

Comparison of Hydrodynamics and Kinetic Transport Theory for $p+A$ and $A+A$ Collisions

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ICNFP2017, Kolymbari, August 2017

Hydrodynamics & BAMPS

Initial state

specific transversal distribution

longitudinal boost invariance

Results and Outlook

What can we learn?

BAMPS

Boltzmann Approach to Multi-Parton Scattering

■ (3+1)D Boltzmann equation

Z.Xu, C.Greiner, PRC 71 (2005) 064901
Z.Xu, C.Greiner, PRC 76 (2007) 024911

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \frac{\partial f}{\partial \vec{r}} = C_{2 \leftrightarrow 2} + C_{2 \leftrightarrow 3}$$

■ Massless particles: partons / quarks & gluons

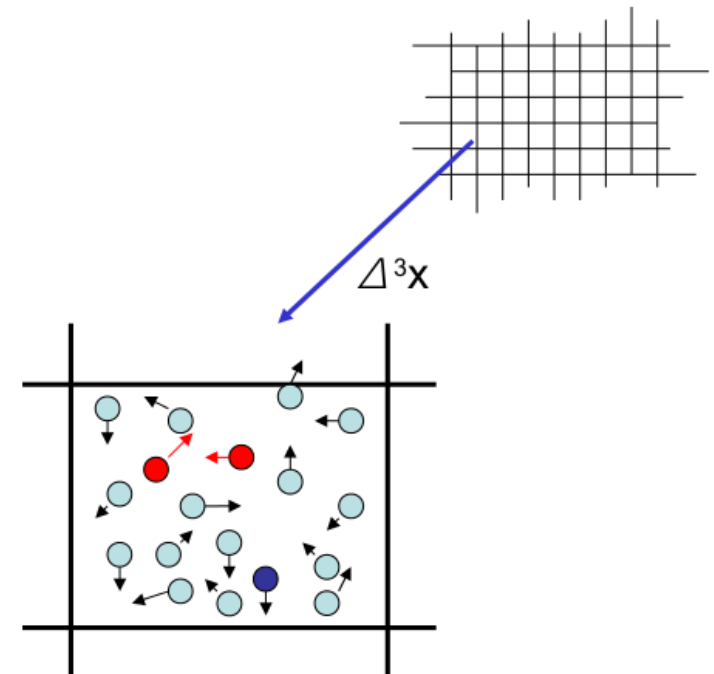
■ Discretized space and time

$$P_{2 \rightarrow 2} = v_{\text{rel}} \sigma_{2 \rightarrow 2} \frac{\Delta t}{\Delta V}$$

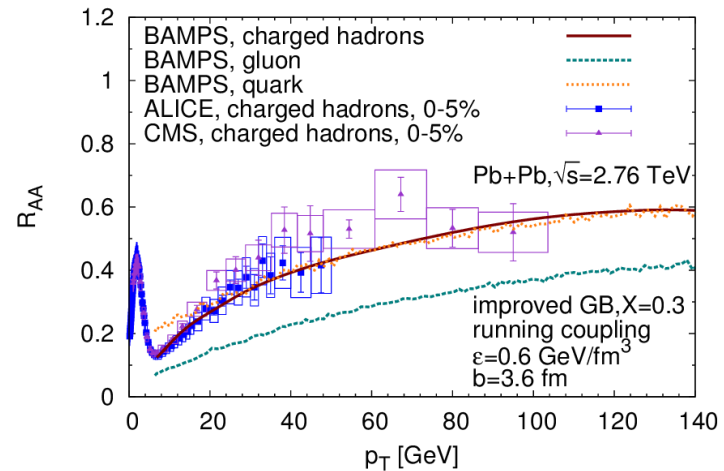
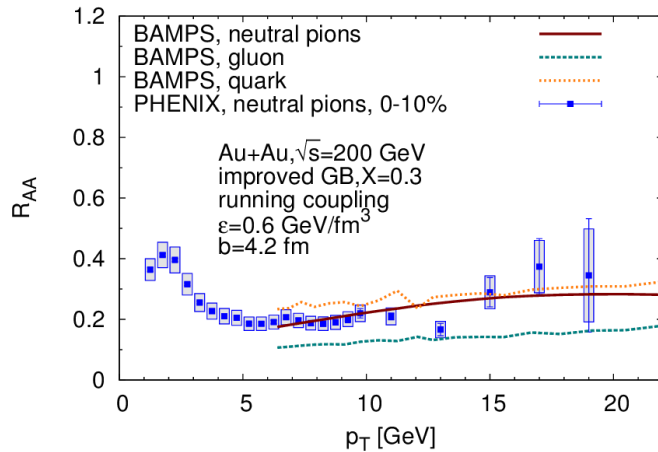
$$P_{2 \rightarrow 3} = v_{\text{rel}} \sigma_{2 \rightarrow 3} \frac{\Delta t}{\Delta V}$$

$$P_{3 \rightarrow 2} = \frac{I_{3 \rightarrow 2}}{8E_1 E_2 E_3} \frac{\Delta t}{(\Delta V)^2}$$

■ Testparticle ansatz: N_{test}

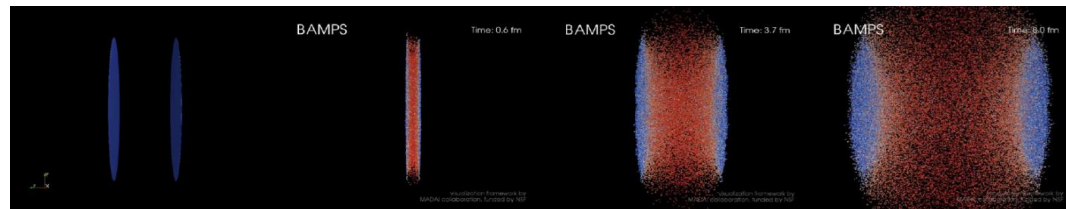


Nuclear modification factor R_{AA}

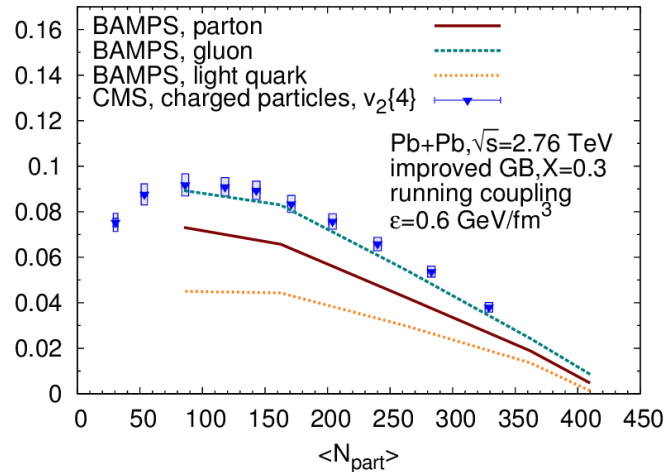
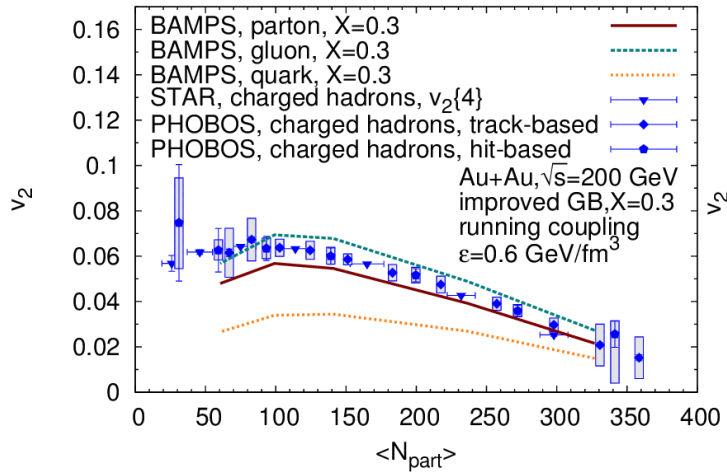


Phys. Rev. Lett. 114 (2015) 112301

- Hadronization of high p_t partons with AKK fragmentation functions
- LPM parameter fixed by comparison to RHIC data
- Realistic suppression both for RHIC and LHC

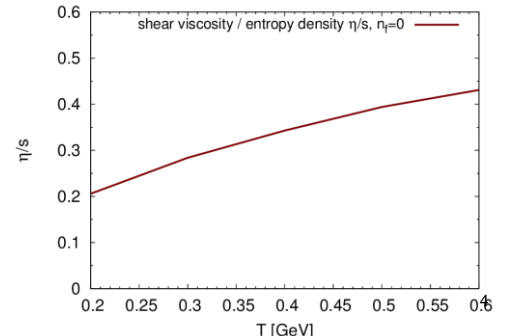


Elliptic flow v_2

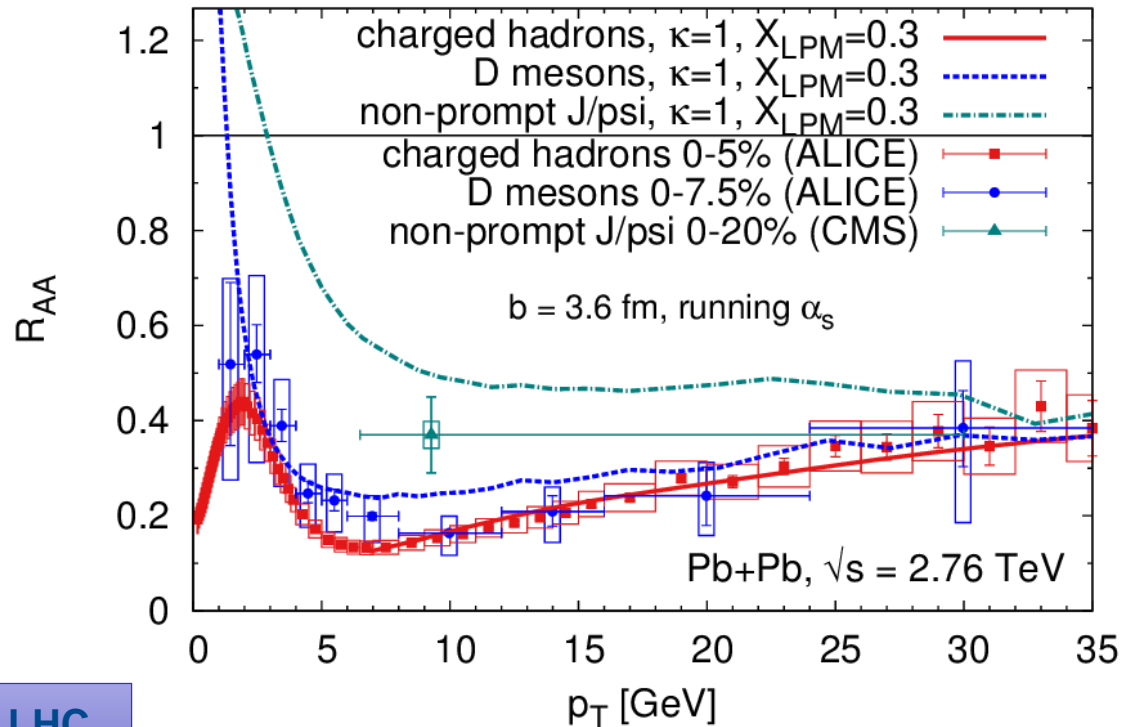


Phys. Rev. Lett. 114 (2015) 112301

- Same pQCD interactions lead to a sizeable elliptic flow for bulk medium
- No hadronization for bulk medium \rightarrow no hadronic after-burner



Heavy flavor and charged hadron R_{AA} at LHC



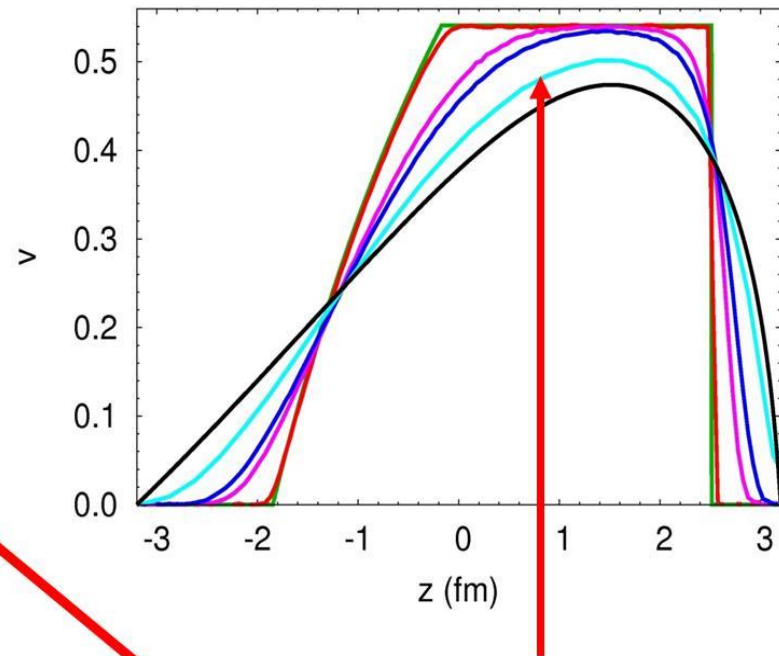
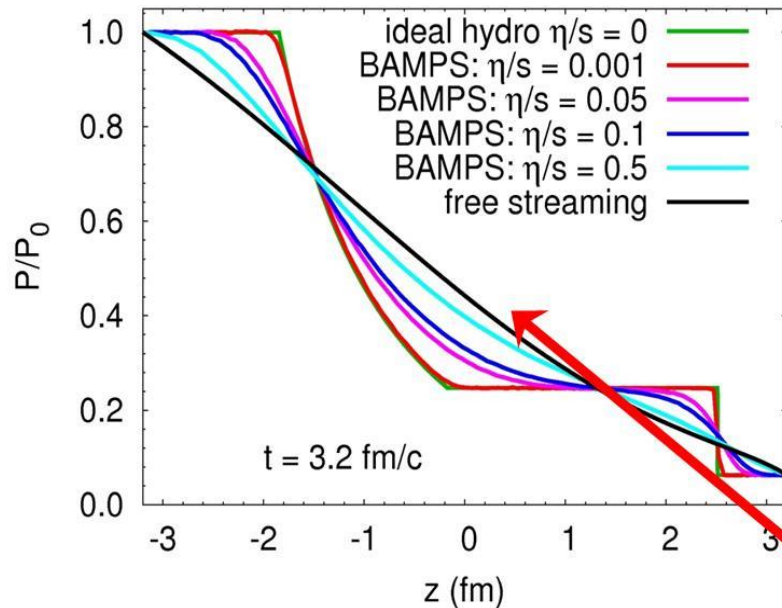
LHC



Riemann problem at finite viscosity

$$p^\mu \partial_\mu f = C$$

I. Bouras et al, PRL 103:032301 (2009)



$T_{\text{left}} = 400 \text{ MeV}$
 $T_{\text{right}} = 200 \text{ MeV}$
 $t = 1.0 \text{ fm}/c$

