Confinement-Deconfinement phase transition in dense two-color QCD

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and new results

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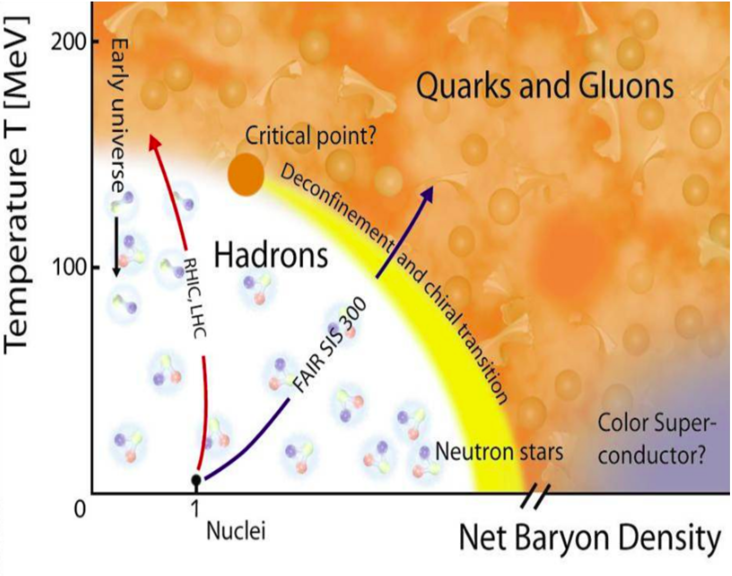
ITEP, Moscow

QCD phase diagram



Sign problem

SU(3)

- $Z = \int DUD\bar{\psi}D\psi\ \exp\left(-S_G - \int d^4x\bar{\psi}(\hat{D}+m)\psi\right) =$
  $= \int DU \exp\left(-S_G\right) \times \det\left(\hat{D}+m\right)$
- Eigenvalues go in pairs $\hat{D}$: $\pm i\lambda \Rightarrow$
  $\det\left(\hat{D}+m\right) = \prod_\lambda(\lambda^2+m^2) > 0$
  i.e. one can use lattice simulation
- Introduce chemical potential: $\det\left(\hat{D}+m\right) \to \det\left(\hat{D}-\mu\gamma_4+m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

SU(2)

- $(\gamma_5 C\tau_2)\cdot D^* = D\cdot(\gamma_5 C\tau_2)$
- Eigenvalues go in pairs $\hat{D}-\mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D}-\mu\gamma_4+m\right) > 0 \Rightarrow$ **free from sign problem**

SU(3) vs SU(2)

Differences between SU(3) and SU(2) QCD

- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons ($N_f = 2$) $\pi^+, \pi^-, \pi^0, d, \bar{d}$

SU(3) vs SU(2)

Similarities:

- There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of observables are equal up to few dozens percent:

Topological susceptibility (Nucl.Phys.B715(2005)461):
$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)),\quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

Critical temperature (Phys.Lett.B712(2012)279):
$T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)),\quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

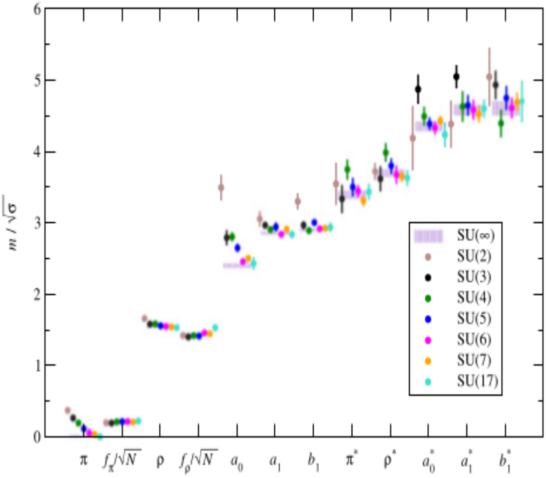
Shear viscosity :
$\eta/s = 0.134(57)\ (SU(2)),\quad \eta/s = 0.102(56)\ (SU(3))$

JHEP 1509(2015)082 Phys.Rev. D76(2007)101701

SU(3) vs SU(2)

Similarities:

- Spectroscopy (Phys.Rep.529(2013)93)



SU(3) vs SU(2)

To summarize:

- Dense SU(2) QCD can be used to study dense SU(3) QCD
  - Calculation of different observables
  - Study of different physical phenomena
- Lattice study of SU(2) QCD contains full dynamics of real system (contrary to phenomenological models)

SU(3) vs SU(2)

To summarize:

