6th International Conference on New Frontiers in Physics

Kolymbari, 17-29 August 2017

Narrow-width tetraquarks in large-N_c QCD

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arXiv: 1706.06003, Phys. Rev. D 96 (2017) 014022.

QCD at large N_c

Framework: $SU(N_c)$ gauge theory, with quarks in the fundamental representation, considered in the limit $N_c \to \infty$ with $\alpha_s \sim 1/N_c$. ('t Hooft (1974).)

At leading order, QCD Green's functions have only non-interacting mesons as intermediate states. (Witten (1979).) Tetraquark bound states, assumed to be essentially made of two quarks and two antiquarks, may emerge only through subleading diagrams. (Coleman (1980).)

For a long time, this fact has been considered as a theoretical proof of the non-existence of tetraquarks as elementary stable particles, surviving in the large-N_c limit, like the ordinary mesons.

Recently, Weinberg (2013) observed that if tetraquarks exist as bound states in the large-N_c limit with finite masses, the crucial point is, even if they contribute to subleading diagrams, the qualitative property of their decay widths: are they broad or narrow? In the latter case, they might be observable. He showed that, generally, they should be narrow, with decay widths of the order of $1/N_c$.

Knecht and Peris (2013) showed that in a particular exotic channel, tetraquarks should even be narrower, with decay widths of the order of $1/N_c^2$.

Cohen and Lebed (2014) showed, in more general exotic channels, with an analysis based on the analyticity properties of two-meson scattering amplitudes, that the decay widths should indeed be of the order of $1/N_c^2$.

Line of approach

Study of exotic and cryptoexotic tetraquark properties, through the analysis of meson-meson scattering amplitudes.

Exotics: contain four different quark flavors.

Cryptoexotics: contain three different quark flavors.

Four-point correlation functions of color-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b,$$

producing a meson M_{ab} from the vacuum:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}; \qquad f_M \sim N_c^{1/2}.$$

Spin and parity ignored; not relevant for the qualitative aspects.

Consider all possible channels where a tetraquark may be present.

To be sure that a QCD diagram may contain a tetraquark contribution, through a pole term, one has to check that it receives a four-quark contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behavior of the diagram.

If the tetraquark contains quarks and antiquarks with masses $m_j,\ j = a, b, c, d$, then the diagram should have a four-particle cut starting at $s = (m_a + m_b + m_c + m_d)^2$.
Its existence is checked with the use of the Landau equations.

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behavior analysis of the tetraquark properties.

Exotic tetraquarks

Four distinct quark flavors, denoted 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2.$$

The following scattering processes are considered:

$$M_{12} + M_{34} \ \to \ M_{12} + M_{34}; \qquad \text{Direct channel I};$$

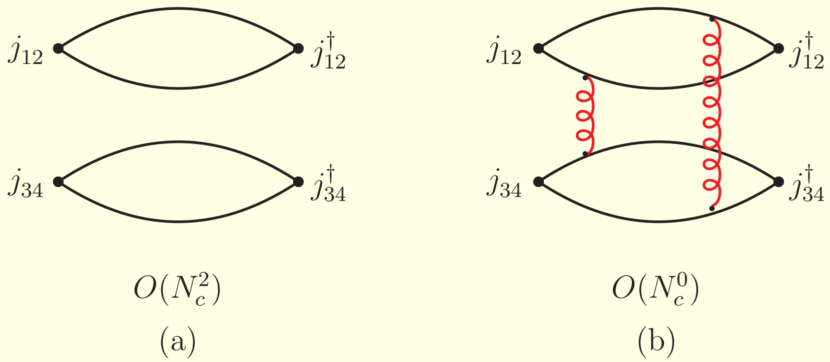
$$M_{14} + M_{32} \ \to \ M_{14} + M_{32}; \qquad \text{Direct channel II};$$

$$M_{12} + M_{34} \ \to \ M_{14} + M_{32}; \qquad \text{Recombination channel}.$$

‘Direct’ 4-point functions

$$\Gamma_I^{(dir)} = \langle j_{12} j_{34} j_{34}^\dagger j_{12}^\dagger \rangle \ , \qquad \Gamma_{II}^{(dir)} = \langle j_{14} j_{32} j_{32}^\dagger j_{14}^\dagger \rangle \ .$$

Leading and subleading diagrams for $\Gamma_I^{(dir)}$:



Similar diagrams for $\Gamma_{II}^{(dir)}$.