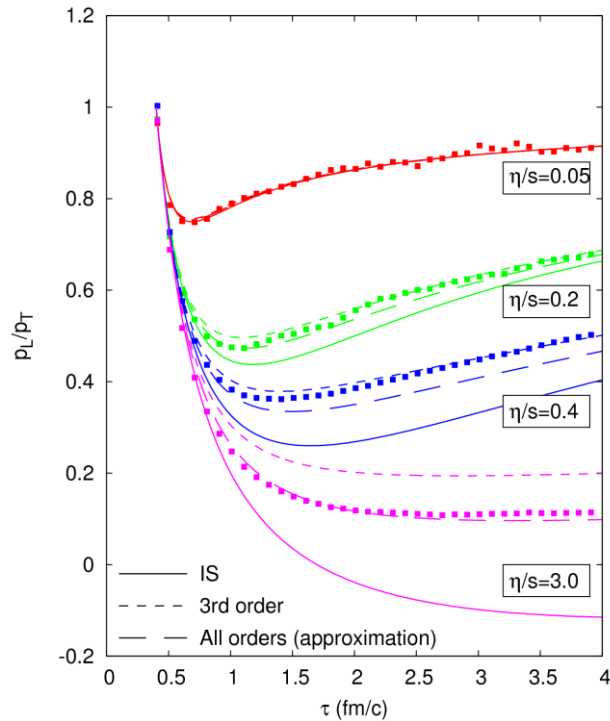
Development of a shock plateau

η/s less than 0.1-0.2

Hydro vs BAMPS in 1D

A. El, Z. Xu, C. Greiner, PRC 81 (2010) 041901

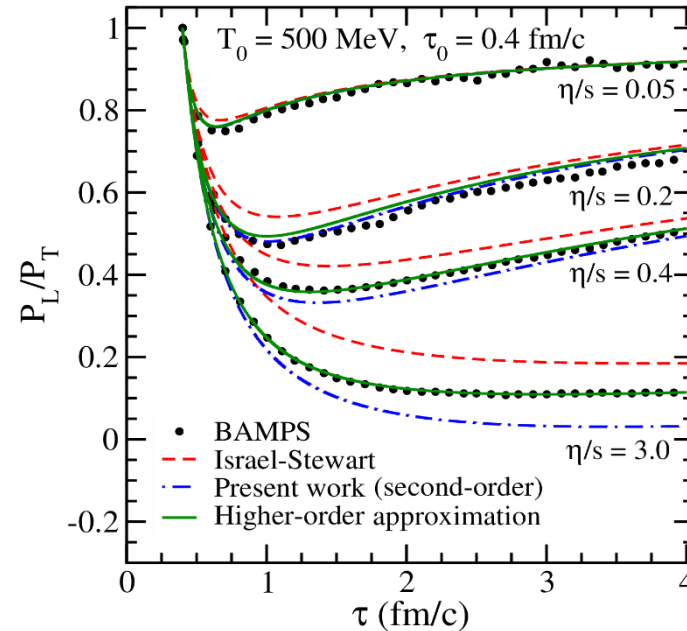
$$\frac{P_L}{P_T} = \frac{p - \pi}{p + \pi/2}$$



x=0: Israel-Stewart

x=3: third-order rel. diss. hydro

x=5/3: approximative 'all-orders'



A. Jaiswal, Phys.Rev.C87:051901,2013

> Resummation works at strong dissipation (large Knudsen number!).

Relativistic Fluid Dynamics

Conservation laws & tensor decompositions

$$\partial_\mu N^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$N^\mu = n u^\mu + V^\mu$$

$$T^{\mu\nu} = e u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$n = u_\mu N^\mu$$

LRF particle density

$$e = u_\mu T^{\mu\nu} u_\nu$$

LRF energy density

$$V^\mu = \Delta^\mu_\alpha N^\alpha$$

particle diffusion current

$$p(e, n) + \Pi = -\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu}$$

isotropic pressure ($p_{eq} + bulk$)

$$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$$

shear stress tensor

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$T^{\langle\mu\nu\rangle} = \left[\frac{1}{2} \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] T^{\alpha\beta}$$

Relativistic Fluid Dynamics

- Transient / second order fluid dynamics (e.g. Israel & Stewart)

$$\tau_\pi \frac{d}{d\tau} \pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \tau_\pi C \pi^{\mu\nu} (\nabla_\alpha u^\alpha) + \dots$$

$$\tau_V \frac{d}{d\tau} V^{\langle\mu\rangle} + V^\mu = \kappa \nabla^\mu \alpha + \tau_V C' V^\mu (\nabla_\alpha u^\alpha) + \dots$$

($\pi^{\mu\nu}$ and V^μ independent variables)

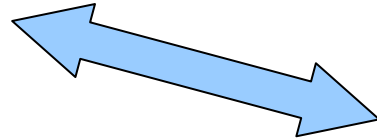
- Second order coefficients from
G.S.Denicol, H.Niemi, E.Molnar, D.H.Rischke, PRD 85, 114047 (2012)
- Expansion in Knudsen and (inverse) Reynolds number

$$\text{Kn} = \frac{\ell_{\text{micr}}}{L_{\text{macr}}} \quad R_V^{-1} \sim \frac{|V^\mu|}{n_0}, \quad R_\pi^{-1} \sim \frac{|\pi^{\mu\nu}|}{P_0}$$

- Hydrodynamical limit: $\text{Kn} \ll \mathcal{O}(1)$ and $R^{-1} \ll \mathcal{O}(1)$

Comparison Hydro / BAMPS

Collectivity in Heavy Ion Collision?
Fast Thermalization?
Flow?



**How small can system be,
how large can gradients be,
until discrepancies occur?**

- Longitudinal: Boost invariant
- Transversal:
 - Radial symmetric, large/small system
 - Glauber; overlapping Woods-Saxon

$$\langle r^2 \rangle = (3\text{fm})^2, (1\text{fm})^2$$

Comparison 1: Radial symmetric

■ Longitudinal: boost invariant

■ Transversal:

■ Rotational symmetric

■ Gaussian density profile, $\langle r^2 \rangle = (3\text{fm})^2$ or $\langle r^2 \rangle = (1\text{fm})^2$

A+A

p+A, p+p

■ Temperature

$$T_0(r), T_0(0) = 500 \text{ MeV}$$

$$\tau_0 = 0.2 \text{ fm}$$

Fugacity

$$\lambda_0(r) = 1$$

start in full equilibrium

■ only gluons

■ Cross section:

■ Elastic

■ Isotropic

■ Constant

$$\sigma = 1, 5, 20, 50, 100 \text{ mb}$$

Comparison 2: Glauber

■ Longitudinal: boost invariant

■ Transversal:

■ Overlapping Woods-Saxon

■ Impact parameter dependence

A+A

selected value: 7.5 fm = ("nBC")

■ Temperature

Fugacity

$$T_0(r), T_0(0) = 500 \text{ MeV}$$

$$\lambda_0(r) = 1$$

$$\tau_0 = 0.2 \text{ fm}$$

start in full equilibrium

■ only gluons

■ Cross section:

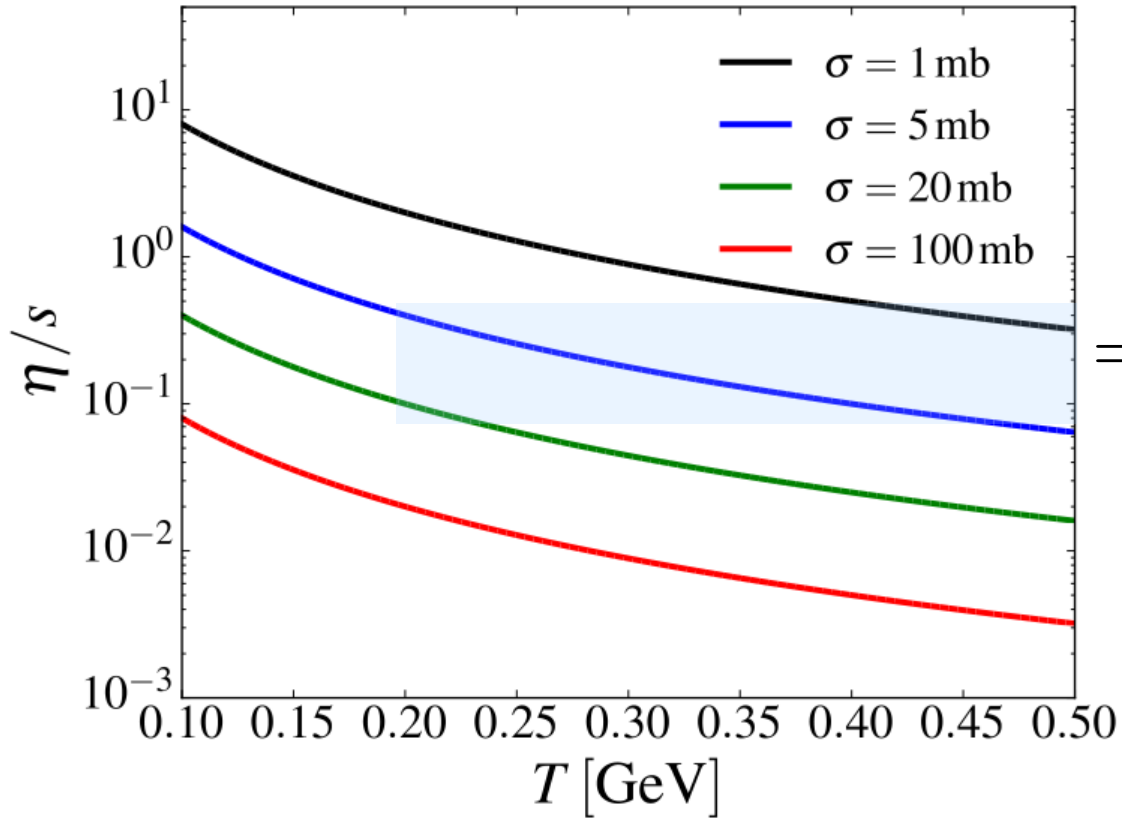
■ Elastic

■ Isotropic

■ Constant

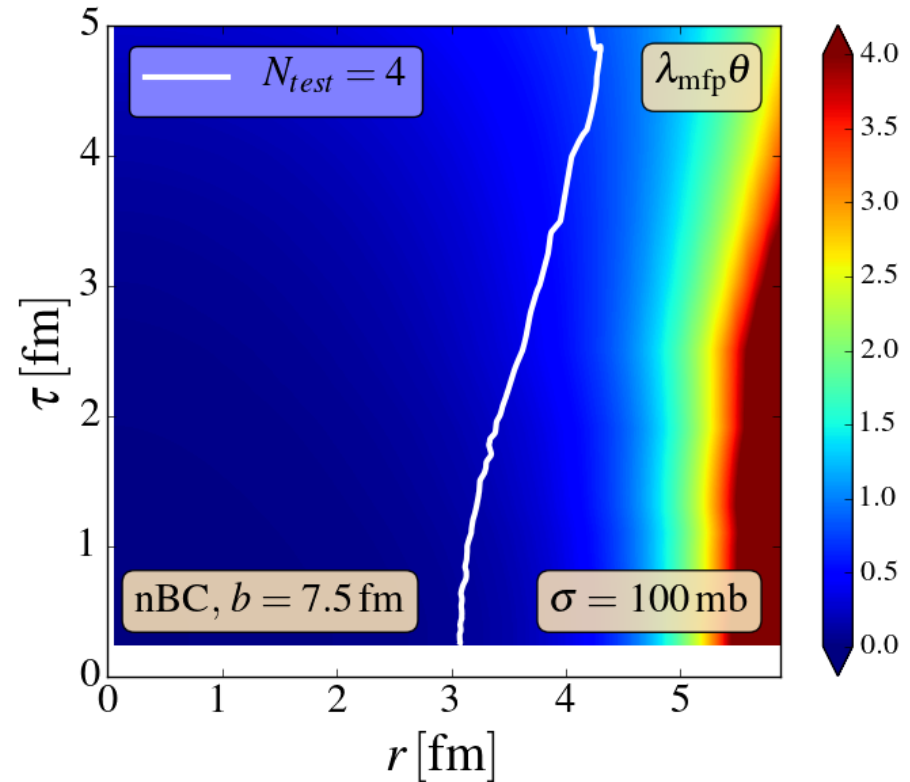
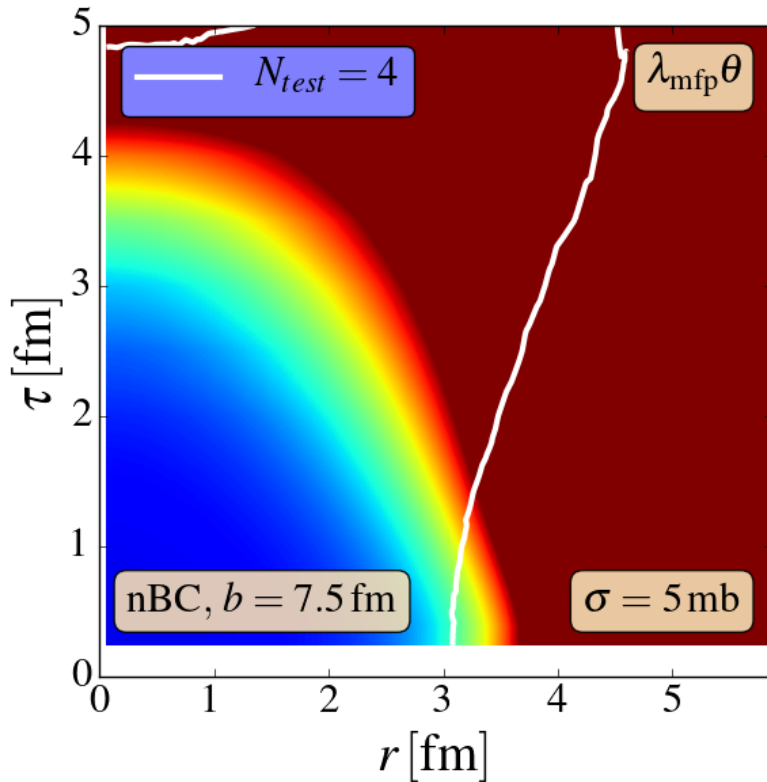
$$\sigma = 1, 5, 20, 50, 100 \text{ mb}$$

