SU(2) Yang-Mills thermodynamics: a priori estimate and radiative corrections

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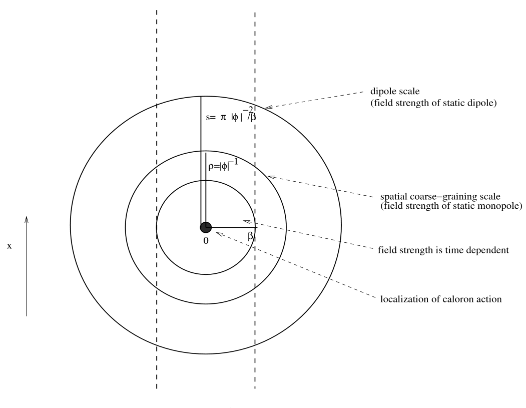

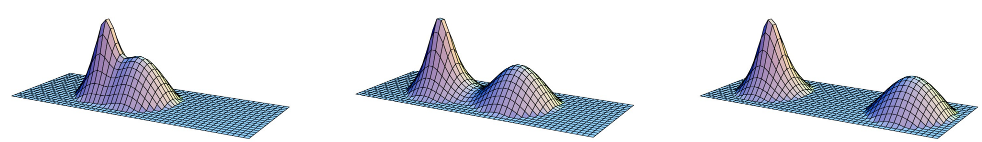

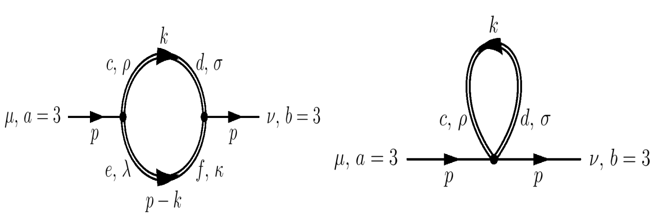

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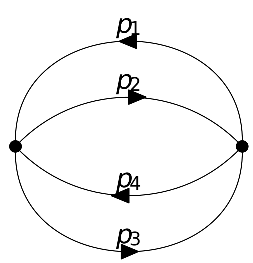

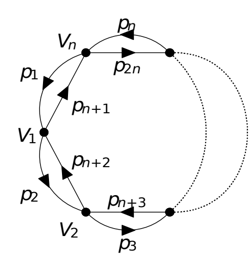

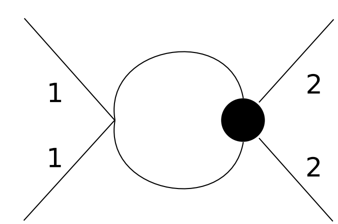

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- $\mathrm{U}(1) \subset \mathrm{SU}(2)$ wave propagation/photonic excitation and deconfining thermal ground state
- radiative corrections: polyhedral loops in massive sector,
  polarisation tensor of massless mode
- some physics implications of deconfining SU(2) Yang-Mills gas

motivation

- Andrei Linde (1980):
„Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability)

  - no „convergence“ of series since kinetic and interaction energies comparable in this sector

- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

  - prejudices on integration constants in „integral method“

  - only approximate knowledge of beta function in „differential method“

nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst & RH (2004), RH (2005-2007), Giacosa & RH (2006), Schwarz, Giacosa & RH (2007), Ludescher & RH (2008), Falquez, Baumbach & RH (2010- 2011), RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)]

thermal ground state at high temperature:

- **Euclidean action:**

$$S = \frac{\mathrm{tr}}{2}\int_0^\beta d\tau \int d^3x \, F_{\mu\nu}F_{\mu\nu} \,, \qquad (\beta \equiv 1/T)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ [Schafer et Shuryak (1996)]

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- **(anti)selfdual gauge fields: [(anti)calorons]**

$$F_{\mu\nu}[A] = \pm \tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.$$

(A_μ periodic)

field configs. stabilized by gauge-field winding: $\partial \mathbf{R}^4 = S_3 \to SU(2) = S_3$

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- **in particular:** (anti)calorons of winding number **unity** with action:

$$S = \frac{8\pi^2}{g^2} \quad\Longrightarrow$$

essential zero of weight $\exp[-S]$ in partition function $\Longrightarrow$ PT ignores these field configs.

Calorons of top. charge unity (selfdual field configs. on $S_1 \times \mathbf{R}_3$):

(singular gauge)
['t Hooft, Rebbi & Jackiw (1977)]

Harrington-Shepard (1977):
(trivial holonomy)

$$A_\mu = \bar{\eta}^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r)$$

with
$$\Pi = \begin{cases} \left(1 + \frac{1}{3}\frac{s}{\beta}\right) + \frac{\rho^2}{x^2} & (|x| \ll \beta) \\ 1 + \frac{s}{r} & (r \gg \beta) \end{cases}$$

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$\Rightarrow$ $F_{\mu\nu}$ that of singular-gauge instanton with $\rho'^2 = \frac{\rho^2}{1+\frac{1}{3}\frac{s}{\beta}}$ $(|x| \ll \beta)$

(action: $S_c = \frac{8\pi^2}{g^2}\int_{S_3^\delta} d\Sigma_\mu K_\mu = \frac{8\pi^2}{g^2}$ localised about instanton center in $S_1 \times \mathbf{R}_3$)

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$$E^a_i = B^a_i = s\,\frac{\delta^a_i - 3\,\hat{x}^a \hat{x}^i}{r^3} \qquad (r \gg s)\,.$$

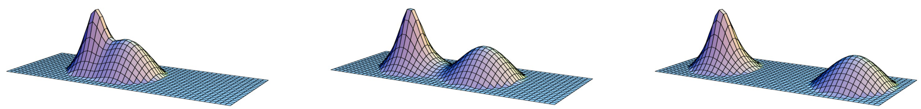
(static selfdual dipole-field with dipole moment: $p^a_i = s\,\delta^a_i$)

Calorons of top. charge unity (selfdual field configs. on $S_1 \times \mathbf{R}_3$):