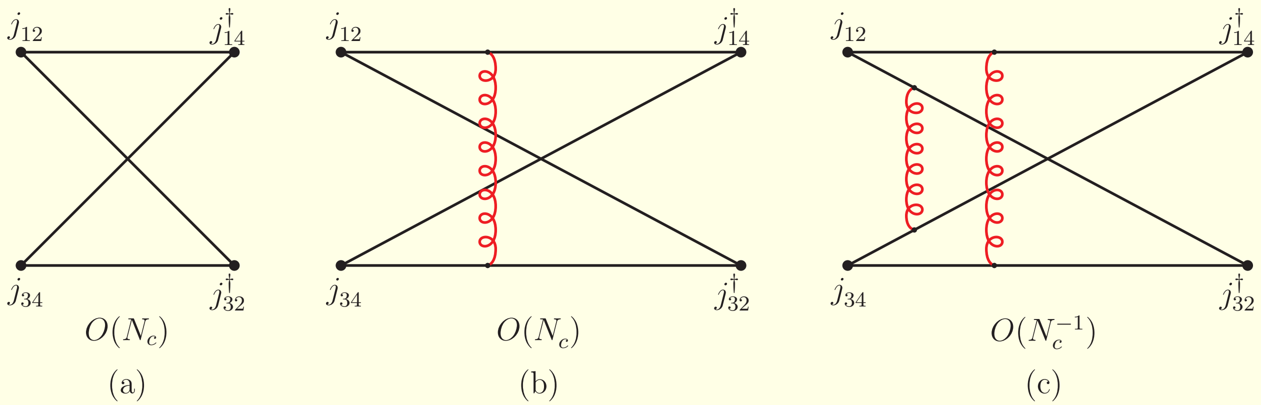
Only diagram (b) may receive contributions from tetraquark states.

$$\Gamma_{I,T}^{(dir)} = O(N_c^0), \qquad \Gamma_{II,T}^{(dir)} = O(N_c^0).$$

'Recombination' 4-point function

$$\Gamma^{(recomb)} = \langle j_{12} j_{34} j_{32}^{\dagger} j_{14}^{\dagger} \rangle \ .$$

Leading and subleading diagrams:



Only diagram (c) may receive contributions from tetraquark states.

$$\Gamma_T^{(recomb)} = O(N_c^{-1}).$$

The fact that the direct and recombination amplitudes have different behaviors in N_c, requires the contribution of two different tetraquarks, T_A and T_B, each having different couplings to the meson pairs.

Factorizing in the correlation functions the external current couplings to the mesons ($\sim f_M \sim N_c^{1/2}$), one obtains

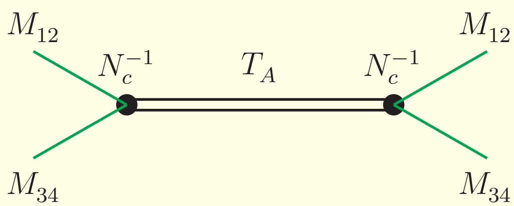
$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}),$$

$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).$$

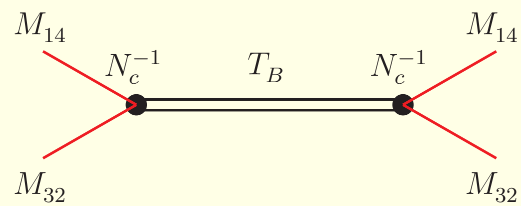
Total widths:

$$\Gamma(T_A) = O(N_c^{-2}), \qquad \Gamma(T_B) = O(N_c^{-2}).$$

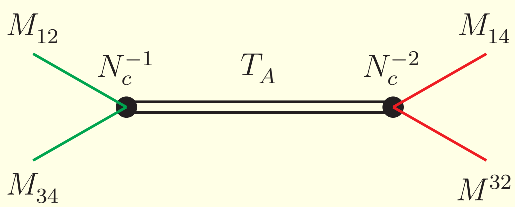
The meson-meson scattering amplitudes at the tetraquark poles (leading contributions):



