

# Vorticity and Lambda polarization in baryon-rich matter

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*Phys. Rev. C88 (2013)*

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*Phys. Rev. C95 (2017)*

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**Joint meeting of JINR PACs for Particle Physics and Nuclear Physics, 26 June 2014**



# Main Topics

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- Rotation in heavy-ion collisions
- $\Lambda$  Polarization in heavy-ion collisions
- Anomalous mechanism: 4-velocity as gauge field
- Chemical potential and Energy dependence
- Rotation in heavy-ion collisions: Vortex sheets, helicity separation (helicity mirror structure and longitudinal vorticity quadrupole structure)
- Baryons vs antibaryons
- Vorticity and chaotic thermalization?
- Conclusions



# Spin Physics in Heavy-Ion Collisions

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- Spin-dependent observables might be manifested in HIC
- No beam polarization but plenty of effects in final state
- Especially interesting is the polarization of hyperons
- Self-analyzing: revealed in weak P-violating decay
- Related to P-odd effects in QCD medium:  
**Vorticity and Hydrodynamic helicity**

# Rotation and polarization in HIC and related quantities

- Non-central collisions – orbital angular momentum is normal to the reaction plane (source of the induced global polarization?)

$$L = \sum r \times p$$

- Differential pseudovector – vorticity

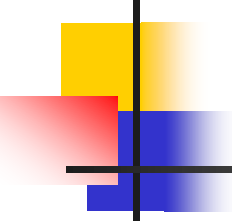
$$\omega = \text{curl } v$$

- Pseudoscalar – helicity

$$H \sim \langle (v \text{ curl } v) \rangle$$

- Maximal helicity – Beltrami chaotic flows

$$v \parallel \text{curl } v$$



# Microworld: where is the fastest possible rotation?

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- Non-central heavy ion collisions (angular velocity  $\sim c/\text{Compton wavelength}$ )
- $\sim 25$  orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd: may lead to various P-odd effects



# $\Lambda$ -polarisation

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- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – no direction normal to the scattering plane
- **But is the randomization complete (smoothly from hadrons to ions)?!**



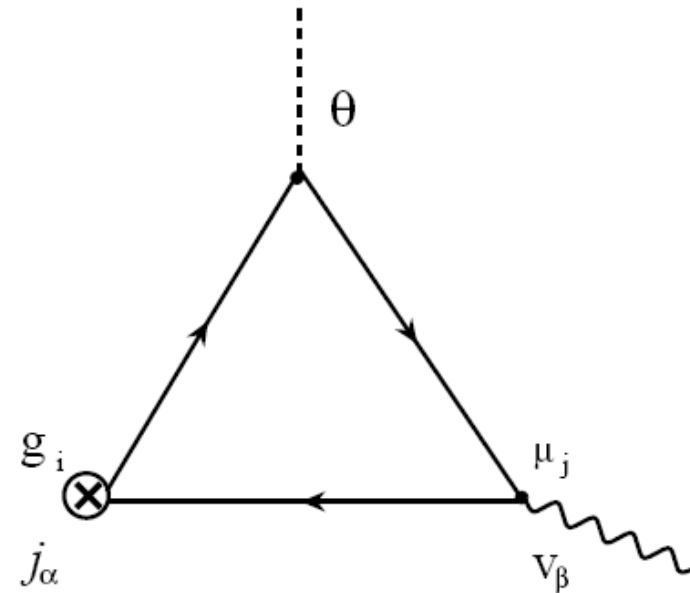
# Global $\Lambda$ polarization

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- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum  $\rightarrow$  large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- **How to transform rotation to spin?**

## Anomaly in medium – new external lines in VVA graph

- Gauge field  $\rightarrow$  4-velocity  $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- CME (Kharzeev, McLerran, Warringa; Fukushima; Vilenkin;...)  $\rightarrow$  CVE
- Kharzeev, Zhitnitsky (2007) – EM current
- Straightforward generalization: any (e.g. baryonic) current – neutron asymmetries@NICA – Rogachevsky, Sorin, Teryaev (2010)
- Vorticity:  $e_j \vec{H} \rightarrow \mu_j \vec{\nabla} \times \vec{V}$
- Induced currents  $J_e^\gamma = \frac{N_c}{4\pi^2 N_f} \varepsilon^{\gamma\beta\alpha\rho} \partial_\alpha V_\rho \partial_\beta (\theta \sum_j e_j \mu_j)$



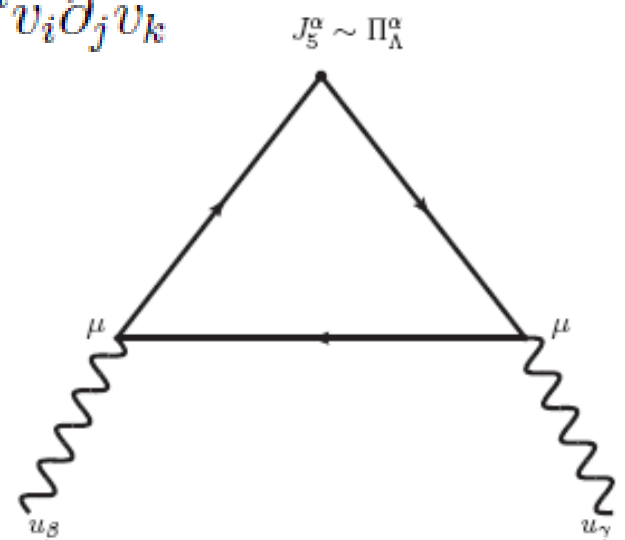
# Axial Vortical Effect (AVE) – anomalous mechanism of polarization

- 4-Velocity is also a "GAUGE" FIELD  $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, Teryaev, 2010)

AVE induced axial current (charge):

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Analogous to anomalous gluon contribution to nucleon spin (Efremov, Teryaev 88)
- 4-velocity instead of gluon field!



# From axial charge to polarization (and from quarks to confined hadrons) – analog of Cooper-Frye

- Analogy of matrix elements and classical averages (account for other charges!)

$$\langle p_n | j^0(0) | p_n \rangle = 2p_n^0 Q_n ; \quad \langle Q \rangle \equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x j_{class}^0(x)}{N}$$

$$\langle p_n, \Pi_n | j_{5,i}^0(0) | p_n, \Pi_n \rangle = 2a_{i,n} m_n \Pi_n^0$$

$$Q_{5,i,n} \rightarrow \frac{m_n a_{i,n} \Pi_n^0}{p_n^0} ; \quad \langle \frac{a_i m \Pi^0}{p_0} \rangle \equiv \frac{\sum_{n=1}^N \frac{m_n a_{i,n} \Pi_n^0}{p_n^0}}{N} = \frac{\int d^3x j_{5,class}^0(x)}{N}$$



# Anomaly for polarization

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- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- T-dependent term - related to gravitational anomaly
- Lattice simulation: suppressed due to collective effects

# Quark mass effects

$$\begin{aligned}\langle p|Q\gamma_\mu\gamma_5 Q|p\rangle &= i\frac{N_c\alpha_s}{2\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\left\{1-\int_0^1 dx\frac{2m_Q^2(1-x)}{m_Q^2-p^2x(1-x)}\right\} \\ &= -i\frac{N_c\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_Q^2}+O\left(\frac{1}{m_Q^4}\right),\end{aligned}$$

