Chaotic Quantum Many-Body Systems and Philosophy of Thermalization

Vladimir Zelevinsky
NSCL/FRIB  Michigan State University

International Conference on New Frontiers in Physics
Crete, August 17, 2017
OUTLINE

1. Quantum chaos in mesoscopic systems
2. Chaos and thermalization (no heat bath!)
3. Thermometers
4. Level density
5. Ground state temperature = limiting temperature?
6. Spin cut-off parameter
7. Problems – Outlook

8. Random parameters explore nuclear landscape
THANKS

- Alex Berlaga (HSHP5S, MSU)
- B. Alex Brown (MSU)
- Fausto Borgonovi (University of Breschia)
- Mihai Horoi (Central Michigan University)
- Felix Izrailev (University of Puebla)
- Sofia Karampagia (MSU)
- Antonio Renzaglia (MSU)
- Lea Santos (Yeshiva University)
- Roman Sen’kov (La Guardia College)
- Alexandr Volya (Florida State University)
MESOSCOPIC SYSTEMS:

MICRO ------ MESO ------ MACRO

- Complex nuclei
- Complex atoms
- Complex molecules (including biological)
- Cold atoms in traps
- Atomic clusters
- Micro- and nano- devices of condensed matter
- Future quantum computers

Common features: quantum bricks, interaction, complexity; quantum chaos, statistical regularities; at the same time – individual quantum states
Chaotic motion in mesoscopic systems

* Mean field (one-body chaos) - classical features
* Strong interaction (many-body chaos)
* High level density
* Mixing of simple configurations
* Destruction of quantum numbers,
  (in nuclei: conserved only energy; J,M; T,T3; parity)
* Local spectral statistics – Gaussian Orthogonal Ensemble
* Correlations between classes of states
* Coexistence with (damped) collective motion
* Thermal equilibrium – without heat bath
* Continuum effects – open system
INSIDE CHAOS


• DISORDERED wave functions

• Any SIMPLE operator has matrix elements of the same order of magnitude between any two of these eigenfunctions

• All typical wave functions of roughly the same energy LOOK ROUGHLY THE SAME being spread over the large region of configuration space

Random matrix canonical ensembles – only as mathematical limit
MANY-BODY QUANTUM CHAOS AS AN INSTRUMENT

SPECTRAL STATISTICS – signature of chaos
- missing levels
- purity of quantum numbers
- **level density without full diagonalization**
- presence of time-reversal invariance

EXPERIMENTAL TOOL – unresolved fine structure
- width distribution
- damping of collective modes

NEW PHYSICS
- **statistical enhancement of weak perturbations**
  (parity violation in neutron scattering and fission)
- mass fluctuations
- chaos on the border with continuum

THEORETICAL CHALLENGES
- order out of chaos
- **chaos and thermalization**
- development of computational tools
- new approximations in many-body problem
From turbulent to laminar level dynamics
Chaos due to particle interactions at high level density

LEVEL DYNAMICS

WAY to CHAOS:
MULTIPLE
AVOIED
CROSSINGS
as a function
of interaction strength

(shell model of 24Mg
as a typical example)

Fraction (%) of realistic strength
4- and 4+ states in the model of Cerium atom

Flambaum et al. 1994, E > 1 eV

Fixed basis!
4+ excited states in the model of random two-body Interaction

/4 particles, 11 levels/

Fixed basis!
MEASURING COMPLEXITY

Eigenstate $|\alpha\rangle$ in a shell model basis $|k\rangle$

$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$

Information entropy

$S^\alpha = -\sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$

No mixing: $S^\alpha \rightarrow 0$

“Microcanonical” mixing: $S^\alpha \rightarrow \ln N$

GOE: $\overline{S}^\alpha = \ln(0.48N)$

Information entropy is basis-dependent
- special role of mean field
INFORMATION ENTROPY AT WEAK INTERACTION

28 Si Shell Model (artificially weak interaction)
INFORMATION ENTROPY of EIGENSTATES
(a) function of energy; (b) function of ordinal number

