Universal Quantum Computing and Quantum Advantage in Continuous Variables

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Continuous Variables QC

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→ **Deterministic generation**.

→ **Efficient** measurement devices with **homodyne detection**.

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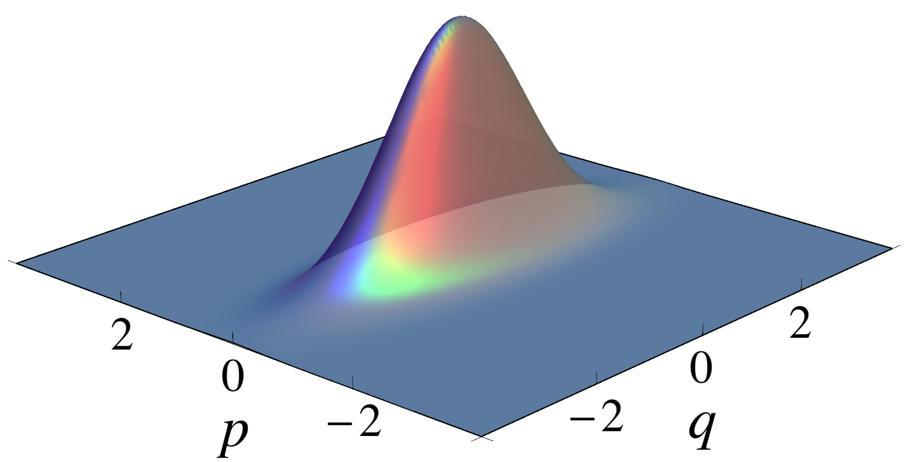

Wigner function of a momentum squeezed state.

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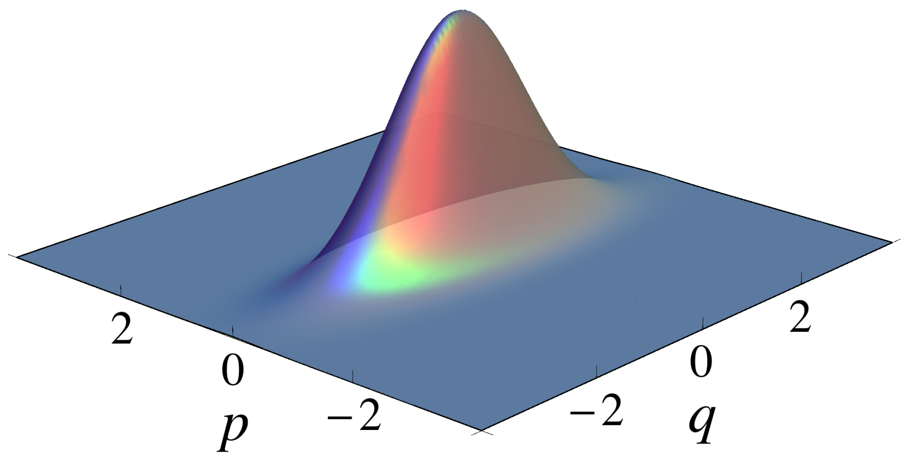

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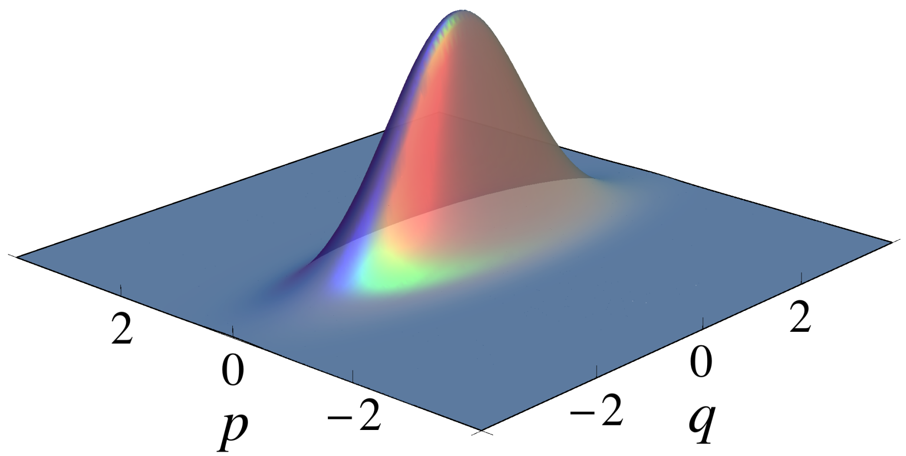

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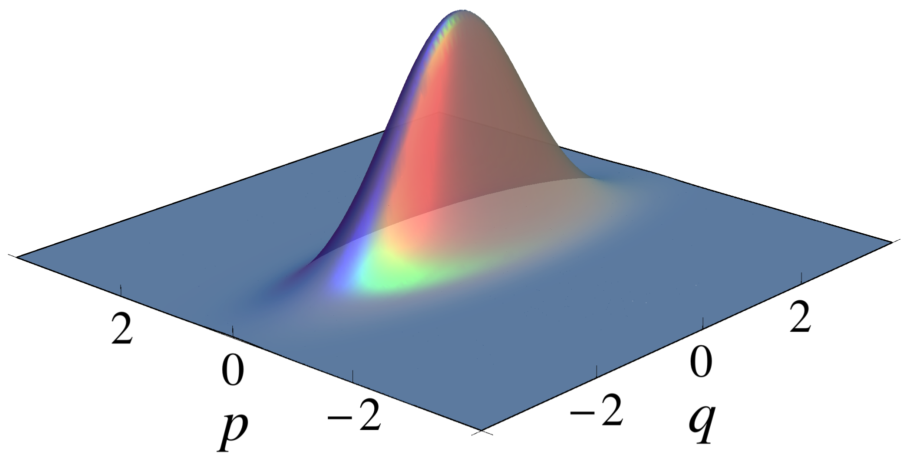

