Understanding $X(3872)$, $X(3862)$ and $X(3930)$ in a Friedrichs-model-like scheme

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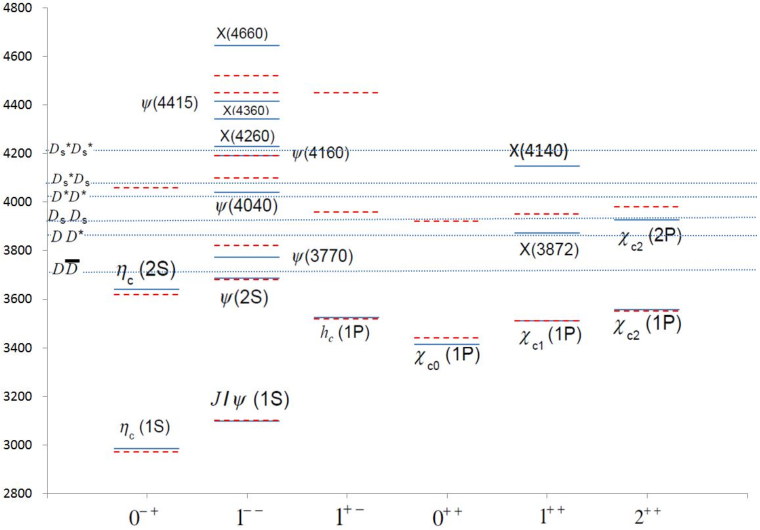
Introduction

Recent years, more and more new X, Y, Z states were found which can not be satisfactorily explained by the naive quark model.

- The quark model such as Godfrey-Isgur can describe the hadron spectrum below the open flavor thresholds. Such as J/ψ, $h_c(1P)$, $\chi_{c0,1,2}(1P)$.
- But for resonances above the open flavor thresholds, general descrepancies are found between the quark model and the experimental observation.
- There are also charged exotic states which do not present in the quark model: $Z_c(3900)$, $Z_c(4020)$

Charmonium-like states



red ones are GI's prediction.

GI's potential model

Godfrey-Isgur's relativized quark potential model,
[PRD32,189,(1985)]

- A relativized quark potential model:

$$H_{GI} = H_0 + V = H_0 + \tilde{H}_{ij}^{\text{conf}} + \tilde{H}_{ij}^{\text{hyp}} + \tilde{H}_{ij}^{\text{so}} + H_A$$

- By diagnalizing the meson-meson matrix $\langle \text{Meson}_i | H | \text{Meson}_j \rangle$, the mass spectra are obtained.
- However, the hadron loop effects in the propagator from the interactions are not included in the spectra. This is the main reason for the discrepancy of the GI's results and the experimentsal ones above the open flavor thresholds.
- We will use a kind of exact solvable model — Friedrichs model, combined with QPC, to incorporate this effect into the spectra.
- We then analyze the spectra of $2P$ charmonium-like states using this method.

Friedrichs Model

A kind of solvable model: The simplist one [Commun. Pure Appl. Math.,1(1948),361]

- Coupling a discrete state $|0\rangle$ with a continuum state $|\omega\rangle$, $\omega > \omega_{th}$, normalization

$$\langle 0|0\rangle = 1, \langle \omega|\omega'\rangle = \delta(\omega - \omega'), \langle 0|\omega\rangle = \langle \omega|0\rangle = 0.$$

- Hamiltonian:

$$H = H_0 + V,$$

H_0 free part, V interaction part,

$$H_0 = \omega_0|0\rangle\langle 0| + \int_{\omega_{th}}^{\infty} \omega|\omega\rangle\langle\omega| d\omega$$

$$V = \lambda \int_{\omega_{th}}^{\infty} \left[f(\omega)|\omega\rangle\langle 0| + f^*(\omega)|0\rangle\langle\omega|\right] \mathrm{d}\omega,$$

Friedrichs Model

- Solution of the Eigenvalue problem:

$$H|\Psi(x)\rangle = (H_0 + V)|\Psi\rangle = E|\Psi(x)\rangle.$$

- Continuum solution: eigenvalues $E > \omega_{th}$.