ORDERING of EIGENSTATES of GIVEN SYMMETRY

SHANNON ENTROPY AS THERMODYNAMIC VARIABLE
CHAOS versus THERMALIZATION

L. BOLTZMANN - *Stosszahlansatz* = MOLECULAR CHAOS

N. BOHR - *Compound nucleus* = MANY-BODY CHAOS

N. S. KRYLOV - *Foundations of statistical mechanics*

L. Van HOVE - *Quantum ergodicity*

M. SREDNICKY - *Chaos and thermalization*

L. D. LANDAU and E. M. LIFSHITZ - “*Statistical Physics*”

Average over the equilibrium ensemble should coincide with the expectation value in a generic individual eigenstate of the same energy – the results of measurements in a closed system do not depend on exact microscopic conditions or phase relationships if the eigenstates at the same energy have similar macroscopic properties

TOOL: MANY-BODY QUANTUM CHAOS
CLOSED MESOSCOPIC SYSTEM

at high level density

Two languages: individual wave functions thermal excitation

* Mutually exclusive ?
* Complementary ?
* Equivalent ?

Answer depends on thermometer
FAMILY OF ENTROPIES FOR A MESOSCOPIOC SYSTEM

- **THERMODYNAMIC** (Boltzmann)

  \[ \rho(E) \propto \exp(S_{th}) \]

- **QUASIPARTICLE** (Landau Fermi-liquid)

  \[ S_{\text{s.p.}}^\alpha = - \sum_i \left\{ n_i^\alpha \ln(n_i^\alpha) + (1 - n_i^\alpha) \ln(1 - n_i^\alpha) \right\} \]

- **INFORMATION** (Shannon)

  \[ |\alpha\rangle = \sum_k C_k^\alpha |k\rangle, \quad S_{\text{inf}}^\alpha = - \sum_k \{|C_k^\alpha|^2 \ln |C_k^\alpha|^2\} \]

  \[ \langle n_i \rangle_E = \left[ e^{(\varepsilon_i - \mu)/T_{\text{s.p.}}} + 1 \right]^{-1} \]

\[ T_{\text{th}} = \left( \frac{dS_{\text{th}}}{dE} \right)^{-1} \]

\[ T_{\text{inf}} = \left( \frac{d\bar{S}_{\text{inf}}}{dE} \right)^{-1} \]

T(s.p.) and T(inf) = for individual states!
$^{28}$Si, parity=$\pm 1$, some $J$, $sd$-shell

Shell Model (solid line) vs. Moments Method (dashed line).
Single – particle occupation numbers

Thermodynamic behavior identical in all symmetry classes

FERMI-LIQUID PICTURE
Artificially strong interaction (factor of 10)

Single-particle thermometer cannot resolve spectral evolution
EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution (dots)
From thermodynamic entropy defined by level density (lines)
Occupation numbers in multicharged ions Au25+ (recombination as analog of neutron resonances in nuclei)

\[ n_s^\alpha = \langle \alpha | \hat{n}_s | \alpha \rangle = \sum_k |C_k^\alpha|^2 \langle k | \hat{n}_s | k \rangle \]

/G. Gribakin, A. Gribakina, V. Flambaum/

Average over individual states is equivalent to a thermal ensemble
TWO-BODY random interaction, 4 fermions, 11 orbitals (s)

LEFT: weak interaction
RIGHT: strong interaction (x10)

\[
\sigma_k^2 = (H^2)_{kk} - (H_{kk})^2 = \sum_{k' \neq k} |H_{kk'}|^2 \\
\sum_s n_s = n, \quad \sum_s \epsilon_s n_s = E + \delta_E, \quad \delta_E = \frac{\sigma_k^2}{\sigma_0^2} (E_E - E)
\]
Moments method

\[ \rho(E, \alpha) = \sum \alpha_{\kappa} \cdot G_{\alpha \kappa}(E) \]