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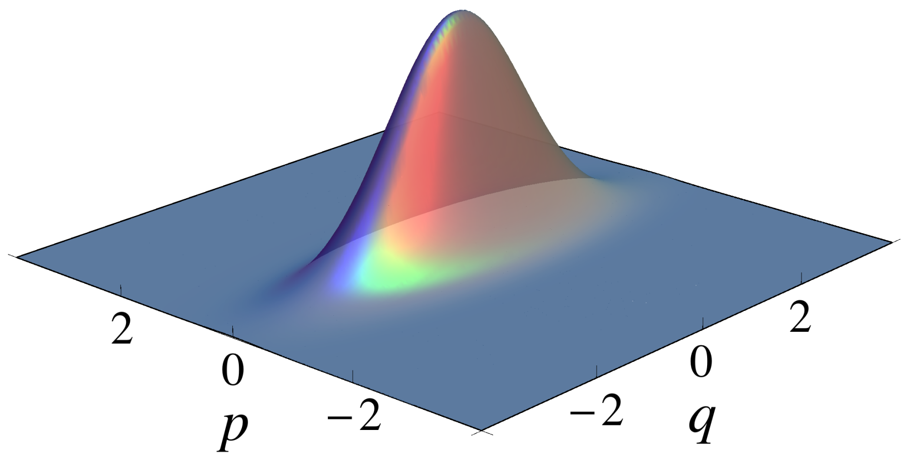

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- → An entangling gate reads: $C_Z = \exp(i\hat{q}_1 \otimes \hat{q}_2)$

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But first… what is **Universal QC in CV?**

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But first… what is **Universal QC in CV?**

→ We need qubits!

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Define **qubits with CV**: GKP states, Gottesman et al. PRA 2001

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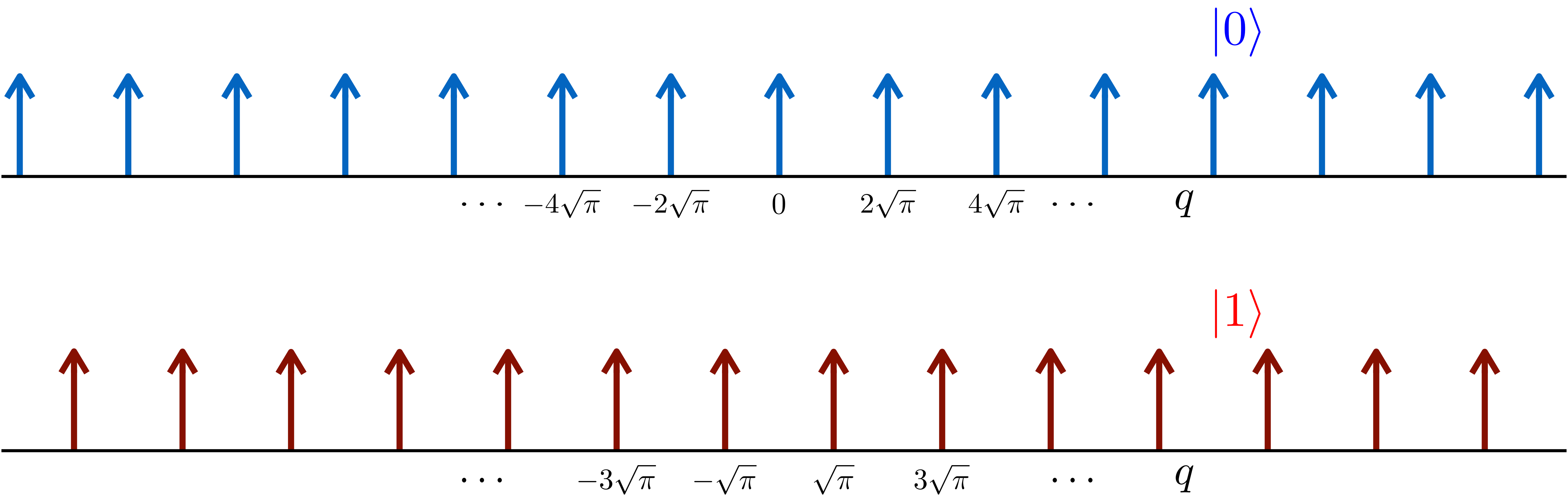

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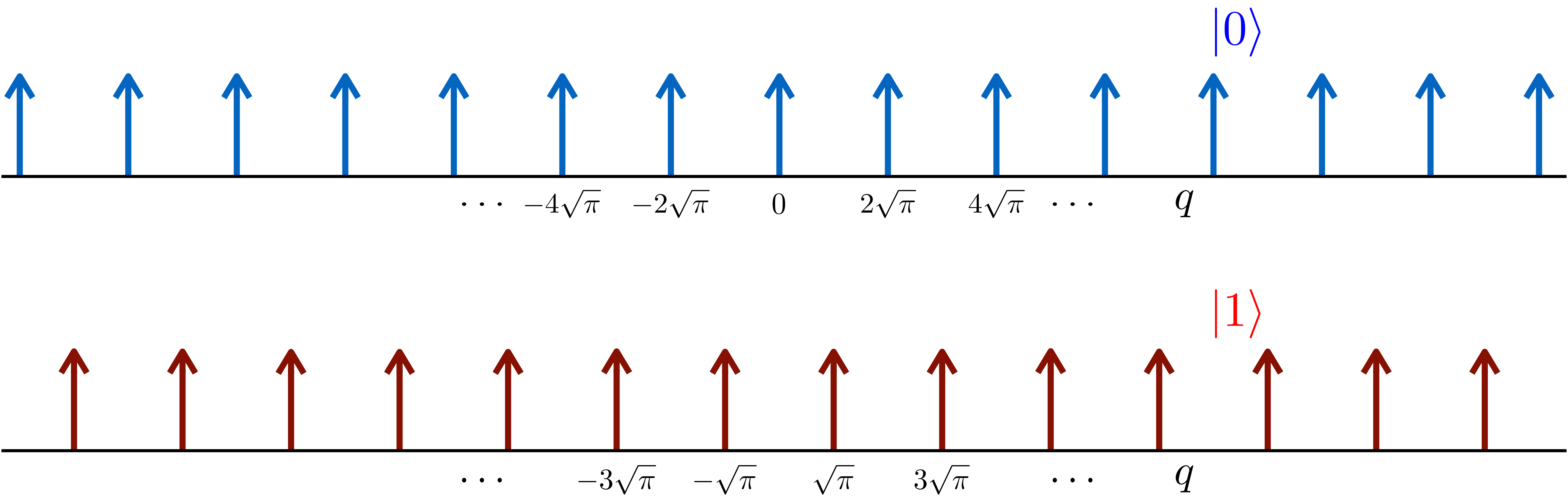