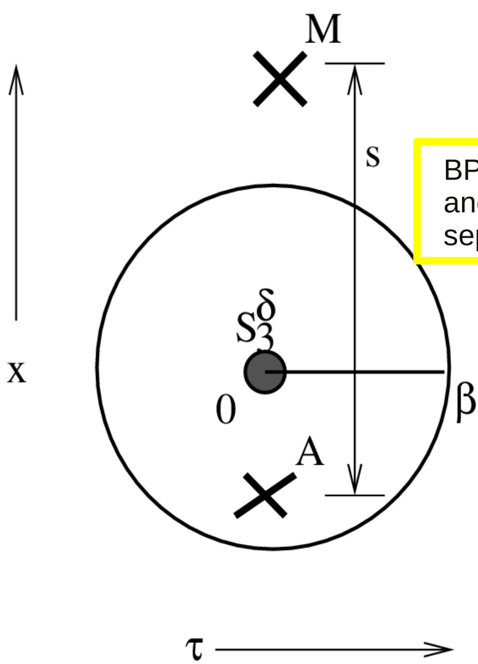
Nahm (1983), Lee-Lu-Kraan-van-Baal (1998):
(nontrivial holonomy)

- M and A of finite mass and extent:

$$m_M = 4\pi u\,, m_A = 4\pi\left(\frac{2\pi}{\beta} - u\right)$$



(action density on spatial slice)



BPS monopole (M)
and antimonopole (A),
separated by distance s

(M-A separation, caloron center)

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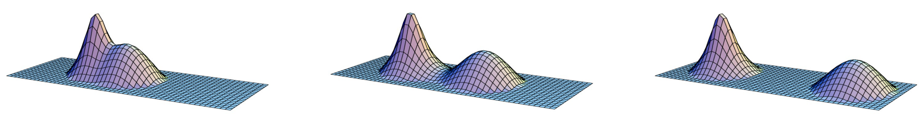

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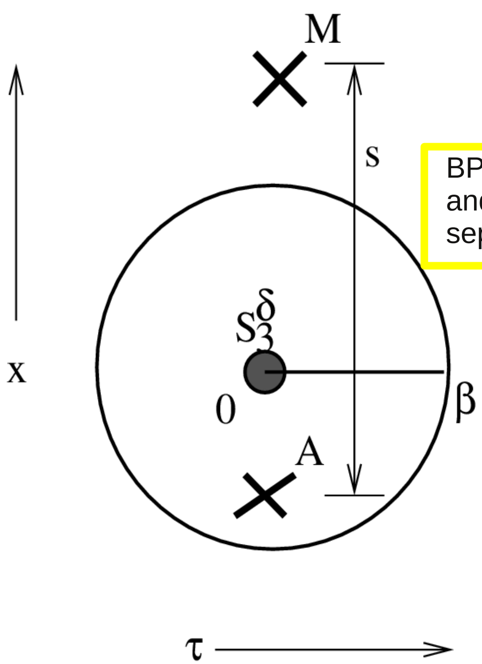

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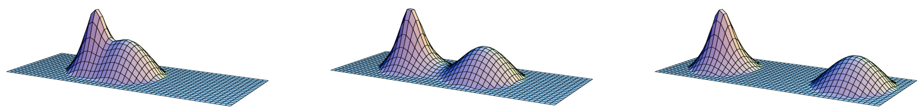

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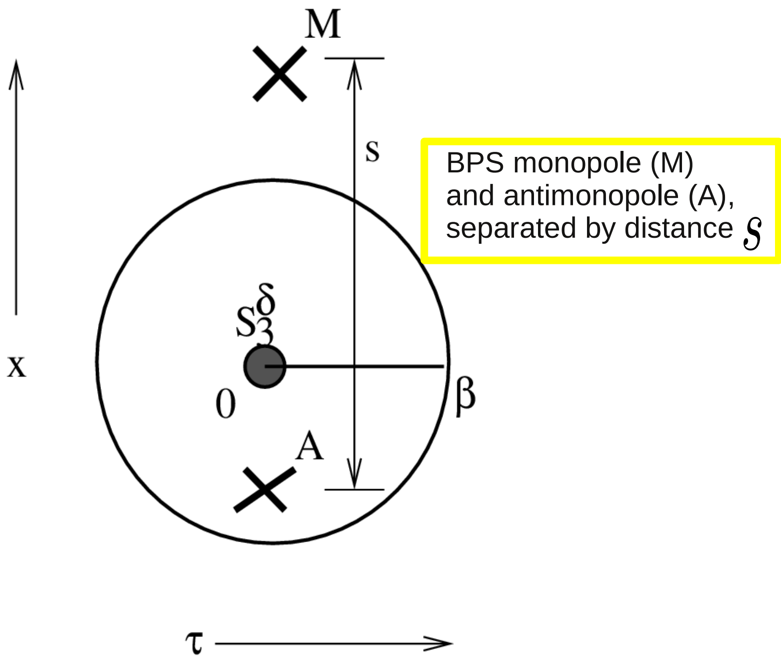

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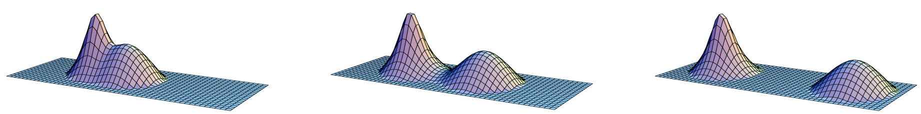

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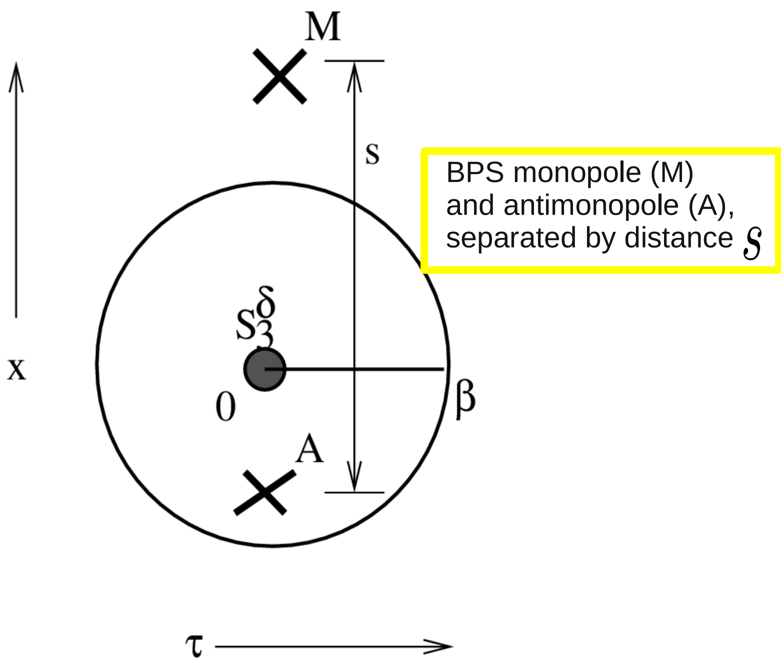

