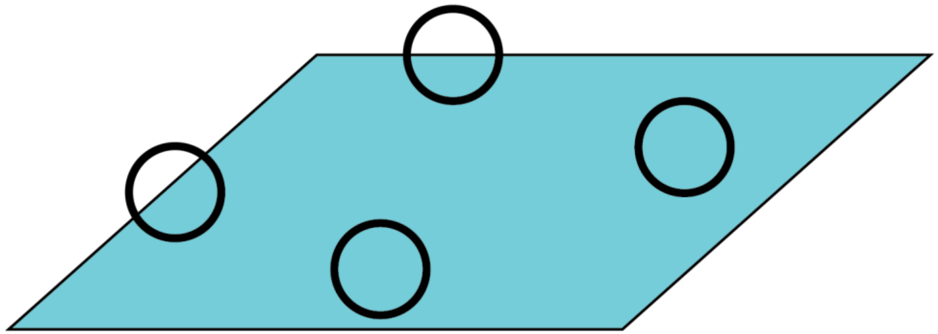
What is the standard model of particle physics trying to tell us?

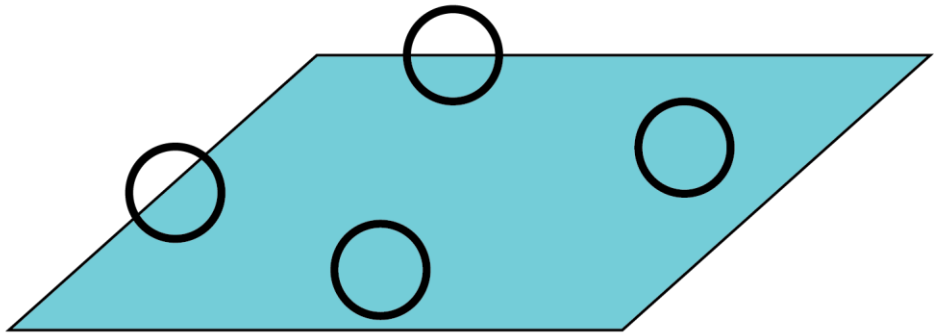
Latham Boyle

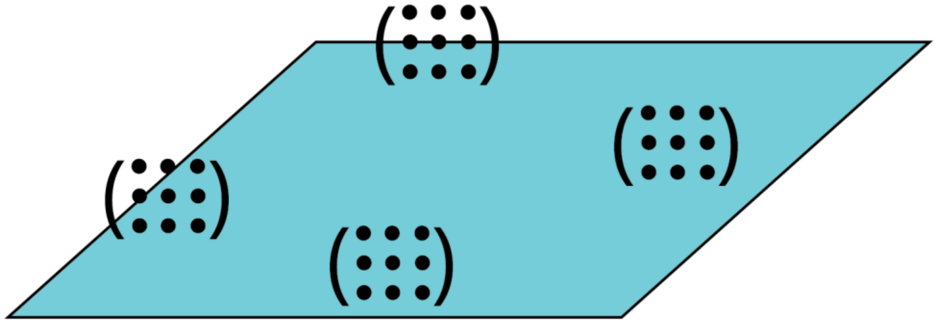
(Perimeter Institute)

builds on work of
Barrett, Bizi, Brouder, Besnard, Chamseddine, Connes, D'Andrea, Dabrowski, Dubois-Violette, Kerner, Krajewski, Landi, Lizzi, Lott, Madore, Marcolli, Martinetti, Schucker, van Suijlekom

based on arXiv:1604.00847 w/ S. Farnsworth
(and another in preparation)







The EFT perspective: the basic input

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Step 1:	$SU(3)$	$SU(2)$	$U(1)$

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Step 2:	q_L^i			

$$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

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Step 1:		$SU(3)$	$SU(2)$	$U(1)$
Step 2:	q_L^i	3		
	u_R^i	3		
	d_R^i	3		
	l_L^i			
	ν_R^i			
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	l_L^i	1		
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Step 3:	h			

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Step 3:	h	1		

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The NCG perspective: an initial thought

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$$\psi(x)$$

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$$\psi(x) \to \psi_A(x)$$

The NCG perspective: an initial thought

$$\psi(x) \to \psi_A(x) = \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{array}\right)$$

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$$z = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$(a, b \in \mathbb{R})$$

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$$z = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \qquad q = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$$

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$$z = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \qquad q = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \qquad q_\lambda = \begin{pmatrix} \lambda & 0 \\ 0 & \bar{\lambda} \end{pmatrix}$$

$$(a, b \in \mathbb{R}) \qquad (\alpha, \beta \in \mathbb{C}) \qquad (\lambda \in \mathbb{C})$$