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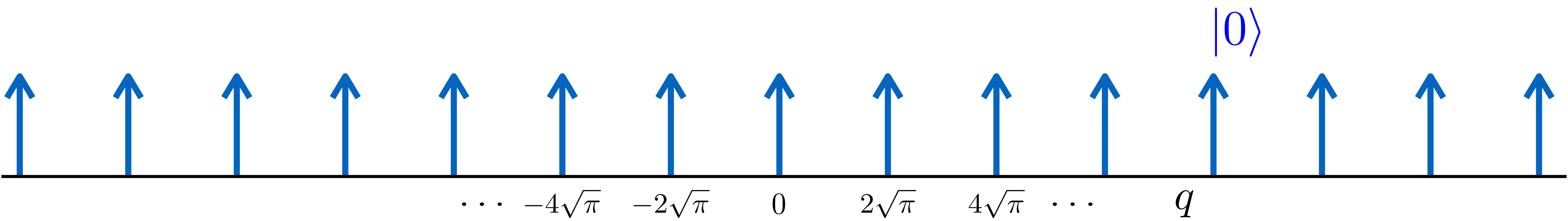

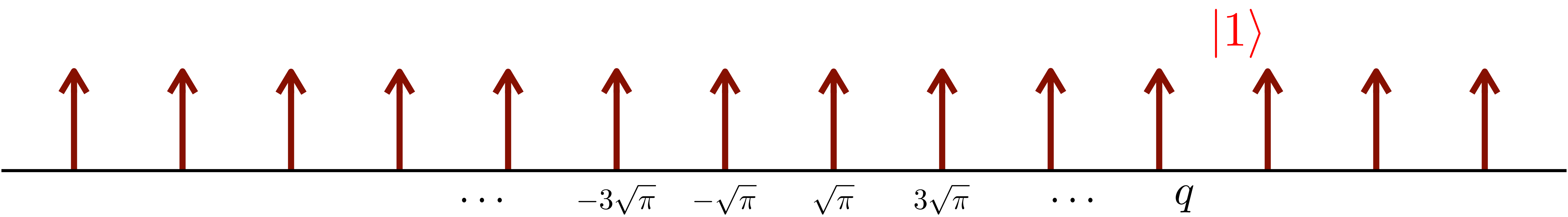

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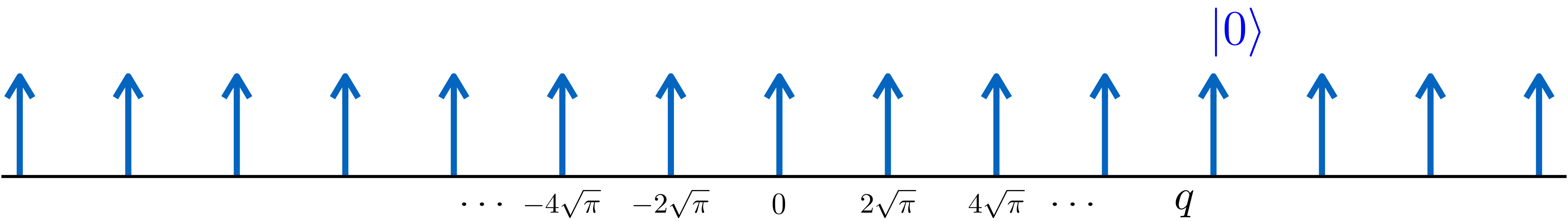

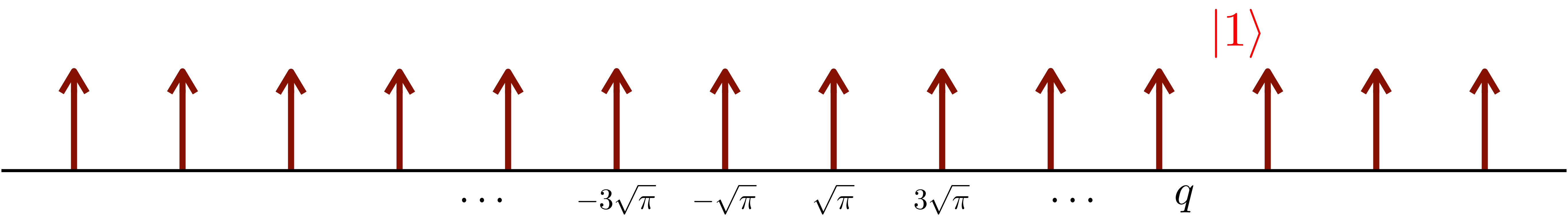

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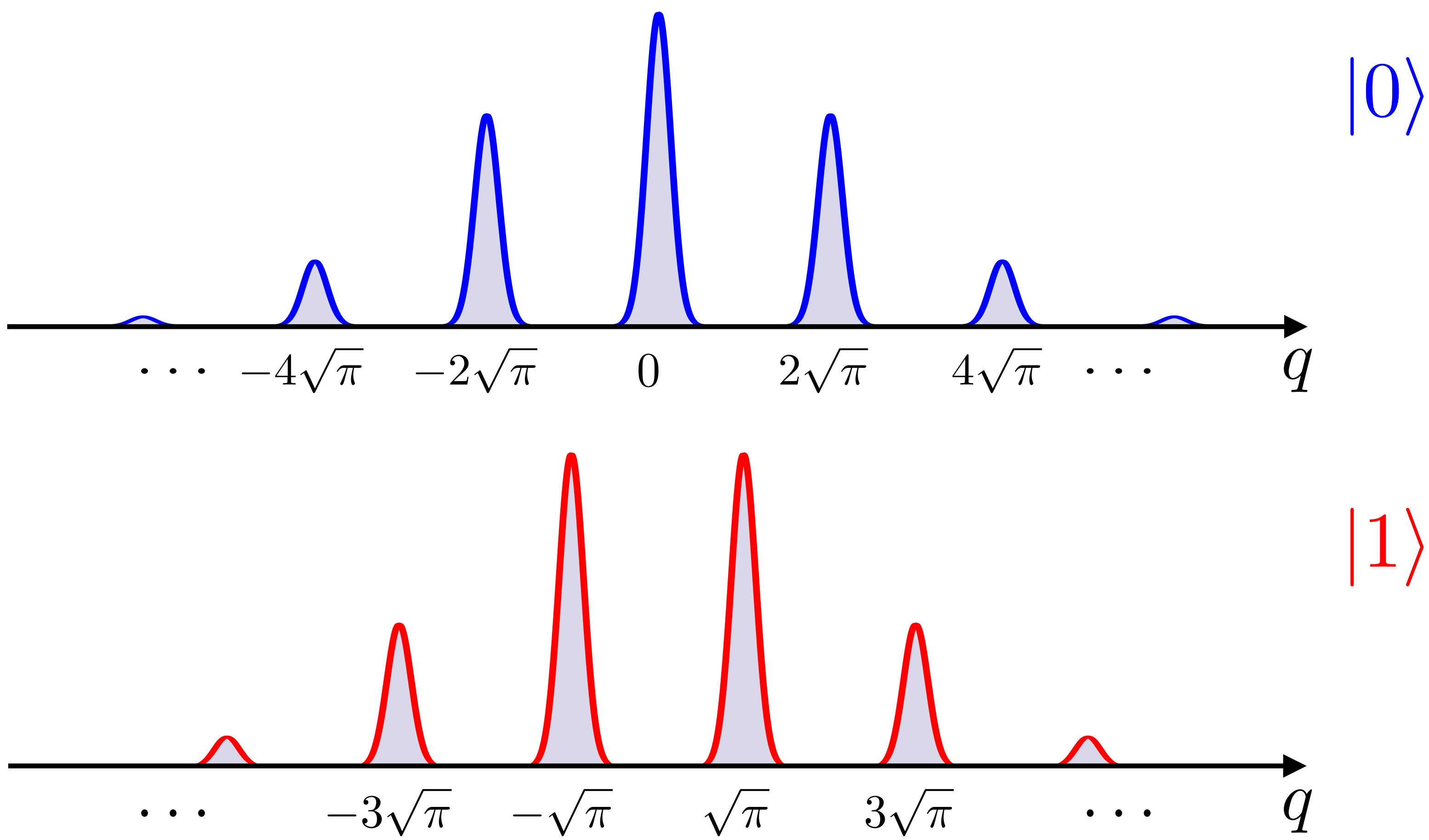

