Mixing of positive and negative frequency waves at the optical horizon

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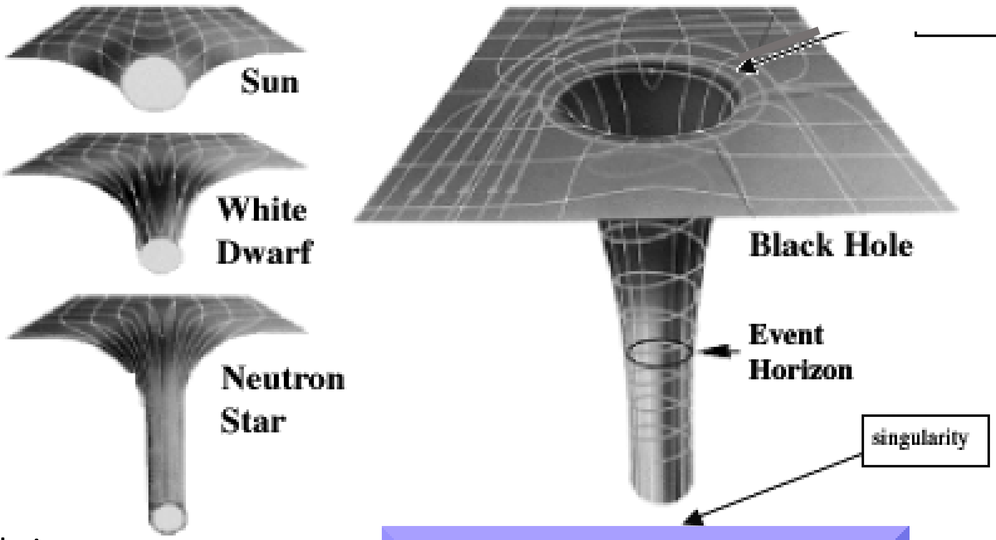
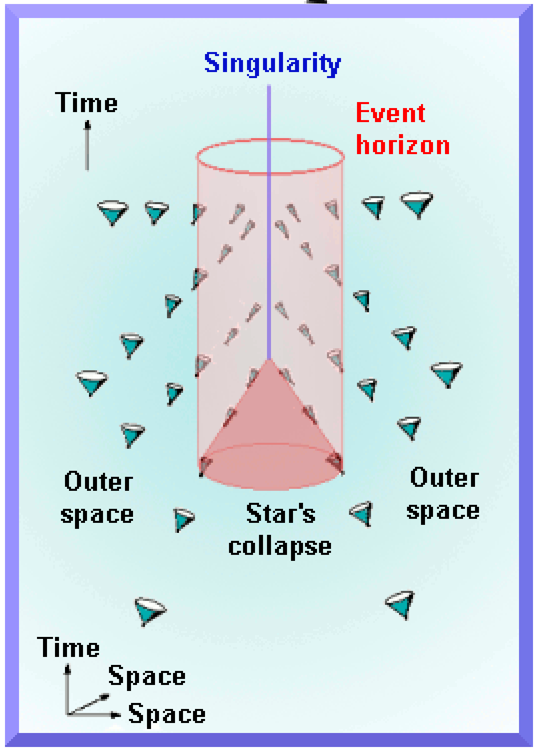
ICNFP’17 – 22/08/17

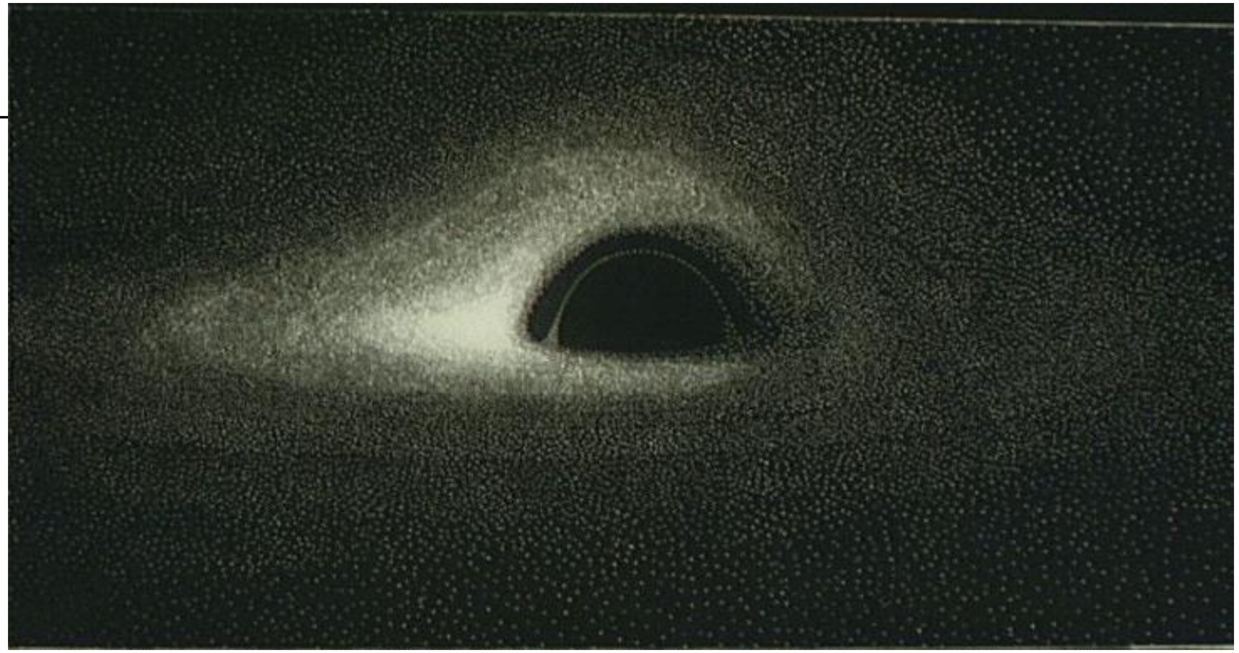
Curvature of spacetime and event horizon