- Dense SU(2) QCD can be used to study dense SU(3) QCD
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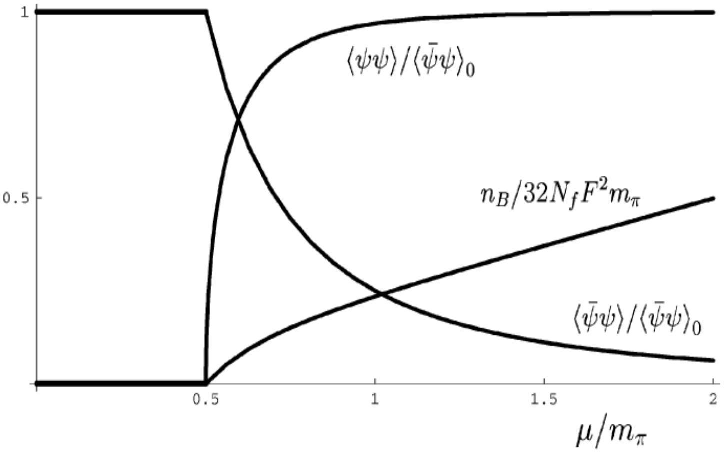
The aim: numerical study of dense SU(2) QCD within lattice simulation

Details of the simulation (previous study)

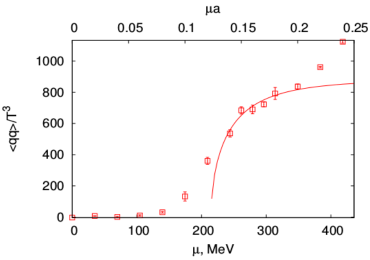
- Staggered fermions with rooting: $N_f = 2$
- Lattice $16^3 \times 32$, $a = 0.11$ fm, $m_\pi = 362(4)$ MeV, $T = 55$ MeV
- Diquark source in the action $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$

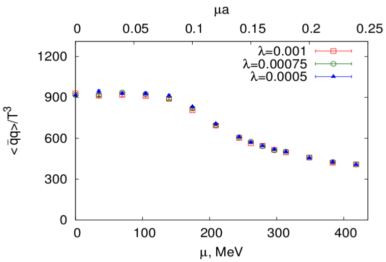
- The symmetry breaking is different
  - Continuum: $SU(2N_f) \to Sp(2N_f)$
  - Staggered fermions: $SU(2N_f) \to O(2N_f)$
- Correct symmetry is restored in continuum limit
  - Naive limit $a \to 0$: two copies of $N_f = 2$ fundamental fermions
  - Correct spectroscopy
  - Correct β function for $a < 0.17$ fm

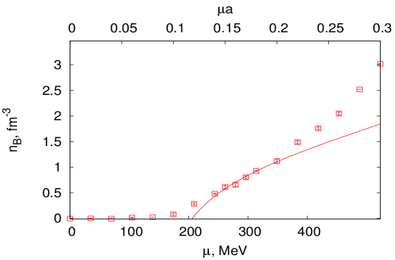
Predictions of CHPT



Diquark & chiral condensate, baryon density $\mu < 350$ MeV







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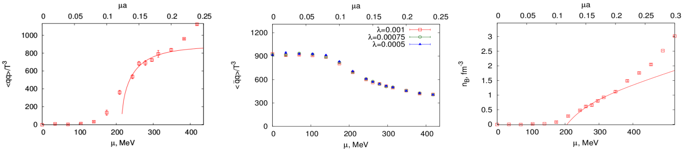


- ChPT: $\langle \psi\psi \rangle / \langle \bar{\psi}\psi \rangle_0 = \sqrt{1 - \frac{m_\pi^4}{\mu^4}}$, $n \sim \mu - \frac{m_\pi^4}{\mu^3}$
- Good agreement with ChPT
- Phase transition at $\mu \sim m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200, 350)$ MeV
- Transition: dilute baryon gas $\rightarrow$ dense matter

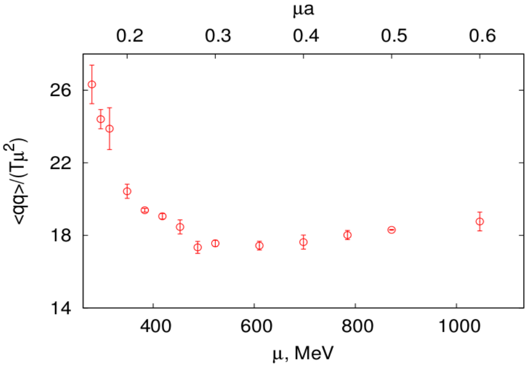
Large chemical potential
$\mu > 350$ MeV

