST symmetry

$$sA^a_\mu = -D^{ab}_\mu c^b \, , \; sc^a = \frac{g}{2} f^{abc} c^b c^c \, , \; s\bar{c}^a = b^a \, , \; sb^a = 0 \, ,$$
$$s\phi^{ab}_\mu = s\omega^{ab}_\mu = s\bar{\omega}^{ab}_\mu = s\bar{\phi}^{ab}_\mu = 0$$

thanks to $sA^h = 0$.

Rewriting GZ action

- Very nonlocal because of A^h $\rightarrow$ hard to discuss renormalizability etc.
- Locality of A^h_μ via introduction of Stückelberg field

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \quad h = e^{ig\xi^a t^a}$$

- Addition of $\int d^4x \tau \partial A^h$ to action gives rise to equivalent action as before upon solving the ξ-EOM
- $sA^h = 0$ under

$$h \rightarrow u^\dagger h\,, \quad h^\dagger \rightarrow h^\dagger u\,, \quad A_\mu \rightarrow u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u$$

or

$$\begin{aligned} sh^{ij} &= -igc^a (T^a)^{ik} h^{kj} \Rightarrow \\ s\xi^a &= -c^a + \frac{g}{2} f^{abc} c^b \xi^c - \frac{g^2}{12} f^{amr} f^{mpq} c^p \xi^q \xi^r + O(g^3) \end{aligned}$$

Rewriting GZ action

- Can be combined with algebraic renormalization formalism, following Dragon et al, NPB Proc.Suppl.56B (1997). A fully renormalizable framework, see Capri et al, PRD94 (2016) & arXiv:1708.01543.
- Important comment about renormalization: Addition of τA^h saves the day.
  - Standard Stückelberg propagator (via addition of gauge invariant mass term $m^2 A^h A^h$):
  $$\langle \xi\xi \rangle \sim \frac{1}{m^2 p^2}$$
  $\rightarrow$ leads to power counting non-renormalizibility!
  - Here (even for $m \neq 0$):
  $$\langle \xi\xi \rangle \sim \frac{1}{p^4}$$
- Final point of interest

$$\langle A \ldots \overline{c} \ldots c \rangle_{\text{new GZ}} \equiv \langle A \ldots \overline{c} \ldots c \rangle_{\text{old GZ}}$$

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The Refined Gribov-Zwanziger action in linear covariant gauge: extra dynamical effects

- We include extra dynamical effects due to nonperturbative gauge invariant $d = 2$ condensates $\to$ lower vacuum energy Dudal et al, PRD78 (2008), PRD84 (2011) & work in progress
- Ghost propagator $G(p^2) \sim \frac{1}{p^2}$ for $p^2 \sim 0$, $\quad G(p^2) \neq \frac{1}{p^4}$.
- Gluon propagator

$$D(p^2) \propto \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4}, \qquad D(0) \neq 0\,.$$

  m^2 and M^2 are mass scales corresponding to condensates, in particular

$$m^2 \sim \left\langle A^h A^h \right\rangle, \qquad M^2 \sim \langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle$$

- Works pretty well to describe Landau lattice data Oliveira et al, PRD81 (2010) 074505; Cucchieri al, PRD85 (2012) 094513; Bornyakov et al, PRD85 (2012) . Massive-type gluon propagator

Lattice Landau gluon propagator for SU(3)

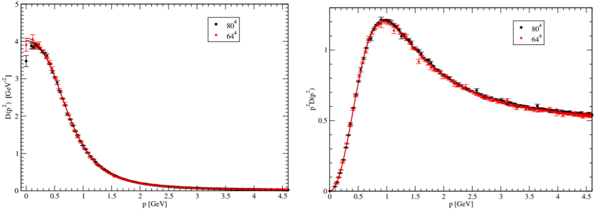

Figure: taken from Dudal et al, PRD86 (2012).