Nasa website



Penrose, 1972



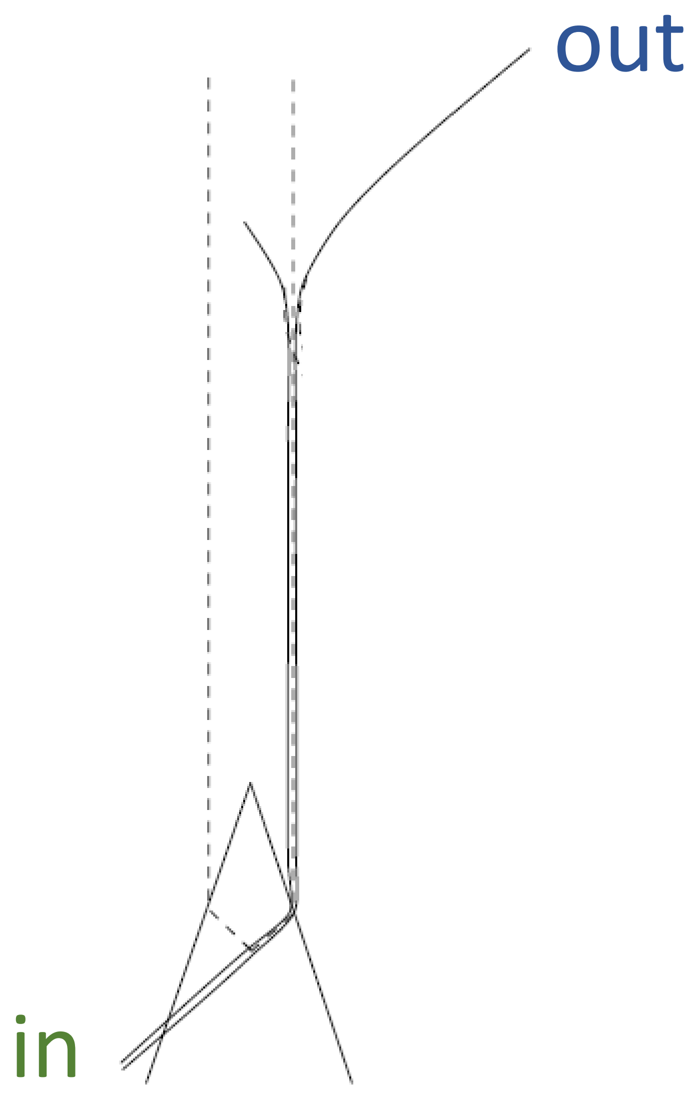
Luminet, 1979

General Relativity identifies gravity with curvature of spacetime

Point of no return: event horizon

General Relativity: nothing can escape a black hole

Spontaneous emission from the vacuum



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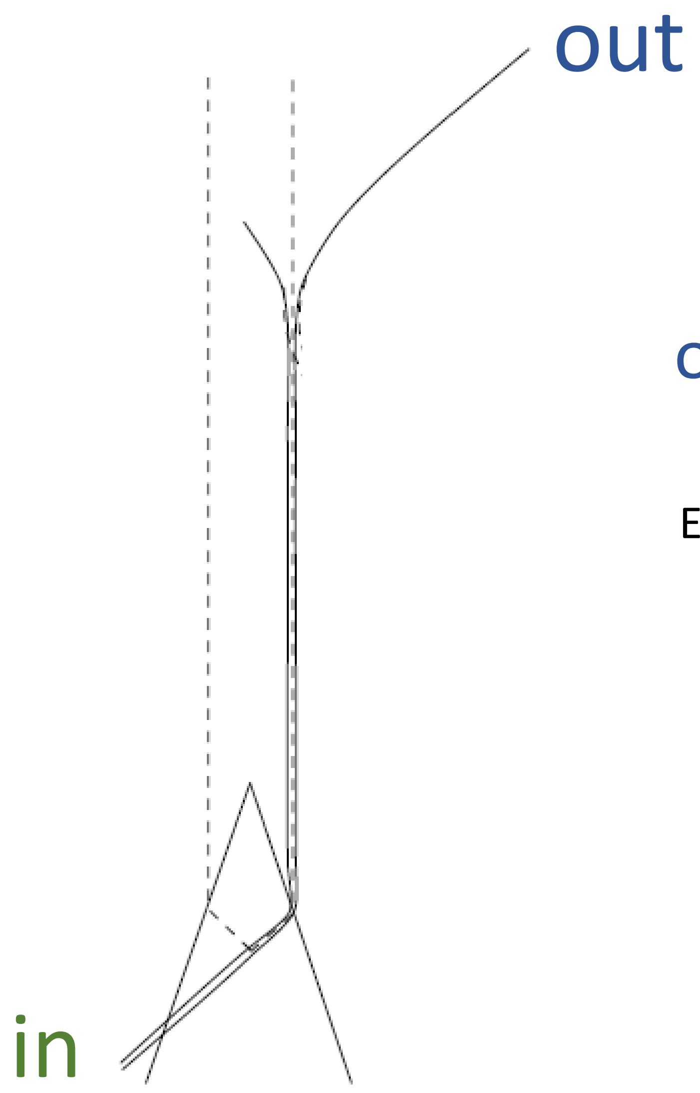