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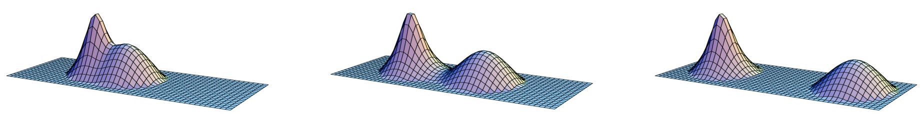

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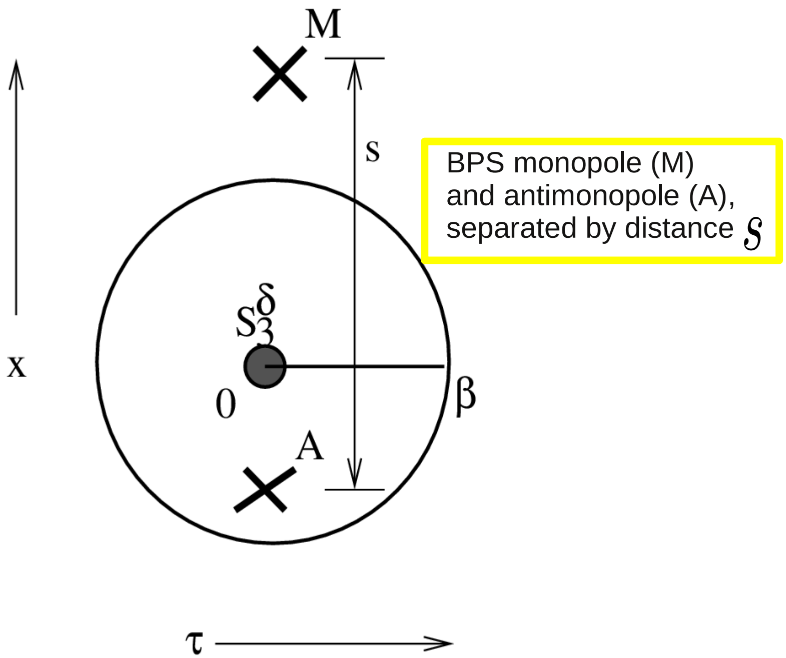

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- locus of action within S_3^δ $(\delta \to 0)$

- trivial-holonomy limit:
  M massless, A still massive, stable

spatial coarse-graining over pair of trivial-hol. (anti-)calorons: inert, adjoint scalar field ϕ

[Herbst & RH (2004)]

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0}) \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} F_{\mu\nu}(\tau,\vec{x}) \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of **family of phases**, where

$$\left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right]$$ and

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- $\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration (integral cubically dependent on cut-off)

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$$V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}$$

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- BPS equation:

$$\partial_\tau \phi = \pm 2i\,\Lambda^3\, t_3\, \phi^{-1} = \pm i\, V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity in V !

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- [Dense packing of (anti)caloron centers only affects (anti)caloron peripheries, packing voids (inhomogeneities) reflected by small imaginary radiative corrections to pressure, later.]

[Bischer, Grandou, RH (2017)]

effective action (deconfining phase), thermal ground state

- $$\mathcal{L}_{\rm eff}[a_\mu] = \mathrm{tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

(i) perturbative renormalizability (G^2 highest power in effect. action,
propagating part of a_μ adiabatic excitation of thermal ground state)
(iil) ϕ 's inertness – no higher dim., mixed operators to mediate
4-momentum transfer between ϕ and a_μ
(iii) gauge invariance

[see also RH (2016)]

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- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

$$a_\mu^{\rm gs} = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

(centers of HS (anti)calorons packed densely, static peripheries overlap to form $a_\mu^{\rm gs}$)

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

← interacting small and transient-holonomy (anti)calorons, (collapsing monopole-antimonopole pairs)

(vanishing entropy density of ground state!)

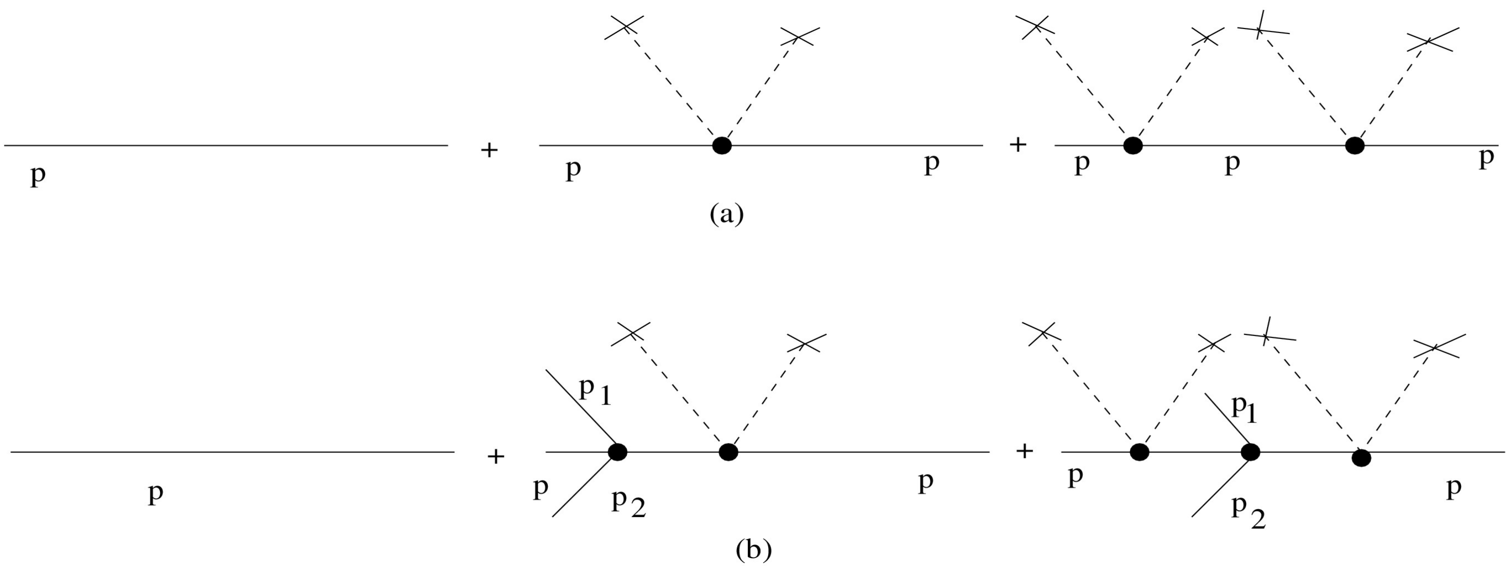
adjoint Higgs mechanism (deconfining phase)