\[ G_{\alpha \kappa}(E) = G(E + E_{g.s} - E_{\alpha \kappa}, \sigma_{\alpha \kappa}) \]

\[ G(x, \sigma) = C \cdot \begin{cases} \exp \left( -x^2 / 2\sigma^2 \right), & |x| \leq \eta \cdot \sigma \\ 0, & |x| > \eta \cdot \sigma \end{cases} \]

\[ E_{\alpha \kappa} = \langle H \rangle_{\alpha \kappa}, \]

\[ \sigma_{\alpha \kappa} = \sqrt{\langle H^2 \rangle_{\alpha \kappa} - \langle H \rangle^2_{\alpha \kappa}} \]

\[ \langle H \rangle_{\alpha \kappa} = \text{Tr}^{(\alpha \kappa)}[H] / D_{\alpha \kappa}, \]

\[ \langle H^2 \rangle_{\alpha \kappa} = \text{Tr}^{(\alpha \kappa)}[H^2] / D_{\alpha \kappa} \]

Quantum numbers

\[ \kappa = \{n_1, n_2, \ldots, n_q\} \]

Partitions

Finite range

Gaussian

Many-body dimension

Centroids – first moment

Widths - second moment

\[ \text{Tr}^{(J)}[\ldots] = \text{Tr}^{(J_z)[\ldots]}_{J_z = J} - \text{Tr}^{(J_z)[\ldots]}_{J_z = J+1} \]
PRACTICAL ALGORITHM

(Ground state energy problem)

- Generate the set of partitions in given orbital space
  Example $^{56}\text{Ni}$ in full $pf$-space:
  1 087 455 228 $m$-scheme states,
  2 581 576 $J^\pi T = 0^+0$ states, 475 partitions

- Calculate partition centroids
  (Hamiltonian traces/ dimension)

- Order partitions by their centroids

- Truncate shell model matrices including consecutively partitions in their entirety

- Calculate the lowest states (Lanczos) with progressive truncation

- Identify onset of the exponential regime

- Obtain the exact energy value
Exponential convergence of ground state energy: 48Cr in (fp)-shell

Full dimension \( n_{\text{max}} = 2 \text{ million} \)

The same convergence of occupancies and simple matrix elements
51Sc

1/2⁻, 3/2⁻

Faster convergence:
E(n) = E + exp(-an)
a ~ 6/N
EXPONENTIAL CONVERGENCE OF SINGLE-PARTICLE OCCUPANCIES

(first excited state J=0)

$^{52}$Cr

Fit with $\gamma' = \gamma$
Analytical results for tridiagonal matrices

\[ H = \begin{pmatrix} 
\varepsilon_1 & V_2 & 0 & 0 & 0 & 0 \\
V_2 & \varepsilon_2 & V_3 & 0 & 0 & 0 \\
0 & V_3 & \varepsilon_3 & V_4 & 0 & 0 \\
0 & 0 & \ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & \ldots & \ldots & V_n \\
0 & 0 & 0 & 0 & V_n & \varepsilon_n 
\end{pmatrix} \]

Assume existence of the limit

\[ \lambda_n \Rightarrow \lambda \]

at

\[ n \Rightarrow \infty \]

Recurrence relation for determinants

\[ D_n(E) = (\varepsilon_n - E)D_{n-1}(E) - V_n^2 D_{n-2}(E) \]

Convergence is determined by

\[ \lambda_n^2 = V_n^2 / (\varepsilon_n \varepsilon_{n-1}) \]
CONVERGENCE REGIMES

- Fast convergence
- Exponential convergence
- Power law
- Divergence

Exponential convergence

\[ \gamma \sim \exp(-\gamma n) \]

\[ \gamma = -\ln \left( \frac{1}{2\lambda^2} \left( 1 - 2\lambda^2 - \sqrt{1 - 4\lambda^2} \right) \right) \]
$^{28}$Si, parity=+1, some $J$, $sd$-shell

Shell Model (solid line) vs. Moments Method (dashed line).

Shell-model level density.

