Magnetic Global Monopoles from Torsion

Sarben Sarkar

King's College London

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KING'S College LONDON

University of London

Magnetic monopoles

$$\vec{\nabla}\cdot\vec{E}=\rho_e \qquad \vec{\nabla}\cdot\vec{B}=0$$

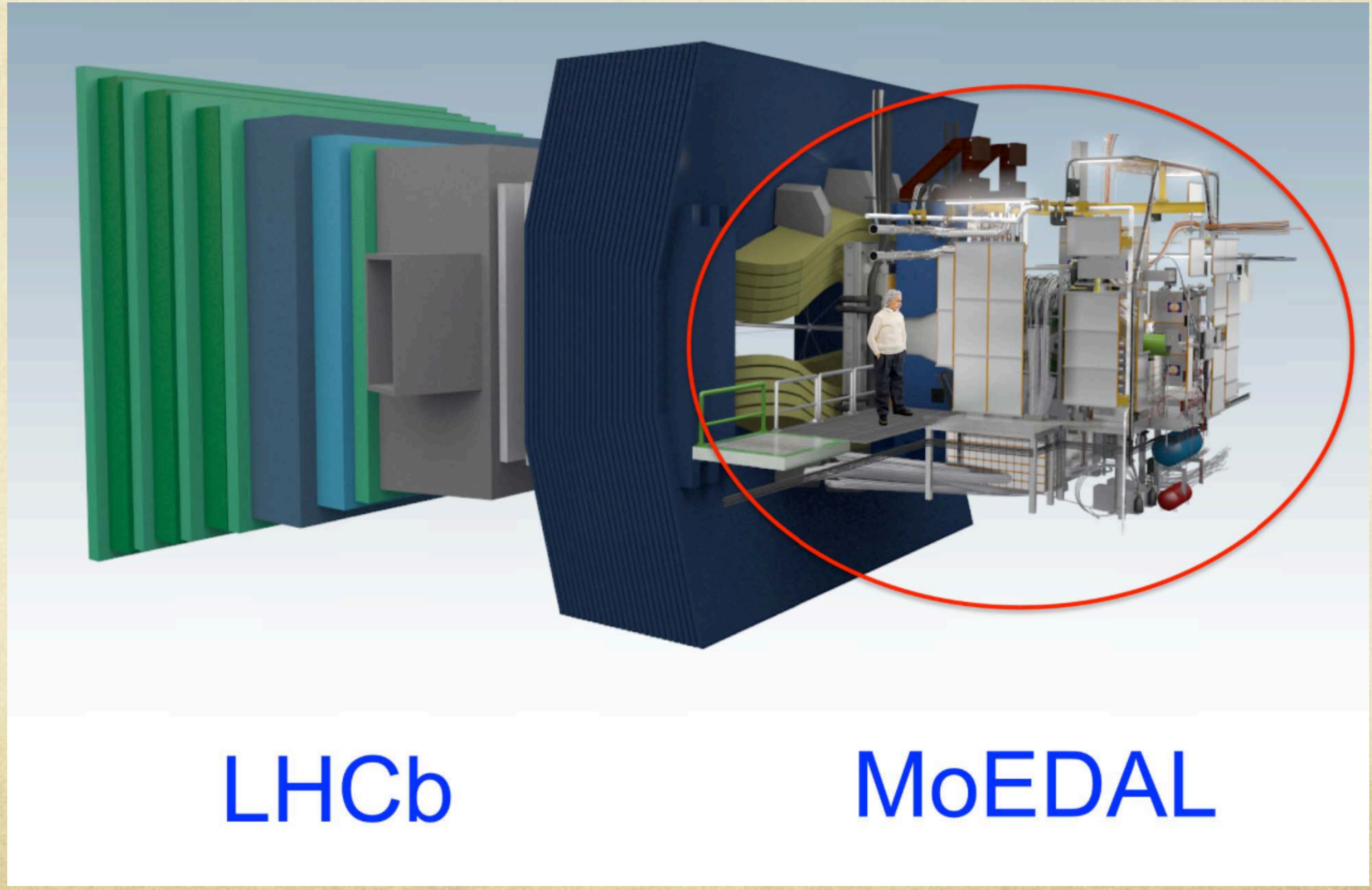
$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0 \qquad \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\vec{j}_e$$

