$SU(2N_F)$symmetry of confinement in QCD and its observation at high T

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What is the origin of hadron mass?

Is it possible to separate confinement and chiral symmetry breaking physics?

What physics is responsible for confinement and for chiral symmetry breaking?

Banks-Casher:

$$< \bar{q}q >= -\pi\rho(0).$$

What we do:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

We use JLQCD $N_f = 2$ overlap gauge configurations.

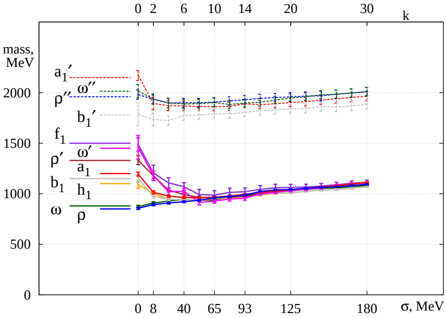
$SU(2)_L \times SU(2)_R$ and $U(1)_A$ partners

What one expects for $J = 1$ mesons:

$$\begin{tikzcd}
(0,0) & \begin{matrix} f_1(0,1^{++}) \\ \overline{\Psi}(\mathbb{1}_{\mathrm{F}} \otimes \gamma^5\gamma^k)\Psi \end{matrix} & \begin{matrix} \omega(0,1^{--}) \\ \overline{\Psi}(\mathbb{1}_{\mathrm{F}} \otimes \gamma^k)\Psi \end{matrix} \\
(1/2,1/2)_a & \begin{matrix} b_1(1,1^{+-}) \\ \overline{\Psi}(\tau^a \otimes \gamma^5\gamma^0\gamma^k)\Psi \end{matrix} \arrow[r, leftrightarrow, "SU(2)_{\mathrm{A}}"] \arrow[d, leftrightarrow, bend right, "U(1)_{\mathrm{A}}"'] & \begin{matrix} \omega(0,1^{--}) \\ \overline{\Psi}(\mathbb{1}_{\mathrm{F}} \otimes \gamma^0\gamma^k)\Psi \end{matrix} \arrow[d, leftrightarrow, bend left, "U(1)_{\mathrm{A}}"] \\
(1/2,1/2)_b & \begin{matrix} \rho(1,1^{--}) \\ \overline{\Psi}(\tau^a \otimes \gamma^0\gamma^k)\Psi \end{matrix} \arrow[r, leftrightarrow, "SU(2)_{\mathrm{A}}"] & \begin{matrix} h_1(0,1^{+-}) \\ \overline{\Psi}(\mathbb{1}_{\mathrm{F}} \otimes \gamma^5\gamma^0\gamma^k)\Psi \end{matrix} \\
(1,0) \oplus (0,1) & \begin{matrix} \rho(1,1^{--}) \\ \overline{\Psi}(\tau^a \otimes \gamma^k)\Psi \end{matrix} \arrow[r, leftrightarrow, "SU(2)_{\mathrm{A}}"] & \begin{matrix} a_1(1,1^{++}) \\ \overline{\Psi}(\tau^a \otimes \gamma^5\gamma^k)\Psi \end{matrix}
\end{tikzcd}$$

M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

$J = 1$



We clearly see a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian. What does it mean !?

L.Ya.G., EPJA 51(2015)27

(i) **(0,0):**

$$|(0,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}R \pm \bar{L}L\rangle_J.$$

(ii) $(1/2,1/2)_a$ **and** $(1/2,1/2)_b$**:**

$$|(1/2,1/2)_a;+;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L + \bar{L}R\rangle_J,$$

$$|(1/2,1/2)_a;-;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L - \bar{L}\boldsymbol{\tau}R\rangle_J,$$

$$|(1/2,1/2)_b;-;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L - \bar{L}R\rangle_J,$$

$$|(1/2,1/2)_b;+;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L + \bar{L}\boldsymbol{\tau}R\rangle_J.$$

(iii) **(0,1)⊕(1,0):**

$$|(0,1)+(1,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}R \pm \bar{L}\boldsymbol{\tau}L\rangle_J,$$