EFT input:					**NCG input:**
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EFT input:					**NCG input:**
Step 1:		$SU(3)$	$SU(2)$	$U(1)$	$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$
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EFT input:

NCG input:

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Step 1:					$\{m, q, \lambda\} \in M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$
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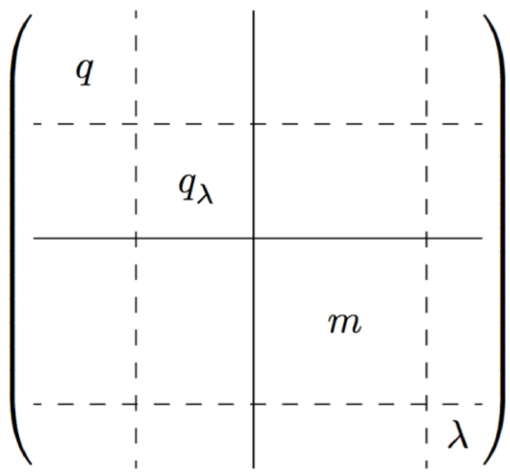

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NCG input:

$$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$$

$$\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix} \begin{pmatrix} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{pmatrix}$$

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Step 3:	h	1	2	1/2

NCG input:

Step 1: $\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$

Step 2:

$$\left[\left(\begin{array}{cc|cc} q & & & \\ & q_\lambda & & \\ \hline & & m & \\ & & & \lambda \end{array}\right), \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{array}\right)\right]$$

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NCG input:

$$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$$

$$\left[\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix}, \begin{pmatrix} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{\nu}_L & \bar{e}_L & \bar{\nu}_R & \bar{e}_R & & & & \end{pmatrix}\right]$$

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NCG input:

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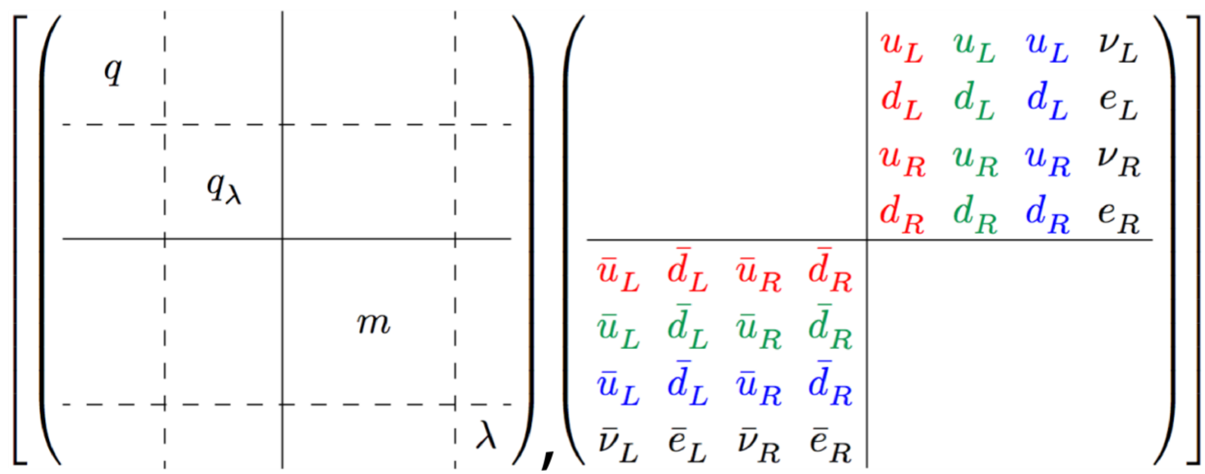

Step 3: ???