- Asymmetric between $\vec{E}$ and $\vec{B}$
- Beyond Standard Model physics

$$\vec{\nabla}\cdot\vec{E}=\rho_e \qquad \vec{\nabla}\cdot\vec{B}=\rho_m$$

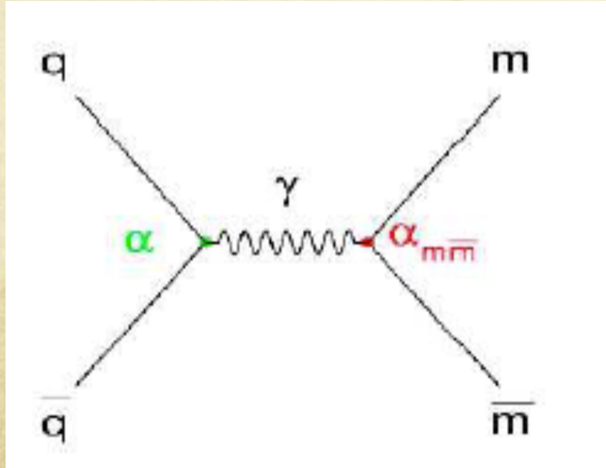
$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=-\vec{j}_m \qquad \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\vec{j}_e$$

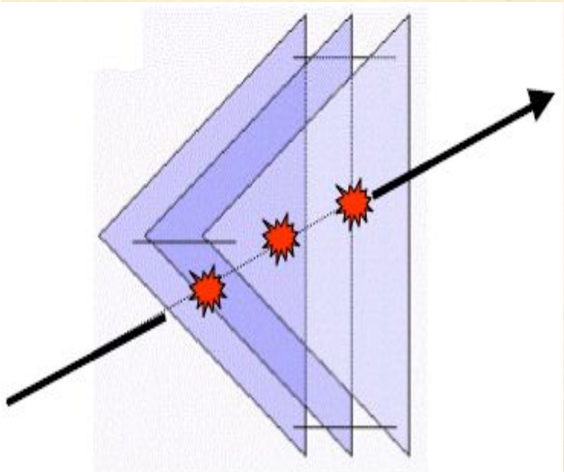
The MoEDAL Experiment



- The search of heavy stable ionizing particles (HIP) avatars of new physics such as magnetic monopoles at LHC energies; the existence of extra dimensions

- MoEDAL uses stacks of passive plastic Nuclear Track-Etch Detectors & Trapping Detectors, with a MediPix chip based online radiation monitor system- an interaction point with the LHCb experiment





Monopoles trapped in the dense material of the trapping detector

Could formation of Black Holes and monopoles be connected?

$$F_{\text{EM}} = \frac{q_1 q_2}{r^2} \qquad F_{\text{gravity}} = G_N \frac{m_1 m_2}{r^2}$$

Gravity is weak, becomes strong at $M_4 \sim 10^{18}$ GeV, far beyond experiment

$$G_N : \frac{1}{M_4^2}$$

In extra dimensions suppose Standard Model interactions confined to $D = 4$ but gravity propagates in n extra dimensions of size L:then

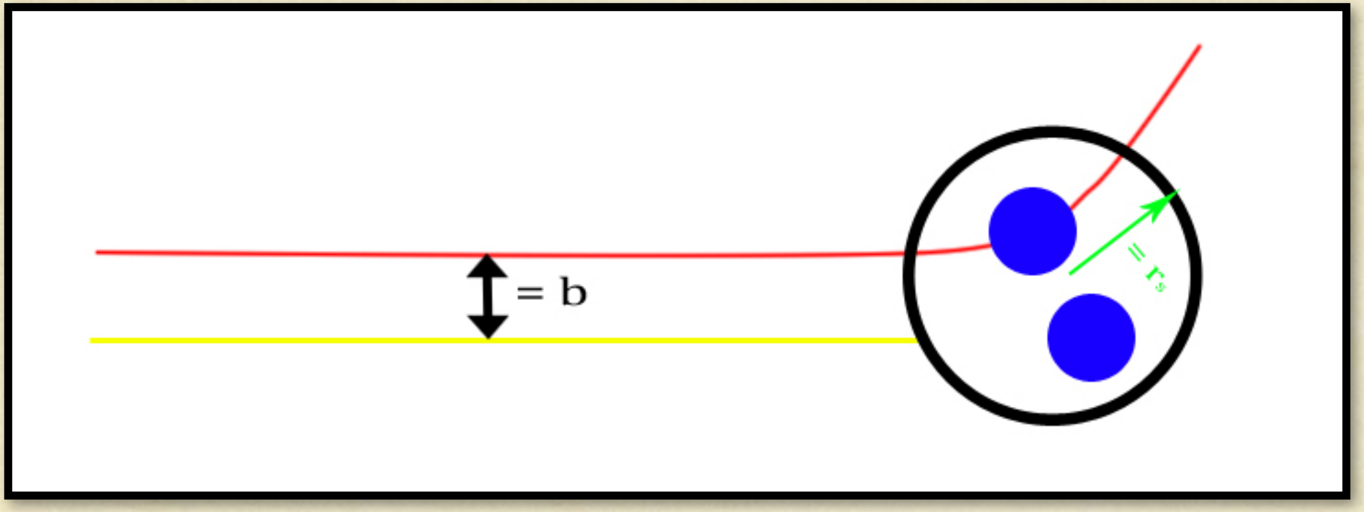
For $r >> L$, $F_{gravity} \sim 1/r^2$
For $r << L$, $F_{gravity} \sim 1/r^{2+n}$