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GKP Quantum Computer

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GKP states $+$ Finite set of CV gates

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Fault Tolerant Universal Quantum Computer

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→ Producing GKP states is extremely challenging.

Universal CV QC

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Using a protocol by Vasconcelos *et al.*, Opt. Lett. (2010), cat states can be used to produce **approximate GKP states**.

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The protocol is **probabilistic**.

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STEP II:

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➡ Cat states

See e.g. Vitali *et al.*, Appl. Phys. B (1997).

Universal CV QC

STEP II: Coherent states + Cross Kerr interaction $\exp\left(it\hat{n}_1\hat{n}_2\right)$ → Cat states

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And the cross Kerr interaction can also be approximated using a finite set of gates:

$$\exp\left(is_1\hat{q}_1\hat{q}_2\right), \exp\left(is_2\hat{q}^2\right), \exp\left(is_3\hat{q}^3\right)$$

… and the Fourier transform.

Universal CV QC

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Fault Tolerant Universal Quantum Computer
(probabilistic)

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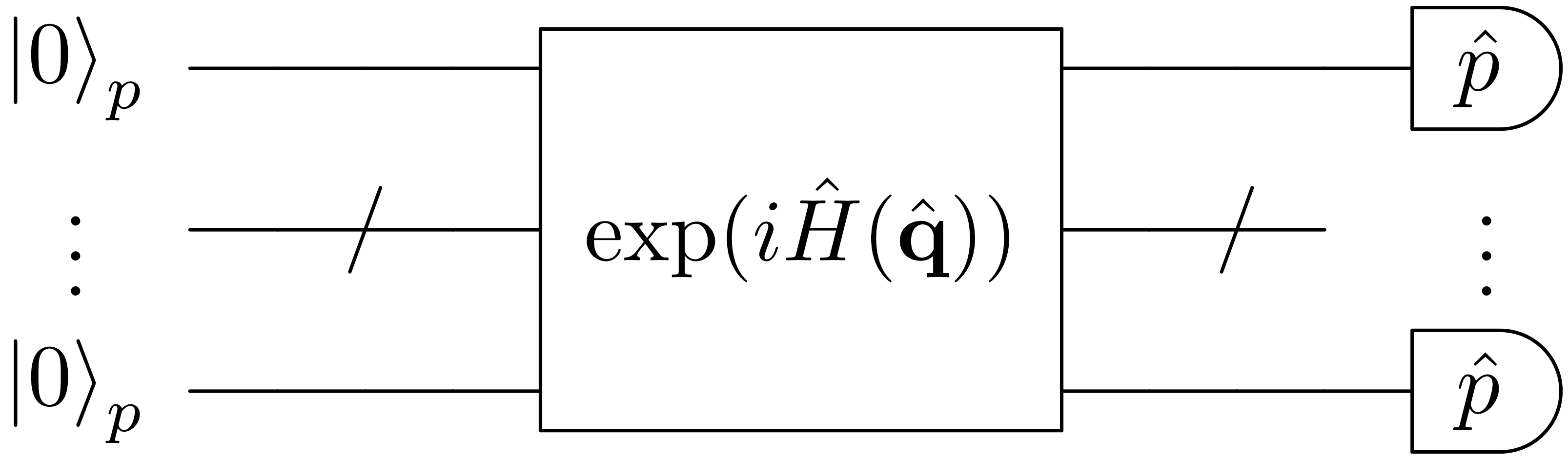