$$|\Psi_{\pm}(E)\rangle = |E\rangle + \lambda \frac{f^*(E)}{\eta^{\pm}(E)}\left[|0\rangle + \lambda \int_{\omega_{th}}^{\infty} \frac{f(\omega)}{E - \omega \pm i\epsilon}|\omega\rangle \mathrm{d}\omega\right]$$
$$\langle\Psi(E)|\Psi(E')\rangle = \delta(E - E')$$

where

$$\eta^{\pm}(x) = x - \omega_0 - \lambda^2 \int_{\omega_{th}}^{\infty} \frac{f(\omega)f^*(\omega)}{x - \omega \pm i\epsilon}\mathrm{d}\omega$$

Friedrichs Model

- Discrete spectrum: eigenvalues are determined by $\eta(x) = 0$, where η is analytically continued — two sheeted Riemann sheet.
- Bound states: $\omega_0 < \omega_{th}$ or some strong coupling cases. Solutions on the first sheet real axis, $E_B < \omega_{th}$, generated from $|0\rangle$ or dynamically generated.

$$|z_B\rangle = N_B\Big(|0\rangle + \lambda\int_{\omega_{th}}^{\infty} \frac{f(\omega)}{z_B - \omega}|\omega\rangle \mathrm{d}\omega\Big)\,, \quad \langle z_B|z_B\rangle = 1$$

- Compositeness X and elementariness Z for bound states:

$$\begin{aligned} X =& N_B^2\lambda^2 \int_{\omega_{th}} \mathrm{d}\omega \frac{|f(\omega)|^2}{(E_B - \omega)^2} \\ Z =& 1 - X \end{aligned}$$

X: Probability of finding a continuum from the bound state
Z: Probability of finding the discrete state $|0\rangle$ from the bound state

Friedrichs Model

Virtual states and Resonant states

- Virtual state: Solutions to $\eta(z)=0$ on the real axis of the second sheet. Similar wave function can be obtained [PRD94,076006;JMP58,062110]

$$|z_V\rangle = N_V\Big(|0\rangle + \lambda\int_{\omega_{th}}^{\infty}\frac{f(\omega)}{z_V-\omega}|\omega\rangle \mathrm{d}\omega\Big),$$

- Resonances: Solutions to $\eta(z)=0$ on the second Riemann sheet, not on the real axis.

$$|z_R\rangle = N_R\Big(|0\rangle + \lambda\int_{\omega_{th}}^{\infty}\mathrm{d}\omega\frac{f(\omega)}{[z_R-\omega]_+}|\omega\rangle\Big),$$

- No well defined norm for these states, in fact for resonances: $\langle z_R|z_R\rangle = 0$. No mathematically rigorously well-defined compositeness and elementariness. There are some physically approximately defined compositeness and elementariness proposed: model dependent, such as [Sekihara,Hyodo, Jido, PTEP, 063D04(2015);Z.H. Guo and J. A. Oller,PRD93,096001].

Friedrichs Model

- The Friedrichs model can be generalized to include more discrete states and more continuum states.
- For continuum two particle states $|\vec{p}, SS_z\rangle$, after the partial wave decomposition, can also be described by a Friedrichs-like model

$$|\vec{p}; SS_z\rangle = \sum_{lm} i^l Y_l^{m*}(\hat{p})|p; lm, SS_z\rangle = \sum_{JM,lm} i^l Y_l^{m*}(\hat{p})\, C^{JM}_{lm,SS_z}|p; JM; lS\rangle$$

- The most generalized Friedrichs-like model:

$$\begin{aligned} H = & \sum_{i=1}^{D} M_i|i\rangle\langle i| + \sum_{i=1}^{C}\int_{M_{i,th}}^{\infty} d\omega_i \omega_i|\omega_i; i\rangle\langle\omega_i; i| \\ & + \sum_{i_2,i_1}\int_{M_{i_1,th}} \mathrm{d}\omega' \int_{M_{i_2,th}} \mathrm{d}\omega\, f_{i_2,i_1}(\omega',\omega)|\omega'; i_2\rangle\langle\omega; i_1| \\ & + \sum_{i=1}^{D}\sum_{j=1}^{C}\int_{M_{j,th}} d\omega\, g_{i,j}(\omega)|i\rangle\langle\omega; j| + h.c. \end{aligned}$$

In some special cases, $f_{i_2 i_1}$ are factorized as $f_{i_2} f_{i_1}$, and $g_{i,j} = \lambda_i f_j$, the model is rigorously solvable.[JMP58,072102]

Friedrichs Model

- If one has the interaction between the discrete states and the continuum states f_i, then the eigenstates for the full Hamiltonian are obtained— the masses, widths and wave functions for the states.
- The interactions can be estimated using differnent models: we will use the QPC model.