Phase diagram for $N_c \to \infty$

- Hadron phase $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
  - Degrees of freedom:
    - Baryons (on the surface)
    - Quarks (inside the Fermi sphere $|p| < \mu$)
  - No chiral symmetry breaking
  - The system is in confinement phase
- Deconfinement

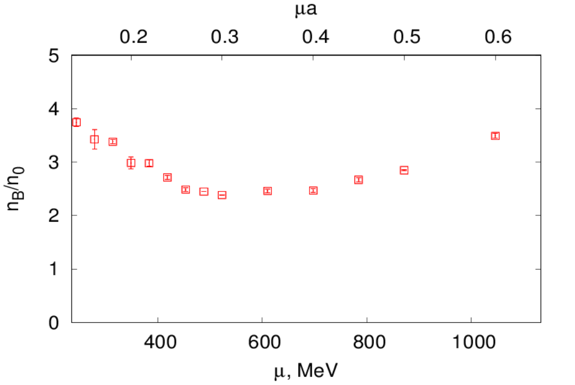
(L. McLerran, R.D. Pisarski, Nucl.Phys. A796 (2007) 83-100)

Diquark condensate



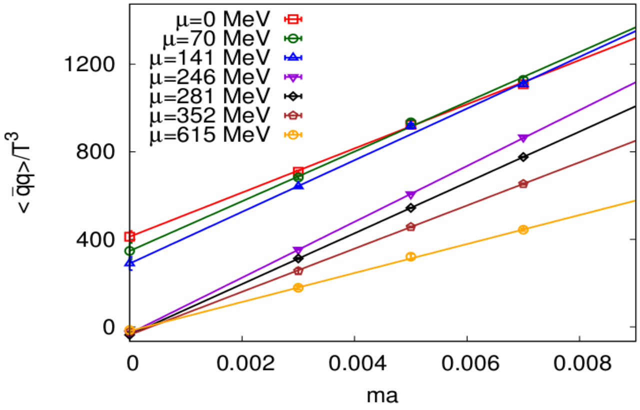
- Bardeen–Cooper–Schrieffer (BCS) phase $\mu > 500$ MeV, $\langle \psi\psi \rangle \sim \mu^2$
- **Baryons (on the surface)**

Baryon density



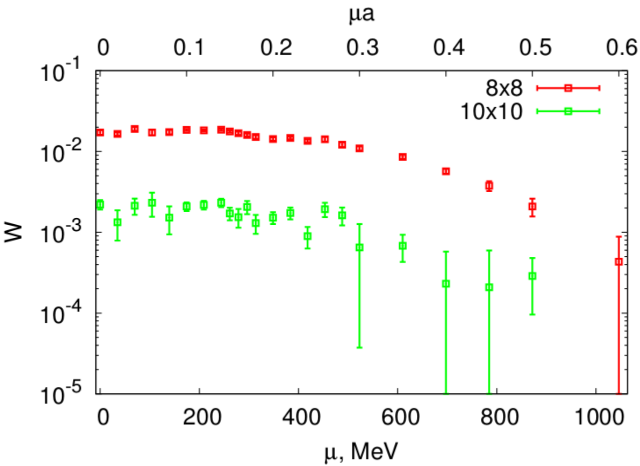
- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3}\theta(|p|-\mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$ $(n \sim (5-10)\times$nuclear density$)$

Chiral condensate (chiral limit $m \to 0$)



No chiral symmetry breaking

Wilson loop



Polyakov loop is zero within the uncertainty of the calculation

Conclusions (intermediate)

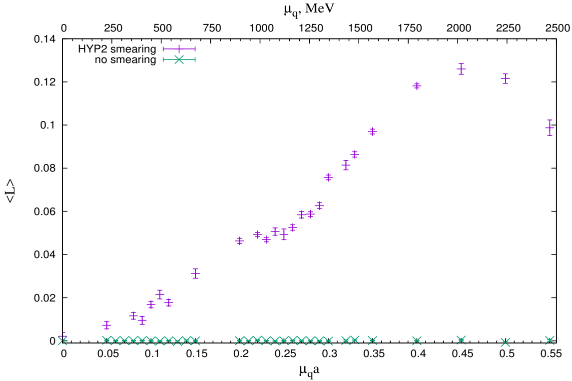
- We observe $\mu < m_\pi/2$ hadronic phase
- Transition at $\mu \simeq m_\pi/2$ (BEC)
- $\mu > m_\pi/2, \mu < m_\pi/2 + 150$ MeV dilute baryon gas
- Hadronic phase and BEC phase are well described by CHPT
- Deviation from CHPT from $\mu > 350$ MeV (dense matter)
- BCS phase $\mu \sim 500$ MeV, transition BEC$\rightarrow$BCS is smooth
- The system is always confined