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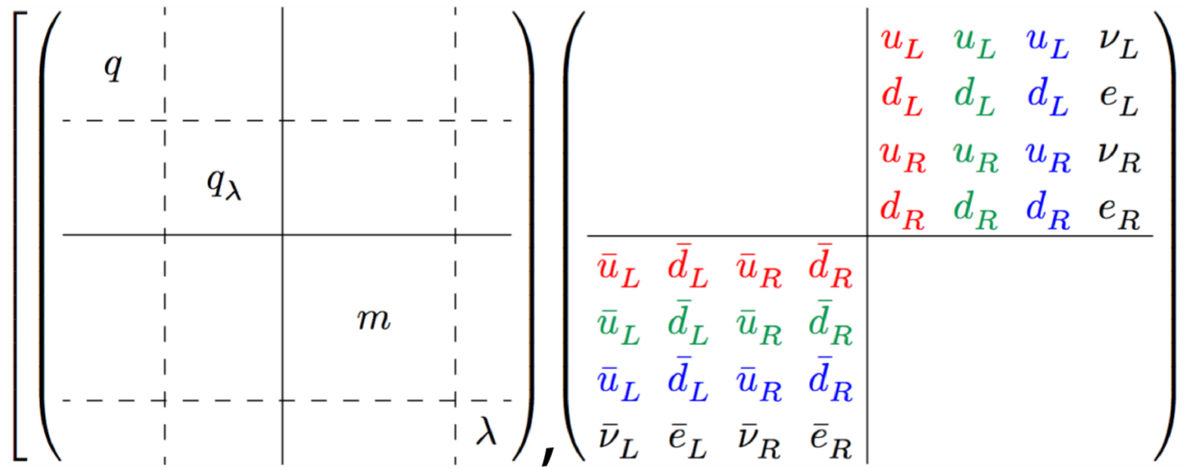

Gauge and Higgs bosons from covariance of D

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- want $D \to D' = UDU^{-1} \qquad$ (i.e. $D\psi \to D'\psi' = UD\psi$)

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- $\psi \to \psi' = U\psi \qquad U = \exp[\alpha^i(x) T_i] \qquad T_i[\psi] = [\hat{a}_i, \psi]$
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- $A^{k\,\prime}_\mu \;=\; A^k_\mu + f_{ij}^{\;k} \alpha^i A^j_\mu - (\partial_\mu \alpha^k) \qquad ([T_i, T_j] = f_{ij}^{\;k} T_k)$

- $\psi \to \psi' = U\psi \qquad U = \exp[\alpha^i(x) T_i] \qquad T_i[\psi] = [\hat{a}_i, \psi]$

- want $D \to D' = U D U^{-1} \qquad$ (i.e. $D\psi \to D'\psi' = U D\psi$)

- $D = i\gamma^\mu (\partial_\mu \otimes 1 + A^k_\mu \otimes T_k)$

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$$D = D_1 \hat{\otimes} 1 + 1 \hat{\otimes} D_2$$

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- $D = i\gamma^\mu(\partial_\mu \otimes 1 \qquad\qquad) + \gamma^5(1 \otimes D_F \qquad\qquad)$?

- $A_\mu^{k\,\prime} \;=\; A_\mu^k + f_{ij}^{\;k}\alpha^i A_\mu^j - (\partial_\mu \alpha^k) \qquad ([T_i, T_j] = f_{ij}^{\;k} T_k)$

- $\psi \to \psi' = U\psi \qquad U = \exp[\alpha^i(x)T_i] \qquad T_i[\psi] = [\hat{a}_i, \psi]$

- want $D \to D' = UDU^{-1} \qquad$ (i.e. $D\psi \to D'\psi' = UD\psi$)

- $D = i\gamma^\mu(\partial_\mu \otimes 1 + A^k_\mu \otimes T_k) + \gamma^5(1 \otimes D^0_F + \Phi^K \otimes \tau_K)$

- $A^{k\,\prime}_\mu \;=\; A^k_\mu + f_{ij}^{\;\;k}\alpha^i A^j_\mu - (\partial_\mu \alpha^k) \qquad ([T_i, T_j] = f_{ij}^{\;\;k} T_k)$

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- $$\begin{aligned} A^{k\,\prime}_\mu &= A^k_\mu + f_{ij}^{\ k} \alpha^i A^j_\mu - (\partial_\mu \alpha^k) \qquad &([T_i, T_j] = f_{ij}^{\ k} T_k) \\ \Phi^{K\,\prime} &= \Phi^K + g_{iJ}^{\ K} \alpha^i \Phi^J \qquad &([T_i, \tau_J] = g_{iJ}^{\ K} \tau_K) \end{aligned}$$

$$D_F[\psi] = \delta_q^\dagger \psi \delta_q + \delta_l^\dagger \psi \delta_l + \delta_m^\dagger \psi \delta_m,$$

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$$\delta_q = \left(\begin{array}{c:c|c:c} 0_{2\times 2} & Y_q & & \\ \hdashline Y_q^\dagger & 0_{2\times 2} & & \\ \hline & & 1_{3\times 3} & \\ \hdashline & & & 0 \end{array}\right)$$

$$\delta_l = \left(\begin{array}{c:c|c:c} 0_{2\times 2} & Y_l & & \\ \hdashline Y_l^\dagger & 0_{2\times 2} & & \\ \hline & & 0_{3\times 3} & \\ \hdashline & & & 1 \end{array}\right)$$

$$\delta_m = \left(\begin{array}{c:c|c:c} & & & \\ \hdashline & & & \begin{matrix} \bar{\mu} \\ 0 \end{matrix} \\ \hline & & & \\ \hdashline & \mu \;\; 0 & & \end{array}\right)$$