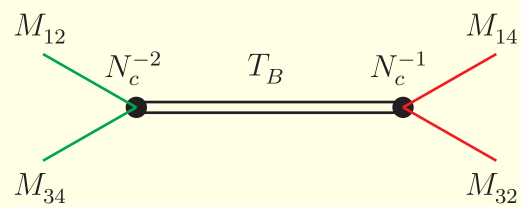
(a) $O(N_c^{-2})$



(b) $O(N_c^{-2})$



(c) $O(N_c^{-3})$



(d) $O(N_c^{-3})$

Cryptoexotic tetraquarks

Three distinct quark flavors, denoted 1,2,3, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{23} = \overline{q}_2 q_3, \quad j_{22} = \overline{q}_2 q_2.$$

The following scattering processes are considered:

$$M_{12} + M_{23} \ \rightarrow \ M_{12} + M_{23}; \qquad \text{Direct channel I};$$

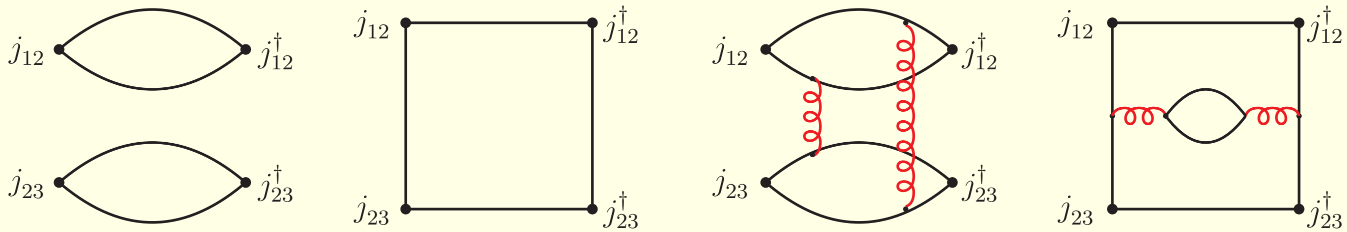
$$M_{13} + M_{22} \ \rightarrow \ M_{13} + M_{22}; \qquad \text{Direct channel II};$$

$$M_{12} + M_{23} \ \rightarrow \ M_{13} + M_{22}; \qquad \text{Recombination channel}.$$

‘Direct’ 4-point functions

$$\Gamma_I^{(dir)} = \langle j_{12} j_{23} j_{23}^\dagger j_{12}^\dagger \rangle \,, \qquad \Gamma_{II}^{(dir)} = \langle j_{13} j_{22} j_{22}^\dagger j_{13}^\dagger \rangle \,.$$

Leading and subleading diagrams for $\Gamma_I^{(dir)}$:



$O(N_c^2)$ (a) $\quad$ $O(N_c)$ (b) $\quad$ $O(N_c^0)$ (c) $\quad$ $O(N_c^0)$ (d)

Diagram (b) receives contributions from one-meson intermediate states.
Diagram (c) may receive contributions from tetraquark intermediate states.
Diagram (d) describes possible mixing of one-meson–one tetraquark states.

$$\Gamma_{I,T}^{(dir)} = O(N_c^0).$$

Leading and subleading diagrams for $\Gamma_{II}^{(dir)}$:

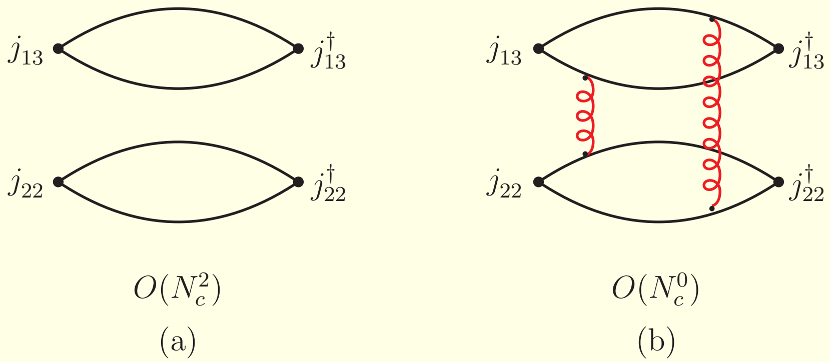


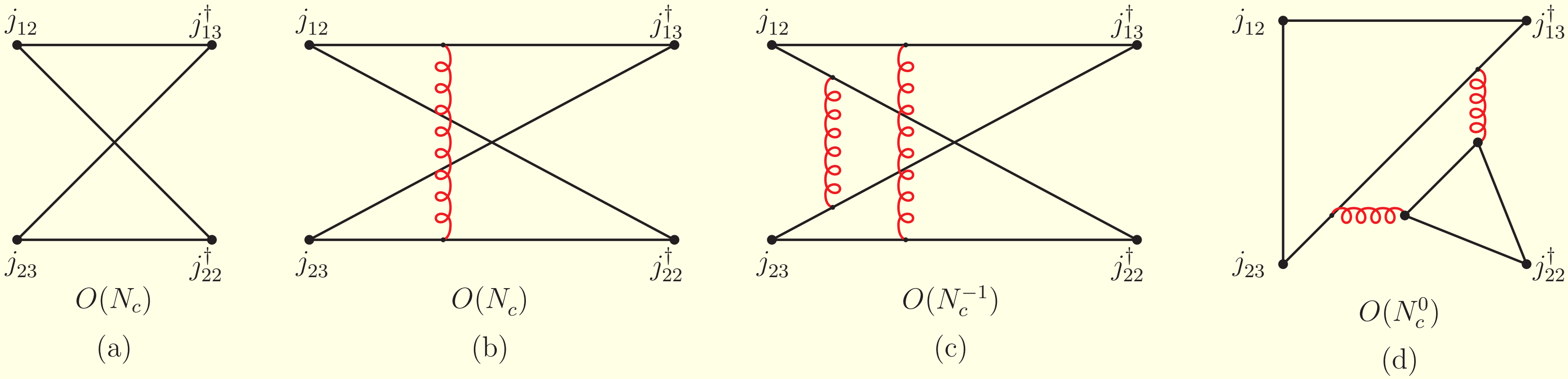
Diagram (b) may receive contributions from tetraquark intermediate states.

$$\Gamma_{II,T}^{(dir)} = O(N_c^0).$$

‘Recombination’ 4-point function

$$\boldsymbol{\Gamma^{(recomb)} = \langle j_{12} j_{23} j_{13}^{\dagger} j_{22}^{\dagger} \rangle} \ .$$

Leading and subleading diagrams:



Diagrams (c) and (d) may receive contributions from tetraquark intermediate states.

$$\boldsymbol{\Gamma_T^{(recomb)} = O(N_c^0).}$$

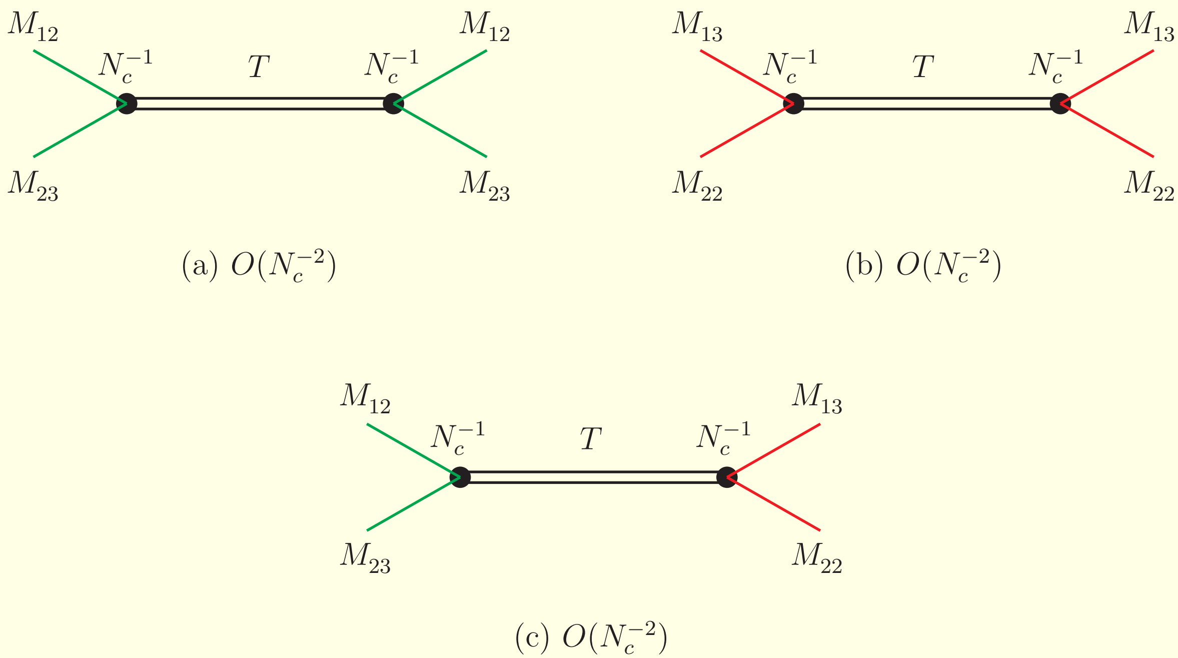
‘Direct’ and ‘recombination’ diagrams have the same N_c-behavior in the present case. A single tetraquark T may accomodate all channels.