Details of the simulation (present study)

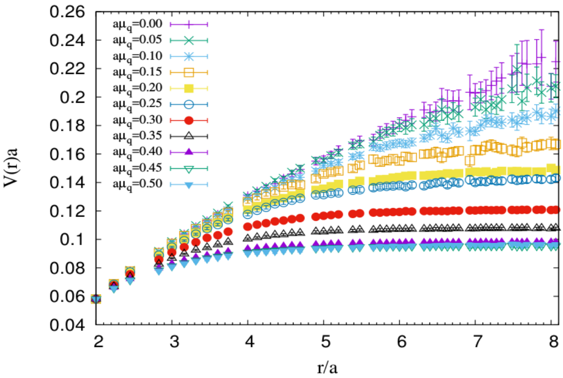
- Tree-level improved gauge action
- $a = 0.044$ fm ($\sqrt{\sigma} = 440$ MeV)
  present study: $\sqrt{\sigma}a = 0.10$ previous study: $\sqrt{\sigma}a = 0.29$
  $\Rightarrow$ **closer to continuum limit**
- $m_\pi = 720(40)$ MeV ($m_\pi L_s \simeq 5$)
- Lattice size $32^3 \times 32$ ($T \simeq 0$)
- Fixed λ parameter

Preliminary results!

Polyakov loop

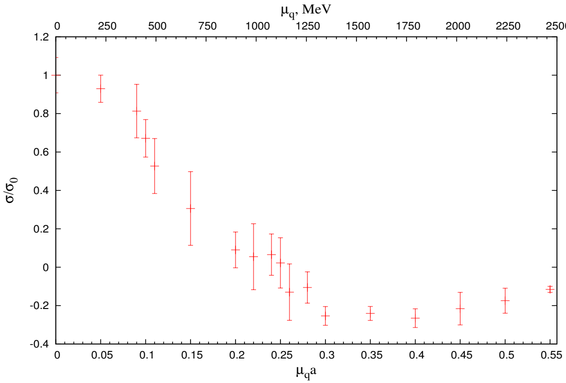


Potential of static charges ($T \simeq 0$)



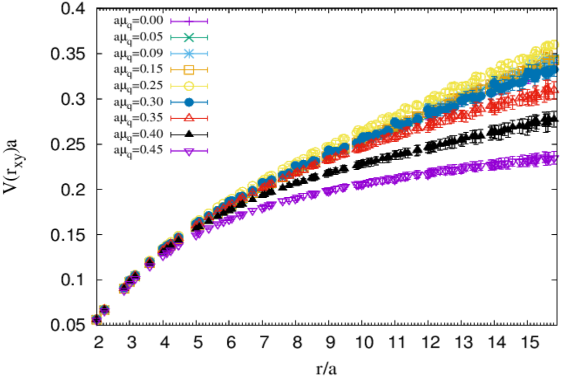
- We observe **deconfinement in dense medium!**

String tension

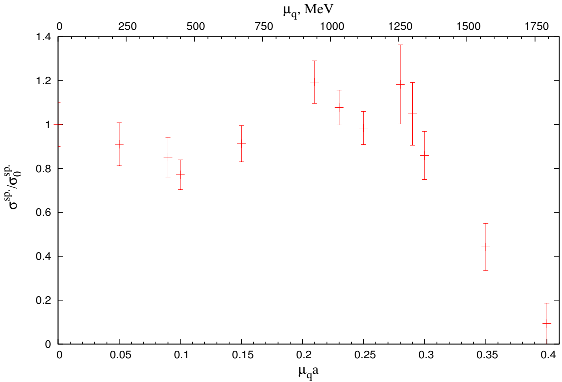


- The Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r$

Spatial potential $V(r)$



Spatial string tension



- Deconfinement at $a\mu > 0.25 - 0.3$?