Moments method (no diagonalization)
$J = 0 - 7$, positive parity level density

$^{52}$Cr, $J^\pi = 1^+$

- exact SM (pf-shell, gx1a)
- moments method
- model of Goriely et al.
$^{52}$Fe, $J^\pi = 1^+$

- exact SM (pf-shell, gx1a)
- moments method
- model of Goriely et al.
Level density and “constant temperature” fit

\[ \text{L.D.}(E) = \text{(const)} \exp\left[\frac{E}{T}\right] \]
“Constant temperature” fit
(different energy ranges)

Red  1 – 5 MeV
Black  5 - 15 MeV
Blue  5 – 25 MeV
Effective temperature for the level density at low energy (up to 6 – 8 Mev)
Even-odd staggering
Clear minima in the vicinity of N=Z
Initial strength function – Gaussian (stronger interaction), \( g(2) > g(1) \)

Linear growth of entropy = exponential growth of the number of principal components

Remote analog of classical Lyapunov exponent?
H = h + k(1)V(pairing) + k(2)V(rest)

**Silicon**: effects in addition to pairing
Cumulative level number

\[ N(E) = \exp(S), \]

Entropy \( S(E) = \ln(N) \)

Thermodynamic temperature

\[ T(t-d) = \frac{dS}{dE} = T[1 - \exp(-E/T)] \]

Parameter \( T \) is \textit{limiting temperature} (\textit{Hagedorn temperature} in particle physics)

\textit{Phase transition? (Moretto) - Chaotization}
"Spin cut-off" parameter

\[
\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-M^2/2\sigma^2}
\]

Markovian random process of angular momentum coupling
$^{52}\text{Fe}$

\[ \frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-M^2/2\sigma^2} \]
$^{44}\text{Ca}$

$^{64}\text{Cr}$

4 valence neutrons

4 proton holes

Space – only $T=2$,
Two-body interaction through $T=1$ channel
"THE BEAUTY OF THIS IS THAT IT IS ONLY OF THEORETICAL IMPORTANCE, AND THERE IS NO WAY IT CAN BE OF ANY PRACTICAL USE WHATSOEVER."
OUTLOOK

• Thermalization by internal interactions (Phys. Rep. 2016)
• Various thermometers
• Comparison with phenomenology
• Odd and odd-odd nuclear systems
• Vicinity of N=Z line
• Spin “cut-off” factors
• Theory for the “ground state temperature”
• Comparison with decay processes
• Beyond shell model; continuum
• Equilibration dynamics
• Equilibration between classes of states
• Quantum analogs for classical “Lyapunov exponents”?
Figure 5: Graph of temperature as a function of $k_1$ in even-even isotopes of sulfur
Matrix elements

9-12: pf mixing,
16 : quadrupole pair transfer,
20-24: quadrupole-quadrupole forces

in particle-hole channel = formation of the mean field

Large fluctuations of non-extensive nature (the same for 10 000 and 100 000 realizations)
Artificially driving shape transitions

<table>
<thead>
<tr>
<th>Shape</th>
<th>Case</th>
<th>Nucleus</th>
<th>$R_{4/2}$</th>
<th>NoL</th>
<th>Renorm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.4$</td>
<td>$^{28}\text{Si}$</td>
<td>3.31</td>
<td>22</td>
<td>60</td>
</tr>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.5$</td>
<td>$^{28}\text{Si}$</td>
<td>3.33</td>
<td>17</td>
<td>54</td>
</tr>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.6$</td>
<td>$^{28}\text{Si}$</td>
<td>3.21</td>
<td>13</td>
<td>49</td>
</tr>
<tr>
<td>Spherical</td>
<td>$k_2 = 1.0$, $k_1 = 0.9$</td>
<td>$^{28}\text{Si}$</td>
<td>2.12</td>
<td>5</td>
<td>34</td>
</tr>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.5$</td>
<td>$^{24}\text{Mg}$</td>
<td>3.20</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.6$</td>
<td>$^{24}\text{Mg}$</td>
<td>3.21</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Spherical</td>
<td>$k_2 = 1.0$, $k_1 = 0.3$</td>
<td>$^{24}\text{Mg}$</td>
<td>2.03</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>Deformed</td>
<td>$k_1 = 1.0$, $k_2 = 0.4$</td>
<td>$^{52}\text{Fe}$</td>
<td>3.07</td>
<td>236</td>
<td>6516</td>
</tr>
<tr>
<td>Spherical</td>
<td>$k_2 = 1.0$, $k_1 = 0.0$</td>
<td>$^{52}\text{Fe}$</td>
<td>2.25</td>
<td>30</td>
<td>2617</td>
</tr>
</tbody>
</table>