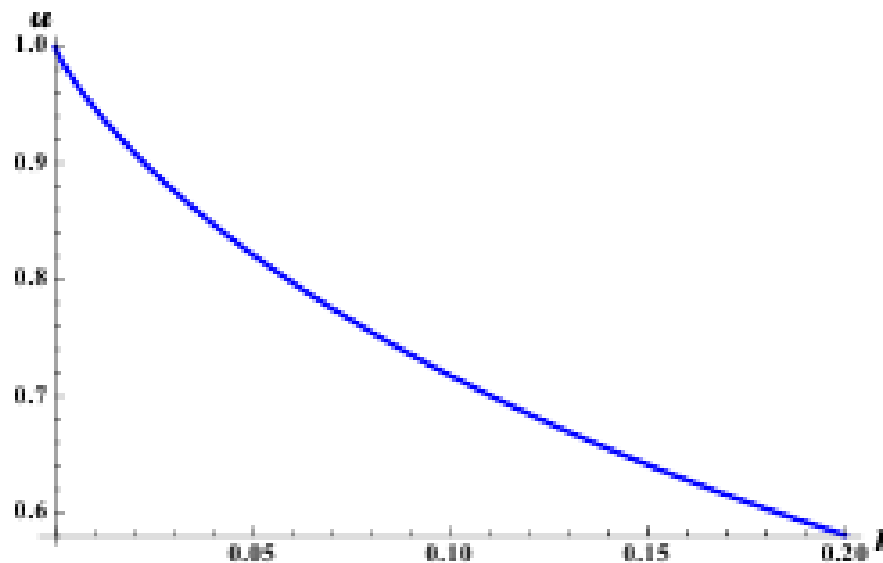


FIG. 2. Dependence of the anomaly coefficient on  $r = 2m^2/k^2$



# Energy dependence

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- Coupling -> chemical potential

$$Q_5^g = \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- **Rapid decrease with energy!**
- Large chemical potential: appropriate for NICA/FAIR energies

One might compare the prediction below with the right panel figures

**O. Rogachevsky, A. Sorin, O. Teryaev**  
**Chiral vortical effect and neutron asymmetries in heavy-ion collisions**  
**PHYSICAL REVIEW C 82, 054910 (2010)**

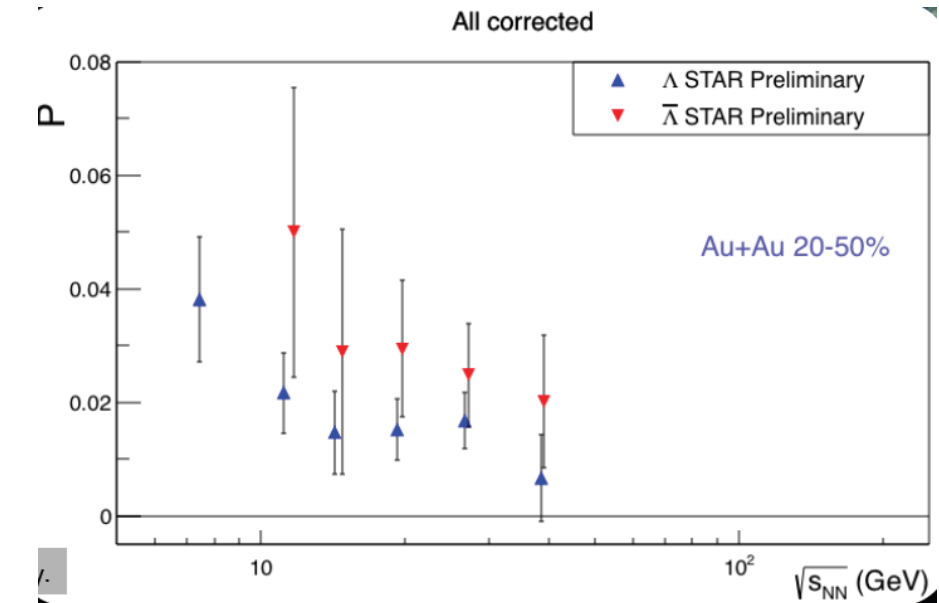
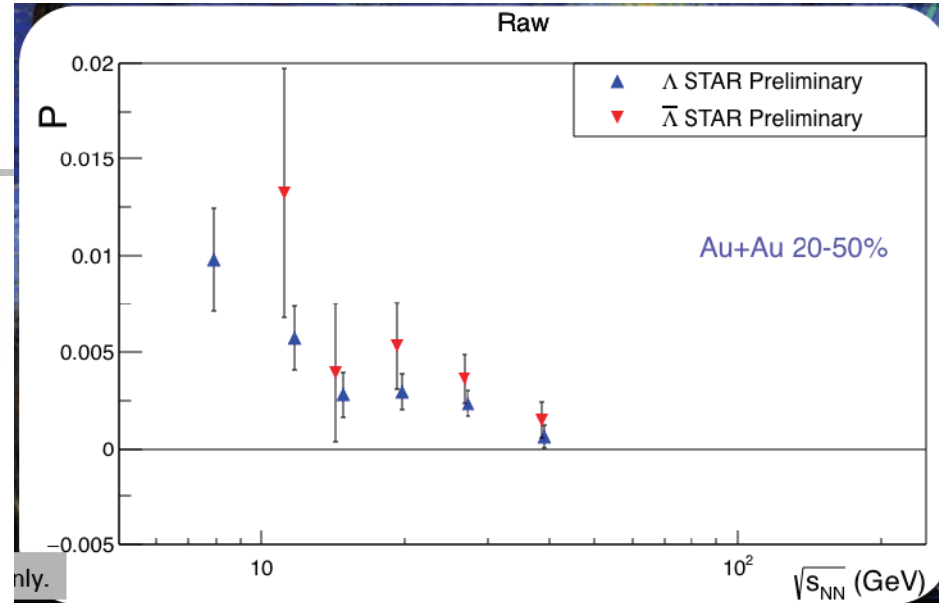
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left( 1 - \frac{2\mu n}{3(\epsilon + P)} \right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \quad (6)$$

where  $n$  and  $\epsilon$  are the corresponding charge and energy densities and  $P$  is the pressure. Therefore, the  $\mu$  dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

**M. Lisa, for the STAR collaboration, QCD Chirality Workshop, UCLA, February 2016;**  
**SQM2016, Berkeley, June 2016**



# Simulation in QGSM (Kinetics -> HD)

Simulations in kinetic Quark-Gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes):  
Baznat, Gudima, Sorin, Teryaev (Phys. Rev. C 2013, 2016)