Available eta/s



$$\eta = \frac{4 T}{3 \sigma} \quad s = \frac{4g}{\pi^2} T^3 \quad (g = 16)$$

Knudsen number



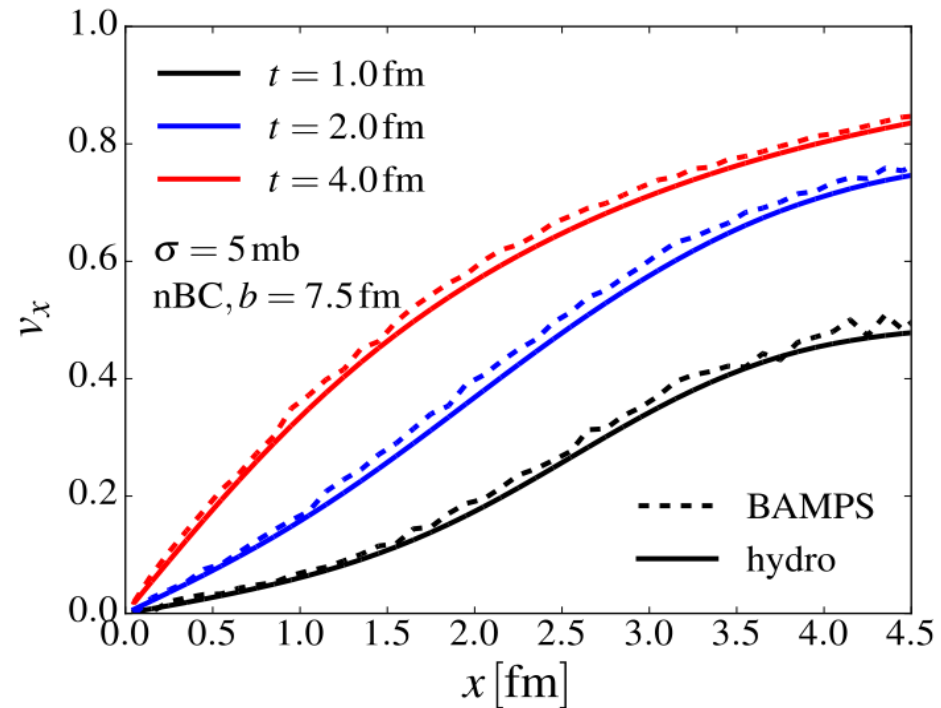
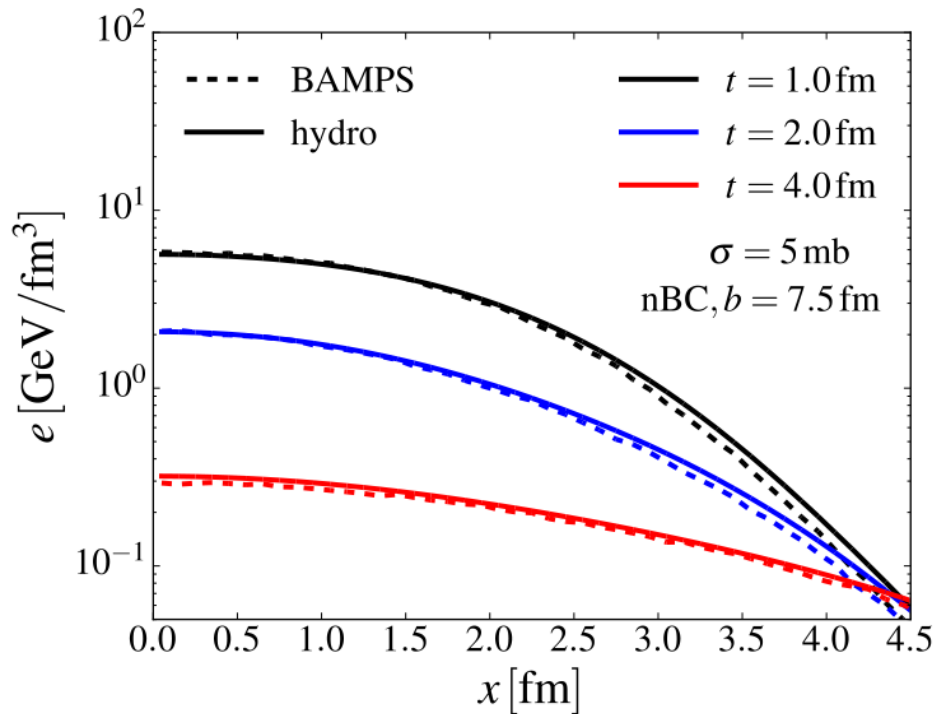
Hydrodynamical limit: $\text{Kn} = \frac{\ell_{\text{micr}}}{L_{\text{macr}}} \ll \mathcal{O}(1)$

$$\text{Kn} \equiv \lambda_{\text{mfp}} \theta \quad , \quad \theta = \partial_{\mu} u^{\mu}$$

Comparison 2: Glauber

A+A

Glauber, 5mb: energy density & velocity

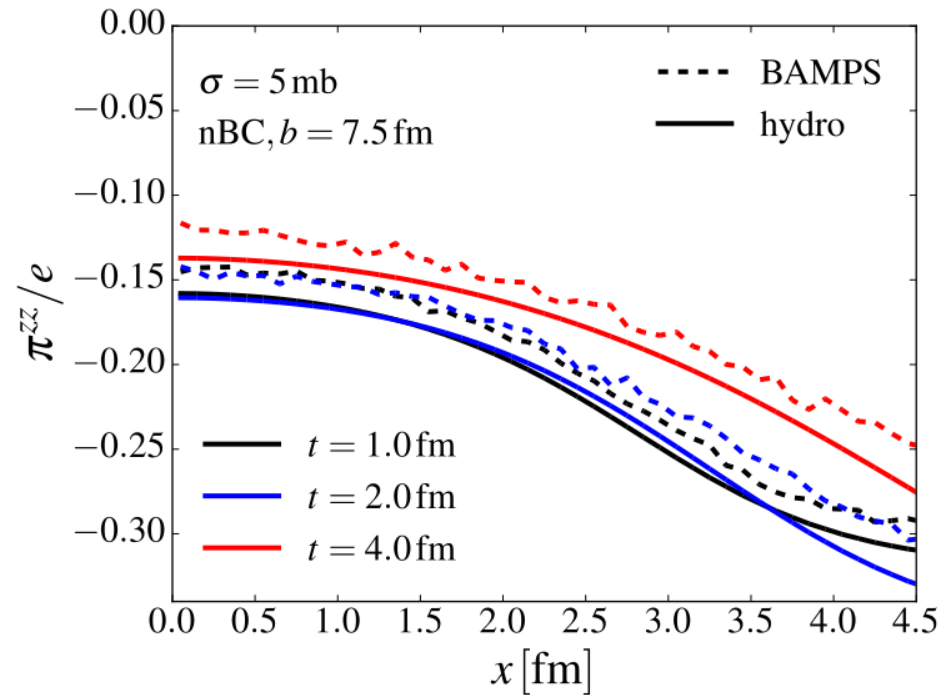
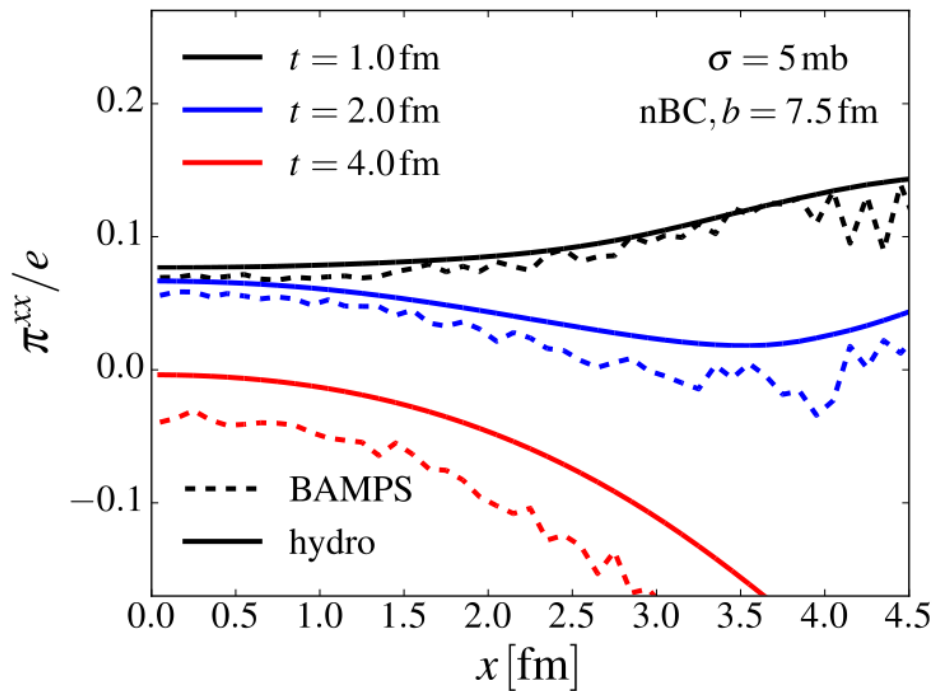


5mb: still very nice agreement

Comparison 2: Glauber

A+A

Glauber, 5mb: shear stress tensor



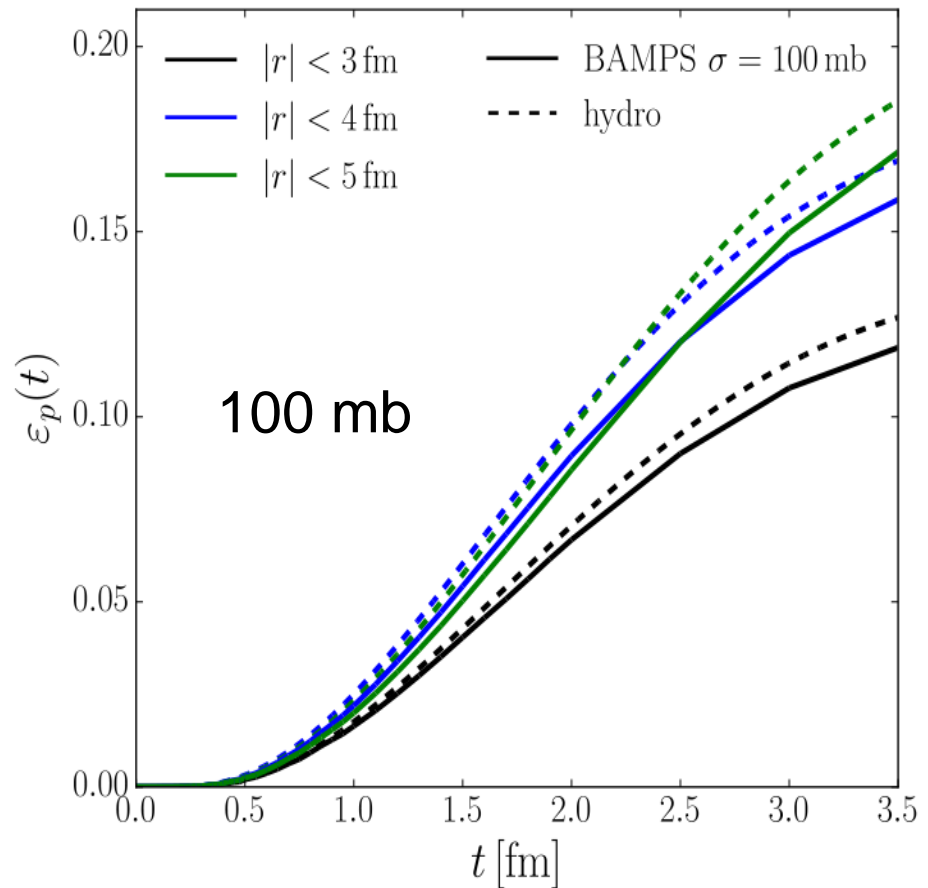
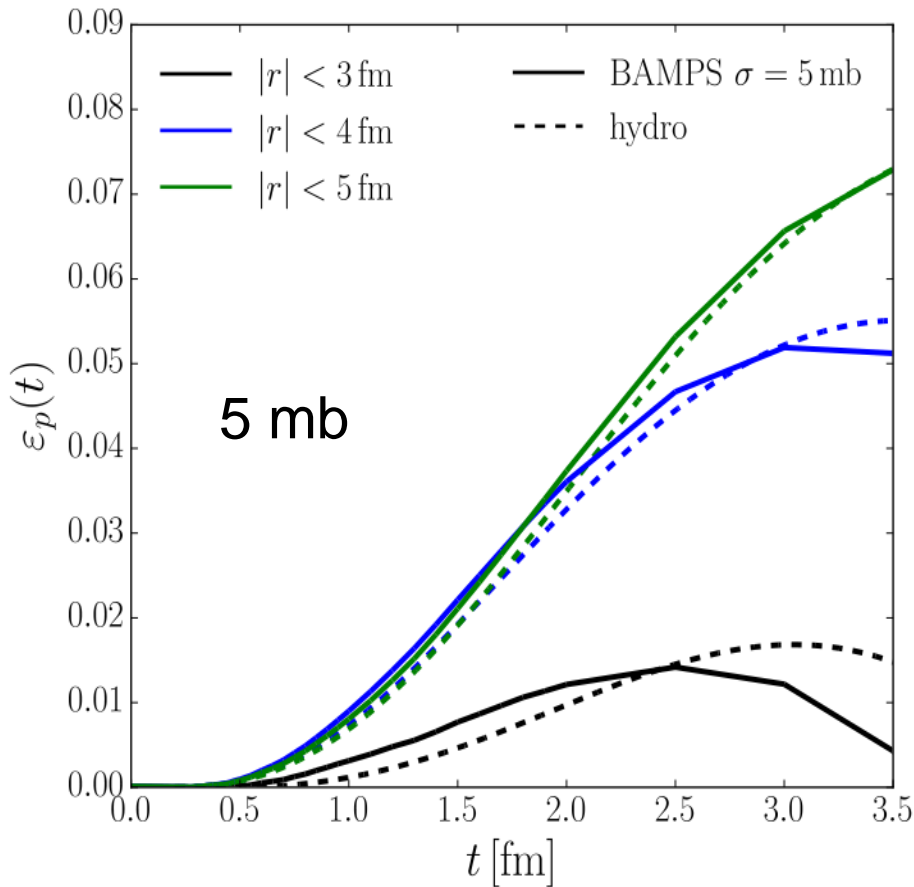
5mb: still very nice agreement