$H = h + k_1 V_1 + k_2 V_2$
Low-lying levels in absolute (a) and rotational (b) units;

Ratio $E(4)/E(2)$ (c)

Transition rates (d)

$H = h + (1 - \lambda)V_1 + \lambda V_2$
\[ H = h + (1 - \lambda)V_1 + \lambda V_2, \]
Amplitudes of the ground state wave functions in terms of \([J(p),J(n)]\)
Number of $0^+$ levels up to energy 10 MeV
J=0 – 10 for 26 Al, 28 Al, 30 P (up to 10 MeV)

J=1/2 – 21/2 for 27 Al (up to 10 MeV)

J=0 – 10 for 50 Mn (up to 60 MeV)
Quadrupole moment of $2^+$ state in $^{30}\text{P}$ as a function of the strength of the mixing interaction strength.
Level density (0+) on two sides of deformation shape transition

//”collective enhancement”//
FIG. 12: (Color online.) Thermodynamic temperature as a function of excitation energy, Eq. (18), calculated in the moments method for $^{28}\text{Si}$, top four curves (black color), and for $^{56}\text{Fe}$, the bottom four curves (blue color).
V. Z., B.A. Brown, N. Frazier and M. Horoi.
The nuclear shell model as a testing ground for many-body quantum chaos.

V. Z.. Quantum chaos and complexity in nuclei.

A.Volya and V. Z.
Invariant correlational entropy as a signature of quantum phase transitions in nuclei.

V. Z. and A. Volya.
Nuclear structure, random interactions and mesoscopic physics.

F. Borgonovi, F.M. Izrailev, L.F. Santos, and V.Z.
Quantum chaos and thermalization in isolated systems of interacting particles.
Physics Reports 626 (2016) 1.

V.Z. and A. Volya.
Chaotic features of nuclear structure and dynamics: Selected topics.


S. Karampagia, R.A. Sen’kov, and V.Z. Level density in the sd-nuclei - statistical shell model predictions. ADNDT /in preparation/. 
Information entropy $S(t)$ for Two-Body Random Interaction model, 6 particles, 12 orbitals

---

Linear dependence $g t \log(K)$, $K$ – number of states directly coupled by random interaction

After the initial stage: kinetic tree avalanche

$\frac{dW(0)}{dt} = -gW(0)$,

$\frac{dW(1)}{dt} = gW(0) - gW(1)$,

$\cdots$

$\frac{dW(j)}{dt} = gW(j-1) - gW(j)$

Solid line:

$S(t) = -W(0) \log W(0) - [1-W(0)] \log [(1-W(0))/(NPC)]$

Initial strength function Breit-Wigner
Time dependence of occupation numbers: \( n(t) = n(0) \ W(0,t) + \ n \ (\text{fin}) \ [1 - \ W(0,t)] \)

Breit-Wigner strength function                                      Gaussian strength function

Solid line – solution of the cascade model
Shape instability in the middle of spectrum?
"Constant temperature" fit to total level density at $E > 5$ MeV

\[
\ln[\rho(E)] = \frac{E}{T} + \text{constant}
\]
Structure of eigenstates in energy representation for spin models: Average over 5 states in the center of spectrum

\[ H_1 = H_0 + \mu V_1, \quad H_0 = \sum_{i=1}^{L-1} J \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right), \quad V_1 = \sum_{i=1}^{L-1} J S_i^z S_{i+1}^z, \]

\[ H_2 = H_1 + \lambda V_2, \quad V_2 = \sum_{i=1}^{L-2} J \left[\left(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y\right) + \mu S_i^z S_{i+2}^z\right], \]