(SU(2) → U(1))

- from effective action:

$$m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a]$$ unitary gauge → $$m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,, \quad m_3 = 0$$

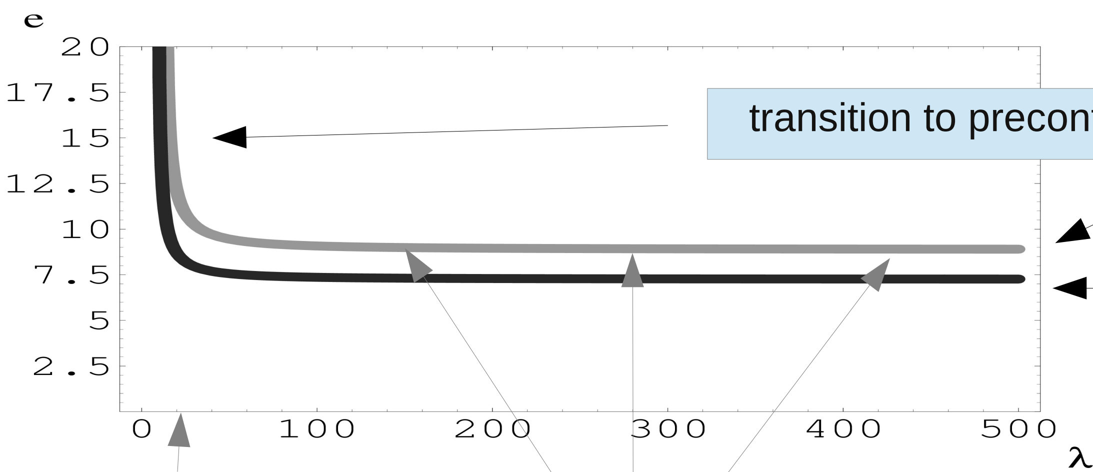
- no momentum transfer to ϕ , **but this infinitely often**
  (Dyson series for mass generation):



- no off-shell propagation of massive modes
  (otherwise: momentum transfer to ϕ !)

effective gauge coupling

e ← thermodynamical consistency



transition to preconfining phase

SU(2)

SU(3)

small-mass expansion, pressure
[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

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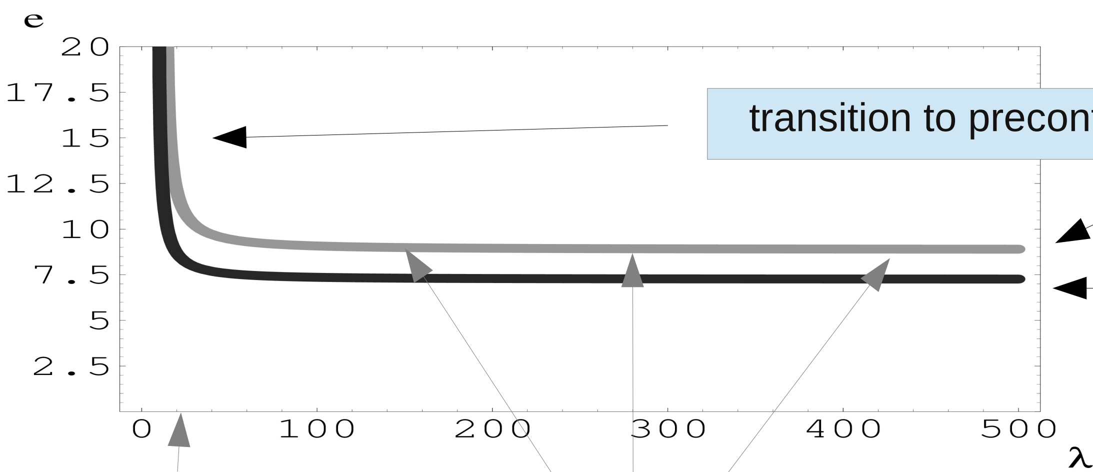

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small-mass expansion, pressure **[Dolan et Jackiw (1974)]**

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coarse-graining dominated by $\rho \sim |\phi|^{-1}$