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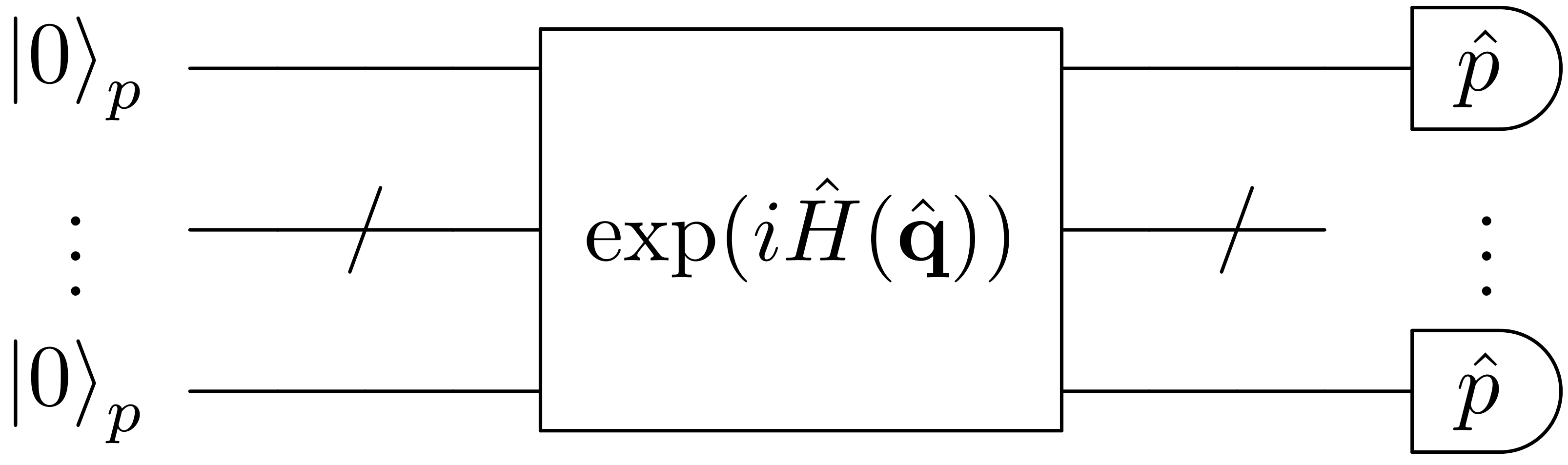

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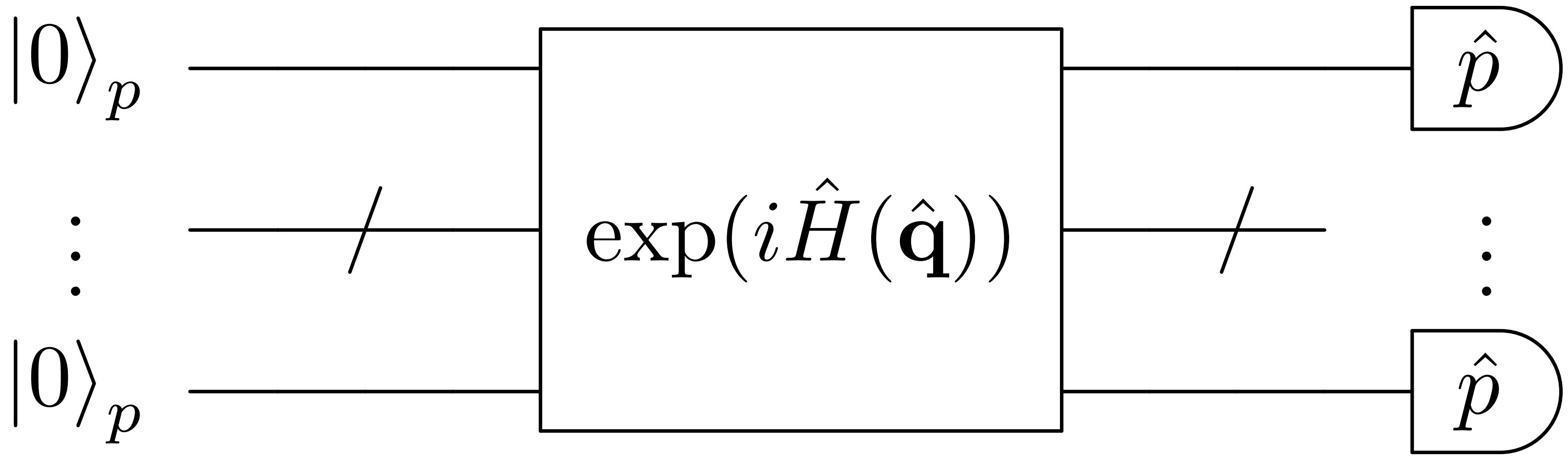

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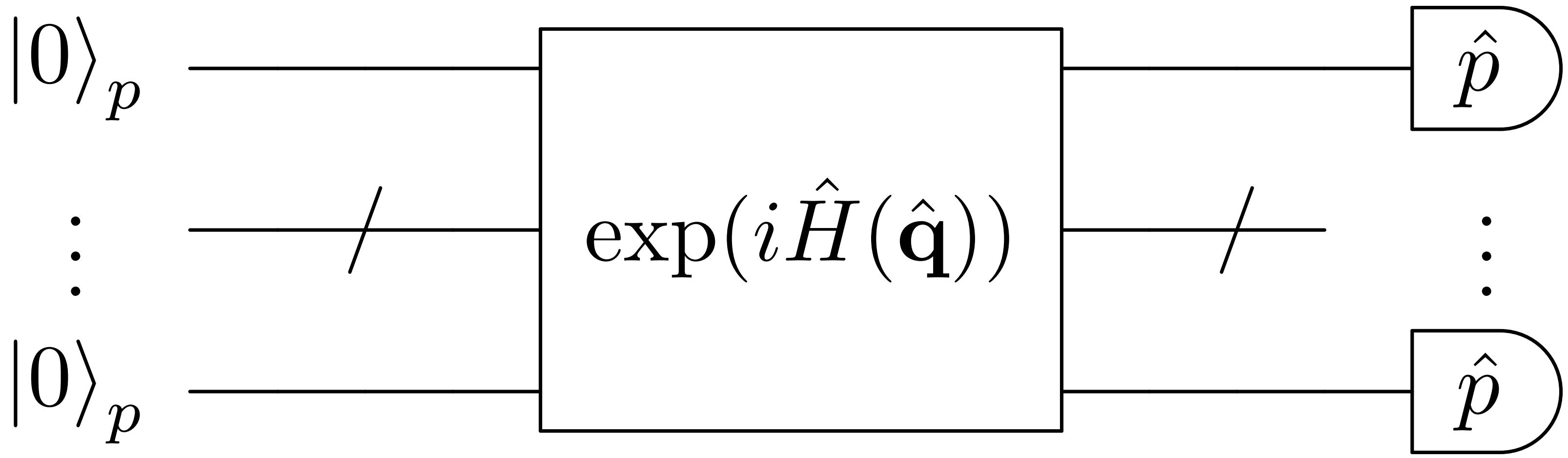

→ **Momentum-squeezed** input states.