**BLUE**: integrable model XXZ

**RED**: chaotic for large V2
* Add random noise to the dynamics
* **Construct the density matrix by averaging**
  * for any individual wave function
* **Calculate the corresponding entropy**
  * measurement of sensitivity of eigenstates
  * quantum phase transitions
  * basis-independent criteria
Invariant correlational entropy as signature of phase transitions

Eigenstates in an arbitrary basis
(Hamiltonian with random parameters)

Density matrix of a given state
(averaged over the ensemble)

Correlational entropy has clear maximum at phase transition (extreme sensitivity)

Pure state: eigenvalues of the density matrix are 1 (one) and 0 (N-1), 
S=0

Mixed state: between 0 and 1, 
S up to \( \ln N \)

For two discrete points 
\[
r^\alpha_{\pm} = \frac{1 \pm |\langle \alpha(\lambda) | \alpha(\lambda') \rangle|}{2}
\]
Model of two levels with pair transfer

Capacity $16 + 16$, $N=16$

Critical value 0.3
(in BCS $\frac{1}{4}$)

Averaging interval 0.01

First excited state
“pair vibration”

No instability in the exact solution

Softening at the same point 0.3
Is there a pairing phase transition in mesoscopic system?

Invariant entropy

Mixing of states $|\alpha\rangle = \sum_k C_k^\alpha(G)|k\rangle$

$|k\rangle$- some reference basis states

Matrix $\rho$ for each state $\alpha$

$\rho_{k k'}^\alpha(G, \delta G) = \frac{1}{\delta G} \int_G^{G+\delta G} C_{k'}^\alpha(G)^* C_k^\alpha(G) dG$

Invariant entropy

$I^\alpha(G, \delta G) = -\text{Tr} (\rho^\alpha \ln(\rho^\alpha))$

- Invariant entropy is basis independent
- Indicates the sensitivity of eigenstate $\alpha$ to parameter $G$ in interval $[G, G+\delta G]$
Shell model $^{48}\text{Ca}$

Ground state invariant entropy; phase transition depends on non-pairing interactions

Occupancy of $f7/2$ shell

Correlation energy $\sim$ 2 MeV
Various measures

Level density

Information Entropy in units of $S(GOE)$

Single-particle entropy of Fermi-gas

Interaction: 0.1 1 10
Two coupled rotors /analog of Lipkin model/

\[ H = L(z) + M(z) + L(x)M(x) \]

Here \( L = M = 10 \), total spin 19

\( \langle a_1, b_1 \rangle \) ground state

\( \langle a_2, b_2 \rangle \) state 10

\( \langle a_3, b_3 \rangle \) state 49

\( \langle a_4, b_4 \rangle \) state 54

Fit by Bose-Einstein distribution

(close to Boltzmann)
Removal of the center-of-mass spurious states

Harmonic oscillator:

\[ N_{\text{spur}}(K \hbar \omega) \sim \sum_{K' = 1}^{K} N_{\text{pure}}((K - K') \hbar \omega), \]

where \( K' \) presents how many times we act with \( A_{\text{cm}} \)


Nuclear level density. Recursive method:

\[ \rho_{\text{pure}}(E, J, K) = \rho(E, J, K) - \sum_{K' = 1}^{K} \sum_{J_{min} = J' = |J - J_{K'}|}^{K_{\text{step}} = 2} \sum_{J_{K'} = J_{min}}^{J + J_{K'}} \rho_{\text{pure}}(E, J', K - K') \]

$^{28}\text{Si, } J=\text{all, } k_1=k_2=\{0.1-1.0\}$
CONCLUSION

• Quantum chaos leads to thermalization in isolated systems of interacting particles.
  * Individual complicated wave functions of stationary states.