BH formation needs gravity with $M_D : 1TeV$

Black Holes produced in particle collisions:

- Collide Masses closer than r_s
  - r_s is dependent on mass $r_s = \frac{2Gm}{c^2}$ $m_r = \frac{m_0}{\sqrt{1-\beta^2}}$

- Planck Mass must be ~ 1 TeV



•Collision with b < r_s,

Regardless of subsequent collisions, this forms an Event Horizon

HOWEVER in our case we are not requiring large extra dimensions

String theory and gravity

- Graviton, dilaton (scalar), Kalb-Ramond field (antisymmetric tensor); model independent sector
- Model-dependent sector: additional (gauge) fields according to type of string theory
- Low energy dynamics from effective action for a truncation to the model independent sector
- Generalisation of general relativity: effect on gravitational (non-magnetic) monopole

Dirac Monopole

- Assume that a magnetic monopole
- with charge q_m exists (at the origin):

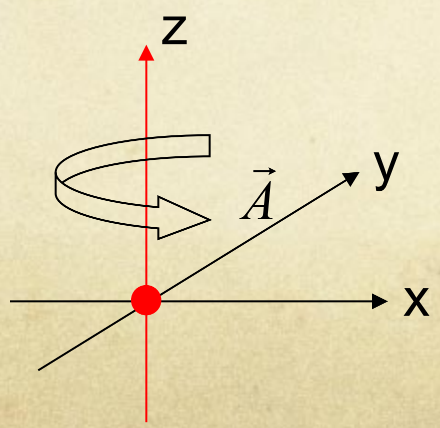
$$\vec{B}(\vec{r}) = \frac{q_m}{4\pi}\frac{1}{r^2}\vec{e}_r$$

q_m is also the flux:

$$q_m = \int_{sphere} \vec{B}d\vec{S}$$

Except for the origin:

$$\vec{\nabla}\vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A}$$



Solutions:

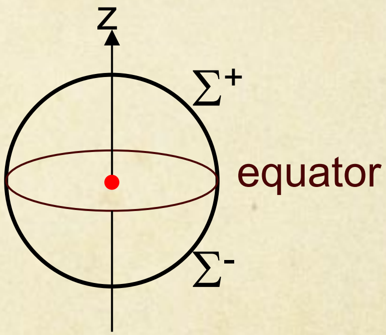
$$\vec{A}_{\pm}(\vec{r}) = \pm\frac{q_m}{4\pi\, r}\frac{1\mp\cos\theta}{\sin\theta}\vec{e}_\phi$$

Properties of solutions

- "+": singular for $\theta = \pi \Rightarrow$ negative z axis
- "-": singular for $\theta = 0 \Rightarrow$ positive z axis

Except for z axis: $$\vec{\nabla} \times (\vec{A}_+ - \vec{A}_-) = 0$$

$$\Rightarrow \exists\, \chi : \vec{A}_+ - \vec{A}_- = \vec{\nabla}\chi$$



$$\vec{A}_+ - \vec{A}_- = \frac{q_m}{2\pi\, r} \frac{1}{\sin\theta} \vec{e}_\phi = \vec{\nabla}\left(\frac{q_m}{2\pi} \phi \right) = \vec{\nabla}\chi$$

$$q_m = \oint \vec{B} d\vec{S} = \int_{\Sigma^+} (\vec{\nabla} \times \vec{A}_+)\, d\vec{S} \; + \int_{\Sigma^-} (\vec{\nabla} \times \vec{A}_-)\, d\vec{S}$$