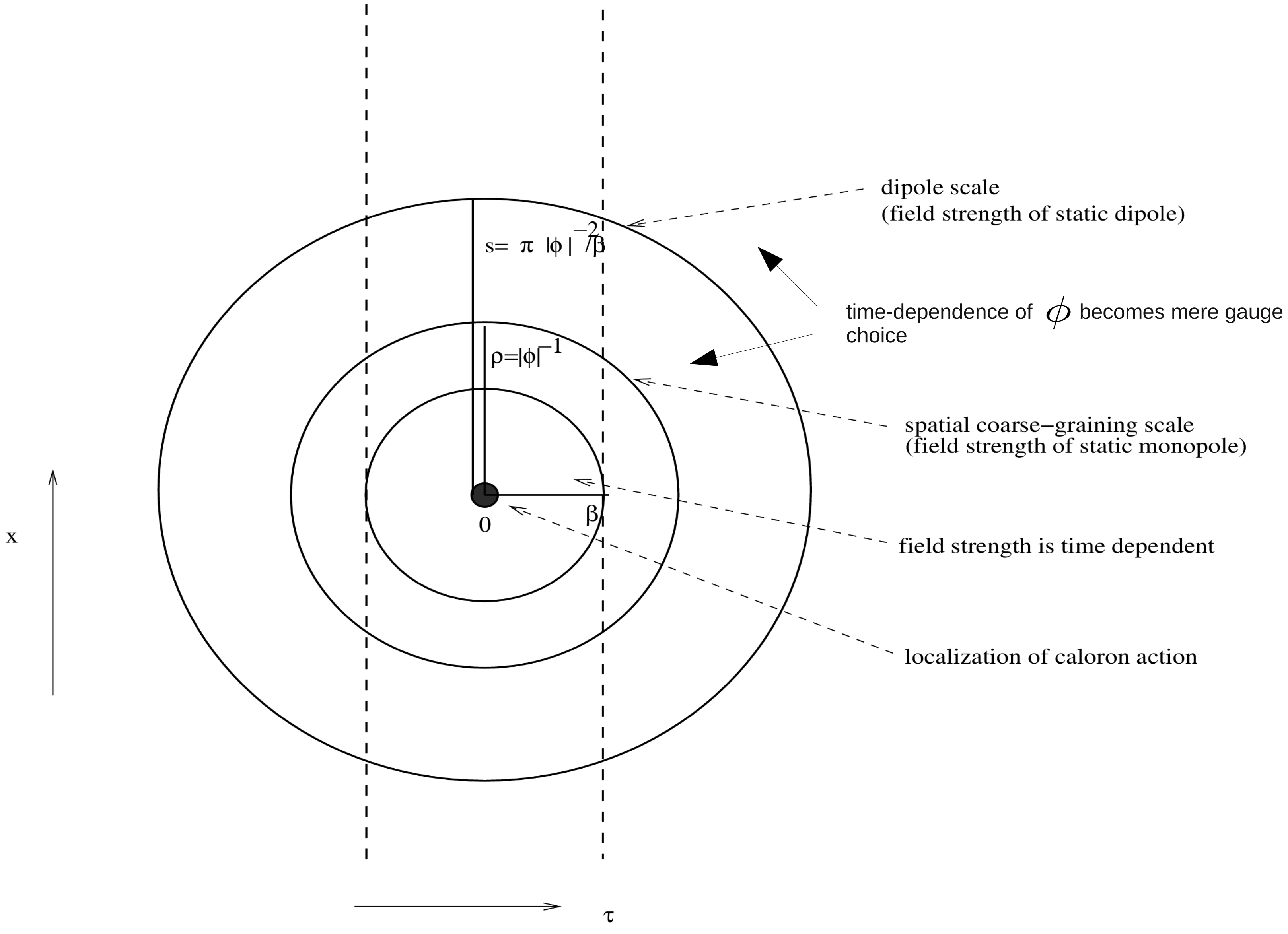
- restore $\hbar$

[Brodsky et al. (2011); Kaviani & RH (2012), RH (2012,2013)]

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

anatomy of caloron, inferred after spatial coarse-graining:



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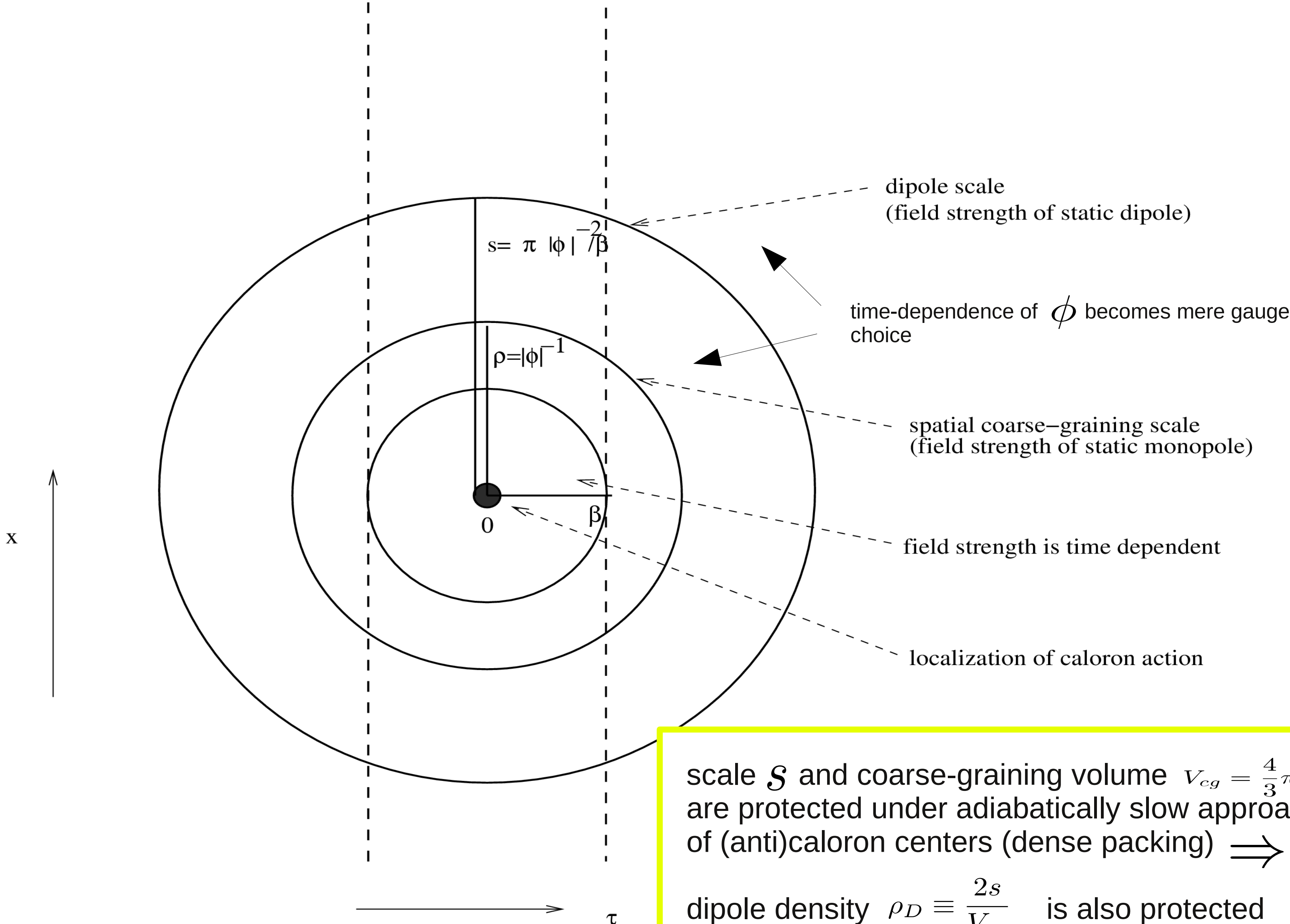