Comparison 2: Glauber

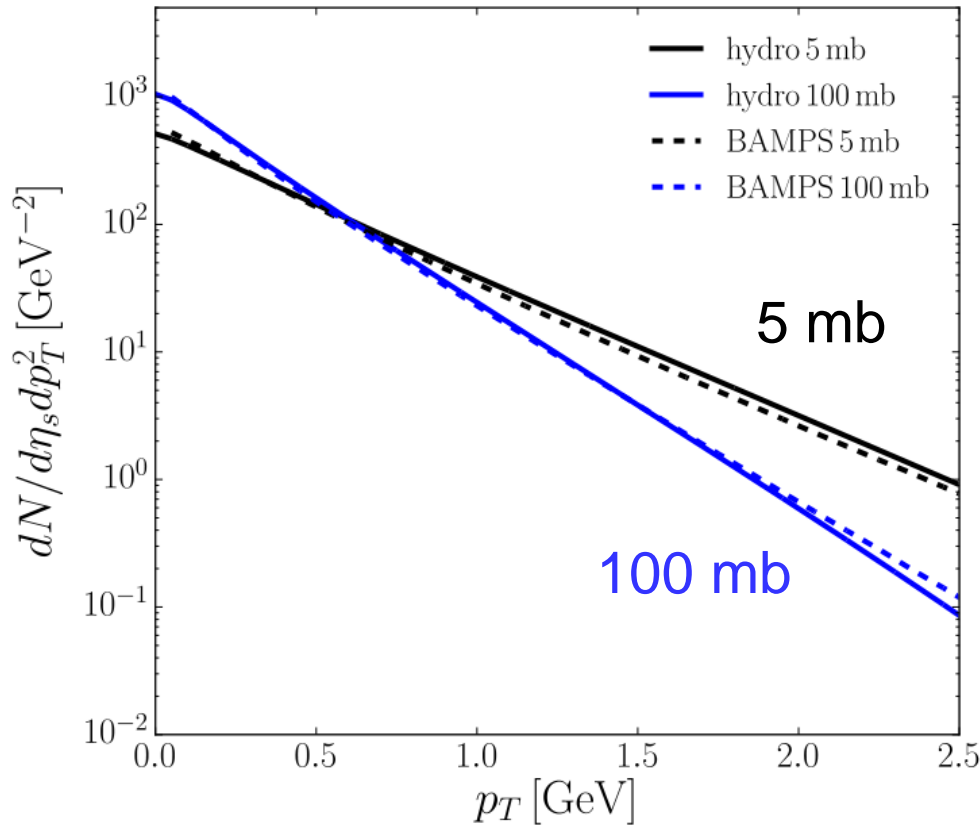
A+A

■ Asymmetry:

$$\varepsilon_P = \frac{\langle T^{xx} - T^{yy} \rangle_{xy}}{\langle T^{xx} + T^{yy} \rangle_{xy}}$$



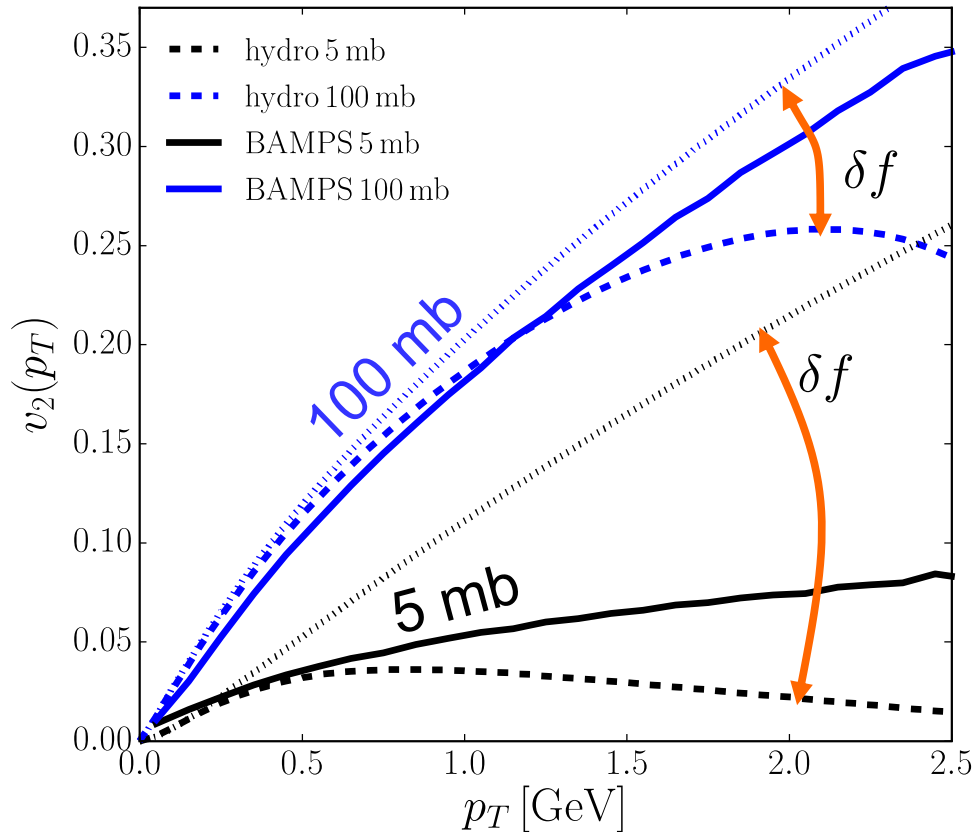
Spectra:



$$\langle p_T \rangle_{\sigma=5 \text{ mb}} = 0.77 \text{ GeV}$$

$$\langle p_T \rangle_{\sigma=100 \text{ mb}} = 0.5 \text{ GeV}$$

Flow:



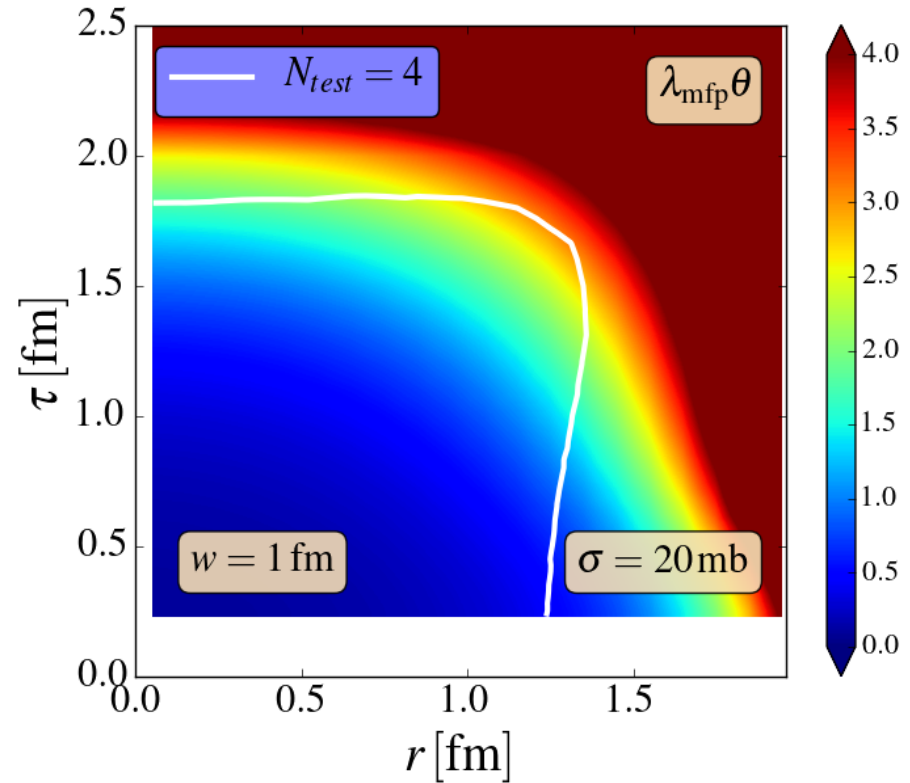
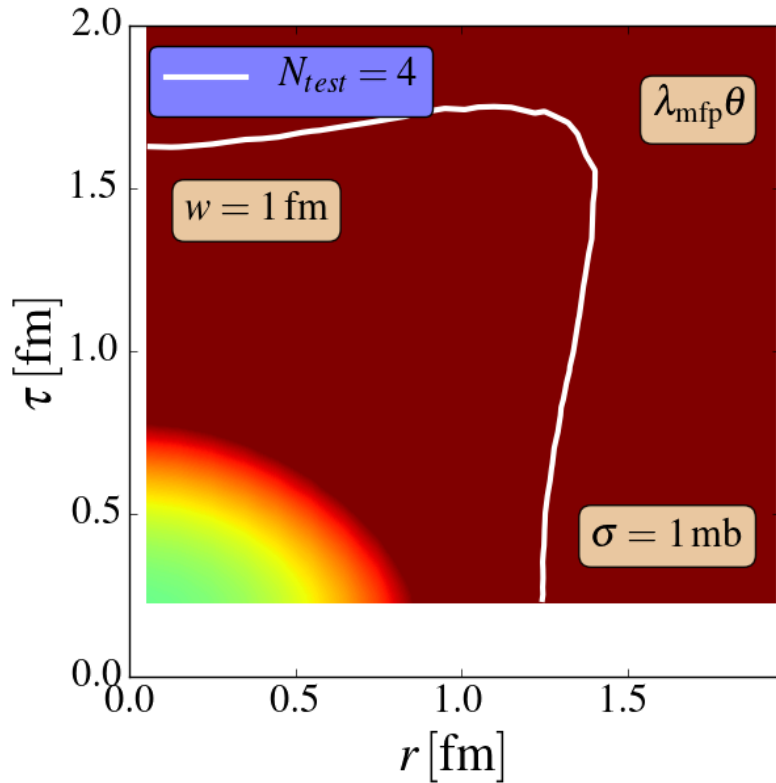
Large uncertainty due to viscous correction terms

Strong dependence on freeze out conditions

$$\delta f_{\mathbf{k}} = f_{0\mathbf{k}} \left(\frac{1}{8p_0 T^2} k_{\langle\mu} k_{\nu\rangle} \pi^{\mu\nu} - \frac{5}{p_0} k_{\mu} n^{\mu} + \frac{1}{p_0 T} E_{\mathbf{k}} k_{\mu} n^{\mu} \right)$$

Comparison 1: Radial symmetric (small)

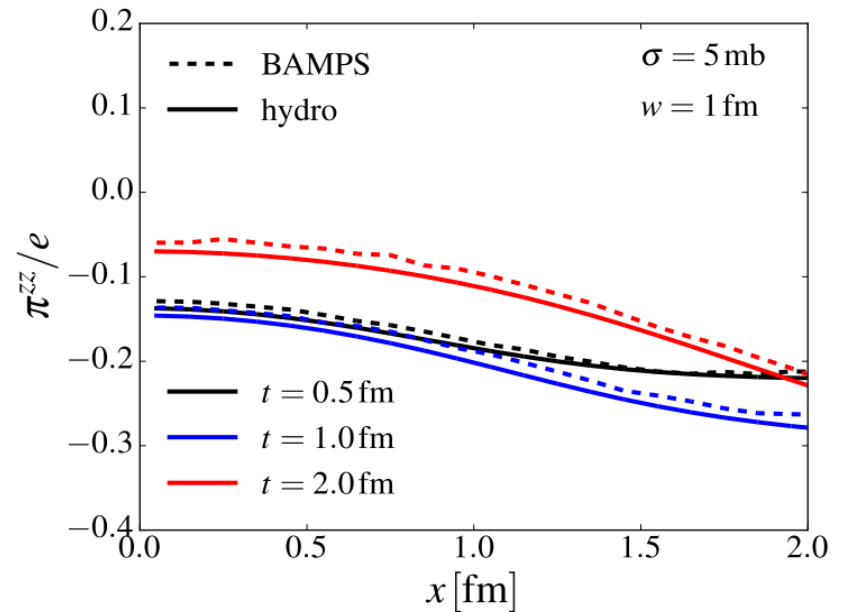
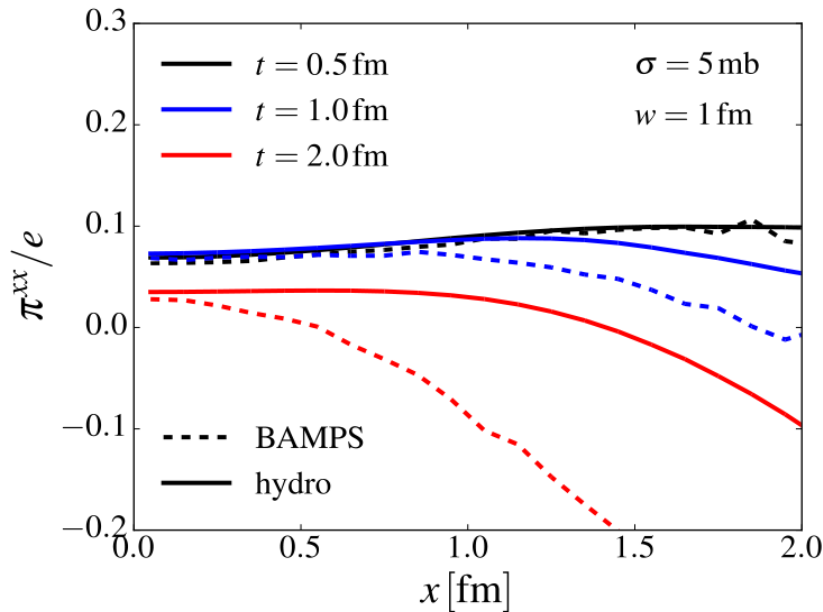
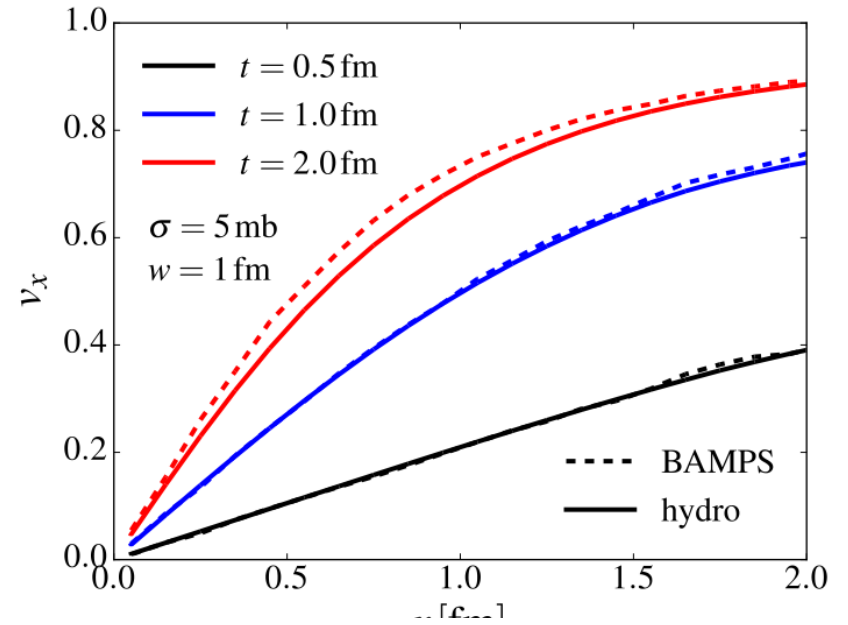
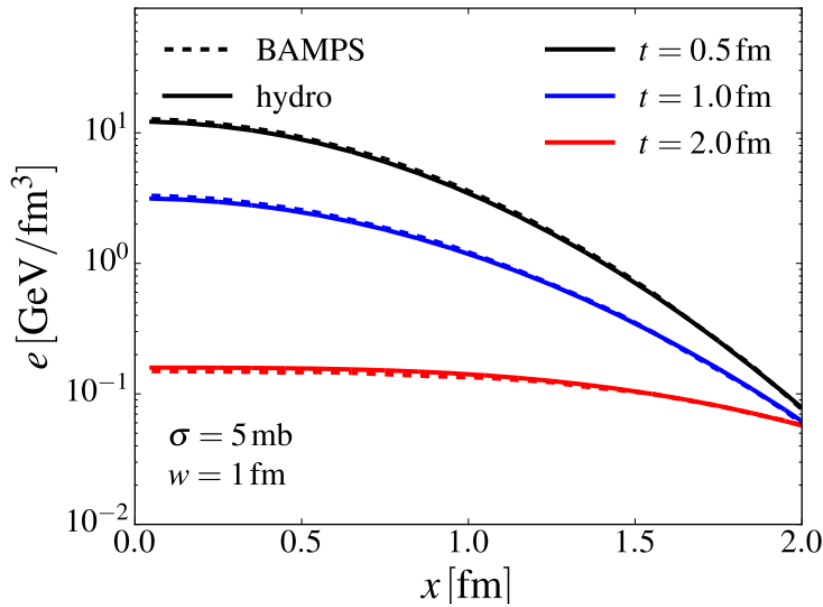
■ Knudsen number



