Comparison of Hydrodynamics and Kinetic Transport Theory for p+A and A+A Collisions

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Hydrodynamics & BAMPS

Initial state
specific transversal distribution
longitudinal boost invariance

Results and Outlook
What can we learn?
BAMPS

Boltzmann Approach to Multi-Parton Scattering

(3+1)D Boltzmann equation

\[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \frac{\partial f}{\partial \vec{r}} = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3} \]

Massless particles: partons / quarks & gluons

Discretized space and time

\[ P_{2\rightarrow 2} = v_{\text{rel}} \sigma_{2\rightarrow 2} \frac{\Delta t}{\Delta V} \]
\[ P_{2\rightarrow 3} = v_{\text{rel}} \sigma_{2\rightarrow 3} \frac{\Delta t}{\Delta V} \]
\[ P_{3\rightarrow 2} = \frac{I_{3\rightarrow 2}}{8E_1E_2E_3} \frac{\Delta t}{(\Delta V)^2} \]

Testparticle ansatz: \( N_{\text{test}} \)
Nuclear modification factor $R_{AA}$

- Hadronization of high $p_t$ partons with AKK fragmentation functions
- LPM parameter fixed by comparison to RHIC data
- Realistic suppression both for RHIC and LHC

Elliptic flow $v_2$

- Same pQCD interactions lead to a sizeable elliptic flow for bulk medium

- No hadronization for bulk medium $\rightarrow$ no hadronic after-burner
Heavy flavor and charged hadron $R_{AA}$ at LHC

![Graph showing $R_{AA}$ vs. $p_T$ for different hadron types and experimental collaborations at LHC at $\sqrt{s} = 2.76$ TeV.](image)

- Charged hadrons, $\kappa=1$, $X_{LPM}=0.3$
- $D$ mesons, $\kappa=1$, $X_{LPM}=0.3$
- Non-prompt $J/\psi$, $\kappa=1$, $X_{LPM}=0.3$

- Charged hadrons 0-5% (ALICE)
- $D$ mesons 0-7.5% (ALICE)
- Non-prompt $J/\psi$ 0-20% (CMS)

$b = 3.6$ fm, running $\alpha_s$

LHC
Riemann problem at finite viscosity

\[ p^\mu \partial_\mu f = C \]

I. Bouras et al, PRL 103:032301 (2009)

Development of a shock plateau

\( \eta/s \) less than 0.1-0.2

T\(_{\text{left}}\) = 400 MeV
T\(_{\text{right}}\) = 200 MeV
t = 1.0 fm/c
Hydro vs BAMPS in 1D

A. El, Z. Xu, C. Greiner, PRC 81 (2010) 041901

\[ \frac{p_L}{p_T} = \frac{p - \pi}{p + \pi/2} \]

\[ T_0 = 500 \text{ MeV}, \quad \tau_0 = 0.4 \text{ fm/c} \]

\[ \eta/s = 0.05, 0.2, 0.4, 3.0 \]

\[ \text{BAMPS} \]
\[ \text{Israel-Stewart} \]
\[ \text{Present work (second-order)} \]
\[ \text{Higher-order approximation} \]

\( x=0 \): Israel-Stewart

\( x=3 \): third-order rel. diss. hydro

\( x=5/3 \): approximative ‘all-orders’


> Resummation works at strong dissipation
(large Knudsen number!).
Relativistic Fluid Dynamics

Conservation laws & tensor decompositions

\[ \partial_\mu N^\mu = 0 \]
\[ \partial_\mu T^{\mu\nu} = 0 \]

\[ N^\mu = n \ u^\mu + V^\mu \]
\[ T^{\mu\nu} = e \ u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \]

\[ n = u_\mu N^\mu \]
\[ e = u_\mu T^{\mu\nu} u^\nu \]
\[ V^\mu = \Delta^\mu_\alpha N^\alpha \]

\[ p(e, n) + \Pi = -\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu} \]
\[ \pi^{\mu\nu} = T^{\langle\mu\nu\rangle} \]