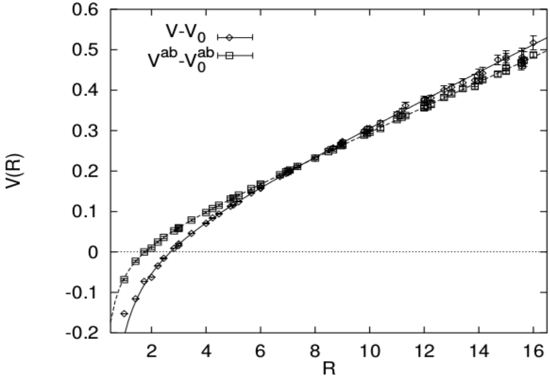
- We observe deconfinement in dense medium $a\mu \sim 0.25 - 0.3$
- Difficult to determine critical chemical potential
- It is not possible to determine the critical chemical potential from susceptibilities

Abelian monopoles

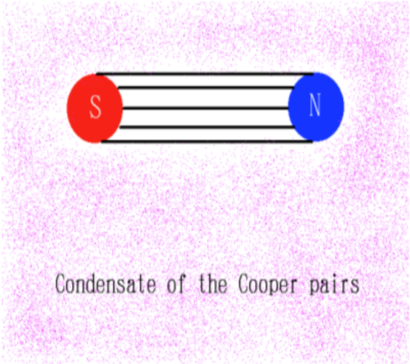
Maximal Abelian gauge

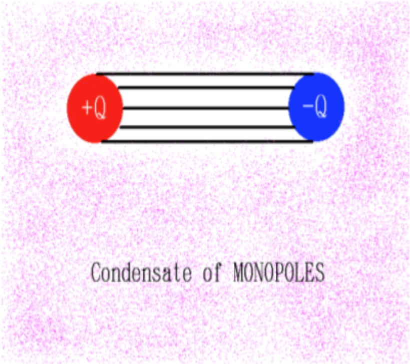
- SU(2) QCD
  $\hat{A} = A_1\hat{\sigma}_1 + A_2\hat{\sigma}_2 + A_3\hat{\sigma}_3$, $\sigma_{1,2,3}$–Pauli matrices
- Choose $\hat{A}$ maximally diagonal:
  $\max_\Omega R(A^\Omega), \quad R(A) = -\int d^4x(A_1^2 + A_2^2)$
- $\Omega_0 = diag(e^{-i\alpha(x)}, e^{i\alpha(x)})$ does not change $R(A)$
- Instead of the $SU(2)$ we study $U(1)$
- In $U(1)$ monopoles can be defined

Abelian dominance

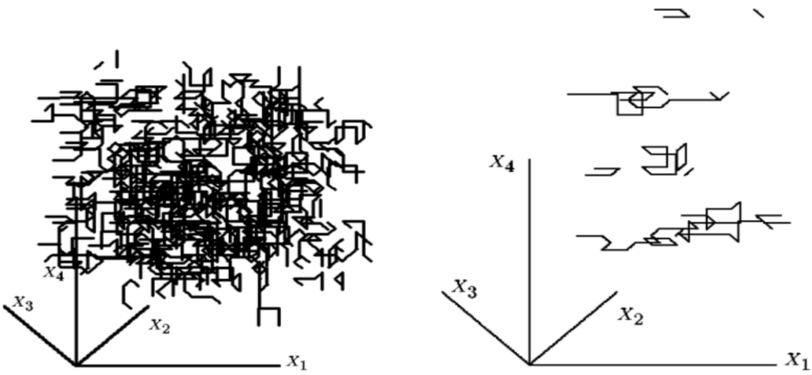


Model of dual superconductor





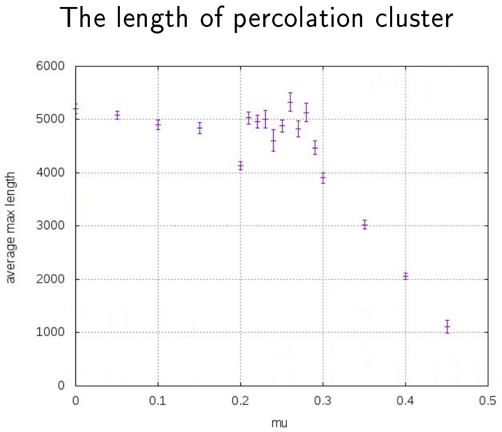
Condensation of monopoles



Percolation cluster (confinement/deconfinement transition)

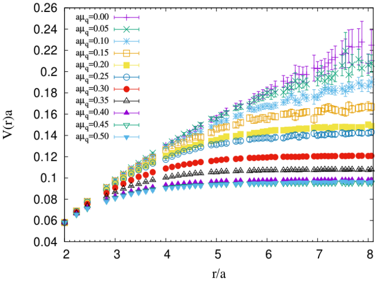
One can use Abelian monopoles to study confinement/deconfinement transition

The length of percolation cluster



- Percolation cluster disappears in the region $a\mu \in (0.2, 0.3)$
- Deconfinement transition $a\mu \in (0.2, 0.3)$

Conclusion



- We observe manifestations of deconfinement in the region $a\mu \in (0.2, 0.3)$
  - String tension
  - Disappearence of percolation cluster

Confinement/deconfinement transition in dense medium!