L.Ya.G., EPJA 51(2015)27

Consider rotations in an imaginary 3-dim space of doublets constructed from the Weyl spinors

$$\mathrm{U} = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \qquad \mathrm{D} = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

$$\mathrm{U} \to \mathrm{U}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{U}\ , \qquad \mathrm{D} \to \mathrm{D}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{D}\ ,$$

where $\boldsymbol{\sigma}$ are the standard Pauli matrices: $[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\,\sigma^k$.
We refer to this imaginary three-dimensional space as the chiralspin space and denote this symmetry group as $SU(2)_{\mathrm{CS}}$

A group that contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS}$ is $SU(4)$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\mathrm{L}} \\ u_{\mathrm{R}} \\ d_{\mathrm{L}} \\ d_{\mathrm{R}} \end{pmatrix}$$

L.Ya.G., M. Pak, PRD 92(2015)016001

Instead of the states constructed with Weyl spinors we can consider the left- and right-handed Dirac bispinors and bilinear operators.
Then the $SU(2)_{\rm CS}$ chiralspin rotations are generated through

$$\mathbf{\Sigma} = \{\gamma^l, -i\gamma^5\gamma^l, \gamma^5\} , \qquad l = 1,2,3,4 \qquad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk}\,\Sigma^k .$$

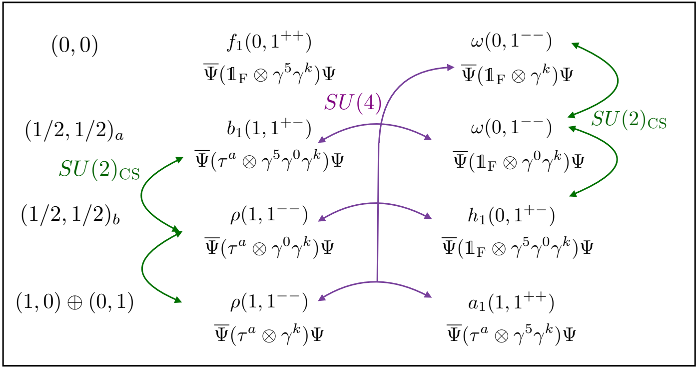
The $SU(4)$ group contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS} \supset U(1)_A$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}$$

and has the following set of generators:

$$\{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}$$

L.Ya.G., M. Pak, PRD 92(2015)016001



$SU(4)$

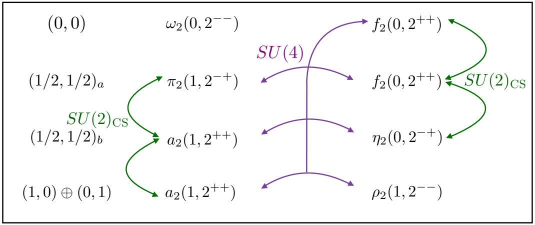
$SU(2)_{\rm CS}$

$SU(2)_{\rm CS}$

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_{\rm F}\otimes\gamma^5\gamma^k)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_{\rm F}\otimes\gamma^k)\Psi$
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a\otimes\gamma^5\gamma^0\gamma^k)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_{\rm F}\otimes\gamma^0\gamma^k)\Psi$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a\otimes\gamma^0\gamma^k)\Psi$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_{\rm F}\otimes\gamma^5\gamma^0\gamma^k)\Psi$
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a\otimes\gamma^k)\Psi$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a\otimes\gamma^5\gamma^k)\Psi$

M. Denissenya, L.Ya.G, M.Pak, PRD 91(2015)114512

J=2 mesons.