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$$\delta_q = \left(\begin{array}{c:c|c:c} 0_{2\times 2} & Y_q & & \\ \hdashline Y_q^\dagger & 0_{2\times 2} & & \\ \hline & & 1_{3\times 3} & \\ \hdashline & & & 0 \end{array}\right) \qquad Y_q = \begin{pmatrix} +y_u \bar{\varphi}_2 & y_d \varphi_1 \\ -y_u \bar{\varphi}_1 & y_d \varphi_2 \end{pmatrix}$$

$$\delta_l = \left(\begin{array}{c:c|c:c} 0_{2\times 2} & Y_l & & \\ \hdashline Y_l^\dagger & 0_{2\times 2} & & \\ \hline & & 0_{3\times 3} & \\ \hdashline & & & 1 \end{array}\right) \qquad Y_l = \begin{pmatrix} +y_\nu \bar{\varphi}_2 & y_e \varphi_1 \\ -y_\nu \bar{\varphi}_1 & y_e \varphi_2 \end{pmatrix}$$

$$\delta_m = \left(\begin{array}{c:c|c:c} & & & \\ \hdashline & & & \begin{matrix} \bar{\mu} \\ 0 \end{matrix} \\ \hline & & & \\ \hdashline & \mu \quad 0 & & \end{array}\right)$$

EFT input:

Step 1:		$SU(3)$	$SU(2)$	$U(1)$
Step 2:	q_L^i	3	2	1/6
	u_R^i	3	1	2/3
	d_R^i	3	1	$-1/3$
	l_L^i	1	2	$-1/2$
	ν_R^i	1	1	0
	e_R^i	1	1	-1
Step 3:	h	1	2	1/2

NCG input:

$$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$$

$$\left[\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix}, \begin{pmatrix} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{\nu}_L & \bar{e}_L & \bar{\nu}_R & \bar{e}_R & & & & \end{pmatrix}\right]$$

Gauge and Higgs bosons from covariance of D

EFT input:

NCG input:

		$SU(3)$	$SU(2)$	$U(1)$
Step 1:				
Step 2:	q_L^i	3	2	1/6
	u_R^i	3	1	2/3
	d_R^i	3	1	−1/3
	l_L^i	1	2	−1/2
	ν_R^i	1	1	0
	e_R^i	1	1	−1
Step 3:	h	1	2	1/2

Step 1: $\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$

Step 2:

$$\left[\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix}, \begin{pmatrix} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{\nu}_L & \bar{e}_L & \bar{\nu}_R & \bar{e}_R & & & & \end{pmatrix}\right]$$

Step 3: Gauge and Higgs bosons from covariance of D

$$D = i\gamma^\mu(\partial_\mu \otimes 1 + A_\mu^k \otimes T_k) + \gamma^5(1 \otimes D_F^0 + \Phi^K \otimes \tau_K)$$

EFT input: | NCG input:

		$SU(3)$	$SU(2)$	$U(1)$
Step 1:				
Step 2:	q_L^i	3	2	1/6
	u_R^i	3	1	2/3
	d_R^i	3	1	−1/3
	l_L^i	1	2	−1/2
	ν_R^i	1	1	0
	e_R^i	1	1	−1
Step 3:	h	1	2	1/2

Step 1: $\{m, q, \lambda\} \in M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$

Step 2:

$$\left[\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix}, \begin{pmatrix} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{\nu}_L & \bar{e}_L & \bar{\nu}_R & \bar{e}_R & & & & \end{pmatrix}\right]$$

Step 3: Gauge and Higgs bosons from covariance of D

$$D = i\gamma^\mu(\partial_\mu \otimes 1 + A_\mu^k \otimes T_k) + \gamma^5(1 \otimes D_F^0 + \Phi^K \otimes \tau_K)$$

$$S_{\text{fermion}} = \langle\psi|D|\psi\rangle$$

$$a\psi = \left(\begin{array}{c|c} \begin{matrix} q & \\ & \\ & q_\lambda \end{matrix} & \\ \hline & \begin{matrix} m & \\ & \\ & \lambda \end{matrix} \end{array}\right) \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar\psi_{11} & \bar\psi_{21} & \bar\psi_{31} & \bar\psi_{41} & & & & \\ \bar\psi_{12} & \bar\psi_{22} & \bar\psi_{32} & \bar\psi_{42} & & & & \\ \bar\psi_{13} & \bar\psi_{23} & \bar\psi_{33} & \bar\psi_{43} & & & & \\ \bar\psi_{14} & \bar\psi_{24} & \bar\psi_{34} & \bar\psi_{44} & & & & \end{array}\right)$$