$$A(T \to M_{12}M_{23}) = O(N_c^{-1}), \qquad A(T \to M_{13}M_{22}) = O(N_c^{-1}).$$

Total width:

$$\Gamma(T) = O(N_c^{-2}).$$

The meson-meson scattering amplitudes at the tetraquark pole:



Open cryptoexotic channels

Three distinct quark flavors, denoted 1,2,3, with meson currents

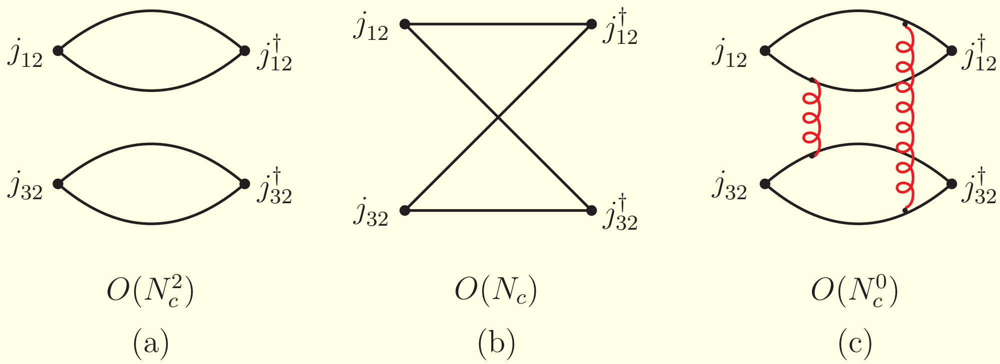
$$j_{12} = \overline{q}_1 q_2, \quad j_{32} = \overline{q}_3 q_2.$$

The following scattering process is considered:

$$M_{12} + M_{32} \ \rightarrow \ M_{12} + M_{32}.$$

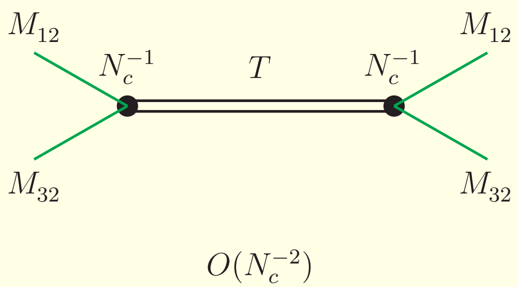
Here, the ‘direct’ and ‘recombination’ channels are identical.

Leading and subleading diagrams:



Only diagram (c) may receive contributions from tetraquark intermediate states.

$$A(T \to M_{12}M_{32}) = O(N_c^{-1}), \quad \Gamma(T) = O(N_c^{-2}).$$



Conclusion

Analysis of the s-channel singularities of Feynman diagrams crucial for the detection of the possible presence of tetraquark intermediate states in correlation functions of meson currents.

If tetraquarks exist as stable bound states of two quarks and two antiquarks in the large-N_c limit, with finite masses, due to the operating confining forces, then they should have narrow decay widths, of the order of N_c^{-2}.

For the fully exotic channel, with four different quark flavors, two different tetraquarks are needed to accomodate the theoretical constraints of the large-N_c limit. In this case, each tetraquark has one predominant decay channel.