$(0,0)$ $\omega_2(0,2^{--})$ $f_2(0,2^{++})$

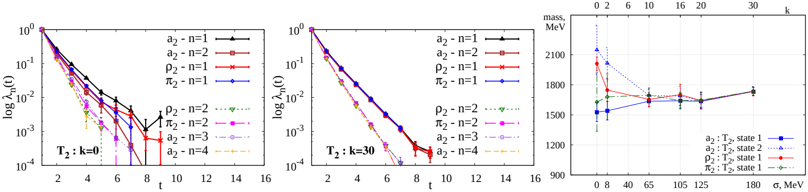
$SU(4)$

$(1/2,1/2)_a$ $\pi_2(1,2^{-+})$ $f_2(0,2^{++})$ $SU(2)_{\rm CS}$

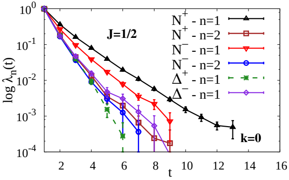
$SU(2)_{\rm CS}$

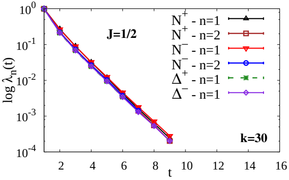
$(1/2,1/2)_b$ $a_2(1,2^{++})$ $\eta_2(0,2^{-+})$

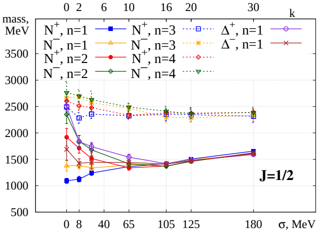
$(1,0)\oplus(0,1)$ $a_2(1,2^{++})$ $\rho_2(1,2^{--})$



$J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, PRD 92 (2015) 074508







L.Ya.G., M. Pak, PRD 92(2015)016001

QCD Hamiltonian in Coulomb gauge (Christ and Lee):

$$H_{QCD} = H_E + H_B + \int d^3x \Psi^\dagger(\mathbf{x})[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta m]\Psi(\mathbf{x}) + H_T + H_C,$$

$$H_T = -g \int d^3x\, \Psi^\dagger(\mathbf{x})\boldsymbol{\alpha} \cdot \mathbf{A}(\mathbf{x})\, \Psi(\mathbf{x})\ ,$$

$$H_C = \frac{g^2}{2} \int d^3x\, d^3y\, J^{-1}\ \rho^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y})\, J\, \rho^b(\mathbf{y})\ .$$

The Coulombic H_C part is a $SU(2)_{CS}$- and $SU(4)$- singlet. It is a confining part of the QCD Hamiltonian. This part generates a $SU(4)$-symmetric spectrum.

The transverse (magnetic) part H_T is not $SU(2)_{CS}$- and $SU(4)$-symmetric and therefore its expectation value vanishes in the $SU(4)$-symmetric hadron wave function.

The chromo-magnetic interaction contributes only to the near-zero modes while the confining chromo-electric interaction is distributed in all modes.

(Near) zero modes of Euclidean QCD and $SU(2)_{CS}$-$SU(4)$ breaking.

The Euclidean $SU(2)_{CS}$ generators:

$$\mathbf{\Sigma} = \{\gamma^k, -i\gamma^5\gamma^k, \gamma^5\} \,. \tag{1}$$

The $SU(2)_{CS}$ generators do not commute with the Dirac operator.

These symmetries are missing in the Lagrangian.
The intrinsic dynamical reason: the zero modes of the Dirac operator

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \tag{2}$$

The zero mode is chiral, L or R, depending on the topological charge $Q \neq 0$.
Atiyah-Singer:

$$Q = n_L - n_R$$

.

At $Q \neq 0$, there is an asymmetry between L and R.
Conclusion: The zero modes break explicitly $SU(2)_{CS}$ and $SU(2N_F)$.

Conclusions to part I and prediction for high T

Observed on the lattice $SU(4)$ symmetry of hadrons upon elimination of the near-zero modes is a symmetry of confinement in QCD that is due to chromo-electric charge-charge interaction.

The chromo-magnetic interaction in QCD contributes only to near-zero modes. It breaks explicitly the $SU(4)$ symmetry of confinement.

The hadron spectra observed in real world can be viewed as a result of splitting of the primary energy levels with the $SU(4)$ symmetry by means of dynamics associated with the near-zero modes.

JLQCD: above Tc both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ get restored and a gap opens in the spectrum of the Dirac operator. Then we expect that the actual symmetry is $SU(4)$ - no deconfinement.

Spatial correlators at high T. Full QCD, no truncation.