scale s and coarse-graining volume $V_{cg}=\frac{4}{3}\pi|\phi|^{-3}$ are protected under adiabatically slow approach of (anti)caloron centers (dense packing) $\Longrightarrow$

dipole density $\rho_D\equiv\frac{2s}{V_{cg}}$ is also protected

electric/magnetic dipole density (permittivity/permeability of vacuum):

[temperature a fictitious quantity]

$$|\mathbf{D}_e| = \frac{2s}{V_{\rm cg}} \propto T^{1/2}$$

R. Hofmann, ITP, Uni Heidelberg

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$$\rho_{\rm gs} = 4\pi T\Lambda^3 = \rho_{\rm EM} = \epsilon_0 \mathbf{E}_e^2 \Rightarrow |\mathbf{E}_e| \propto T^{1/2}$$

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$$\Longrightarrow \quad \epsilon_0[Q(\mathrm{Vm}^{-1})] \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} = \frac{9}{32\pi^2}\frac{\Lambda[\mathrm{m}^{-1}]}{\Lambda[\mathrm{eV}]}(\xi Q)^2 \neq f(T)$$

$(\xi = 19.56)$

similarly for magnetic permeability μ_0 .

$\Longrightarrow$ Lorentz invariance of thermal ground state.

[Grandou & RH (2015)]

electric/magnetic dipole density (permittivity/permeability of vacuum):

[temperature a fictitious quantity]

However:

$$\mathbf{E}_e^4 \nu \ll 8\Lambda^9$$

(due to wavelength/frequency not probing (anti)caloron centers)

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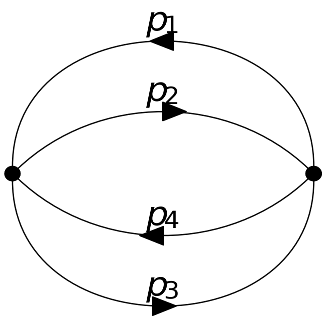
$\Longrightarrow$

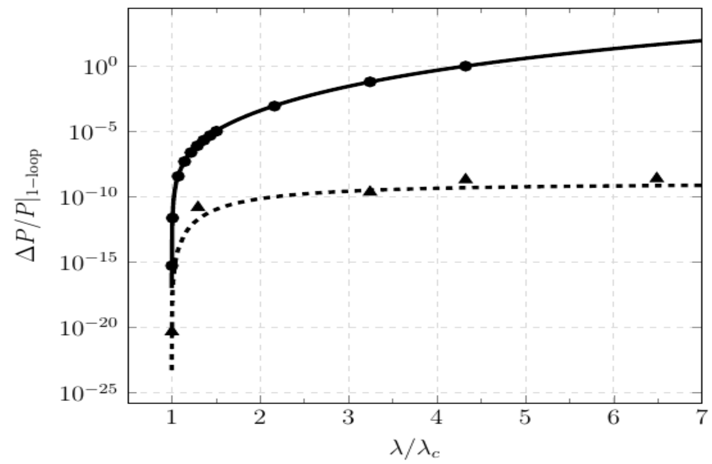
- if em wave propagation indeed occurs by undulating repolarisations of dipole densities in SU(2) deconfining thermal ground state then nature must make use of s**everal, mixing SU(2) YM factors of hierarchical YM scales**

e.g.: $\Lambda_{\rm CMB} \sim 10^{-4}$ eV, $\Lambda_e \sim 10^5$ eV, etc.

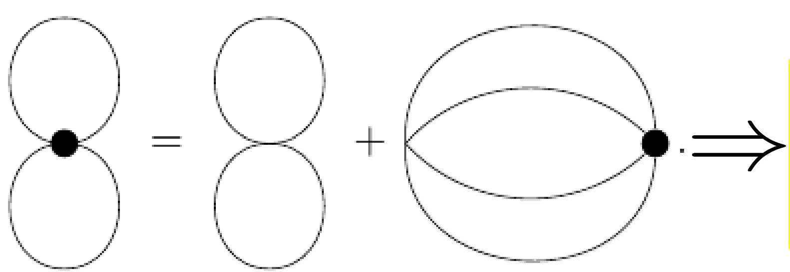
radiative corrections:
(purely massive sector, unitary-Coulomb gauge)



subject to vertex constraints $\Rrightarrow$



resummation in terms of form factor $f(\lambda)$



$$f^2(\lambda)\Delta P|_{3-\text{loop}} \approx \left(-0.94\cdot 10^{15} i\lambda^{-11.6}\right)^2 \cdot 5.3\cdot 10^{-20} i\Lambda^4\lambda^{13}$$
$$= -4.7\cdot 10^{10} i\Lambda^4\lambda^{-10.2}$$

Resummation transmutes λ^{13} towards $\lambda^{-10.2}$!