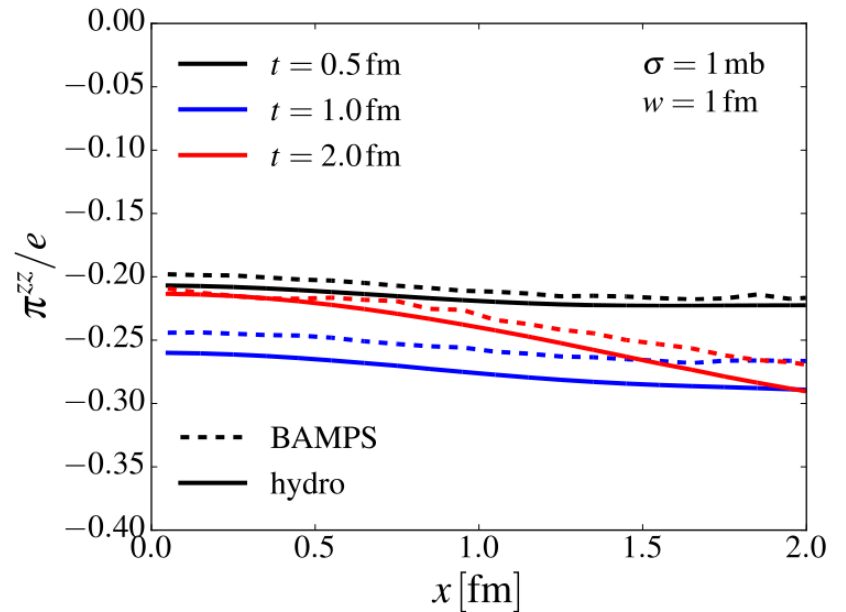
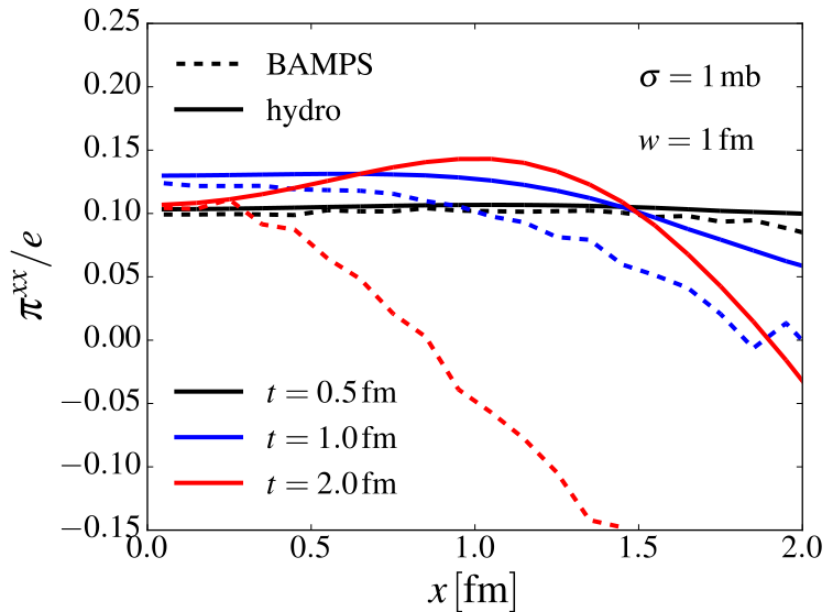
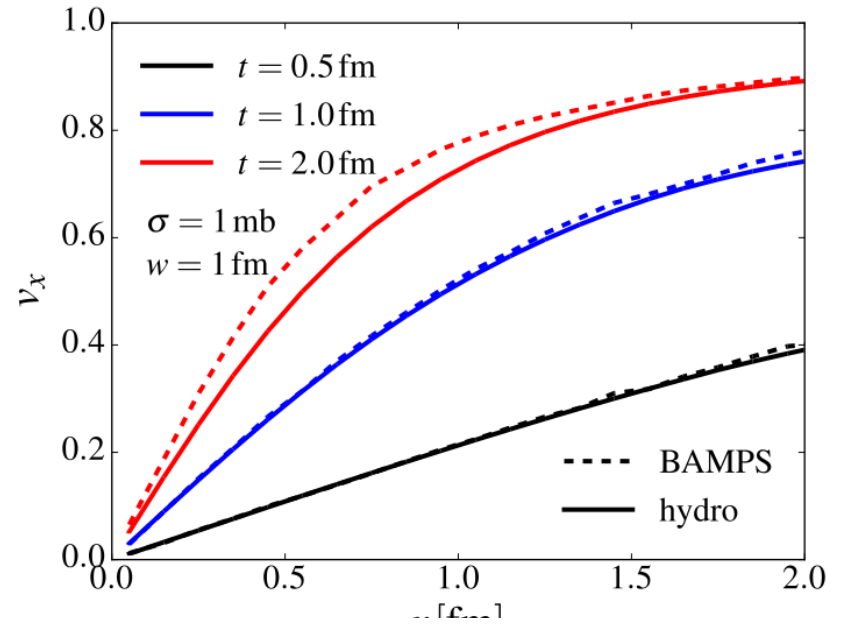
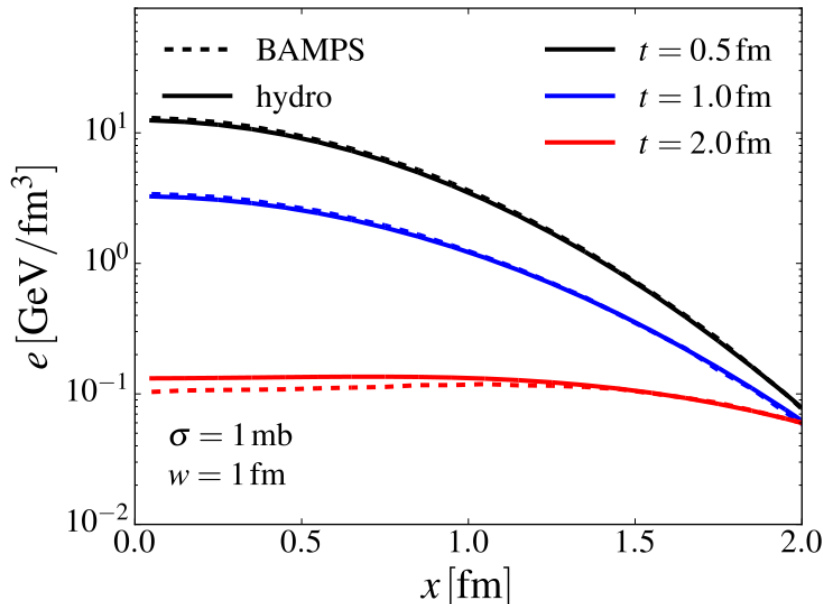
Hydrodynamical limit:
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$$\text{Kn} \equiv \lambda_{\text{mfp}} \theta \quad , \quad \theta = \partial_{\mu} u^{\mu}$$

Comparison 1: Radial symmetric (small, 5mb)



Comparison 1: Radial symmetric (small, 1mb)

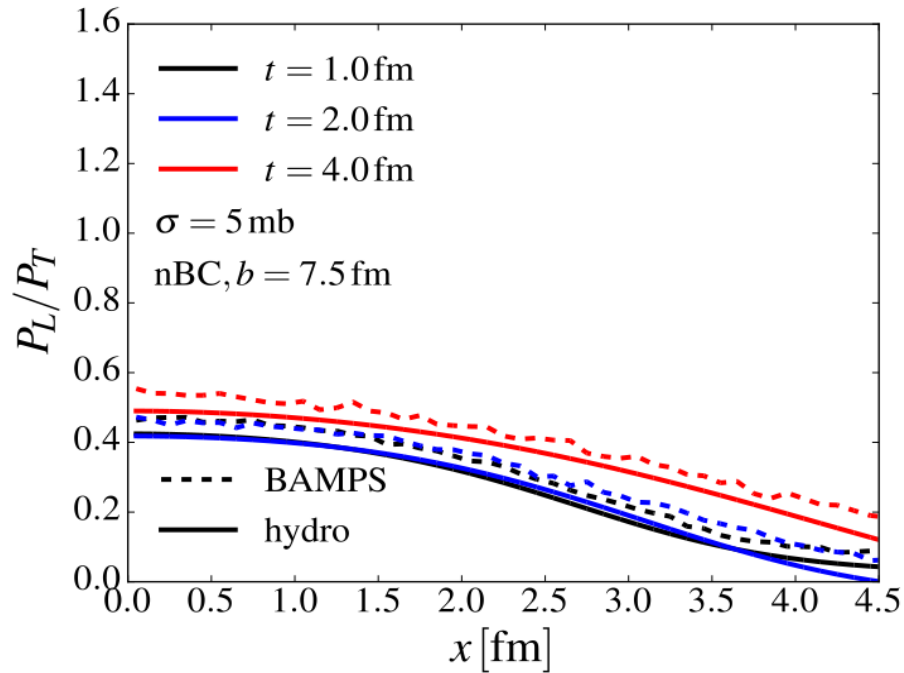


Comparison 2: Glauber

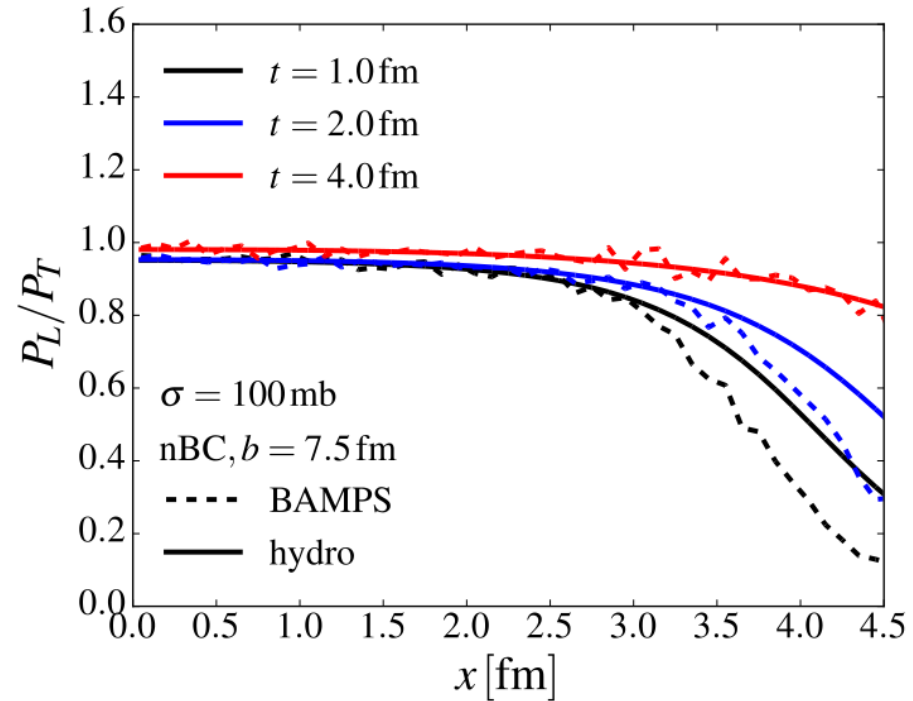
A+A

■ Pressure ratio: P_L/P_T (in the LRF)

5 mb



100 mb

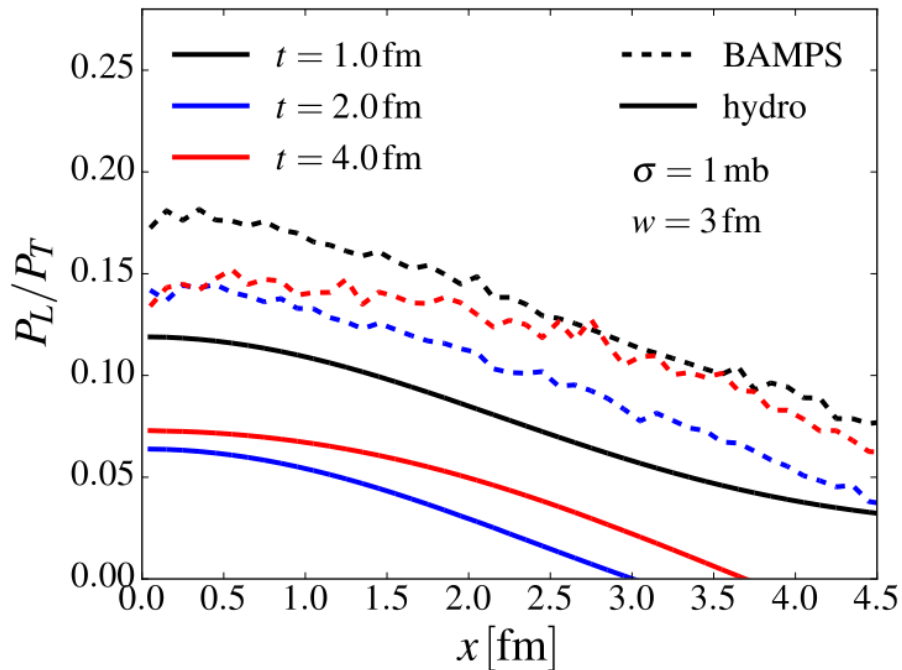


Comparison 1: Radial symmetric (large)