χ Discontinuous function

$$= \int_0^{2\pi} \vec{A}_+ d\vec{l} - \int_0^{2\pi} \vec{A}_- d\vec{l} = \int_0^{2\pi} \vec{\nabla}\chi\, d\vec{l} = \chi(2\pi) - \chi(0)$$

Dirac monopole quantised

- If one non-relativistic monopole in the world

$$-\frac{\hbar^2}{2m}\left(\vec{\nabla} - ie\vec{A}\right)^2 \psi = i\hbar\frac{\partial\psi}{\partial t}$$

Invariance under gauge transformation:

$$\vec{A} \to \vec{A} + \vec{\nabla}\chi\,, \qquad \psi \to \exp(-ie\chi)\,\psi$$

ψ Must be single valued function $\Rightarrow eq_m = 2\pi\, n$

e quantised
Monopole pointlike

Alternative non-singular monopole

$$L(\vec{x},t) = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}\left(D_\mu\phi^a\right)\left(D^\mu\phi^a\right) - \frac{1}{4}\lambda\left(\phi^a\phi^a - F^2\right)^2$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc}W^b_\mu W^c_\nu$$

$$D_\mu\phi^a = \partial_\mu\phi^a + g\varepsilon^{abc}W^b_\mu\phi^c$$

Georgi-Glashow model: $SU(2)$ Gauge symmetry

Search for solutions which minimize the energy E

$$E = \int d^3\vec{x}\;\; \varepsilon(\vec{x}) = \int d^3\vec{x}\left[\frac{1}{4}G^a_{ij}G^{aij} + \frac{1}{2}\left(D_i\phi^a\right)\left(D^i\phi^a\right) - \frac{1}{4}\lambda\left(\phi^a\phi^a - F^2\right)^2\right]$$

$E \neq 0 \quad \text{but} \quad \varepsilon(\vec{x}) \to 0 \;\; \text{for} \;\; r \to \infty$

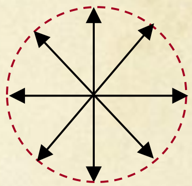
it follows for $r \to \infty$: $\quad r^{3/2}\, D_i\phi^a \to 0 \quad \forall a$

$$\phi^a\phi^a \to F^2$$

'tHooft-Polyakov (HP) monopole

$$D_i\phi^a = \partial_i\phi^a + g\varepsilon^{abc}W_i^b\phi^c \to 0$$

$$\Rightarrow \partial_i\phi^a \to -g\varepsilon^{abc}W_i^b\phi^c$$



Non-trivial topology

$$Q = \int d^3\vec{x}\, k_0 = \ldots = \frac{1}{8\pi}\int_{S_2^{phys}} d^2\sigma_i\left(\varepsilon_{ijk}\varepsilon_{abc}\hat{\phi}^a\partial^j\hat{\phi}^b\partial^k\hat{\phi}^c\right)$$

where

$$k_\mu = \frac{1}{8\pi}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{abc}\partial^\nu\hat{\phi}^a\partial^\rho\hat{\phi}^b\partial^\sigma\hat{\phi}^c$$

which is conserved: $\partial^\mu k_\mu = 0$

Gauge invariant EM tensor

$$F_{\mu\nu} = \hat{\phi}^a G^a_{\mu\nu} - \tfrac{1}{g}\,\varepsilon_{abc}\hat{\phi}^a D_\mu \hat{\phi}^b D_\mu \hat{\phi}^c$$

$$\partial^\nu \left(\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \right) = \frac{4\pi}{g} k_\mu$$

with $\tfrac{1}{2}\,\varepsilon_{ijk}\, F^{jk} = B_i$

$$\vec{\nabla}\vec{B} = 4\pi\, k_0 \,/\, g$$

Magnetic monopole charge: $q_m = \int d^3\vec{x}\,\frac{k_0}{g} = \frac{Q}{g}$

Q = topological charge
= 0, 1, 2, …

HP monopole in curved space-time:

$\Rightarrow$ *Metric tensor Reissner-Nordstrom form corresponding to a point magnetic charge g=1/e*

- **Global monopole: gravitational field + scalar triplet (** spontaneous symmetry breaking of global $O(3)$ symmetry)
- Metric tensor similar to Schwarzschild spacetime with solid angle deficit
- Non-zero scalar curvature though
- **Can the global monopole transform to a HP type of monopole in a generalised general relativity (from string theory) and standard electromagnetism**?