[Bischer, Grandou, RH (2017)]

radiative corrections: 2PI 3-loop bubble with resummed vertices

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- spatially homogeneous a priori estimate neglects **packing voids** for densely packed ball-like (anti)caloron **centers**

- radiative corrections re-introduce these inhomogeneities in terms of well controlled **imaginary correction** to free quasiparticle pressure from resummed n-gon diagrams

[Bischer, Grandou, RH (2017)]

real-world implications

electric-magnetically dual interpretation of U(1) charge:

if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc]
then **electric-magnetically dual** interpretation required:
in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar}\,,$$

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$(e = \frac{\sqrt{8\pi}}{\sqrt{\hbar}}\,.)$

$$Q \propto \frac{1}{e}\,.$$

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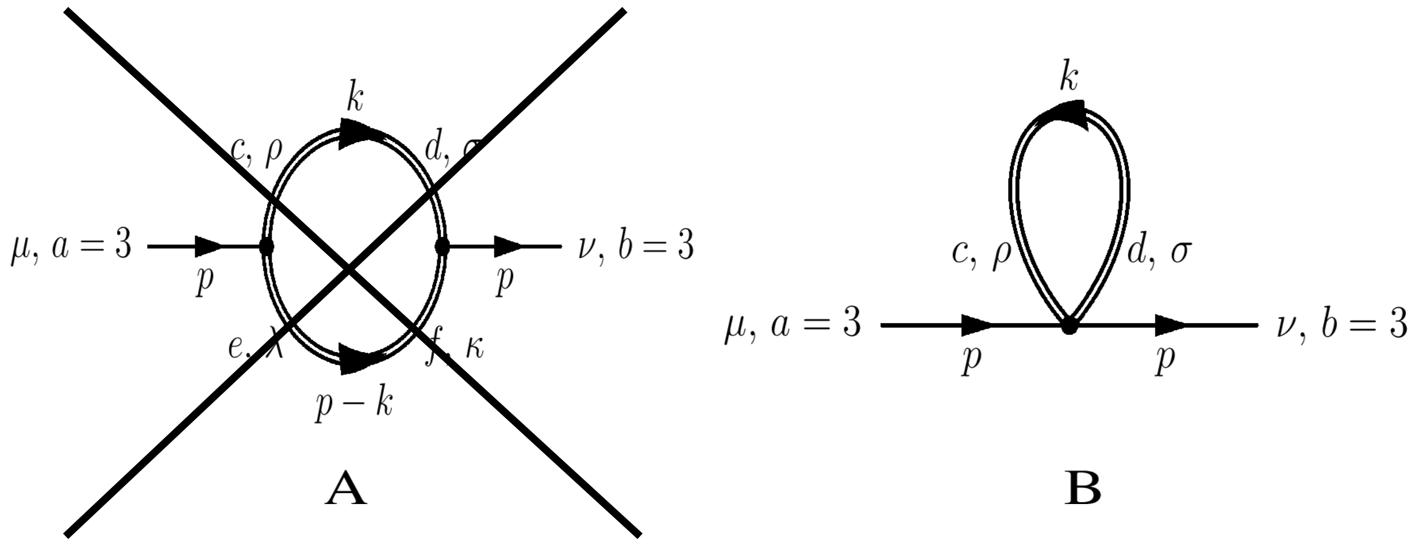
But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e}\,.$$

$\Rightarrow$ SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

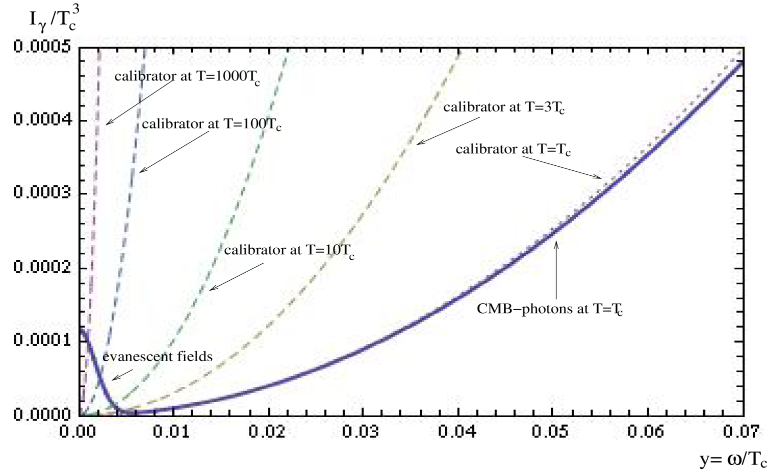
radiative corrections: polarization tensor of massless mode (unitary-Coulomb gauge)



→ gap equations for **transverse and longitudinal** parts of polarization tensor

→ screening functions G, F

$SU(2)_{CMB}$ radiative effects: blackbody spectral anomaly



deeply Rayleigh-Jeans part of CMB spectrum:

- cosmic radio background (UEGE),
- in SU(2) YMTD **critical** onset of Meissner effect at deconfining-preconfining phase boundary **evanescent modes at low frequencies**
- sharp fixation of

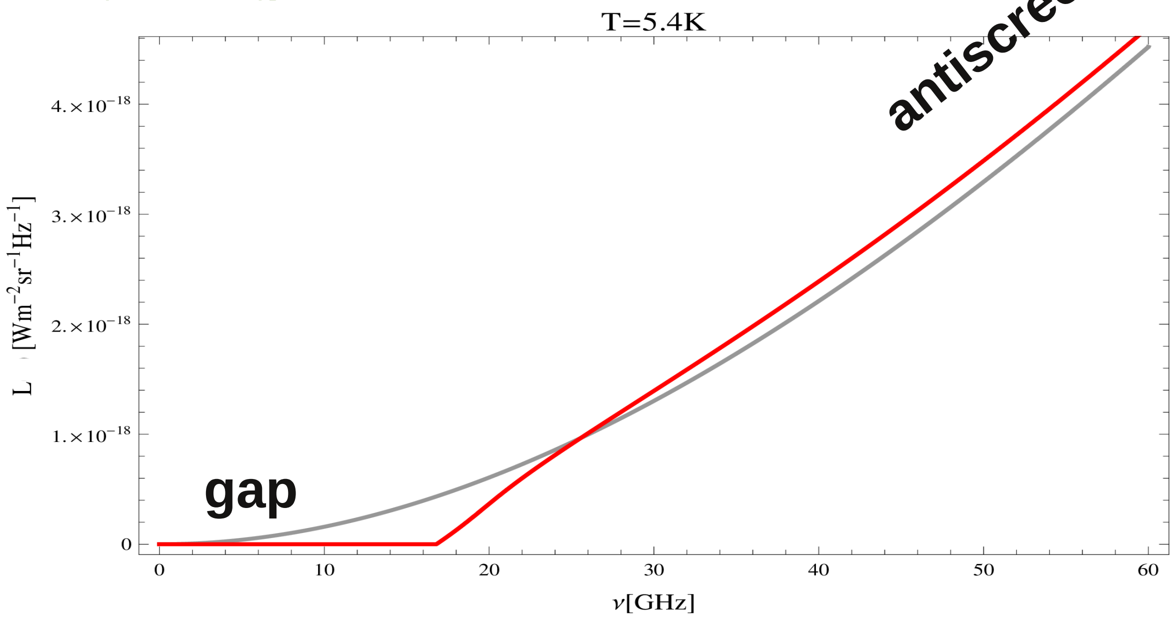
$T_c = T_0 = 2.725$ or $\Lambda = 10^{-4}$ **eV**