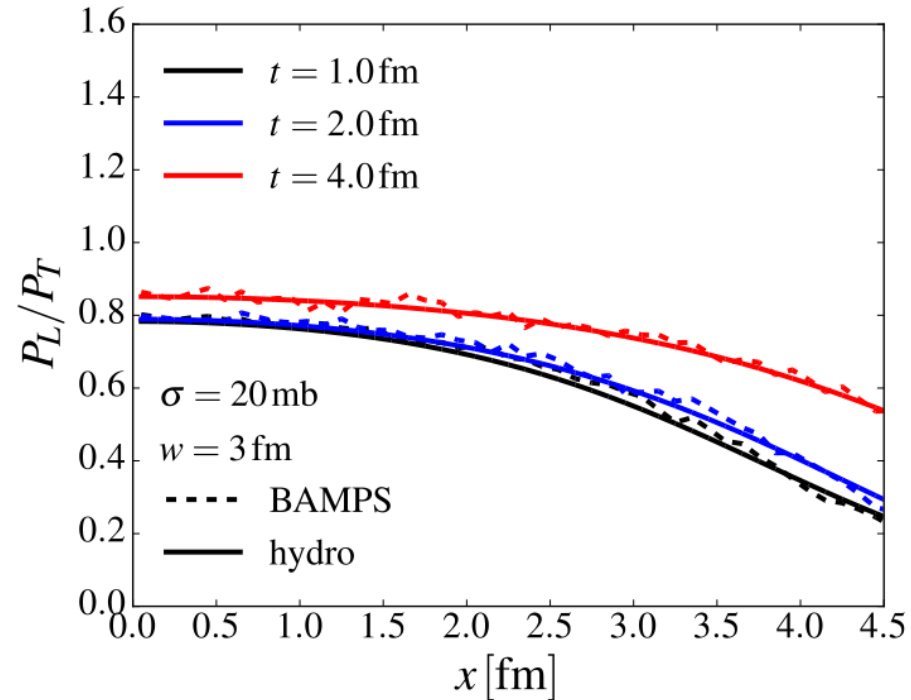
■ Pressure ratio: P_L/P_T (in the LRF)

$$\langle r^2 \rangle = (3\text{fm})^2$$

1 mb



20 mb

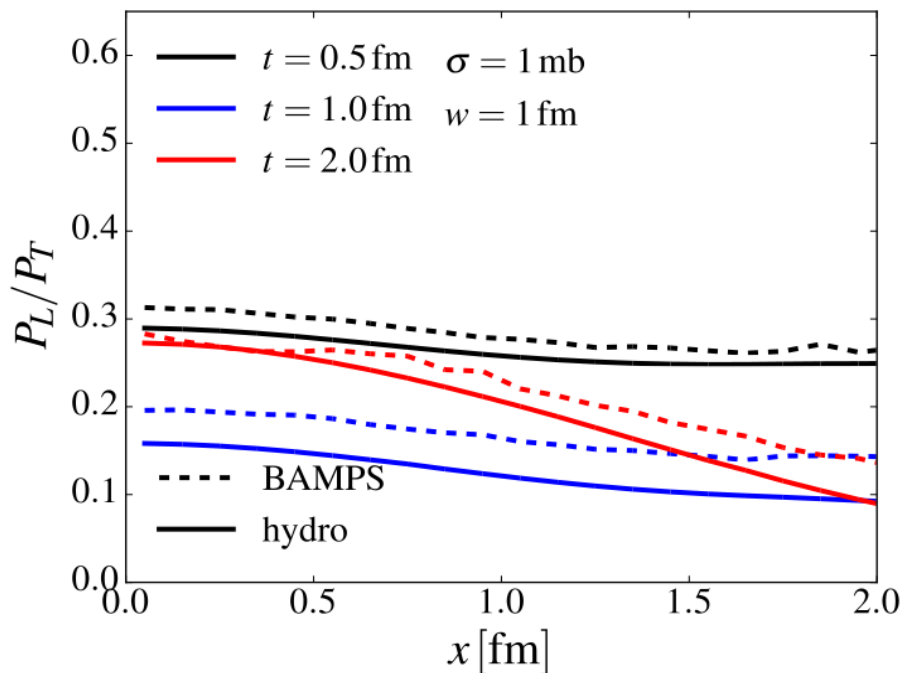


Comparison 1: Radial symmetric (small)

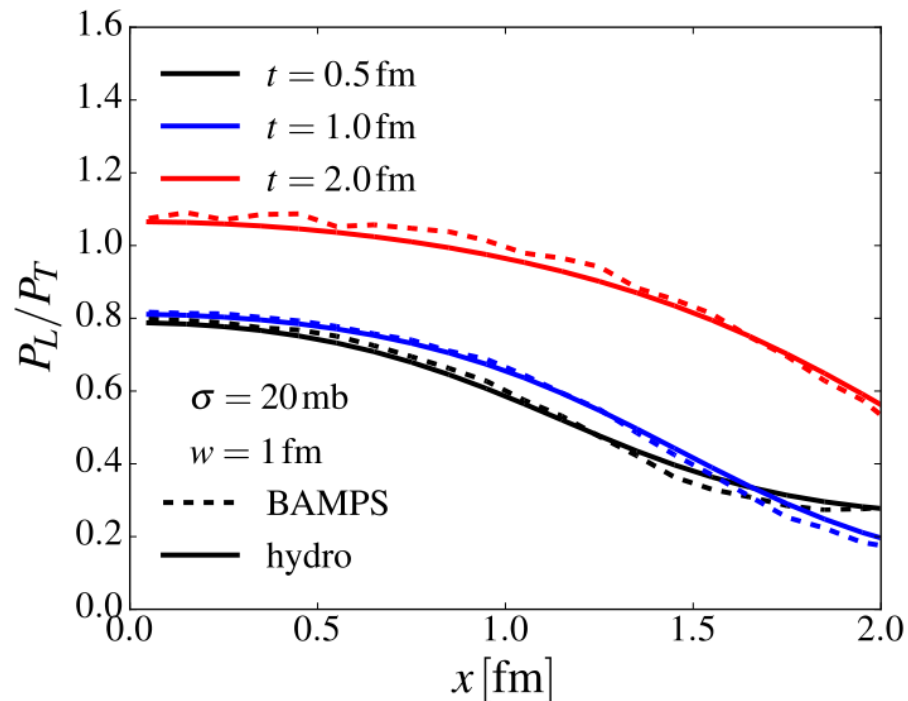
■ Pressure ratio: P_L/P_T (in the LRF)

$$\langle r^2 \rangle = (1\text{fm})^2$$

1 mb



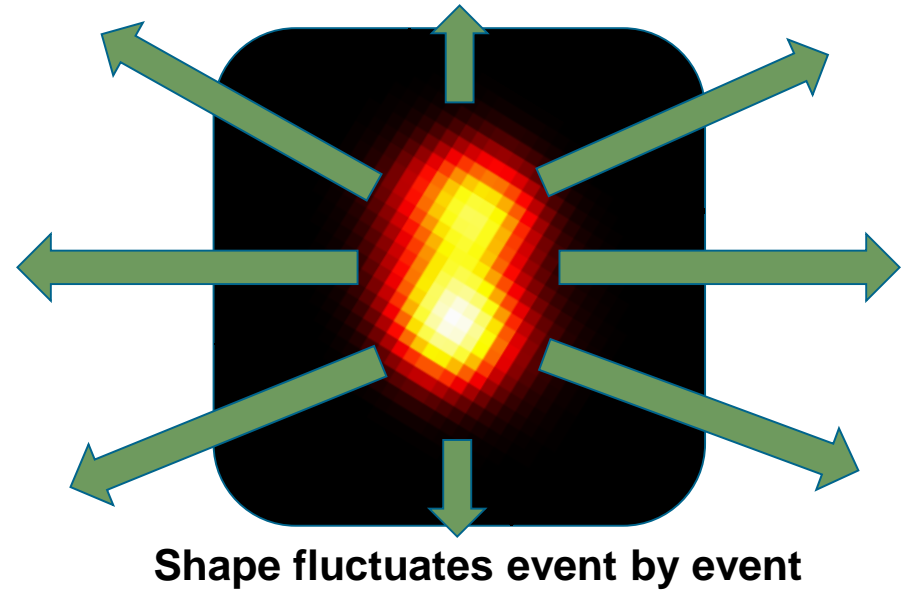
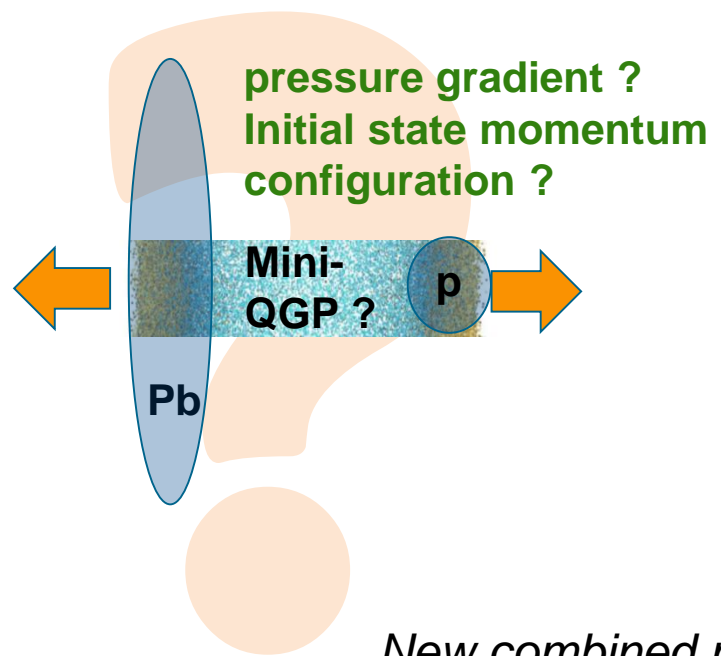
20 mb



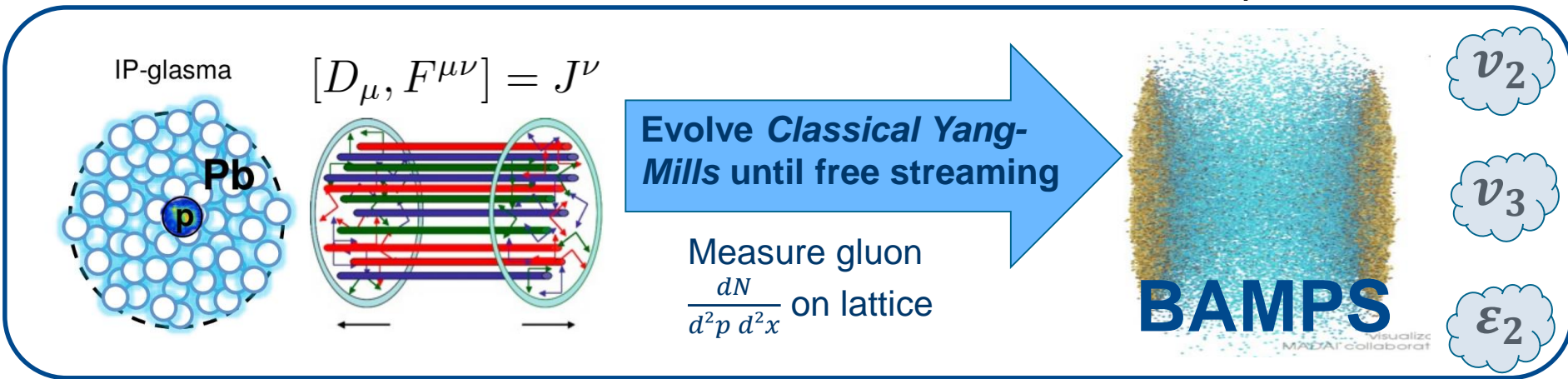
Conclusions

- Comparison of 3D Bjorken Scenario
- Radial symmetric configuration
 - Nice agreement ($\sim 10\%$) for densities, temperatures, velocities
 - Systematic deviation of fugacities
 - Deviations in components of shear-stress tensor
 - No difference between large and small system
- Asymmetric configuration
 - Same agreement as in radial symmetric case
 - ε_p and flow v_2 : nice agreement, dependence on freeze-out
- Work in progress: quantify deviation as function of Knudsen number
- ToDo: hot spots, anisotropic hydro, ...
- Work in progress: Greif, Schenke, ...; IP-Glasma for p+A

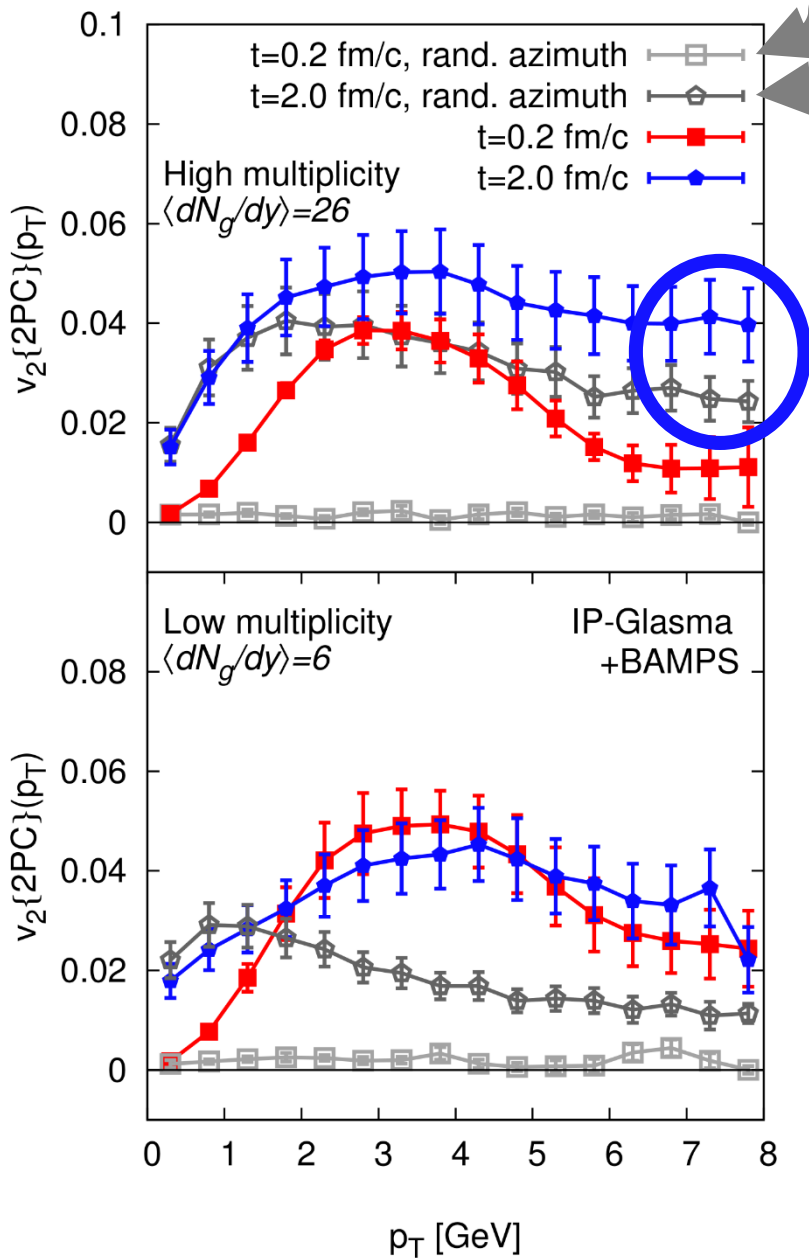
Initial and final state effects on correlations in Proton-Lead collisions