LRF particle density
LRF energy density
particle diffusion current
isotropic pressure \((p_{eq} + \text{bulk})\)
shear stress tensor

\[ \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \]

\[ T^{\langle\mu\nu\rangle} = \left[ \frac{1}{2} \left( \Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] T^{\alpha\beta} \]
Relativistic Fluid Dynamics

Transient / second order fluid dynamics (e.g. Israel & Stewart)

\[
\tau_\pi \frac{d}{d\tau} \pi^{(\mu\nu)} + \pi^{\mu\nu} = 2\eta \sigma^{\mu\nu} + \tau_\pi C^{\mu\nu} (\nabla_\alpha u^\alpha) + \cdots
\]

\[
\tau_V \frac{d}{d\tau} V^{(\mu)} + V^{\mu} = \kappa \nabla^\mu \alpha + \tau_V C'' V^{\mu} (\nabla_\alpha u^\alpha) + \cdots
\]

(\pi^{\mu\nu} and V^{\mu} independent variables)

Second order coefficients from


Expansion in Knudsen and (inverse) Reynolds number

\[
K_n = \frac{\ell_{micr}}{L_{macr}} \quad R_V^{-1} \sim \frac{|V^{\mu}|}{n_0}, \quad R_\pi^{-1} \sim \frac{|\pi^{\mu\nu}|}{P_0}
\]

Hydrodynamical limit: \( K_n \ll O(1) \) and \( R^{-1} \ll O(1) \)
Comparison Hydro / BAMPS

Collectivity in Heavy Ion Collision?
Fast Thermalization?
Flow?

How small can system be, how large can gradients be, until discrepancies occur?

- **Longitudinal:** Boost invariant
- **Transversal:**
  - Radial symmetric, large/small system
  - Glauber; overlapping Woods-Saxon

\[ \langle r^2 \rangle = (3\text{fm})^2, (1\text{fm})^2 \]
Comparison 1: Radial symmetric

- Longitudinal: boost invariant
- Transversal:
  - Rotational symmetric
  - Gaussian density profile, \( \langle r^2 \rangle = (3 \text{fm})^2 \) or \( \langle r^2 \rangle = (1 \text{fm})^2 \)

- Temperature
- Fugacity

\[ T_0(r), \ T_0(0) = 500 \text{ MeV} \]
\[ \lambda_0(r) = 1 \]
\[ \tau_0 = 0.2 \text{ fm} \]

only gluons

Cross section:
- Elastic
- Isotropic
- Constant

\[ \sigma = 1, 5, 20, 50, 100 \text{ mb} \]

A+A  \hspace{2cm} p+A, p+p
Comparison 2: Glauber

- Longitudinal: boost invariant
- Transversal:
  - Overlapping Woods-Saxon
  - Impact parameter dependence selected value: 7.5 fm
- Temperature
- Fugacity
  \[ T_0(r), \ T_0(0) = 500 \text{ MeV} \]
  \[ \lambda_0(r) = 1 \]
- \( \tau_0 = 0.2 \text{ fm} \)
- only gluons
- Cross section:
  - Elastic
  - Isotropic
  - Constant
  \[ \sigma = 1, 5, 20, 50, 100 \text{ mb} \]

start in full equilibrium
Available eta/s

\[ \eta = \frac{4}{3} \frac{T}{\sigma} \]

\[ s = \frac{4g}{\pi^2} T^3 \quad (g = 16) \]

\[ \implies \sigma = 1 \ldots 20 \text{ mb} \]
Comparison 2: Glauber

Knudsen number

Hydrodynamical limit:

\[ \text{Kn} = \frac{\ell_{\text{micr}}}{L_{\text{macr}}} \ll \mathcal{O}(1) \]

\[ \text{Kn} \equiv \lambda_{\text{mfp}} \theta , \quad \theta = \partial_{\mu} u^\mu \]
Comparison 2: Glauber