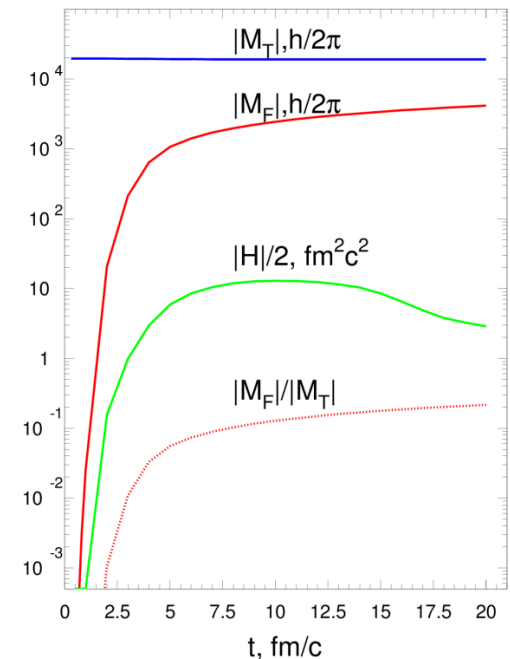
**50 × 50 × 100 cells**

$dx = dy = 0.6 \text{ fm}, dz = 0.6/\gamma \text{ fm}$

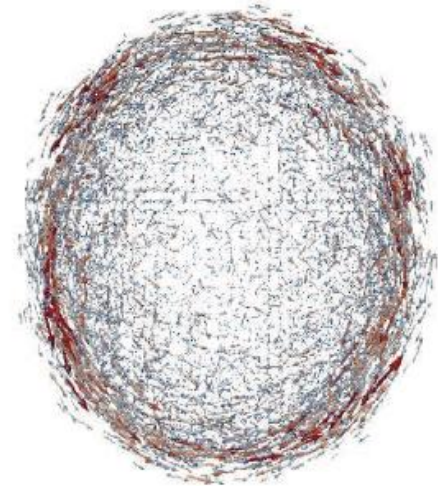
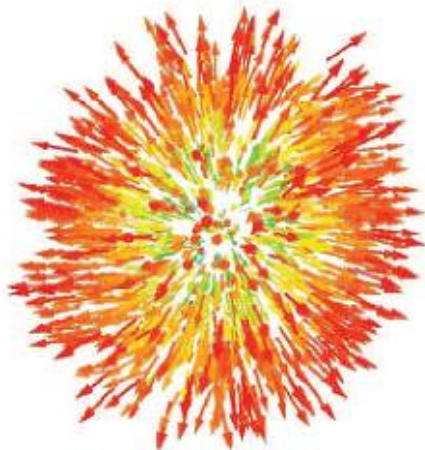
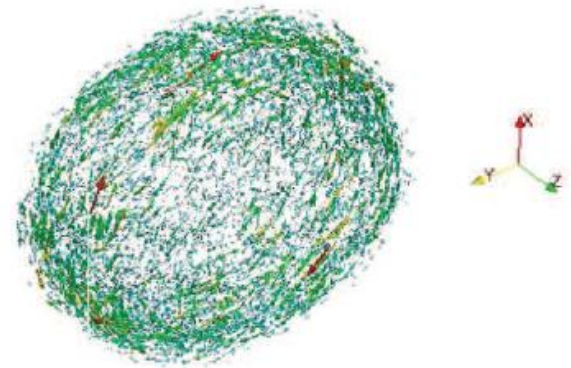
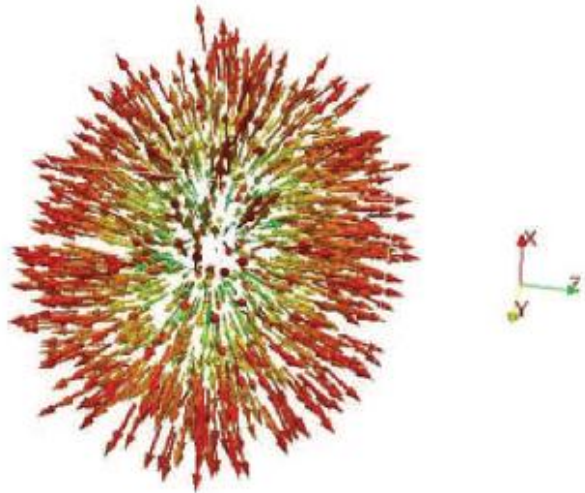
- Velocity 
$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$
- Vorticity – from discrete partial derivatives

# Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- ( $\sim 10\%$  of total)
- Conservation of OAM with a good accuracy!



# Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)

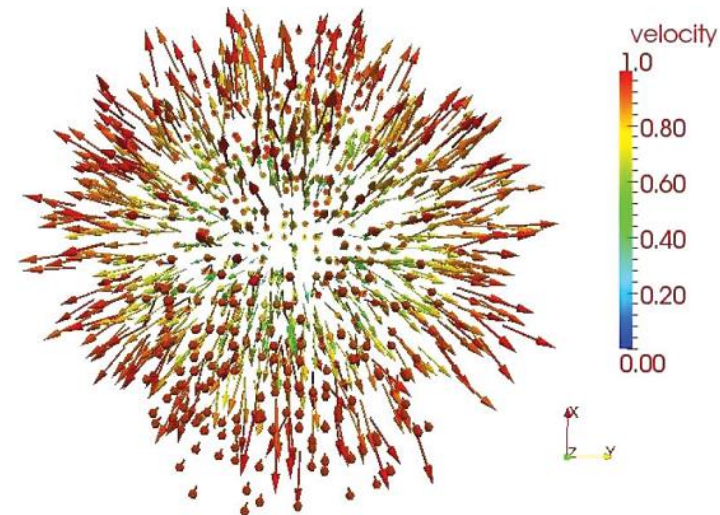
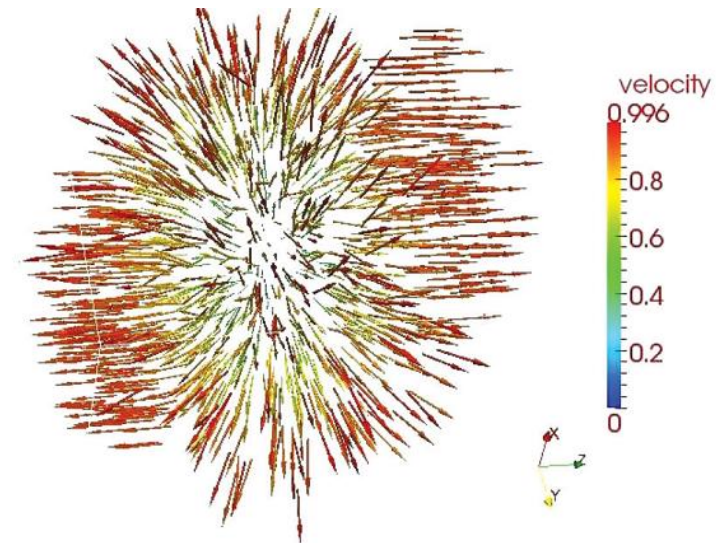


# Distribution of velocity ("Little Bang")

- 3D/2D projection
- z-beams direction
- x-impact parameter
- Little Hubble law

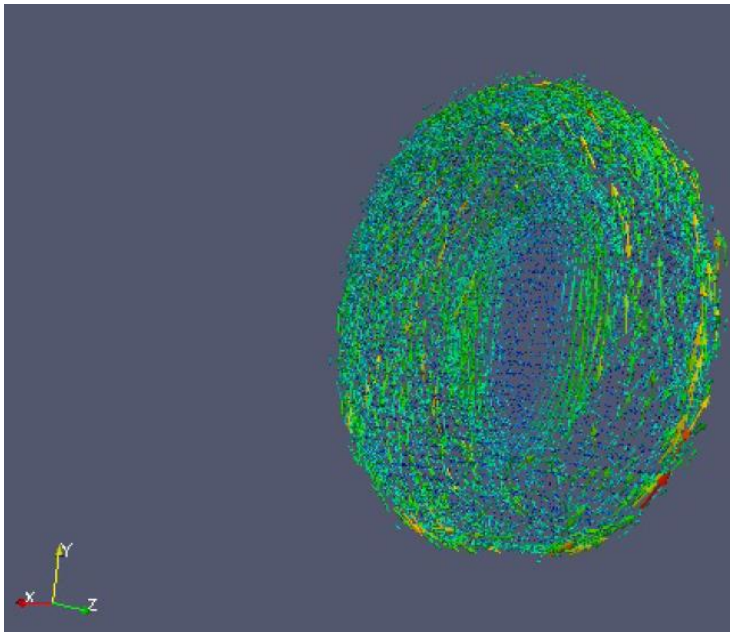
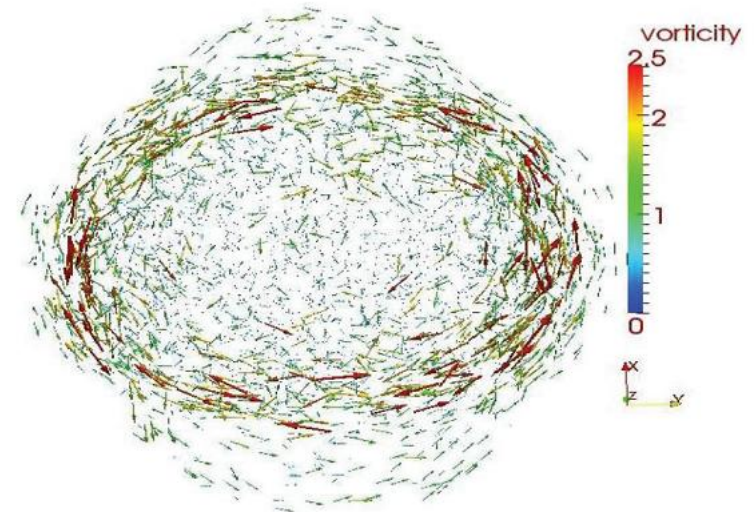
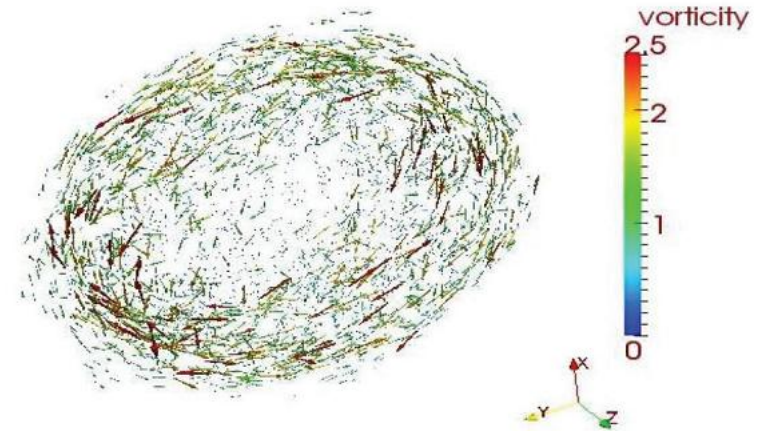
$$\langle v/c \rangle = v_0/c + H\rho.$$