• Variety of systems: nuclei, complex atoms, molecules (including biological), devices of condensed matter and quantum optics on nano- and micro-scale, cold atoms, spin systems, ion traps, ...

• Qubits in quantum computers
  * Appropriate thermometer?

• Information entropy and thermodynamic entropy
  * On the way to quantum kinetics

• Artificial chaos as research instrument
\[ H = k(1)V(1) + k(2)V(2) \]

V(1) – matrix elements of single-particle transfer
Level density (0+) on two sides of deformation shape transition

"collective enhancement"
Predicted long ago:

Rosenzweig effect
Classical definitions

For given energy $E$ – **linear interpolation** determines the spin cut-off parameter

\[ \rho(E, M) = \rho(E, 0) G(M, \sigma^2) \]

\[ \rho(E, 0) = \frac{\sqrt{\pi}}{12 a^{1/4} E^{5/4}} e^{-2\sqrt{E}} \]

\[ \ln[\rho(E, M)] = X(E) - \frac{M^2}{2\sigma^2} \]

\[ X(E) = 2\sqrt{aE} - \frac{5}{4} \ln E + \text{const} \]

**Good**: $^{28}\text{Si}$, $^{52}\text{Fe}$ (all isospin parts of interaction)

**Bad**: $^{44}\text{Ca}$ - only 4 valence neutrons, $^{64}\text{Cr}$ - pf-space occupied (40 neutrons), only 4 active protons (isospin 1 part of interaction)

**Isospin spoils the random spin coupling**

Energy dependence of spin cut-off parameter:

\[ \sigma^2 \text{ proportional to } T. \]

To extract this parameter:

\[ \sigma^2 = \alpha \sqrt{E} [1 + \beta E] \]

in the appropriate energy regions
Two-body random ensemble, 4 particles, 11 orbitals

Difference goes down at high temperature

Thermodynamic temperature

\[ T(\text{therm}) = \frac{\sigma^2}{E_c - E} \]

Canonical temperature

\[ T(\text{can}) = \frac{\sigma^2}{E_c - E + \Delta} \]

Temperature of the ground state

/ V. Flambaum, F. Izrailev /
PREFACE

Quantum chaos in a quantum many-body system:
   a. theoretical,
   b. experimental, and
   c. computational tool

Examples from nuclear physics:
Spectral chaos – level statistics and fluctuations
   level density
   invisible fine structure
   exponential convergence
Chaotic wave functions – information entropy
   quantum phase transitions
   thermalization
   chaotic enhancement of weak interactions

Correlations between the sectors of Gilbert space governed by the same Hamiltonian
Exploration of the nuclear landscape by random interactions
NSCL

- 700 Employees
- Faculty of 40+ physicists and chemists
- Technical staff of over 400 employees
- 90 graduate and 100+ undergraduate students
- Over 700 separate users to date from over 140 organizations worldwide

The largest university-based laboratory in the country

- Nuclear physics graduate program ranked #1 in nation

NSCL becomes NSCL/FRIB
MSU’s Plans for a Facility for Rare Isotope Beams

- 200 MeV/u, 400 kW superconducting heavy-ion driver linac
- Initial capabilities includes fragmentation of fast heavy-ion beams combined with gas stopping and reacceleration
- Capable of scientific research program at start of operation; future upgrades possible
- World premiere nuclear physics facility
- Research opportunities for more than 150 undergraduate and undergraduate students
- Important for the state of Michigan
- Supported by DOE and MSU ($730 M) – 2018 ? (<17%)
Properties of nucleonic matter (+ new exotic isotopes near the drip line)
- Many-body quantum problem: intellectual overlap to mesoscopic science – how to understand the world from simple building blocks

Nuclear processes in the universe
- Nuclei determine the chemical history of the Universe
- Connection of models of supernovae, X-ray bursts, …

Tests of fundamental symmetries
- Effects of symmetry violations are amplified in certain nuclei

Societal applications and benefits
- Bio-medicine, energy, material sciences, national security