→ **Homodyne detection** of the **momentum quadrature.**

→ **Hamiltonian** evolution function of the **position operators,** also includes **non-Gaussian gates.**

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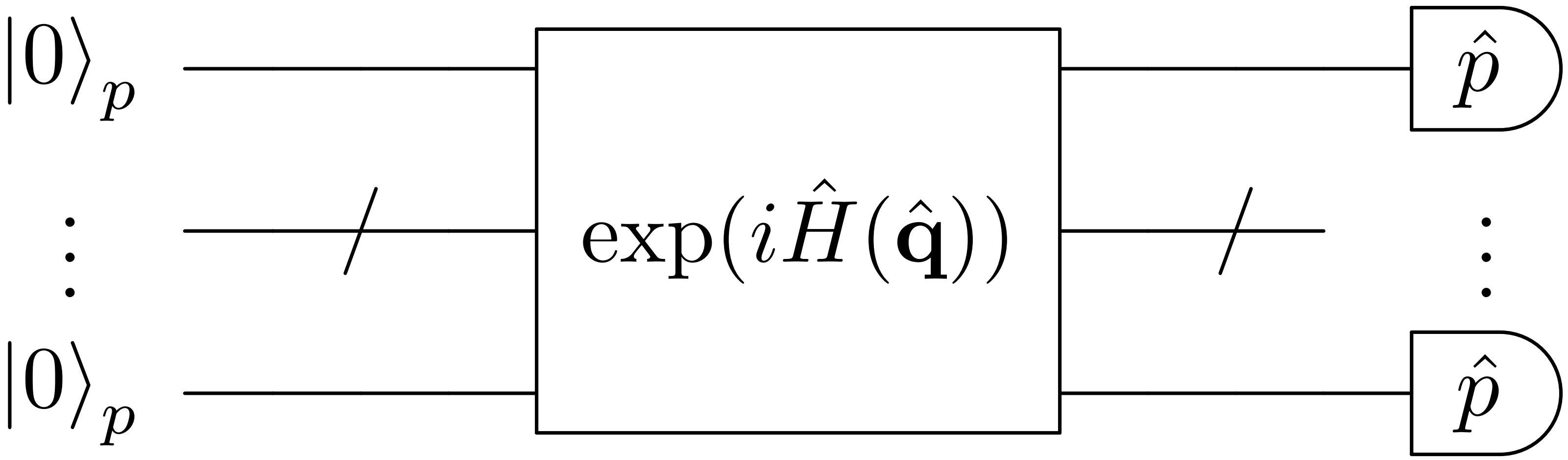

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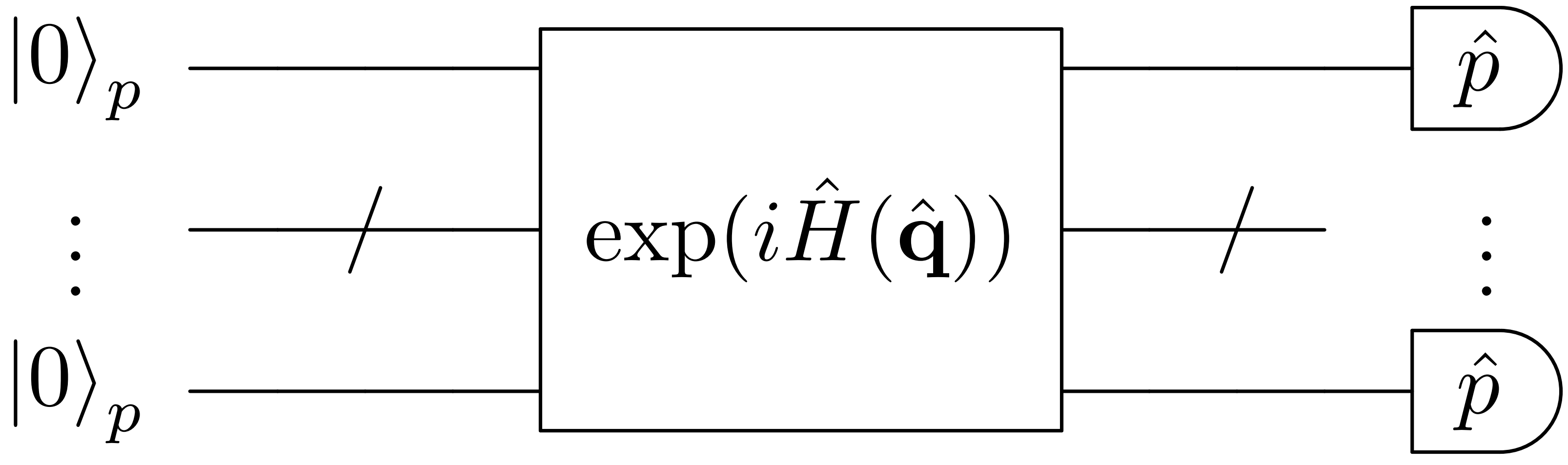

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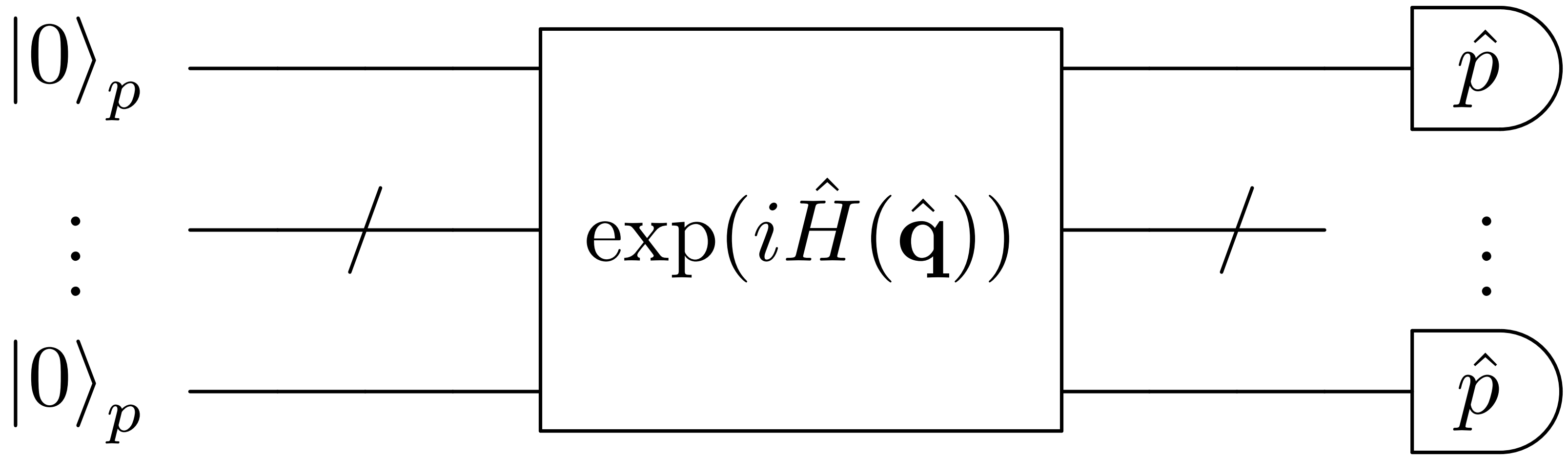