$$H = 0.024 \div 0.028 \text{ (fm/c)}^{-1}$$



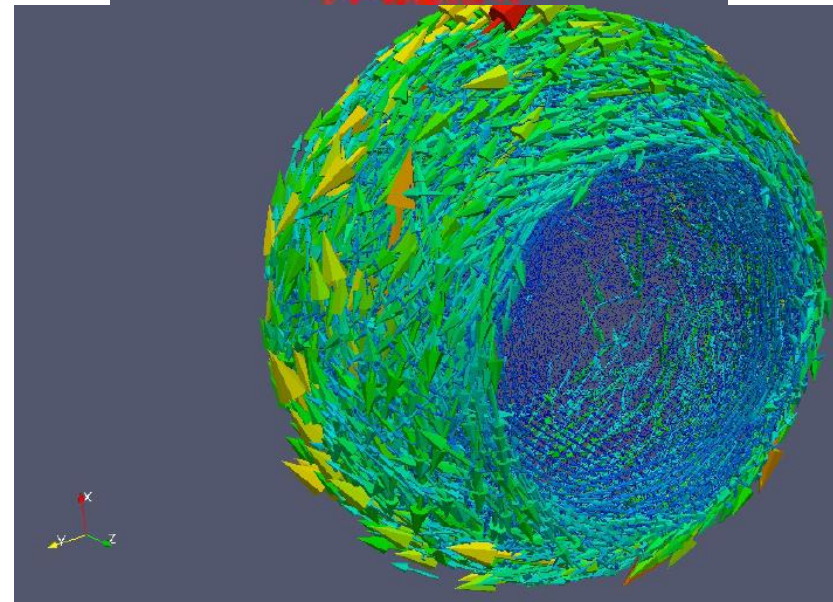
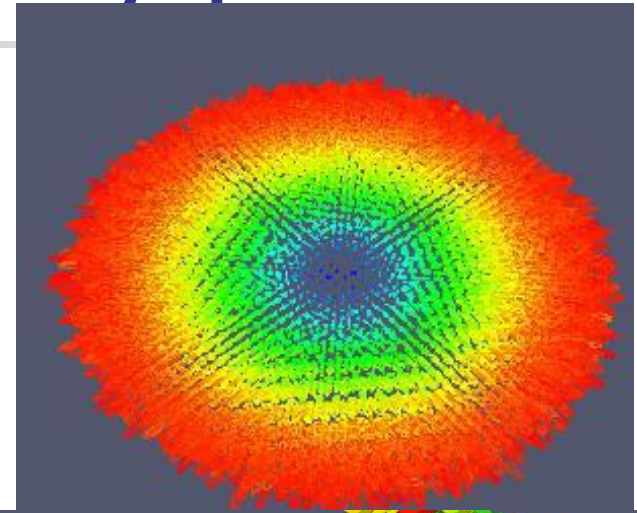
# Distribution of vorticity ("Little galaxies")