Gravitational Multiplet Model

- Inspired by bosonic sector of string theory:

$$L=(-g)^{1/2}\left\{\frac{1}{2}\partial_\mu\chi^a\partial^\mu\chi^a-\frac{\lambda}{4}\left(\chi^a\chi^a-\eta^2\right)^2-R\right.$$

$$+\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-V(\Phi)-\frac{1}{12}e^{-2\gamma\Phi}H_{\rho\mu\nu}H^{\rho\mu\nu}$$

$$\left.-\frac{1}{4}e^{-\gamma\Phi}f_{\mu\nu}f^{\mu\nu}\right\}$$

$g_{\mu\nu}$ $g=\det\left(g_{\mu\nu}\right)$

Φ Scalar $\gamma=0$ (dilaton $\gamma=1$)

$H_{\rho\mu\nu}=\partial_{[\rho}B_{\mu\nu]}$

$f_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ Electromagnetic field tensor

γ Labels Lagrangian

What could be the form of the solutions? Special cases

$$L = -\ \left(-g\right)^{1/2}\left(R + \frac{1}{4} f_{\mu\nu} f^{\mu\nu}\right)$$

$$ds^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega^2$$

$$\Delta = 1 - \frac{2GM}{r} + \frac{G}{r^2}\left(Q^2 + P^2\right)$$

P magnetic charge

Q electric charge

Both undetemined

Reissner-Nordstrom metric

Self-gravitating $O(3)$ global monopole

$$L=(-g)^{1/2}\left\{\frac{1}{2}\partial_\mu\chi^a\partial^\mu\chi^a-\frac{\lambda}{4}\left(\chi^a\chi^a-\eta^2\right)^2-R\right\}$$

Spontaneous symmetry breaking Goldstone bosons: energy densities which scale as $1/r^2$; linear divergence of mass

$$L=(-g)^{1/2}\left\{\frac{1}{2}\partial_\mu\chi^a\partial^\mu\chi^a-\frac{\lambda}{4}\left(\chi^a\chi^a-\eta^2\right)^2-R\right\}$$

$$\chi^a=\eta f(r)\frac{x^a}{r}$$

$$ds^2=\left(1-8\pi G_N\eta^2-\frac{2G_N M_{core}}{r}\right)dt^2-\left(1-8\pi G_N\eta^2-\frac{2G_N M_{core}}{r}\right)^{-1}dr^2+r^2d\Omega^2$$

$r=\sqrt{x^a x^a}>>1$

$\eta\to 0$ Unbroken phase → Schwarzschild metric

Conical deficit angle

$$t \to \left(1-8\pi G_N \eta^2\right)^{-1/2} t' \qquad r \to \left(1-8\pi G_N \eta^2\right)^{1/2} r'$$

Asymptotically NON FLAT

$$ds^2 = dt'^2 - dr'^2 - \left(1-8\pi G_N \eta^2\right) r'^2 d\Omega^2$$

Scalar curvature: $R \sim 16\pi G_N \eta^2 / r^2$

Scattering amplitude: very large in forward direction: angular regions of the order of the deficit angle

Torsion vs Electromagnetism

	Torsion	Electromagnetism
Potential	$\vec{a}, \vec{A}$	$\phi, \vec{A}$
Field from potential	$\vec{b} = \vec{\nabla} \times \vec{A}$ $\phi = \vec{\nabla}.\vec{a}$	$\vec{E} = -\nabla\phi - \frac{\partial}{\partial t}\vec{A}$ $\vec{B} = \vec{\nabla} \times \vec{A}$
Gauge invariance	$\vec{a} \to \vec{a} + \vec{\nabla} \times \vec{V}$ $\vec{A} \to \vec{A} + \nabla\lambda + \frac{\partial}{\partial t}\vec{V}$	$\phi \to \phi - \frac{\partial}{\partial t}\lambda$ $\vec{A} \to \vec{A} + \nabla\lambda$