QPC MODEL

- The meson coupling $A \to BC$ can be defined using the transition matrix element

$$\langle BC|T|A\rangle = \delta^3(\vec{P}_f - \vec{P}_i)M^{ABC}$$

where the transition operator T is the one in the QPC model

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d^3\vec{p_3}\, d^3\vec{p_4}\, \delta^3(\vec{p_3}+\vec{p_4}) \times \mathcal{Y}_1^m\left(\frac{\vec{p_3}-\vec{p_4}}{2}\right)\chi^{34}_{1-m}\phi^{34}_0\omega^{34}_0 b^\dagger_3(\vec{p_3})d^\dagger_4(\vec{p_4}).$$

γ parameterize the strength of creating a quark-antiquark pair from the vacuum which is fixed to be a typical standard value 6.9 [Godfrey & Isgur, PRD32,189(1985)].

- The wave function of A, B, C are GI's results.

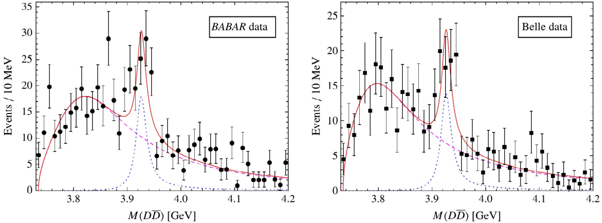
After we have the interaction between the discrete states and the continuum states, we can use the solution to the Friedrichs model to find out the different eigenstates of the full Hamiltonian.

$2P$ Charmonium-like states

Current status:

- 2^3P_2 is well established: $X(3930)$, [Belle,PRL96,082003;BaBar,PRD81,092003]
- 2^3P_1 channel: $X(3872)$ [Belle,PRL91,262001;] molecular state or $c\bar{c}$? Mixture of molecule and $c\bar{c}$, and which is the dominant component?
- If $X(3872)$ is a dynamically generated molecular state, where is the charmonium $\chi_{c1}(2P)$ state.
- 2^1P_1 channel: The $h_c(2P)$ state still has not been seen by experiments.

- 2^3P_0 channel: $X(3915)$ [Belle,PRL104,092001;BaBar,PRD86,072002] used to be assigned to χ_{c0}. However there are some arguement against this assignment [Guo,Meissner,PRD,86,091501; Olsen,PRD91,057501].



The real χ_{c0} state may lie far below, around 3860, and may be a wide one.

- A reanalysis of the BABAR and Belle Data shows that the 2^{++} assignment to $X(3915)$ can not be excluded and it may possibly be the same tensor state as $X(3930)$ [Zhou, Xiao, Zhou, PRL115,022001].
- Belle's recent analysis also found a possible alternative $\chi_{c0}(2P)$ candidate around 3860MeV. [PRD95,112003].

Our scheme

- Using the masses and wave functions from the Godfrey-Isgur model, in the free part of the Friedrichs model and the QPC model.
- Bare discretes states: $\chi_{c0}(2P)$ at 3917 MeV, $\chi_{c1}(2P)$ at 3953 MeV, $\chi_{c2}(2P)$ at 3979 MeV. $h_c(2P)$ at 3956MeV.
- Continuum states: $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$ threshold, upto D-wave.
  $\chi_{c0}(2P)$ couples to $D\bar{D}$ (S-wave), $D^*\bar{D}^*$ (S-wave, D-wave).
  $\chi_{c1}(2P)$ couples to $D\bar{D}^*$(S,D-wave), $D^*\bar{D}^*$ (D-wave)
  $\chi_{c2}(2P)$ couples to $D\bar{D}$ (D-wave), $D\bar{D}^*$ (D-wave), $D^*\bar{D}^*$ (S,D-wave)
  $h_c(2P)$ couples to $D\bar{D}^*$ (S,D-wave), $D^*\bar{D}^*$ (S,D-wave).
- Parameterize the interaction between the bare states and the continuum using the QPC model.