→ $SU(2)_{CMB}$

transverse polarizations:

max. gap in Rayleigh-Jeans reg. at T=5.4 K

[Schwarz, Giacosa & RH (2006), Ludescher & RH (2008), Falquez, RH & Baumbach (2010,2011)]

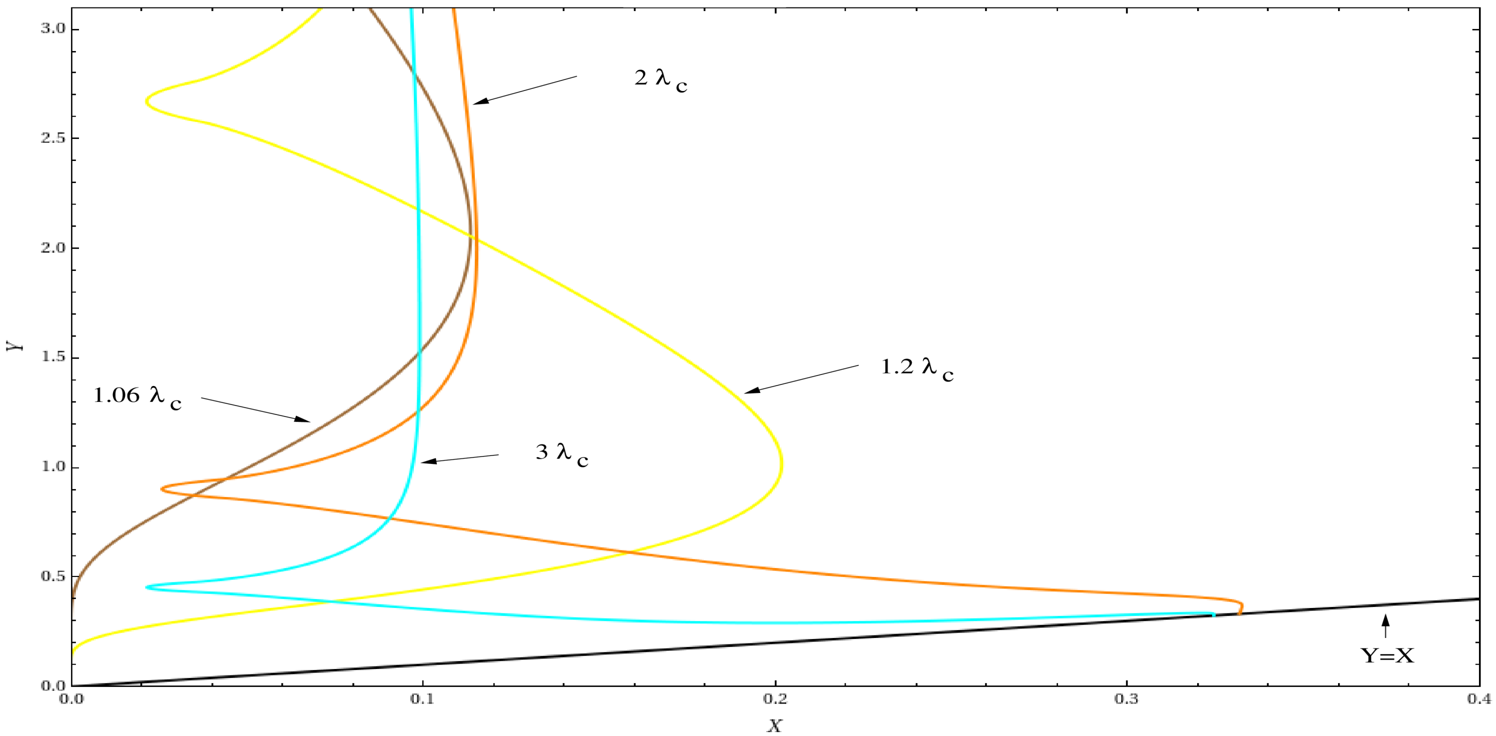


(CMB large-angle anomalies)

[Ludescher (2009) , RH, Nature Physics (2014)]

$\mathrm{SU}(2)_{\mathrm{CMB}}$ radiative effects: blackbody spectral anomaly

longitudinal polarizations: - low-momentum support of magnetic branches (dual interpretation) massless mode – longitudinal polarization



$\Longrightarrow$ intergalactic magnetic fields, seed fields for galactic dynamos [Falquez et al. (2011)]

(astrophysical/cosmological coherence lengths through local breaking of isotropy by biasing negative temperature fluctuations of CMB through blackbody anomaly)

Summary (outlook):

- nonperturbative approach to Yang-Mills theory

- deconfining thermal ground state in terms of densely packed HS (anti)caloron centers and overlapping peripheries

- (anti)caloron structure → static dipole densities (periphery, classical regime)
  → static monopole densities (transition region, radiative corr.)
  → Euclidean time dependence of field strength (deep inside centers, pure quantum behavior, n_B)

- action of an (anti)caloron, e-m dual interpretation

- radiative corrections in massive sector

- pol. tensor of massless mode: blackbody anomaly, extragalact. magn. Fields

- two more phases: preconfining, confining with implications for particle physics and cosmology