In 4 space-time dimensions $\vec{a}, \vec{A} \leftrightarrow B_{\mu\nu}$ Antisymmetric tensor

Tensor formulation

	Torsion	Electromagnetism
Field from potential	$H_{\mu\nu\sigma} = B_{[\mu\nu,\sigma]}$	$F_{\mu\nu} = 2A_{[\nu,\mu]}$
Gauge invariance	$B_{\mu\nu} \to B_{\mu\nu} + \xi_{[\mu,\nu]}$	$A_\mu \to A_\mu + \partial_\mu \lambda$

Torsion from string:

Point on string $x^\mu(\tau,\varsigma)$

Tangent vectors

$$dv_0 = \frac{\partial x^\mu(\tau,\varsigma)}{\partial \tau} d\tau$$

$$dv_1 = \frac{\partial x^\mu(\tau,\varsigma)}{\partial \varsigma} d\varsigma$$

Area element in target space:

$$d\sigma^{\mu\nu} = \left(\frac{\partial x^\mu}{\partial \tau}\frac{\partial x^\nu}{\partial \varsigma} - \frac{\partial x^\mu}{\partial \varsigma}\frac{\partial x^\nu}{\partial \tau} \right) d\tau d\varsigma$$

Torsion coupling to strings

- String torsion action:

$$S=-\frac{1}{2}\int d\sigma^{\mu\nu}B_{\mu\nu}\left(x\left(\tau,\varsigma\right)\right)-\frac{1}{6\kappa^{2}}\int d^{4}x\,H_{\mu\nu\rho}H^{\mu\nu\rho}$$

String is the source of torsion:

$$\frac{1}{\kappa^{2}}\frac{\partial H^{\mu\nu\rho}}{\partial x^{\rho}}=j^{\mu\nu}$$

$$j^{\mu\nu}\left(x'\right)=\frac{1}{2}\int d\sigma^{\mu\nu}\,\delta^{4}\left(x'-x\left(\tau,\varsigma\right)\right)$$

$\gamma = 1$ model

In 4 dimensions introduce pseudoscalar axion field

b

$$H_{\mu\nu\rho} = e^{2\Phi}\varepsilon_{\mu\nu\rho}{}^{\sigma}\partial_{\sigma} b$$

Dilaton equation of motion:

$$e^{2\Phi}\partial_{\mu}b\partial^{\mu}b + \frac{1}{4}e^{-\Phi}f_{\mu\nu}f^{\mu\nu} - \frac{\partial V(\Phi)}{\partial\Phi} + O(\partial\Phi) = 0$$

Consider the dilaton stabilised to a constant

$$\left.\frac{\partial V(\Phi)}{\partial\Phi}\right|_{\Phi=\Phi_0} = 0 \qquad V(\Phi_0) = 0$$

$$e^{2\Phi}\partial_{\mu}b\partial^{\mu}b + \frac{1}{4}e^{-\Phi}f_{\mu\nu}f^{\mu\nu} = 0$$

Torsion and electromagnetism related

Kalb-Ramond and Christoffel symbols

- Scherk and Schwarz showed that
- the action

$$I=\int d^4x\sqrt{-g}\left({}^0R-f(\phi)H_{\mu\nu\sigma}H^{\mu\nu\sigma}-\frac{1}{2}\phi^{,\sigma}\phi_{,\sigma}\right)$$

- (where $f(\phi)$ is arbitrary and $\nabla_\sigma g_{\mu\nu}\sim g_{\mu\nu}\phi_{,\sigma}$) is equivalent to

$$I=\int d^4x\sqrt{-g}R$$

$$\left\{\begin{matrix} & & \sigma \\ \mu & \nu & \end{matrix}\right\}\equiv\left\{\begin{matrix}\sigma & \\ \mu & \nu\end{matrix}\right\}+\sqrt{\frac{4\pi G}{3}}\left(\delta^\sigma_\mu\phi_{,\nu}+\right.$$

$$\left.\delta^\sigma_\nu\phi_{,\mu}-g_{\mu\nu}\phi^{,\sigma}\right)+\sqrt{\frac{8\pi Gf(\phi)}{3}}H_{\mu\nu}{}^{\sigma}$$