New combined model: **IP-Glasma + BAMPS** for pPb

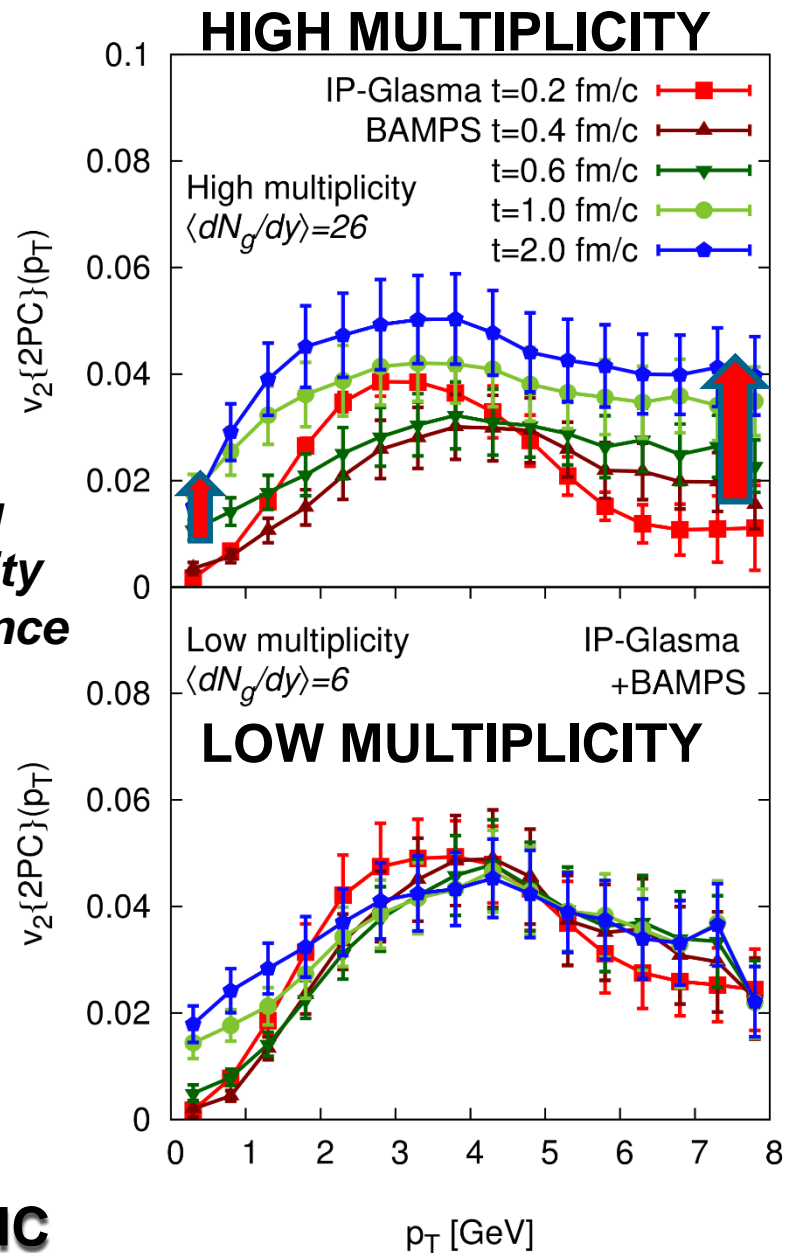


Randomized angle
~ like „hydro“

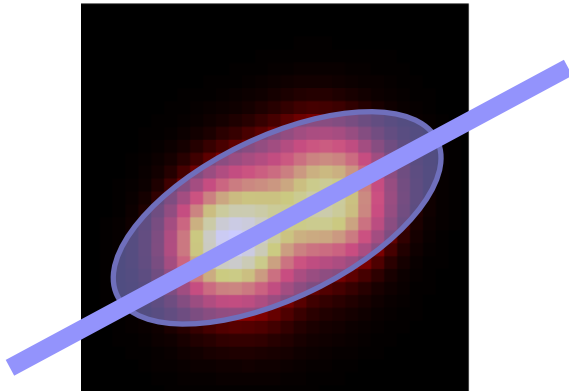


→ **Strong multiplicity dependence**

pPb@LHC

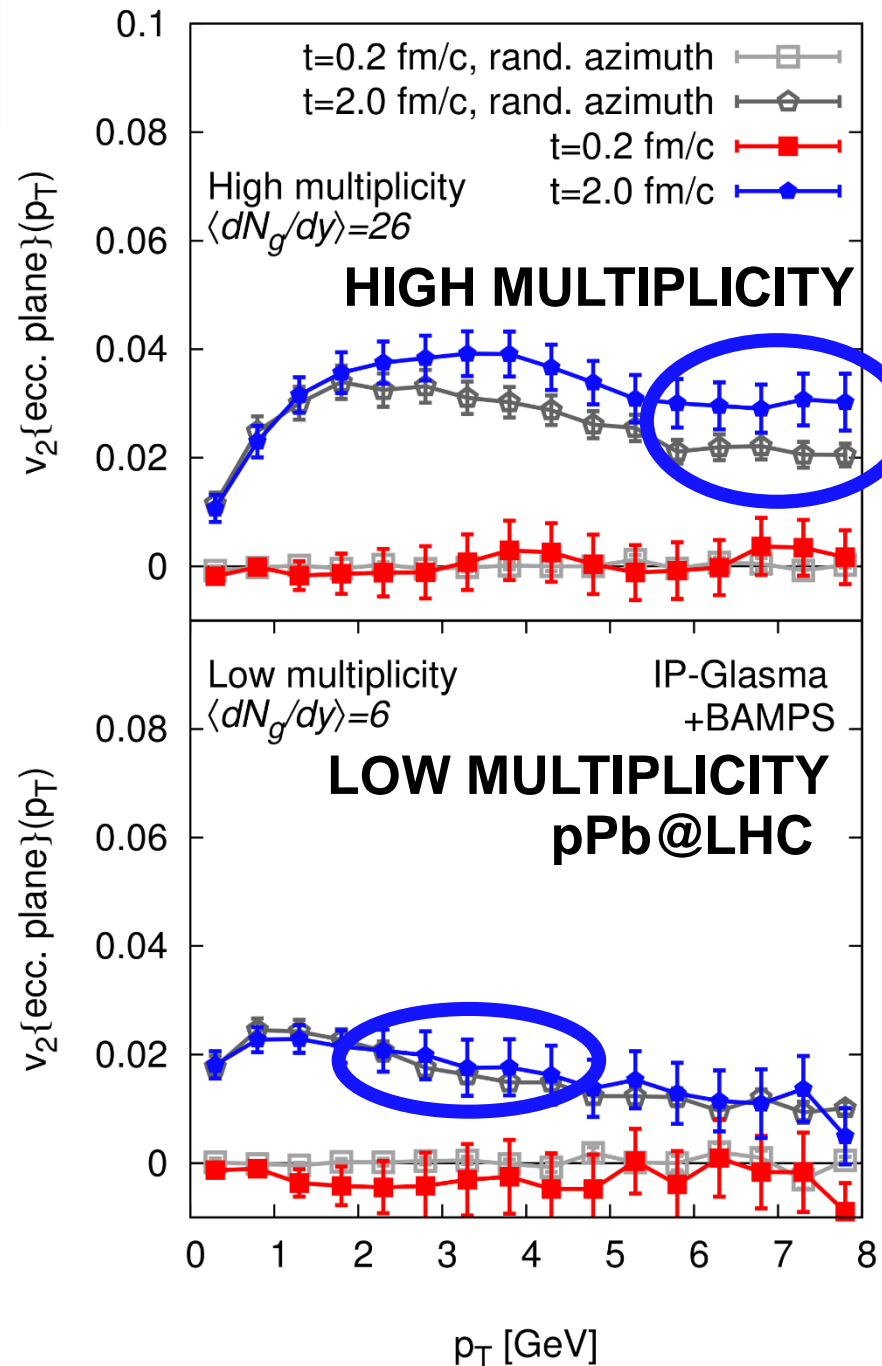


Eccentricity plane v_2



- Momenta and geometry uncorrelated initially
- Partonic cascade builds up flow from pressure gradient

→ Here: Weak dependence on initial momentum correlations



Conclusions & Outlook **arXiv: 1708.02076**

What ?

- First combined **initial** and **final** state calculation for p+Pb collisions
- **Initial state: IP-Glasma** (Impact parameter saturation, CGC, Glasma)
- **Final state: BAMPS** (pQCD parton cascade)

Results

- Strong difference for high and low multiplicities
- High Multiplicities: Large final state elliptic flow buildup
- Low Multiplicities: Initial state correlations very important
- Triangular flow: final state destroys initial flow
- Eccentricity plane: weak dependence on initial momenta

Conclusion

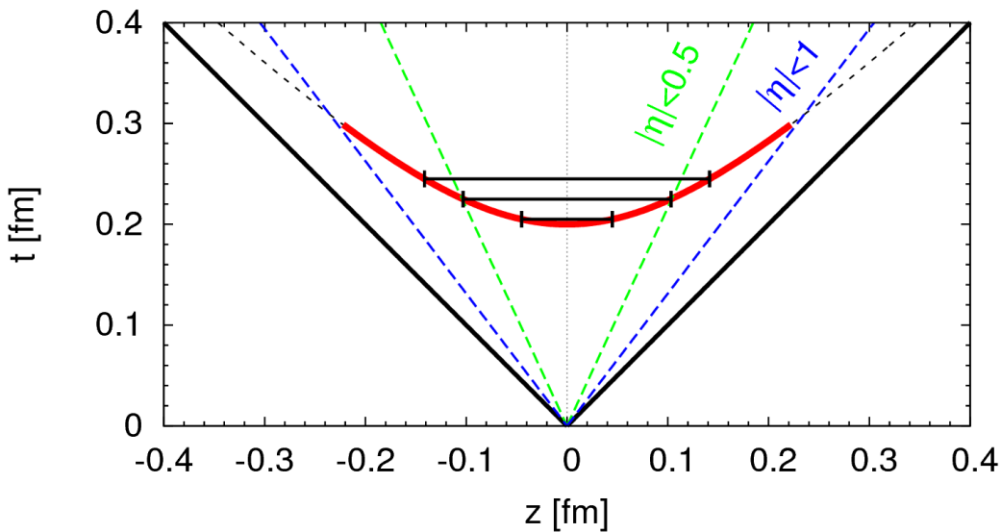
- Differential study of az. correlations across large range of multiplicities and transverse momenta will give insight to the initial state

Possibilities

- Survival of flow: Hadrons and Photons? Dynamical hadronization? 3D-IP-Glasma initial state ? Jet correlations?

Backup Slides

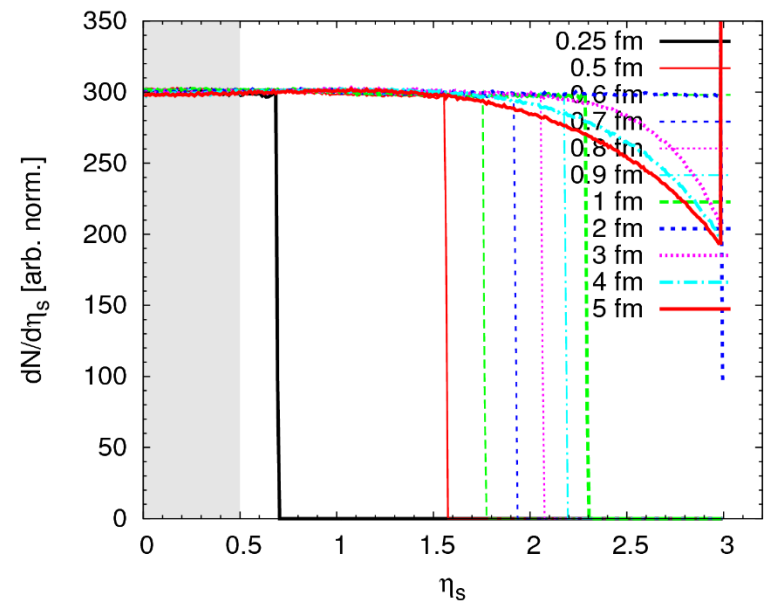
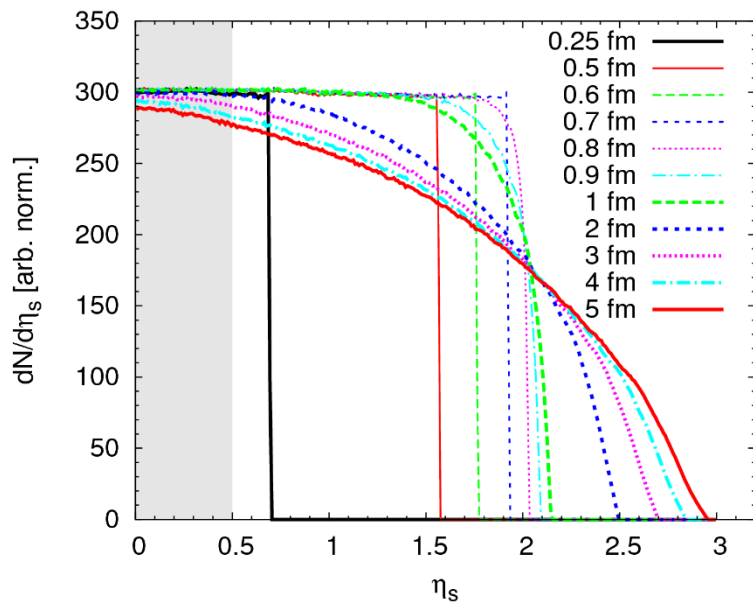
BAMPS Challenge #1: Boost invariance



- BAMPS runs with (z,t)
- Initialized in (η_s, τ)