- Layer (on core - corona borderline) patterns

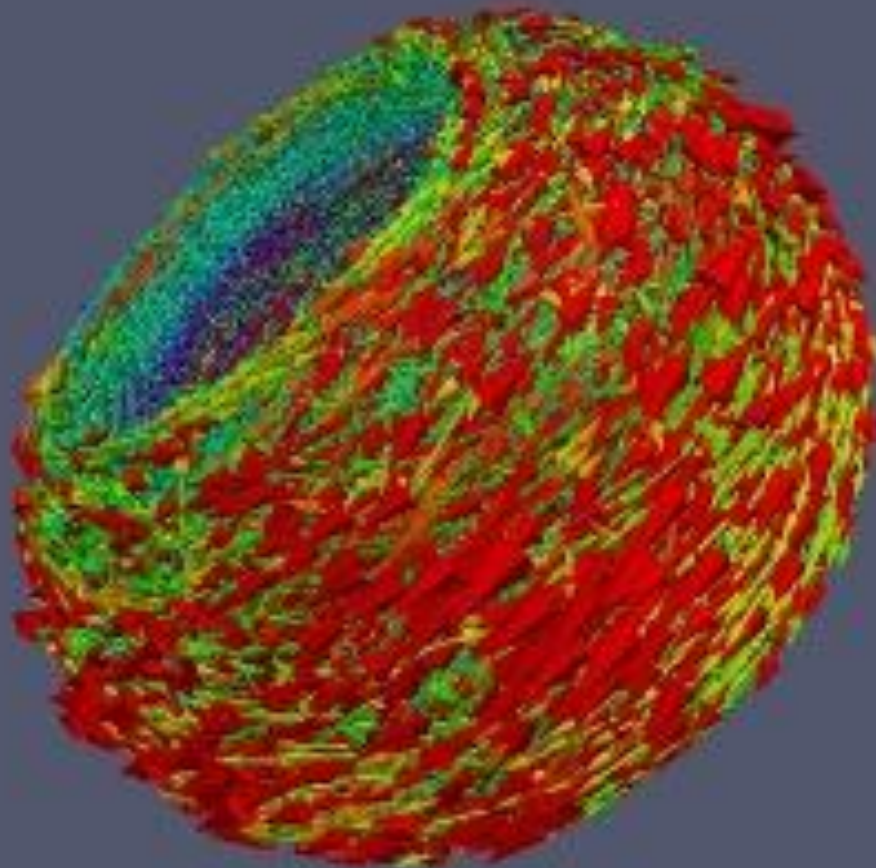


# Velocity and vorticity patterns

- Velocity
- Vorticity pattern – vortex sheets

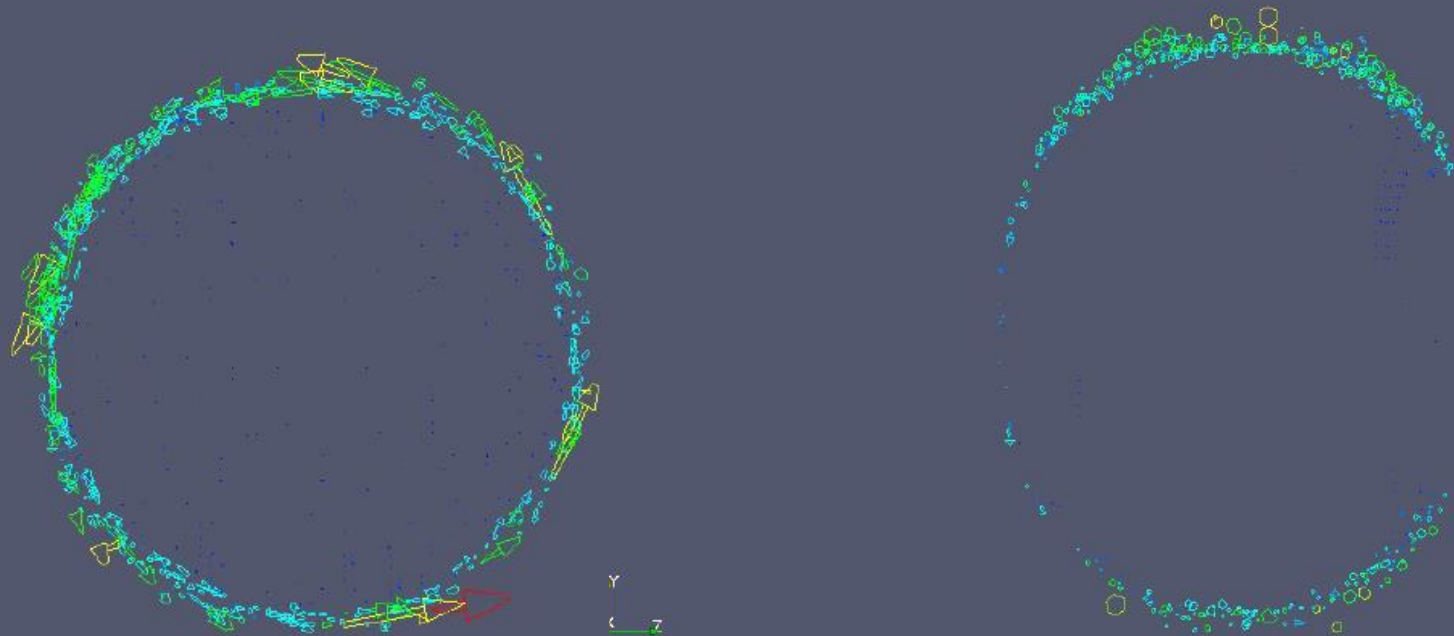


# Vortex sheet



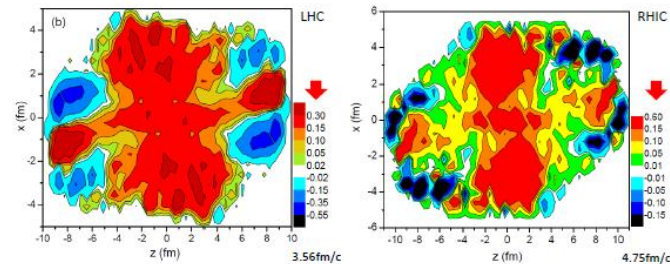
# Sections of vorticity patterns

- Front and side views

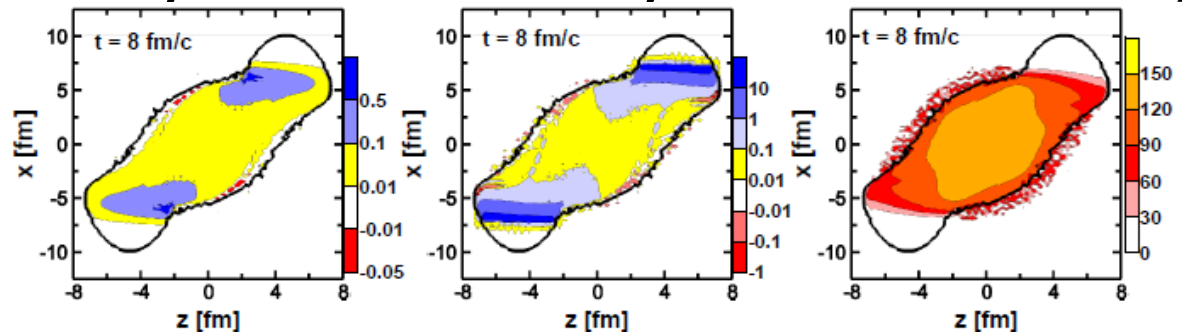


# Vortex sheets

- Naturally appears in kinetic models
- Absent in viscous HD (L. Csernai et al)



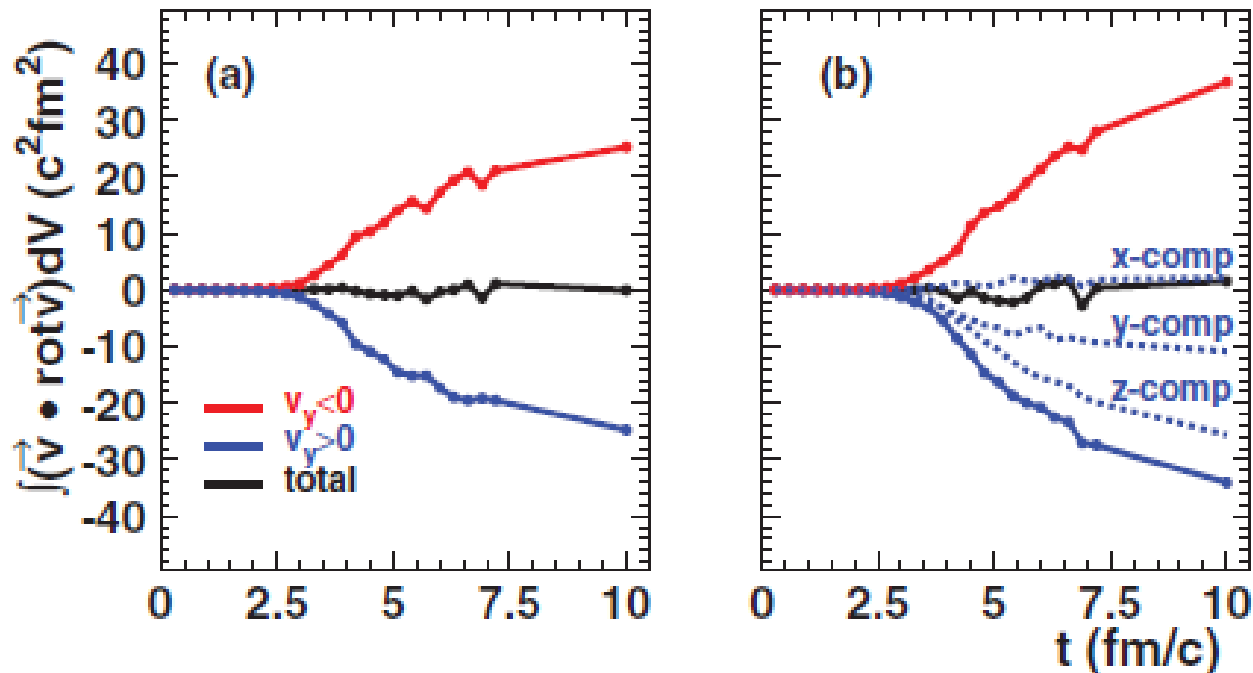
- Appears in 3 fluid dynamics model (Yu. Ivanov, A. Soldatov, [arXiv:1701.01319](https://arxiv.org/abs/1701.01319))



# Helicity separation in QGSM

## PRC88 (2013) 061901

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (Teryaev, Usubov, PRC92 (2015) 014906)



# Transverse and longitudinal vorticity

- Transverse vorticity – the same sign on the two sides of reaction plane
- Change of velocity sign ( $v_y \sim \text{sign}(y)$ ) leads to helicity separation  $h_y \sim \text{sign}(y)$
- Longitudinal vorticity – must have **quadrupole** structure to provide **mirror** structure of helicity

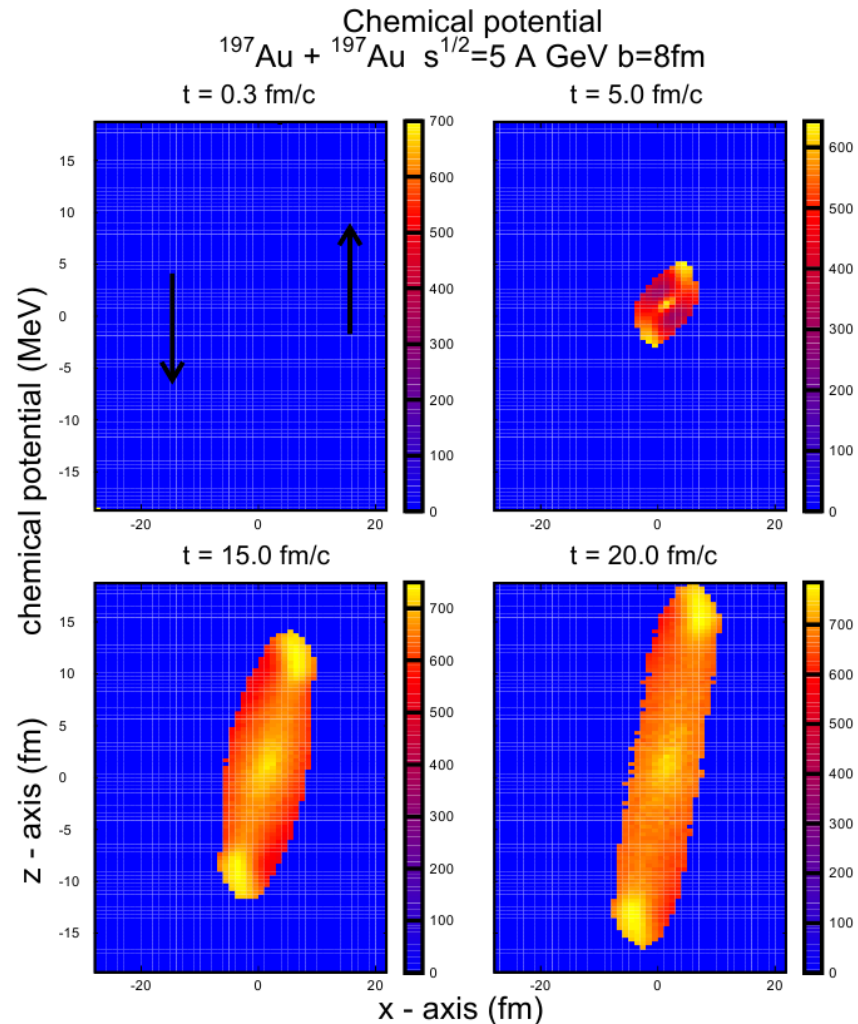
$$h = h_x + h_y + h_z \sim \text{sign}(y); v_x \sim \text{sign}(x); \omega_x \sim \text{sign}(x)\text{sign}(y)$$