RGZ fit $D(p^2) = \frac{Z(p^2+M^2)}{p^4+(m^2+M^2)p^2+\lambda^4}$ in MOM scheme at $\mu = 4.3\ GeV$

N	Z	M^2 (GeV^2)	m^2+M^2 (GeV^2)	λ^4 (GeV^4)
80	0.7838(17)	4.303(50)	0.526(14)	0.4929(39)
64	0.7800(29)	4.442(86)	0.576(24)	0.4964(64)

Starting from here, we can add (small) perturbative corrections on top of nonperturbative RGZ vacuum. One loop corrections are in progress. How about situation for $\alpha \neq 0$?

Lattice gluon propagator in the linear covariant gauge

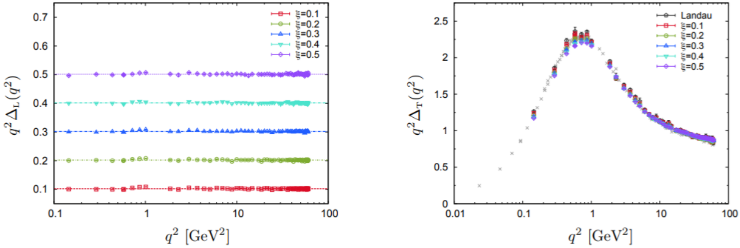

Figure: taken from Bicudo et al, PRD92 (2015)

We notice almost no dependence on gauge parameter. Nonperturbatively, the longitudinal propagator still fixed by $\frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}$.

Lattice gluon propagator in the linear covariant gauge

We notice almost no dependence on gauge parameter. Compare with Dyson-Schwinger equations (DSE) output, with much more prominent dependence. Less good modeling of vertices for $\alpha \neq 0$ than in Landau?

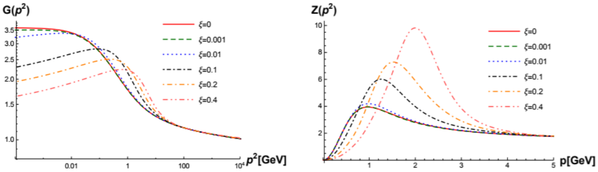

Figure: taken from Huber, PRD91 (2015). Similar results as in Aguilar et al, PRD 91 (2015). Left: ghost form factor $p^2 G(p^2)$; right: transversal gluon form factor $p^2 D(p^2)$.

Outside Landau gauge, the ghost is suppressed in IR with gauge parameter?

What does GZ teach us?

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How to tame Gribov copies in generic linear covariant gauge

- We will restrict to

$$\Omega^h = \{ A_\mu^a | \, \partial_\mu A_\mu^a = -i\alpha b^a, \, \mathcal{M}^{ab}(A^h) > 0 \}$$

  Due to $\partial A^h = 0$, $\mathcal{M}^{ab}(A^h) > 0$ makes sense.
- In Dudal et al, PRD92 (2015), it was shown that this removes all infinitesimal gauge copies that are connected via Taylor expansion in α to Landau gauge zero modes.
- The (BRST symmetric) RGZ action was derived to be

$$\begin{aligned} S &= S_{YM} + \int d^4x \left(\alpha \frac{b^a b^a}{2} + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab}(A) c^b \right) + \int d^4x \, \tau^a \partial_\mu (A^h)_\mu^a + \int d^4x \, \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} \\ &+ \int d^4x \left(-\bar{\varphi}_\mu^{ac} \mathcal{M}(A^h)^{ab} \varphi_\mu^{bc} + \bar{\omega}_\mu^{ac} \mathcal{M}(A^h)^{ab} \omega_\mu^{bc} + g\gamma^2 f^{abc} (A^h)_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) + M^2 (\bar{\varphi}_\mu^{bc} \varphi_\mu^{bc} - \bar{\omega}_\mu^{bc} \omega_\mu^{bc}) \right) \end{aligned}$$

Intermediate summary

Gauge invariant GZ action

- Constructed a local (non-polynomial) GZ action
- Kills off (large class of) infinitesimal copies in generic linear covariant gauge
- Fully gauge invariant and renormalizable to all orders, including gauge invariant Gribov mass
- $$\left\langle g f^{abc} A_{\mu}^{h,a} (\varphi + \overline{\varphi})_{\mu}^{bc} \right\rangle = 2d(N^2-1)\gamma^2$$

  and other gauge invariant $d=2$ condensates.