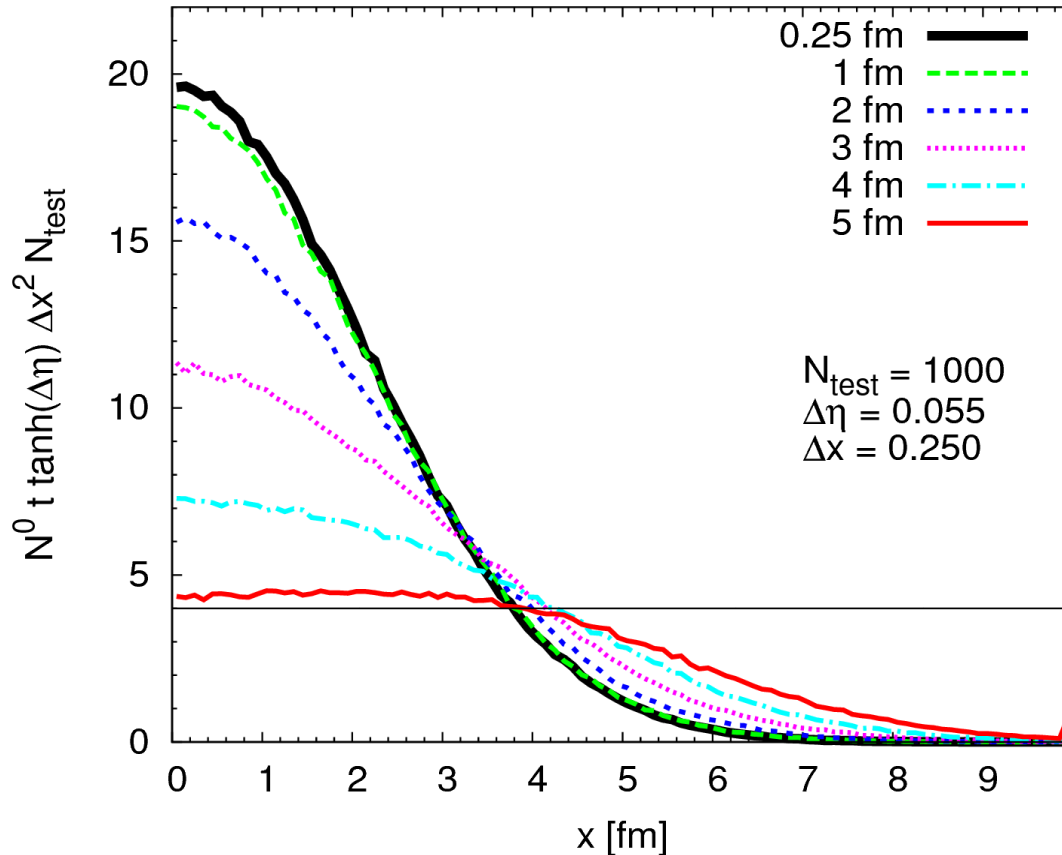
$$\eta_s > 2 \text{ for } t > 2 \text{ fm}$$

$$\eta_s > 3 \text{ for } t > 5 \text{ fm}$$



BAMPS Challenge #2: Cell content

- BAMPS: cells need at least 4 particles for interactions



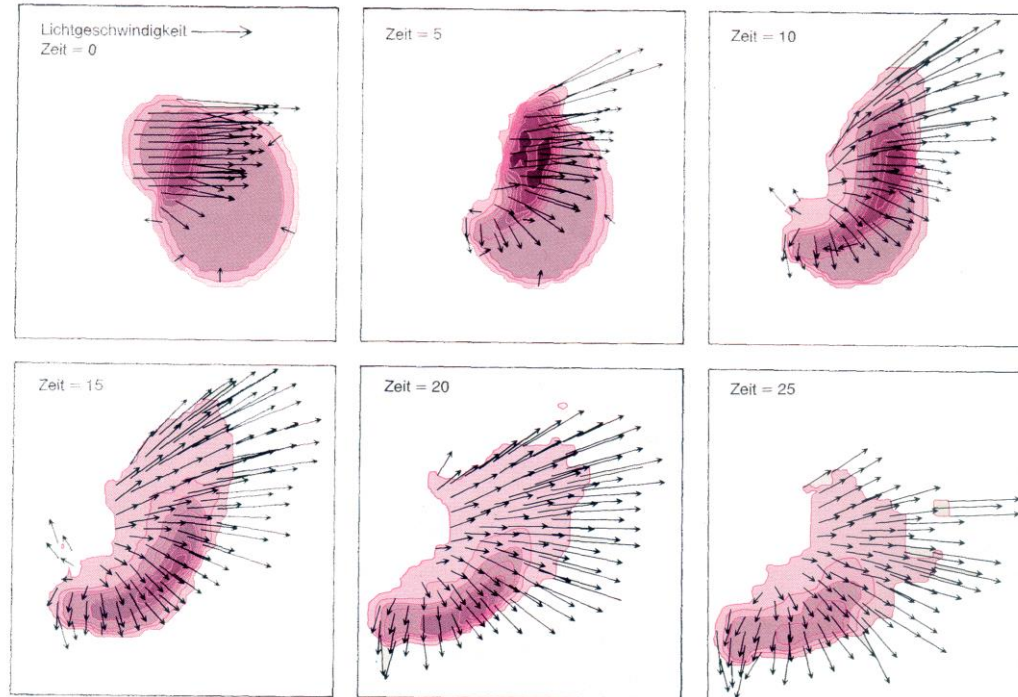
Attention:

RAM on LOEWE-CSC
= 2.7 GB/CPU (AMD)
= 6.4 GB/CPU (Intel)

Alternative:
geometrical collision criterion
(too slow!)

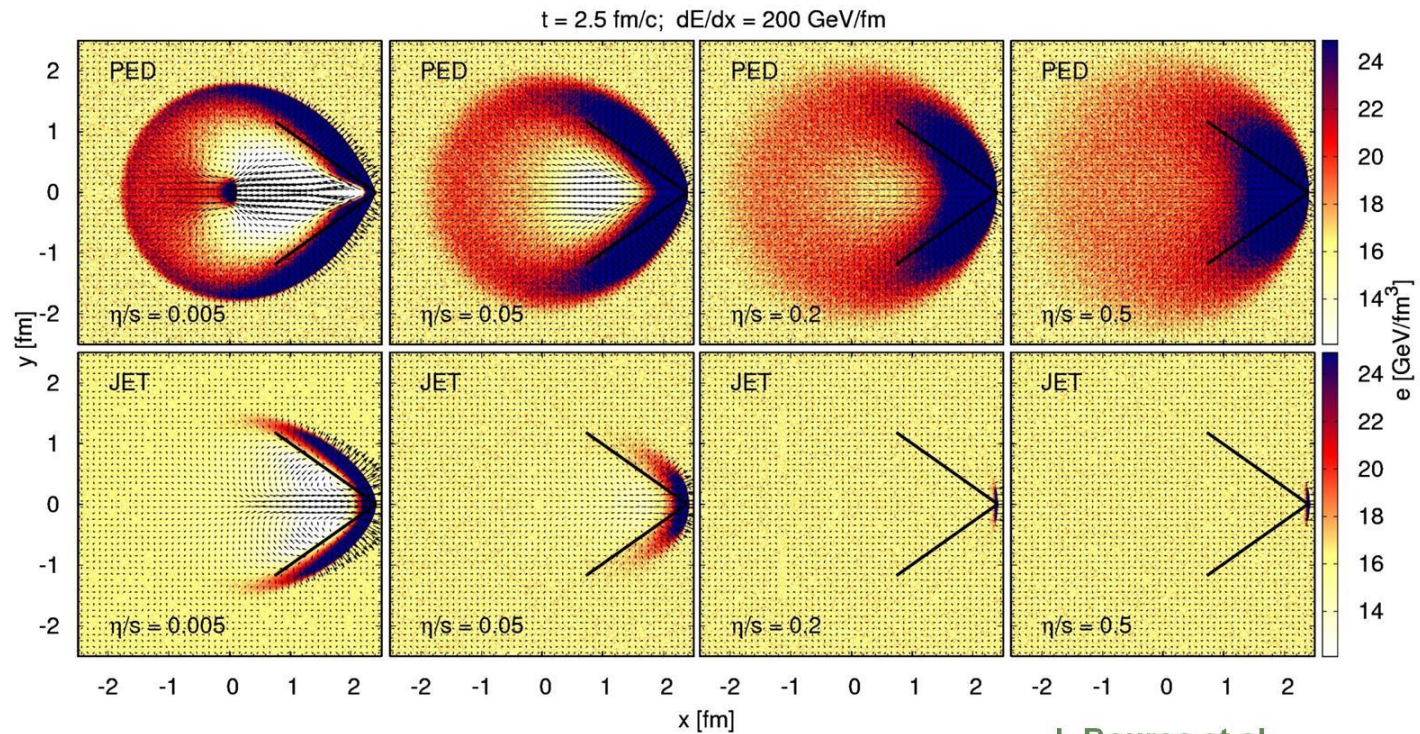
Free streaming

Mach Cones at Bevalac



W. Scheid, H. Muller & W. Greiner, Phys.Rev.Lett., 1974
H. Stöcker

VISCOUS Solutions

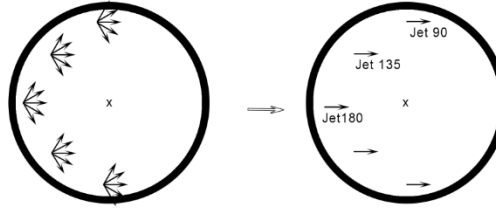


I. Bouras et al,
PLB 710 (2012) 641

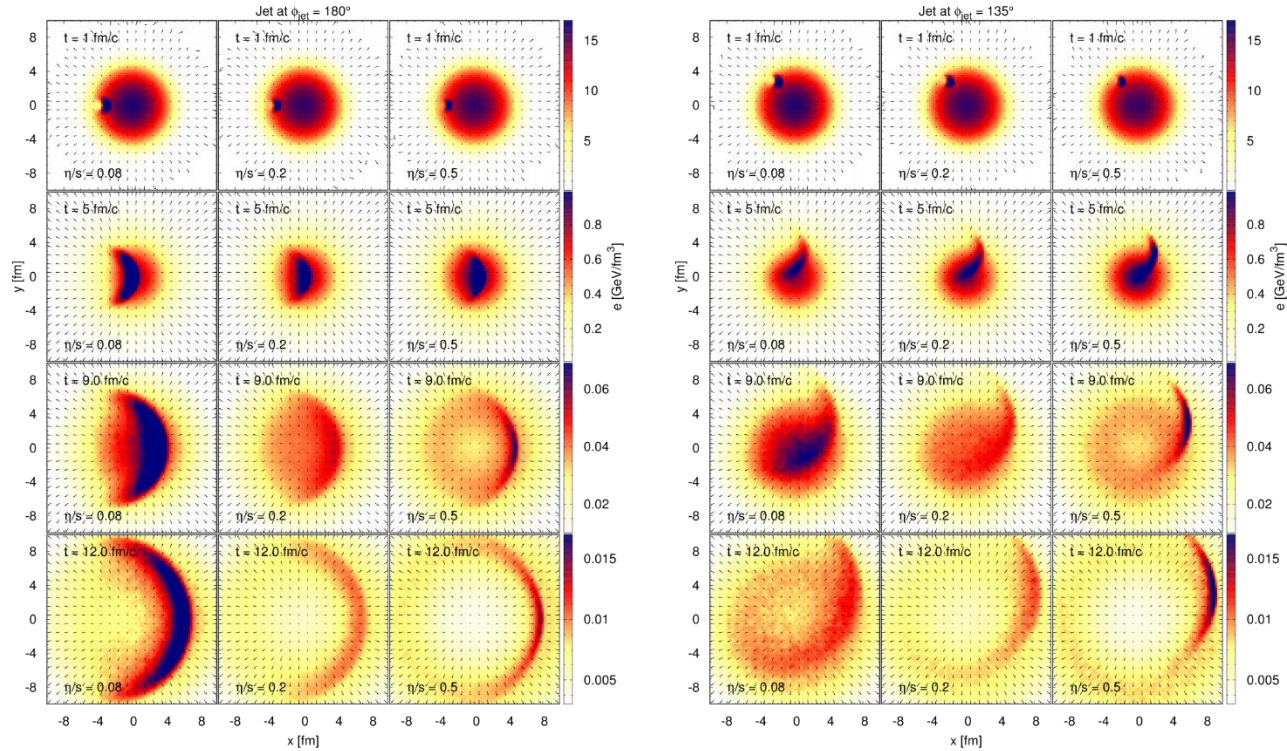
... the death of Mach Cones ?

Mach cones at relativistic HIC

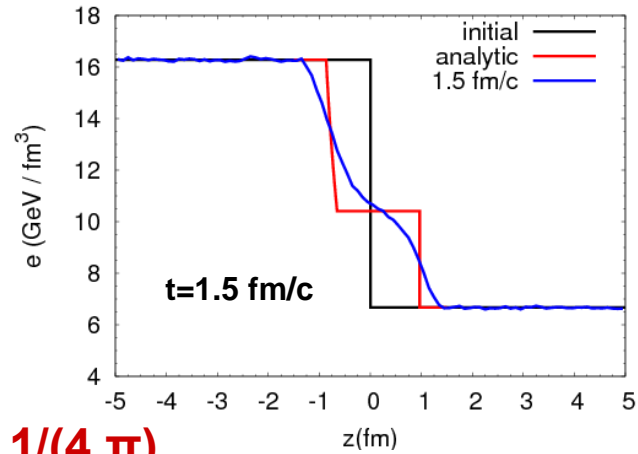
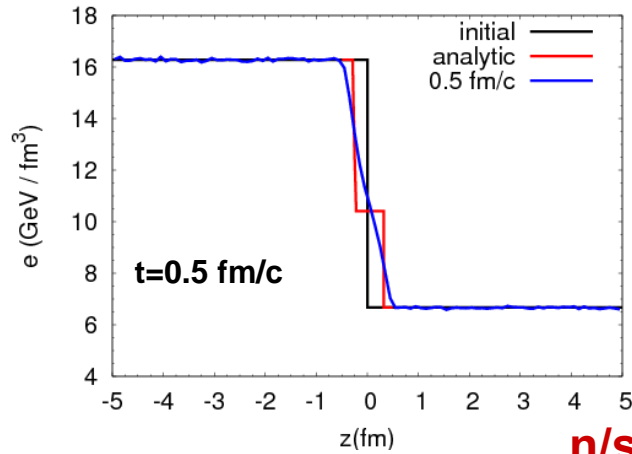
Central collisions and
Smooth initial conditions
at RHIC energies



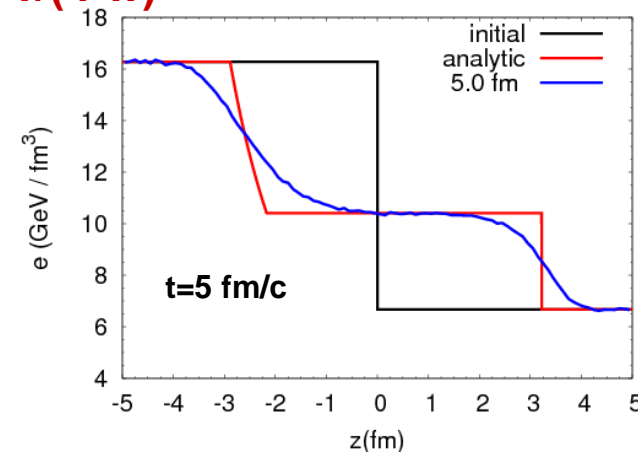
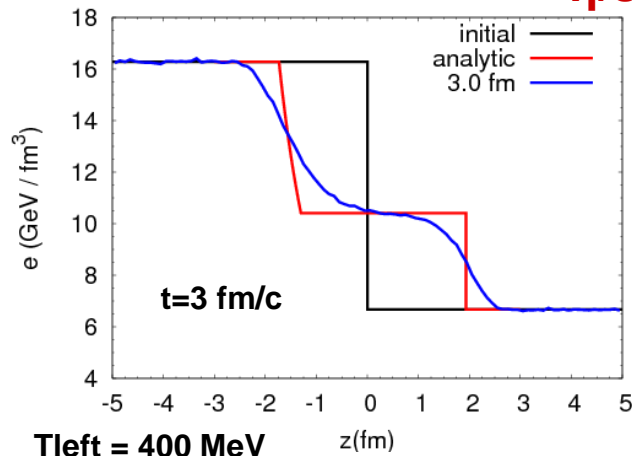
$E_{\text{jet}} = 20 \text{ GeV}$



time evolution of viscous shocks



$$\eta/s = 1/(4\pi)$$



$T_{left} = 400$ MeV
 $T_{right} = 320$ MeV