$$h_x = \omega_x v_x = (\text{sign}(x))^2 \text{sign}(y) = \text{sign}(y)$$

# Chemical potential : Kinetics

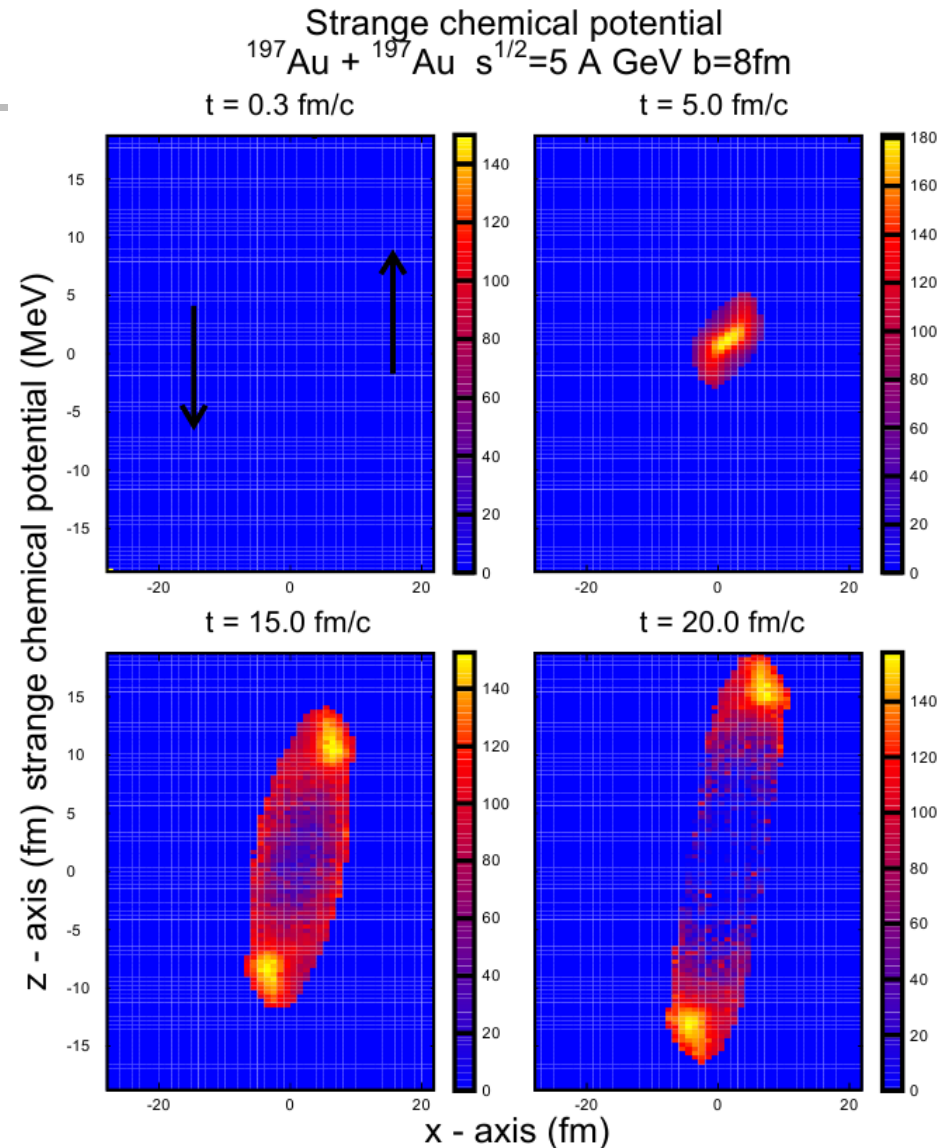
-> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section:  $y=0$

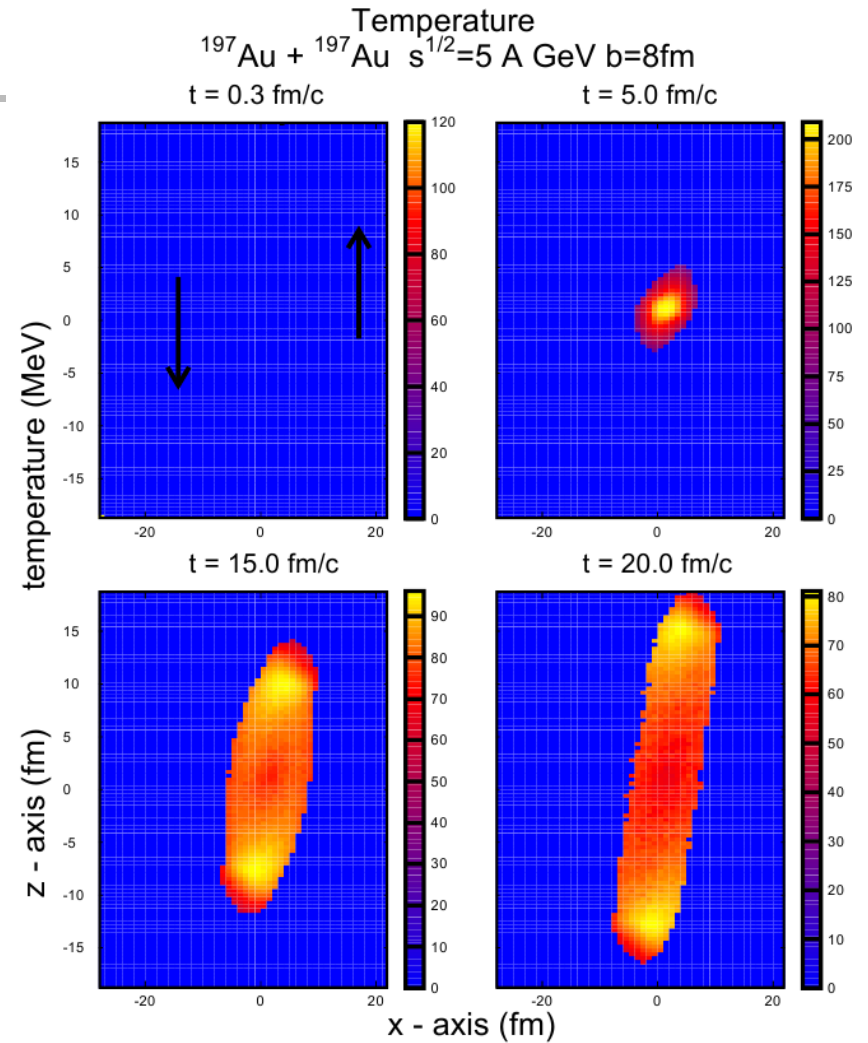


# Strange chemical potential (polarization of Lambda is carried by strange quark!)

- Non-uniform in space and time



# Temperature



# From axial charge to polarization (and from quarks to confined hadrons) – analog of Cooper-Frye

- Analogy of matrix elements and classical averages (account for other charges!)

$$\langle p_n | j^0(0) | p_n \rangle = 2p_n^0 Q_n \quad \langle Q \rangle \equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x j_{class}^0(x)}{N}$$