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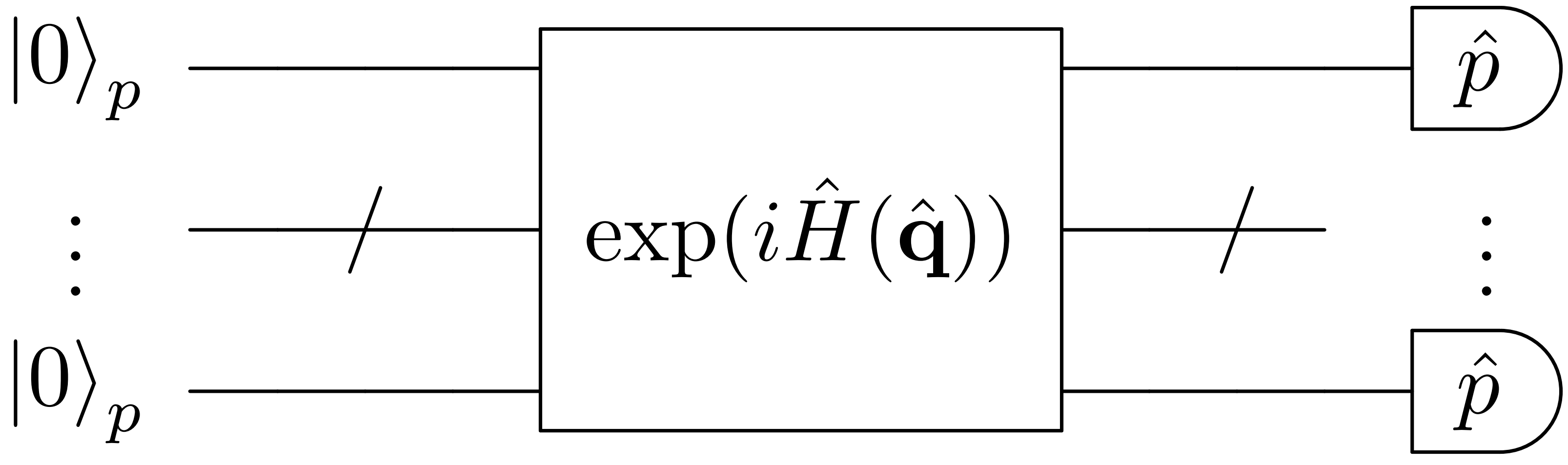


→ **finite squeezing** σ for the input states.

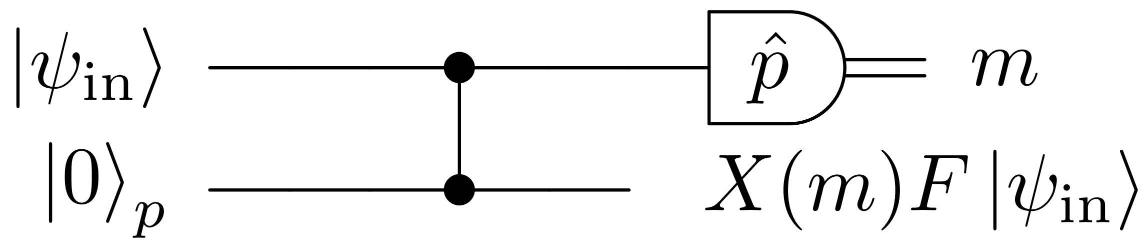
→ **finite resolution** η for the homodyne detection.

→ Contains all gates diagonal in the position basis… only the Fourier transform is missing!

More details in Douce *et al.*, PRL (2017)

IQP in CV

STEP I: RECOVERING THE FOURIER TRANSFORM.



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IQP in CV

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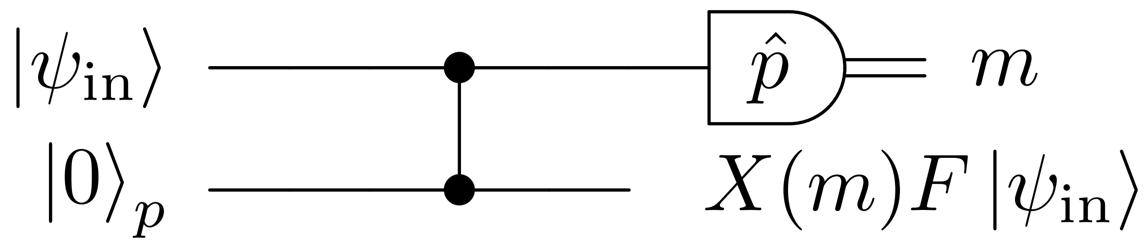

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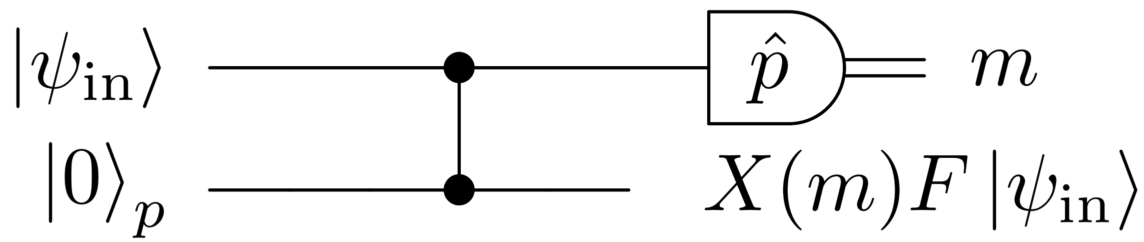