Glauber, 5mb: energy density & velocity

5mb: still very nice agreement
Comparison 2: Glauber

Glauber, 5mb: shear stress tensor

\[ \pi^{xx}/e \]

\[ \pi^{zz}/e \]

5mb: still very nice agreement
Comparison 2: Glauber

Asymmetry:

$$\varepsilon_P = \frac{\langle T_{xx} - T_{yy} \rangle_{xy}}{\langle T_{xx} + T_{yy} \rangle_{xy}}$$

![Graph showing the comparison between different scenarios for 5 mb and 100 mb.](image)
Comparison 2: Glauber

Spectra:

\[ \langle p_T \rangle_{\sigma=5 \text{ mb}} = 0.77 \text{ GeV} \]
\[ \langle p_T \rangle_{\sigma=100 \text{ mb}} = 0.5 \text{ GeV} \]
Comparison 2: Glauber

Flow:

- Large uncertainty due to viscous correction terms
- Strong dependence on freeze out conditions

\[ \delta f_k = f_{0k} \left( \frac{1}{8p_0 T^2} k_{\mu} k_{\nu} \pi^{\mu\nu} - \frac{5}{p_0} k_{\mu} n_{\mu} + \frac{1}{p_0 T} E_k k_{\mu} n_{\mu} \right) \]
Comparison 1: Radial symmetric (small)

Knudsen number

Hydrodynamical limit:

\[ \text{Kn} = \frac{\ell_{\text{micr}}}{L_{\text{macr}}} \ll O(1) \]

\[ \text{Kn} \equiv \lambda_{\text{mfp}} \theta, \quad \theta = \partial_\mu u^\mu \]
Comparison 1: Radial symmetric (small, 5mb)
Comparison 1: Radial symmetric (small, 1mb)
Comparison 2: Glauber

Pressure ratio: $P_L/P_T$ (in the LRF)

- 5 mb
- 100 mb

Graphs showing the pressure ratio $P_L/P_T$ as a function of $x$ for different times $t$ and pressures $\sigma$. The graphs are divided into two panels: 5 mb and 100 mb, with different lines representing $t = 1.0$ fm, $t = 2.0$ fm, and $t = 4.0$ fm. The graph also includes the notation $nBC, b = 7.5$ fm and $\sigma = 5$ mb, and $\sigma = 100$ mb.
Comparison 1: Radial symmetric (large)

Pressure ratio: $P_L/P_T$ (in the LRF)

$$\langle r^2 \rangle = (3 \text{fm})^2$$
Comparison 1: Radial symmetric (small)

Pressure ratio: $P_L/P_T$ (in the LRF)

$\langle r^2 \rangle = (1\text{fm})^2$

1 mb

\[
\begin{align*}
P_L/P_T & \quad x[\text{fm}] \\
0.6 & \quad 0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \\
0.5 & \quad 0.4 \quad 0.3 \quad 0.2 \quad 0.1 \quad 0.0 \\
0.4 & \quad 0.3 \quad 0.2 \quad 0.1 \quad 0.0 \\
0.3 & \quad 0.2 \quad 0.1 \quad 0.0 \\
0.2 & \quad 0.1 \quad 0.0 \\
0.1 & \quad 0.0 \\
0.0 & \quad 0.0 \\
\end{align*}
\]

- $t = 0.5\text{fm}$, $\sigma = 1\text{mb}$
- $t = 1.0\text{fm}$, $\omega = 1\text{fm}$
- $t = 2.0\text{fm}$