Our scheme

Try to understand the mass spectrum and width using our method

- Solution of $\eta(z) = 0$ gives the mass and width of the resonances $z_R = M + i\Gamma/2$, or masses of the bound states.
- We also give a Breit-Wigner mass for narrow resonances to compare with the experimental results, i.e. from Real part of $\eta(z) = 0$,

$$M_{BW} - \omega_0 - \lambda^2 \sum_n \mathcal{P} \int_{\omega_{th,n}}^{\infty} \frac{\sum_{SL} |f^n_{SL}(\omega)|^2}{M_{BW} - \omega} \mathrm{d}\omega = 0,$$

$$\Gamma^n_{BW} = 2\pi \sum_{S,L} |f^n_{SL}(M_{BW})|^2,$$

Numerical results

Table: Comparison of the experimental masses and the total widths (in MeV) [PDG2016] with our results.

$n^{2s+1}L_J$	M_{expt}	Γ_{expt}	M_{BW}	Γ_{BW}	pole	GI
2^3P_2	3927.2 ± 2.6	24 ± 6	3910	12	3908-5i	3979
2^3P_1	3942 ± 9	37^{+27}_{-17}			3917-45i	3953
	3871.69 ± 0.17	< 1.2	3871	0	3871-0i	
2^3P_0	3862^{+66}_{-45}	201^{+242}_{-149}	3860	25	3861-11i	3917
2^1P_1			3890	26	3890-22i	3956

Numerical results

- The there is a narrow 2^3P_2 state which can be assigned to the well-established χ_{c2}.
- The 2^3P_0 state is found to be around 3860 MeV, which is consistent with mass of the experimental reanalysis of the Belle data. Our result prefers an unexpected narrow width, whereas the experimental result has a large uncertainty 201^{+242}_{-149} MeV.
  There are also other predictions with small width, [Barnes et.al., PRD72,054026;Eichten et.al, PRD69,094019]

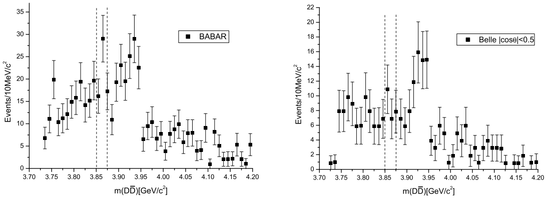


Figure: $\gamma\gamma \to D\bar{D}$ from BABAR [PRD81,092003] and Belle [RPL,96,082003]. The two dashed lines are set at $m(D\bar{D}) = 3850$ MeV and 3875 MeV.

Numerical results

(2^3P_1) : $X(3872)$ & χ_{c1}

- A dynamically generated bound state , located just at 3871 is naturally assigned to $X(3872)$.
- The $X(3872)$ is not originated from the bare state χ_{c1}, but from the interaction between the bare state and the continuum.
- If we reduce the γ parameter, the $X(3872)$ pole will move to the second sheet becoming a virtual state.
- The bare state pole is shifted to about 3917 MeV with a large width — may be related to $X(3940)$ observed by the experiment.
- $X(3872)$:

$$\frac{\text{elementariness}}{\text{compositeness}} = 1 : 2.7.$$

A large portion of continuum state $D\bar{D}^*$ — more molecular component than the $c\bar{c}$ component.

Numerical results

2^1P_1: a prediction of h_c,

- Mass: shifted from GI's result 3956 to 3890.
- $J^{PC} = 1^{+-}$: need a negetive C-parity channel to look for it, such as $\eta_c\gamma$, $J/\psi\eta$.

Summary

- We study the first excited P-wave charmonium-like states in a Friedrichs-like model method. This method can also be generalized to study other states above the open flavor thresholds.
- The $X(3872)$ is naturally dynamically generated by the interaction of χ_{c1} state and the continuum states. The continuum components constitute a larger portion of the $X(3872)$ than the $c\bar{c}$.
- With the compositeness and the elementariness, the further properties of $X(3872)$ can also be studied which is a work in progress.
- In our scheme, the χ_{c0} state is a narrow one around 3860.
- We also give a prediction of the position and width of the $h_c(2P)$.

Thank you !