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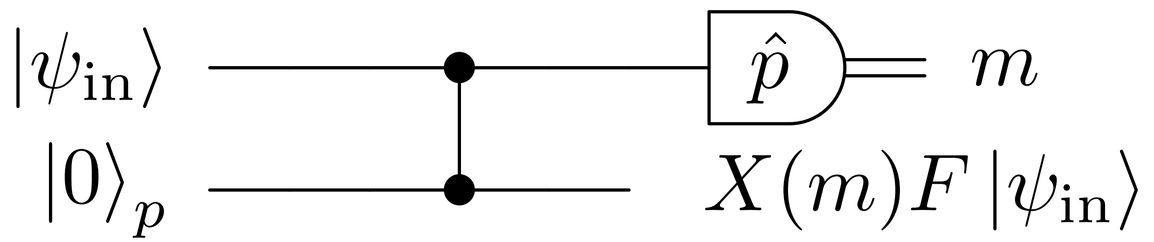

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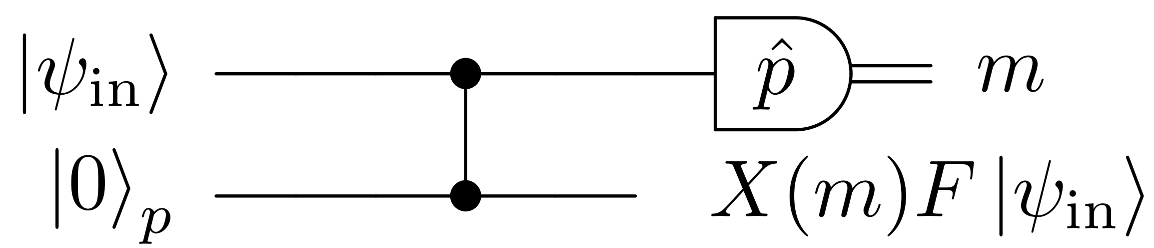

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- → **squeezing decreases** with the protocol.
- → **finite resolution** turns the quantum state into a **mixed state**.

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IQP in CV

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So postselected CVIQP circuits are as powerful as a postselected fault tolerant universal QC.

IQP in CV

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It would imply a **collapse of the Polynomial Hierarchy** to its third level.

INCONCEIVABLE!

IQP in CV

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Reductio ad abusrdum: the original assumption was wrong and **CVIQP is hard** for classical computers.

IQP in CV

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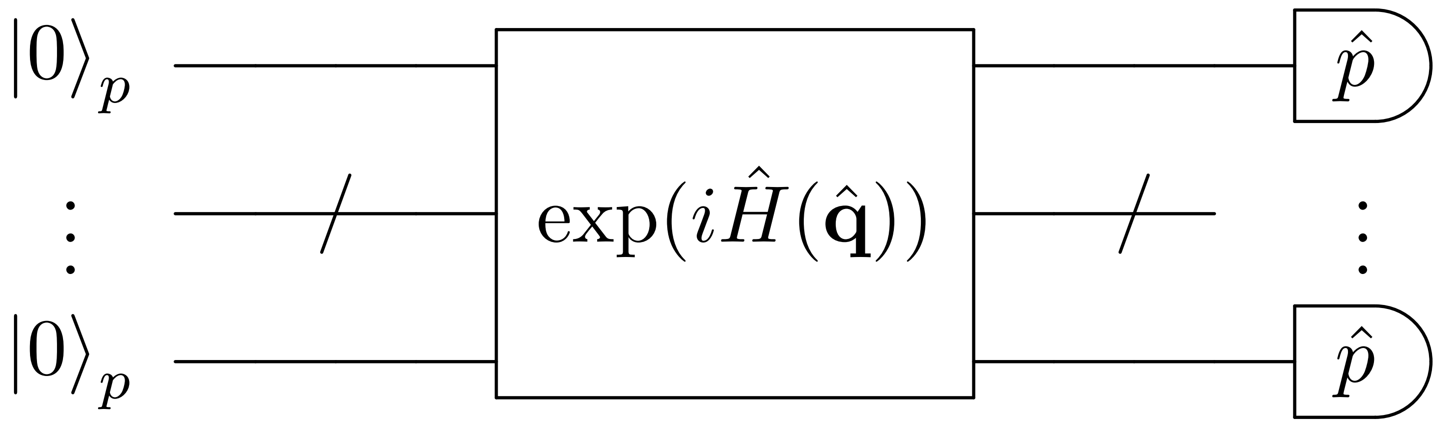
→ We defined a model for Instantaneous Quantum Computing in CV.



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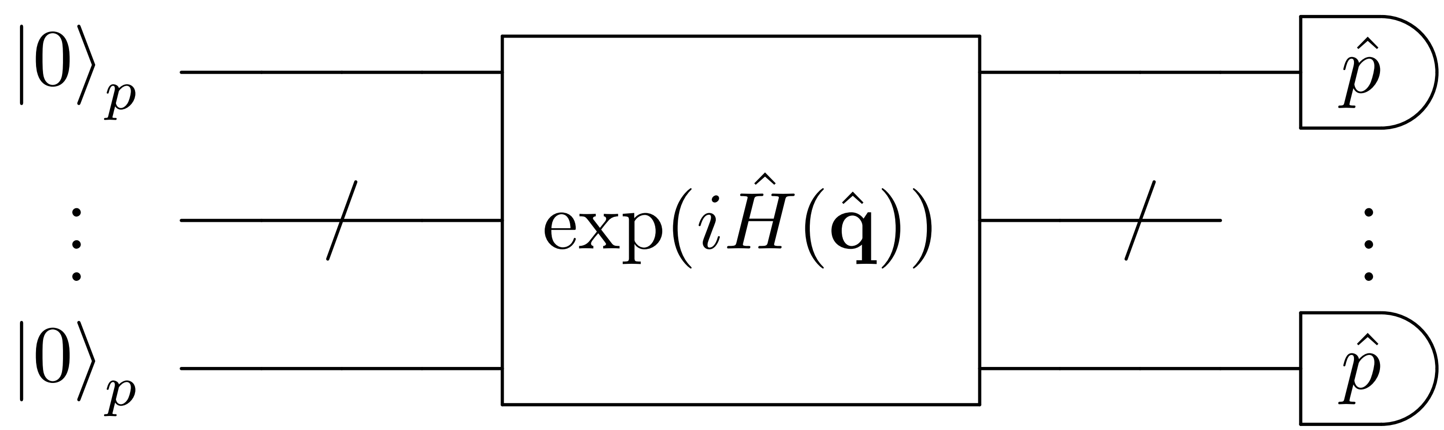

→ Using a new approach to Universal QC in CV, we showed that this model could not be efficiently simulated classically (unless the Polynomial Hierarchy collapses).

IQP in CV

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THANK YOU!!!