← Antisymmetric piece

Solution Ansatze

$$g_{\mu\nu} = \begin{pmatrix} B(r) & & & \\ & -A(r) & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix}$$

$$f_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2r\sin\theta W(r) \\ 0 & 0 & 2r\sin\theta W(r) & 0 \end{pmatrix}$$

satisfies $\nabla_\lambda \left(e^{-\gamma\Phi} f^{\lambda\kappa} \right) = 0$

Kalb-Ramond Bianchi identity

$$\varepsilon^{\mu\nu\lambda\rho} \partial_\rho H_{\mu\nu\lambda} = 0 \implies b'(r) = \frac{\varsigma}{r^2} \sqrt{\frac{A(r)}{B(r)}}$$

where ς is a constant

Asymptotic results: near field $(r \to 0)$ and far field $r \longrightarrow \infty$

For small r from Einstein's equations

$$B = 1 - \frac{2M}{r} + \frac{p_0}{r^2}$$

$$A(r)B(r) = 1 + \varepsilon(r)$$

$$\varepsilon(r) = \left(\frac{\varsigma^2}{4p_0^{\,2}} + \frac{\eta^2 f_0^2}{2} \right) r^2 + \ldots$$

$$f(r) = f_0 r + \ldots$$

$$W^2(r) \sim \frac{p_0}{2r^2}$$

Without the use of the dilaton equation p_0

is undetermined

Dilaton equation case for all r

$\gamma = 1$

$\frac{1}{4} f_{ij} f^{ij} = \frac{1}{2} \left(\mathbf{B}^r\right)^2 A$ where $\mathbf{B}^r$ is the radial component

$$\frac{1}{4} f_{ij} f^{ij} = \frac{\varsigma^2}{r^4} \frac{1}{B} = \frac{1}{2} \left(\mathbf{B}^r\right)^2 A$$

$$\Rightarrow \mathbf{B}^r = \sqrt{2} \frac{1}{\sqrt{AB}} \frac{\varsigma}{r^2}$$

As $r \to 0$ $AB \simeq 1$ and so

$$\mathbf{B}^r \simeq \frac{g}{r^2} \qquad g = \sqrt{2}\varsigma$$

For large r

$$A(r)B(r) \simeq 1 + \frac{\varepsilon_0}{r^2}$$

and so also

$$\mathbf{B}^r \simeq \frac{g}{r^2}$$

Large coupling λ

- The scalar triplet takes on classical vacuum expectation value
- Reissner-Nordstrom form for B both large and small r
- *Monopole mass cannot be calculated from asymptotic analysis alone BUT it is proportional to torsion charge*

Torsion quantisation when $\gamma = 1$

$$\mathbf{B}^r \simeq \frac{g}{r^2} \quad \text{with} \quad g = \sqrt{2}\varsigma$$

Since g is quantised, ς is quantised from

Dirac quantisation

The model and experiments

1. Need large λ to have high enough probability P of producing monopole-antimonopole pairs

$$P \propto \exp\left(-\frac{const}{\lambda}\right)$$

2. Monopole mass M estimate

$$M \sim 4\pi\xi\lambda^{-1/2}\eta$$

3. Since assumed $\eta \ll 1$ $M \ll M_P$ no Reissner_Nordstrom horizons (no black holes)

4. Scalars are non-propagating

Issues for phenomenology

- Large λ: losses due to production of Goldstone bosons need to be estimated for monopole-antimonopole pairs
- Gravitational coupling to photons and Kalb-Ramond modes of monopoles
- Perturbative Drell-Yan process estimates for scalar monopole mass not valid in the presence of strong magnetic fields
- Collision of Standard Model particles and production of global monopoles
- Interpolating solutions and estimate of ξ

Relevant Papers on Monopoles

1. Magnetic monopoles from global monopoles in the presence of a Kalb-Ramond field, Phys Rev D **95, 104025** (2017) N E Mavromatos and S Sarkar
2. The Physics Programme of the MoEDAL Experiment at the LHC, hep-ph 1405.7662 (with B. Acharya et al.) Intnl J Mod Phys A 29 1430050 (2014)
3. Search for magnetic monopoles with the Moedal prototype trapping detector in 8 TeV proron-proron collisions at the LHC, arXiv 1604.06645 (Moedal collaboration) JHEP 1608 (2016) 067

4. Search for magnetic monopoles with the MoEDAL forward trapping detector in 13 TeV proton-proton collisions at the LHC
Phys Rev Lett 118 061801 (2017) ArXiv 1611.06817

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