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Back to RGZ propagator in Landau gauge

- $D(p^2) = \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4} \rightarrow$ 2 complex conjugate mass poles
  Cannot correspond to physical gluon $\rightarrow$ sort of "effective confinement" , see e.g. Kharzeev & Levin, PRL 114 (2015)
- What about situation in other linear gauges?
- cc poles are RG and gauge independent $\rightarrow$ follows from Nielsen identities
- Much more can be learnt from GZ Nielsen identities!

Nielsen identities

- BRST invariance $\rightarrow$ functional Slavnov-Taylor identity (STI) on 1PI generator Γ:

$$\mathcal{S}(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta\Omega_\mu^a}\frac{\delta\Gamma}{\delta A_\mu^a} + \frac{\delta\Gamma}{\delta L^a}\frac{\delta\Gamma}{\delta c^a} + \frac{\delta\Gamma}{\delta K^a}\frac{\delta\Gamma}{\delta\xi^a} + ib^a\frac{\delta\Gamma}{\delta\bar{c}^a} \right) + \chi\frac{\partial\Gamma}{\partial\alpha}$$

  Ω is source coupled to BRST variation of A (Dc), L to that of c (Lcc) and $s\alpha = \chi$, χ couples to $\bar{c}b$.

- Acting with test operators on STI allows to derive functional relations between n-point functions.

  In particular, deriving w.r.t. χ allows to control α-dependence!

- After a long technical analysis in GZ theory (due to mixed propagators):

$$\frac{\partial}{\partial\alpha}[\text{gluon poles}] = 0$$

Nielsen identities

- $$\frac{\partial}{\partial\alpha}[\text{gluon poles}] = 0$$

  gives an *a posteriori* argument why it works to fit lattice Landau gauge propagator with

  $$D(p^2) = Z\frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4}$$

  and the observed α-independence of lattice gluon propagator. Indeed, same (tree level propagator) fit will work for any α.

Longitudinal gluon propagator

- Slavnov-Taylor identity, next to Ward identity $\frac{\delta\Gamma}{\delta b} = \alpha\partial A$ (gauge condition) allows to prove, also in GZ, that

$$\frac{p_\mu p_\nu}{p^2}\left\langle A_\mu A_\nu\right\rangle \equiv \frac{\alpha}{p^2}$$

  (consistent with lattice)
- Interesting comment: gluon self energy, $\langle AA\rangle_{1PI}$, is *not* transverse, despite exact BRST invariance. This is only true in absence of propagator mixing!

GZ ghost propagator in the linear covariant gauge

$$G(k^2) = \frac{1}{k^2}\frac{1}{1-\omega(k^2)}$$

where

$$\omega(k^2) = \frac{Ng^2}{k^2(N^2-1)}\int \frac{d^4q}{(2\pi)^4}\frac{k_\mu(k-q)_\nu}{(k-q)^2}\langle A^a_\mu(q)A^a_\nu(-q)\rangle = \omega^T(k^2)+\omega^L(k^2)$$

$\omega^T(k^2)$ comes from transverse component of gluon propagator, with

$$\omega^T(k^2) = c + \mathcal{O}(k^2)$$

$\omega^L(k^2)$ stems from the (standard perturbative) longitudinal component, at 1-loop

$$\omega^L(k^2) = \alpha\frac{Ng^2}{64\pi^2}\log\frac{k^2}{\bar{\mu}^2}$$

as we have exactly $D_L(k^2) = \frac{\alpha}{k^2}k_\mu k_\nu k^2$. So, logarithmic IR suppression is there, and will remain to be there at higher orders.