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L.Ya.G., S. Hashimoto, C. B. Lang, S. Prelovsek, arXiv:1707.01881

$N_f = 2$ QCD with the chirally symmetric Dirac operator.

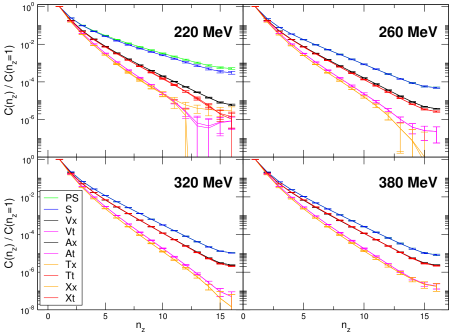
$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(\mathbf{0}, 0)^\dagger \rangle \, .$$

where $\mathcal{O}_\Gamma(x) = \bar{q}(x)\Gamma\frac{\tau}{2}q(x)$ are all possible $J = 0$ and $J = 1$ local operators:

Name	Dirac structure	Abbreviation	
Pseudoscalar	γ_5	PS	$]\ U(1)_A$
Scalar	$\mathbb{1}$	S	
Axial-vector	$\gamma_k\gamma_5$	**A**	$]\ SU(2)_A$
Vector	γ_k	**V**	
Tensor-vector	$\gamma_k\gamma_3$	**T**	$]\ U(1)_A$
Axial-tensor-vector	$\gamma_k\gamma_3\gamma_5$	**X**	

Table : Operators considered in this work and their transformation properties. The open vector index k denotes the components 1, 2, 4, *i.e.* x, y, t.

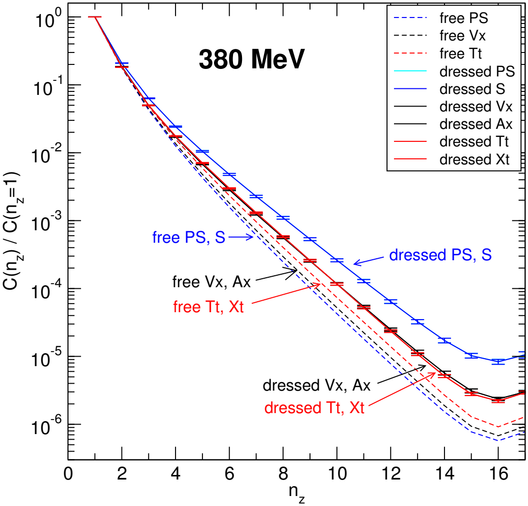
Spatial correlators at high T. Full QCD, no truncation.



In total we observe three different multiplets:

$$E_1(U(1)_A): \quad PS \leftrightarrow S$$
$$E_2(SU(4)): \quad V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x$$
$$E_3(SU(4)): \quad V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t$$

Spatial correlators at high T. Full QCD, no truncation.



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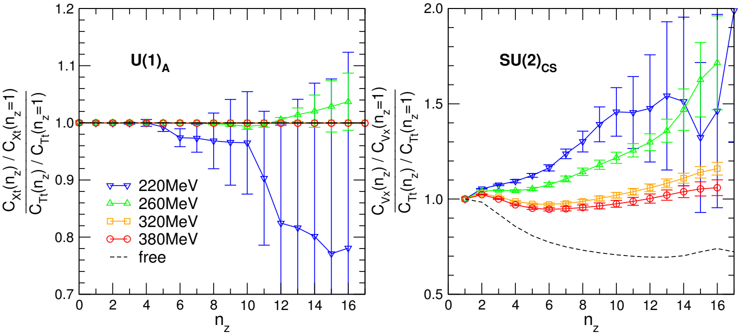

At T=380 MeV we observe approximate $SU(2)_{CS}$ and $SU(4)$ symmetries at the level of 5%.

Conclusions to part II

We observe emergence of approximate $SU(2)_{CS}$ and $SU(4)$ symmetries with increasing temperature.

It seems that we do not approach the free quark limit.

Emergence of these symmetries could happen if the chromo-magnetic interaction were screened while the chromo-electric interaction were still active.