- Lorentz boost: compensate the sign of helicity

$$\langle p_n, \Pi_n | j_{5,i}^0(0) | p_n, \Pi_n \rangle = 2a_{i,n} m_n \Pi_n^0 \quad Q_{5,i,n} \rightarrow \frac{m_n a_{i,n} \Pi_n^0}{p_n^0}$$

$$\Pi^{\Lambda,lab} = (\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}) = \frac{\Pi_0^{\Lambda}}{m_{\Lambda}} (p_y, 0, p_0, 0)$$

$$\langle \Pi_0^{\Lambda} \rangle = \frac{m_{\Lambda} \Pi_0^{\Lambda,lab}}{p_y} = \langle \frac{m_{\Lambda}}{N_{\Lambda} p_y} \rangle Q_5^s \equiv \langle \frac{m_{\Lambda}}{N_{\Lambda} p_y} \rangle \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

# Axial charge and properties of polarization



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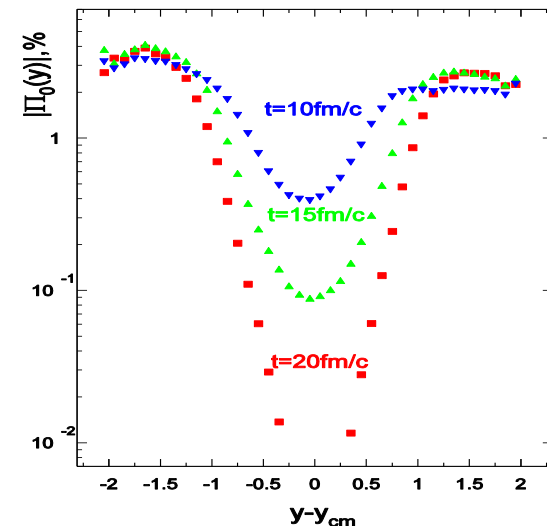
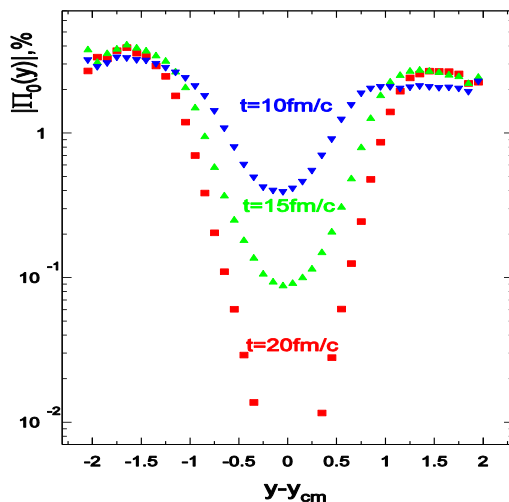
- Polarization is enhanced for particles with small transverse momenta – azimuthal dependence naturally emerges
- Antihyperons: same sign (C-even axial charge) and larger value (smaller N)
- More pronounced at lower energy BUT
- Baryon/antibaryon splitting due to magnetic field – increase with energy
- Recent STAR data support polarization for particles with angles close to reaction plane and closeness of baryons and antibaryons polarization at 200 GeV

# QGSM numerics for polarization

- Helicity  $\sim$  0th component of polarization in lab. frame + effect of boost to Lambda rest frame

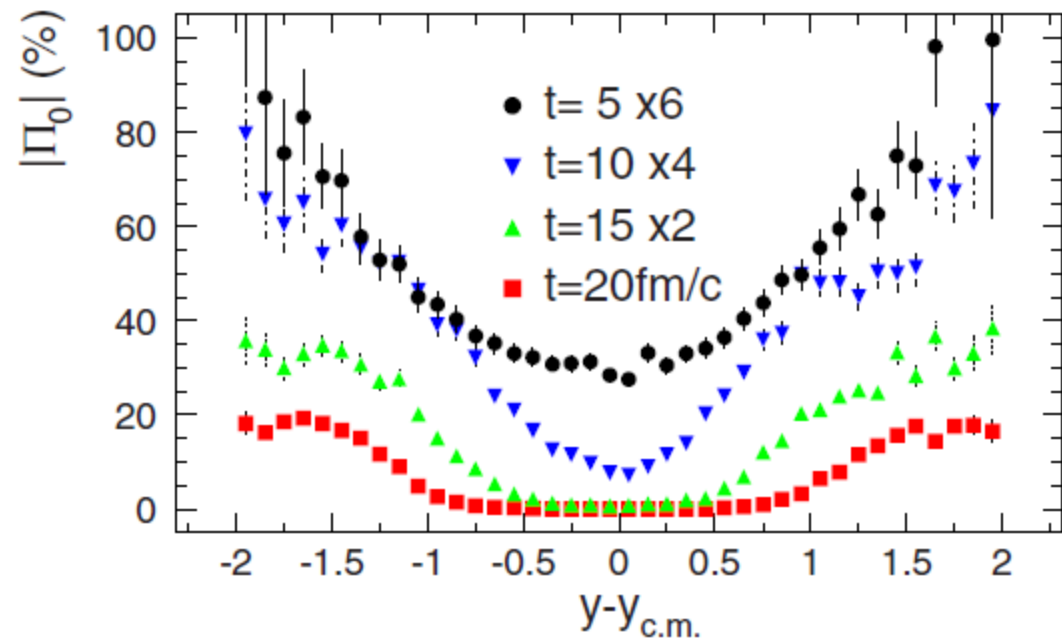
$$\Pi_0(y) = \frac{1}{(4\pi^2)} \int \gamma^2(x) \mu_s^2(x) |\mathbf{v} \cdot \text{rot}(\mathbf{v})| n_\Lambda(y, \mathbf{x}) w_1 d^3x / \int n_\Lambda(y, \mathbf{x}) w_2 d^3x$$