Lattice ghost propagator in the linear covariant gauge

- No predictions yet, because the FP operator $-\partial D$ is non-Hermitian for $\alpha \neq 0 \rightarrow$ numerical not so easy nut to crack how to invert this FP operator.
- Work in progress by Roelfs et al, based on lattice linear covariant gauge minimizing functional Cucchieri & Mendes, PRL103 (2009), Binosi et al, PRD92(2015).

$$\min_U \mathrm{Tr} \int d^4x \left(A_\mu^U A_\mu^U + \frac{2}{g} \mathrm{Re}(iU\Lambda) \right)$$

Λ^a Gaussian sampled with width related to gauge parameter.

- In conjunction with Gribov copy question based on this functional.

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LKF transformations

Nielsen identities

- In a compact notation, for a propagator the Nielsen identity looks like
$$\frac{\partial}{\partial\alpha}G_{\phi\phi} = G_{\phi\phi}M$$
where M corresponds to a composite operator correlation function like $\Gamma_{\chi\Omega\phi}$ (insertion of Dc and $\int \overline{c}b$) This leads to
$$G_{\phi\phi}^{(\alpha)} = G_{\phi\phi}^{(\alpha=0)}e^{\int_0^\alpha d\alpha' M(\alpha')}$$
- This relates propagator in one gauge to that of Landau gauge.

(Abelian) Landau-Khalatnikov-Fradkin transformations

- Landau & Khalatnikov, Sov. Phys. JETP **2** (1956) & Fradkin, idem, see also Johnson & Zumino, PRL3 (1959)
- (Abelian!) transformation that relates photon and electron propagator in different gauges,

$$\langle\bar{\psi}(x)\psi(y)\rangle_{\alpha} = \langle\bar{\psi}(x)\psi(y)\rangle_{\alpha=0} \left\langle e^{-ig\xi(x)} e^{ig\xi(y)} \right\rangle_{\alpha'=0}$$

  with

$$\langle\xi(p)\xi(-p)\rangle_{\alpha} = -\alpha\frac{1}{p^4}$$

- Useful to check validity of e.g. vertex Ansätze: consistent with gauge (LKF) transformation law, see e.g. Bashir & Raya, PRD66 (2002)
- Would be most interesting to apply to QCD, to see how "gauge invariance" in physical results (meson spectra etc) manifests itself based on employed vertex modelling.
- Problem: only Abelian LKFs are known!

(non-Abelian) Landau-Khalatnikov-Fradkin

- The Abelian LKF can be derived via path integral manipulations.
- An alternative proof was given in Sonoda, PLB 499 (2001) using Stückelberg-like trick (to construct gauge invariant photon).
- Since we have by now access to non-Abelian gauge invariant A^h, without destroying renormalizability $\to$ possibility to derive non-Abelian LKF? $\to$ YES
- Gauge invariant A^h, but also gauge invariant

$$\psi^h = h^\dagger \psi$$

with $h = e^{ig\xi^a t^a}$, t^a in adjoint or fundamental rep. For the moment, we solve for ξ:

$$\xi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + \mathcal{O}(A^3)$$

(non-Abelian) Landau-Khalatnikov-Fradkin

work in progress by De Meerleer et al

- Crux of the matter

$$\left\langle A^h \ldots \overline{\psi}^h \ldots \psi^h \right\rangle = \text{gauge invariant}$$
$$= \langle A \ldots \overline{\psi} \ldots \psi \rangle_{\alpha=0}$$
$$= \langle A \ldots \overline{\psi} \ldots \psi \rangle_{\alpha} + \text{ series in } \xi \text{ (or } \partial A)$$

Application: LKF for quark propagator

Using the Landau gauge properties, we found

$$\langle \bar{\psi}(x)\psi(y) \rangle_{\alpha} = \langle \bar{\psi}(x)\psi(y) \rangle_{\alpha=0} \left\langle h^{\dagger}(x) h(y) \right\rangle_{\alpha=0}$$

We also managed to confirm the non-Abelian LKF relations via a path integral reasoning. Can they also be derived from Nielsen identities? And can we use them to “transform” Landau gauge Ansätze to other gauges?

The End!

Thanks!