$$
a\psi = \left(\begin{array}{cc|cc} q & & & \\ & q_\lambda & & \\ \hline & & m & \\ & & & \lambda \end{array}\right)\left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar\psi_{11} & \bar\psi_{21} & \bar\psi_{31} & \bar\psi_{41} & & & & \\ \bar\psi_{12} & \bar\psi_{22} & \bar\psi_{32} & \bar\psi_{42} & & & & \\ \bar\psi_{13} & \bar\psi_{23} & \bar\psi_{33} & \bar\psi_{43} & & & & \\ \bar\psi_{14} & \bar\psi_{24} & \bar\psi_{34} & \bar\psi_{44} & & & & \end{array}\right)
$$

	$SU(3)$	$SU(2)$	$U(1)_Y$
q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	$-1/3$
l_L	1	2	$-1/2$
ν_R	1	1	0
e_R	1	1	-1

$$
a\psi = \left(\begin{array}{cc|cc} q & & & \\ & q_\lambda & & \\ \hline & & m & \\ & & & \lambda \end{array}\right) \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar\psi_{11} & \bar\psi_{21} & \bar\psi_{31} & \bar\psi_{41} & & & & \\ \bar\psi_{12} & \bar\psi_{22} & \bar\psi_{32} & \bar\psi_{42} & & & & \\ \bar\psi_{13} & \bar\psi_{23} & \bar\psi_{33} & \bar\psi_{43} & & & & \\ \bar\psi_{14} & \bar\psi_{24} & \bar\psi_{34} & \bar\psi_{44} & & & & \end{array}\right)
$$

	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_{B-L}$
q_L	3	2	$1/6$	$1/3$
u_R	3	1	$2/3$	$1/3$
d_R	3	1	$-1/3$	$1/3$
l_L	1	2	$-1/2$	-1
ν_R	1	1	0	-1
e_R	1	1	-1	-1

$$a.\psi = \frac{1}{2}\left\{\left(\begin{array}{cc|cc} c_2 & & & \\ & r_2 & & \\ \hline & & c_3 & \\ & & & r \end{array}\right), \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{array}\right)\right\}$$

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	$SU(3)$	$SU(2)$	$U(1)_X$
q_L	3	2	0
u_R	3	1	$1/2$
d_R	3	1	$-1/2$
l_L	1	2	0
ν_R	1	1	$1/2$
e_R	1	1	$-1/2$

$$a.\psi = \frac{1}{2}\left\{\left(\begin{array}{cc|cc} c_2 & & & \\ & r_2 & & \\ \hline & & c_3 & \\ & & & r \end{array}\right), \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{array}\right)\right\}$$

	$SU(3)$	$SU(2)$	$U(1)_X$	$U(1)_{B-L}$
q_L	3	2	0	$1/3$
u_R	3	1	$1/2$	$1/3$
d_R	3	1	$-1/2$	$1/3$
l_L	1	2	0	-1
ν_R	1	1	$1/2$	-1
e_R	1	1	$-1/2$	-1

$$ab = \begin{cases} +(-1)^{ab} ba & \text{(graded Jordan)} \\ -(-1)^{ab} ba & \text{(graded Lie)} \end{cases}$$

$$(-1)^{ac}[a, b, c] + (-1)^{ba}[b, c, a] + (-1)^{cb}[c, a, b] = 0$$

$$(-1)^{ac}(L_a L_{bc} - L_{ab} L_c) + (-1)^{ba}(L_b L_{ca} - L_{bc} L_a) + (-1)^{cb}(L_c L_{ab} - L_{ca} L_b)$$

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$$\begin{aligned} d[f+g] &= d[f] + d[g] \\ d[cf] &= cd[f] \text{ (for constant } c\text{)} \\ d[fg] &= d[f]g + fd[g] \\ fd[g] &= d[g]f \\ d[f]d[g] &= -d[g]d[f] \end{aligned}$$

Filling in the back story: briefly!

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- Recently: Non-Commutative → Jordan!

Some topics for the future

- A better bosonic action (F^2?)
- Relation to fine-tuning problems in SM
- Relation to grand unification
- Relation to exceptional Jordan algebra