BAMPS

hydro

20 mb

\[
\begin{align*}
P_L/P_T & \quad x[\text{fm}] \\
1.6 & \quad 0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \\
1.4 & \quad 1.2 \quad 1.0 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0 \\
1.2 & \quad 1.0 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0 \\
1.0 & \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0 \\
0.8 & \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0 \\
0.6 & \quad 0.4 \quad 0.2 \quad 0.0 \\
0.4 & \quad 0.2 \quad 0.0 \\
0.2 & \quad 0.0 \\
0.0 & \quad 0.0 \\
\end{align*}
\]

- $t = 0.5\text{fm}$
- $t = 1.0\text{fm}$
- $t = 2.0\text{fm}$

$\sigma = 20\text{mb}$

$\omega = 1\text{fm}$

BAMPS

hydro
**Conclusions**

- **Comparison of 3D Bjorken Scenario**
  - Radial symmetric configuration
    - Nice agreement (~10%) for densities, temperatures, velocities
    - Systematic deviation of fugacities
    - Deviations in components of shear-stress tensor
    - No difference between large and small system

- **Asymmetric configuration**
  - Same agreement as in radial symmetric case

  - $\varepsilon_{p}$ and flow $v_2$: nice agreement, dependence on freeze-out

- **Work in progress**: quantify deviation as function of Knudsen number
- **ToDo**: hot spots, anisotropic hydro, …
- **Work in progress**: Greif, Schenke, …; IP-Glasma for p+A
Initial and final state effects on correlations in Proton-Lead collisions

Pressure gradient? Initial state momentum configuration?

Mini-QGP? p

New combined model: **IP-Glasma + BAMPS** for pPb

Greif, Schenke, Schlichting, C.G., Xu, arXiv:1708.02076
Randomized angle ~ like "hydro"

→ Strong multiplicity dependence
**Eccentricity plane $v_2$**

- Momenta and geometry uncorrelated initially
- Partonic cascade builds up flow from pressure gradient

→ Here: Weak dependence on initial momentum correlations
Conclusions & Outlook

What?
• First combined initial and final state calculation for p+Pb collisions
• Initial state: IP-Glasma (Impact parameter saturation, CGC, Glasma)
• Final state: BAMPS (pQCD parton cascade)

Results
• Strong difference for high and low multiplicities
• High Multiplicities: Large final state elliptic flow buildup
• Low Multiplicities: Initial state correlations very important
• Triangular flow: final state destroys initial flow
• Eccentricity plane: weak dependence on initial momenta

Conclusion
• Differential study of az. correlations across large range of multiplicities and transverse momenta will give insight to the initial state

Possibilities
• Survival of flow: Hadrons and Photons? Dynamical hadronization?
• 3D-IP-Glasma initial state? Jet correlations?
BAMPS Challenge #1: Boost invariance

BAMPS runs with (z,t)

Initialized in (\(\eta_s, \tau\))

\(\eta_s > 2\) for \(t > 2\) fm

\(\eta_s > 3\) for \(t > 5\) fm
BAMPS Challenge #2: Cell content

BAMPS: cells need at least 4 particles for interactions

Attention:

RAM on LOEWE-CSC
= 2.7 GB/CPU (AMD)
= 6.4 GB/CPU (Intel)

Alternative:
geometrical collision criterion
(too slow!)

Free streaming
Mach Cones at Bevalac

H. Stöcker
VISCIOUS Solutions

$t = 2.5 \text{ fm/c}; \quad dE/dx = 200 \text{ GeV/fm}$

... the death of Mach Cones?
Mach cones at relativistic HIC

Central collisions and Smooth initial conditions at RHIC energies

\[ E_{\text{jet}} = 20 \text{ GeV} \]
time evolution of viscous shocks

\[ \eta/s = 1/(4\pi) \]

$t=0.5 \text{ fm/c}$

$e (\text{GeV/fm}^3)$ vs $z (\text{fm})$

$t=1.5 \text{ fm/c}$

$t=3 \text{ fm/c}$

$t=5 \text{ fm/c}$

$T_{\text{left}} = 400 \text{ MeV}$

$T_{\text{right}} = 320 \text{ MeV}$