$w_1 = 1, w_2 = 1$ 
 $w_1 = 1, w_2 = p_y/m$



# Combining QGSM (thermal)vorticity with TD mechanism

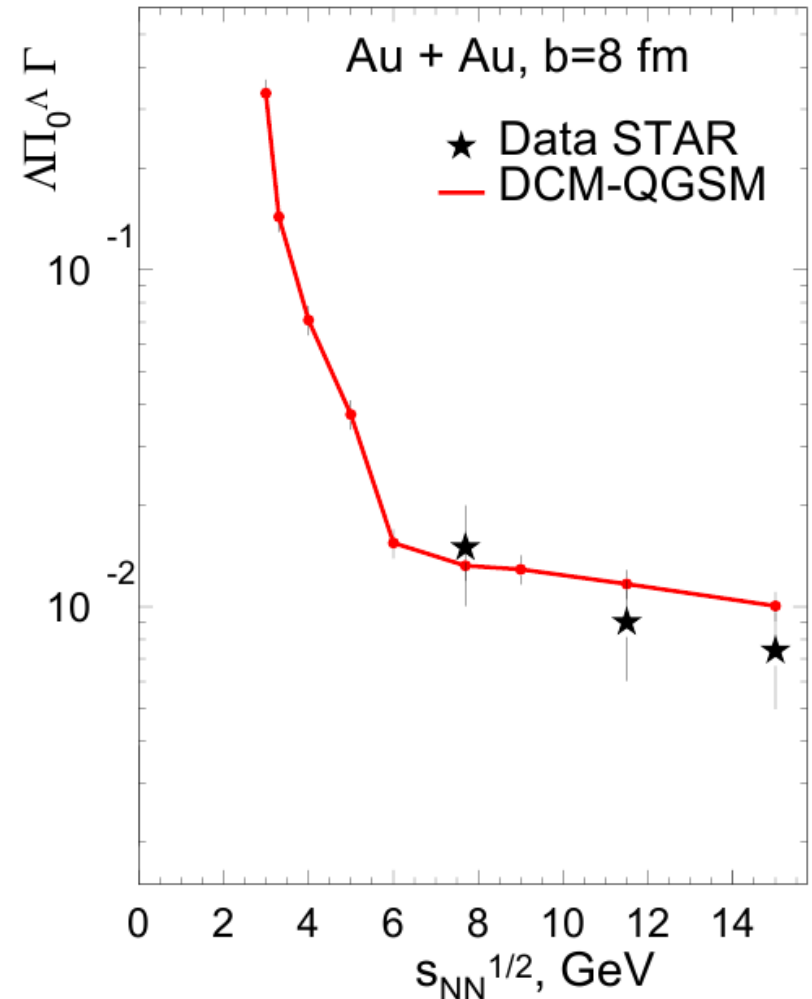
- Thermal vorticity + axial charge



- Similar polarization pattern

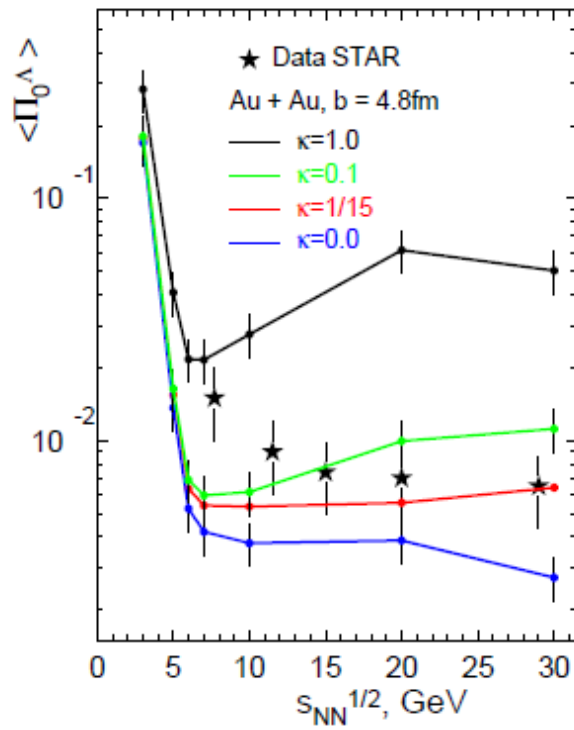
# Energy dependence

- Growth at low energy
- Close to STAR data
- Structure – due to low number of Lambdas



# The role of (gravitational anomaly related) $T^2$ term

- Different values of coefficient probed



$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + k \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- LQCD suppression by collective effects supported

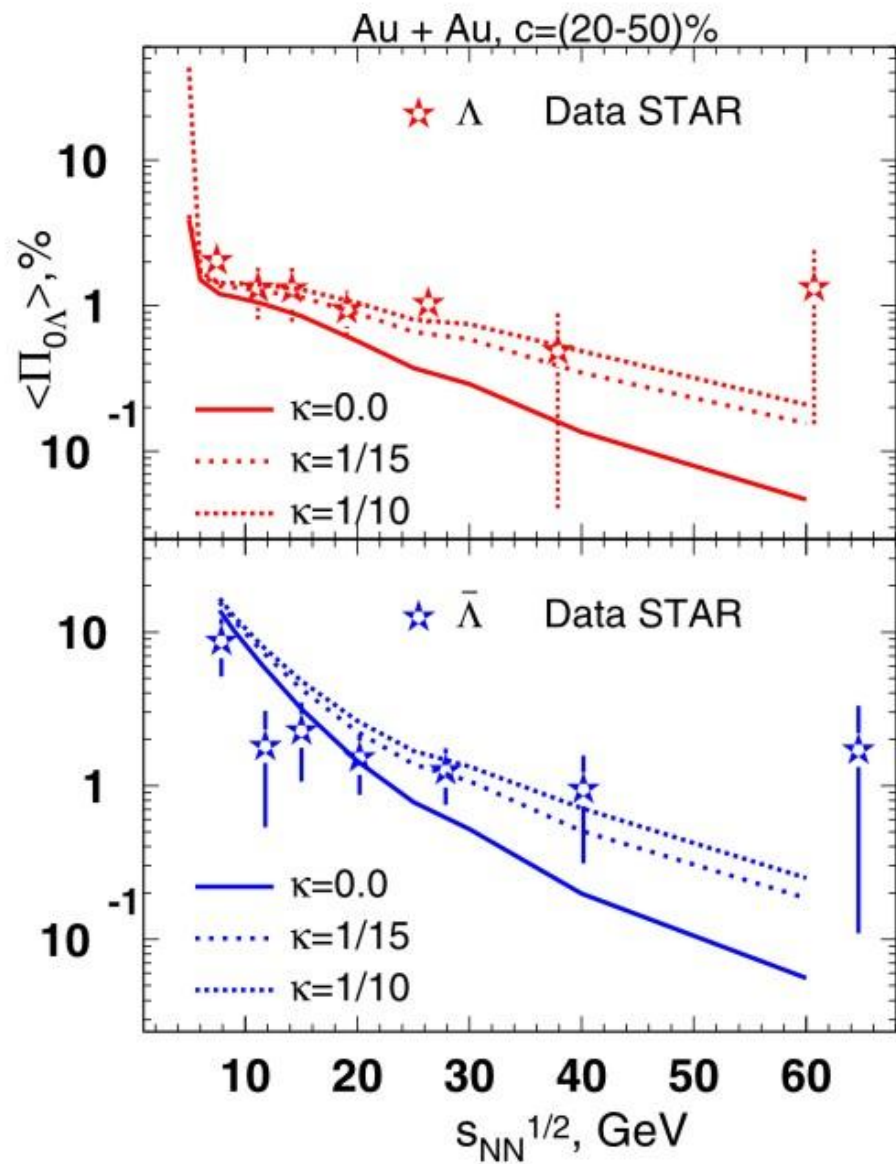


# Lambda vs Antilambda and role of vector mesons

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- Difference at low energies too large – same axial charge carried by much smaller number of hadrons
- Strange axial charge may be also carried by  $K^*$  mesons
- $\Lambda$  - accompanied by  $(-, \text{anti } 0)$   $K^*$  mesons with two sea quarks – small corrections
- Anti  $\Lambda$  – accompanied by more numerous  $(+, 0)$   $K^*$  mesons with single (sea) strange antiquark

# $\Lambda$ vs Anti $\Lambda$





# Arnold-Beltrami flows

- Nonrelativistic incompressible fluids with vorticity parallel to velocity

$$\omega_i \equiv \epsilon_{ijk} \partial_j v_k = m v_i$$

- Compatible with Euler equation for steady flows

$$v_j \partial_j v_i = -\frac{1}{\rho} \partial_i p$$

- Bernoulli condition is valid in the whole volume of the fluid

$$\partial_i \phi = 0 \qquad \phi = \frac{p}{\rho} + \frac{v^2}{2}$$



# Chaotic streamlines

$$\frac{d}{dt}x^i(t) = u^i(x(t), t)$$

## Arnold's theorem:

For flows taking place on compact three manifolds, the only velocity fields able to produce chaotic streamlines are those satisfying Beltrami equation.

&

Topological conception of contact structures, each of which admits a representative contact vector field also satisfying Beltrami equation.



# Chaotic thermalization?

- Arnold-Beltrami flows – Lagrangian turbulence
- Simple explanation: Bernoulli in the volume – the streamlines come close to each other
- Chaotic advection: laminar flows result in the chaotic motion of passive admixture
- Fast Dynamo problem, the spontaneous generation of a exponential growing magnetic field in a flow of conducting fluid with vorticity
- Possible role in the fast thermalization (complementary description)?!



# Conclusions/Outlook

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- Polarization – new probe of anomaly in quark-gluon matter (to be studied at NICA)
- Generated by **femto-vortex sheets**
- Energy dependence **predicted** and confirmed
- Same sign and larger magnitude of antihyperon polarization: splitting decreases with energy (contradicts to explanation due to magnetic moment/field ; supported by the data)
- T-dependent term due to gravitational anomaly may be extracted from the data
- All known tests seem to be passed – but further studies required

# THANK YOU FOR ATTENTION!

# WELCOME TO DUBNA!



XVII Workshop on High Energy Spin Physics

## *DSPIN - 17*

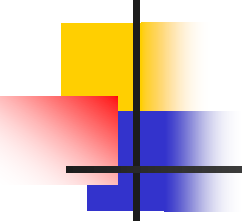
Dubna, Russia, September 11 - 15, 2017

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### **Topics and scope**

*Recent experimental data on spin physics*  
*The nucleon spin structure and GPD's*  
*Spin physics and QCD*  
*Spin physics in Standard Model and beyond*  
*T-odd spin effects*  
*Polarization and heavy ion physics*  
*Spin in gravity and astrophysics*  
*The future spin physics facilities*



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Spin effects in heavy ion collisions may be used as a complementary probe spin physics program involving all the NICA detectors (MPD, SPD, BM@N),

# Polarization at NICA/MPD (A. Kechechyan)

- QGSM Simulations and **recovery**  
**accounting for MPD acceptance effects**