$$\omega_{in} \sim e^{\kappa t}\omega_{out}$$

surface gravity of the black hole

in: $\phi = \int d\omega(a_\omega f_\omega + a_\omega^\dagger f_\omega^*)$ $\quad a|0\rangle = 0$

out: $\phi = \int d\omega(\bar{a}_\omega F_\omega + \bar{a}_\omega^\dagger F_\omega^*)$ $\quad \bar{a}|\bar{0}\rangle = 0$

Express out modes in terms of in modes: $F_\omega = \int d\omega'(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*)$

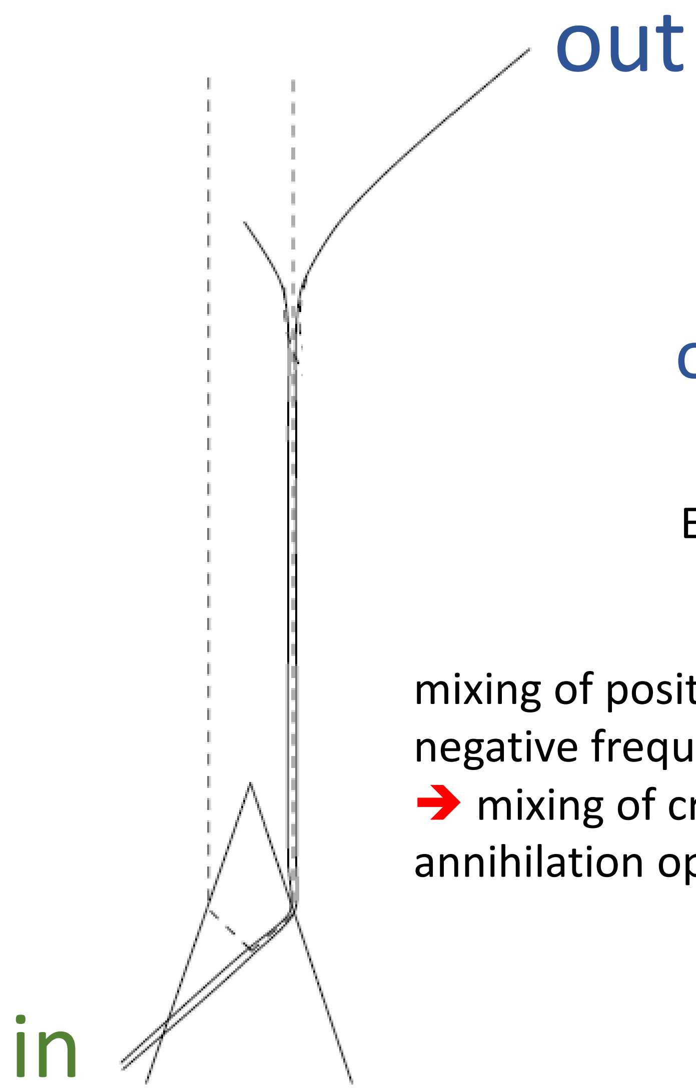
$\Rightarrow |\bar{0}\rangle \neq |0\rangle$

$$a|\bar{0}\rangle = \sum_{\omega'} \beta_{\omega\omega'} |\bar{1}\rangle > 0$$

Spontaneous emission from the vacuum!

Black hole ➔ Hawking radiation

Negative frequency waves



positive frequency wave — negative frequency wave

in: $\phi = \int \mathrm{d}\omega (a_\omega f_\omega + a^\dagger_\omega f^*_\omega)$ $\qquad a|0\rangle = 0$

out: $\phi = \int \mathrm{d}\omega (\bar{a}_\omega F_\omega + \bar{a}^\dagger_\omega F^*_\omega)$ $\qquad \bar{a}|\bar{0}\rangle = 0$

positive frequency wave — negative frequency wave

Express out modes in terms of in modes: $F_\omega = \int \mathrm{d}\omega' (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'})$

positive frequency wave

➔ $|\bar{0}\rangle \neq |0\rangle$

mixing of positive and negative frequency waves
➔ mixing of creation and annihilation operator

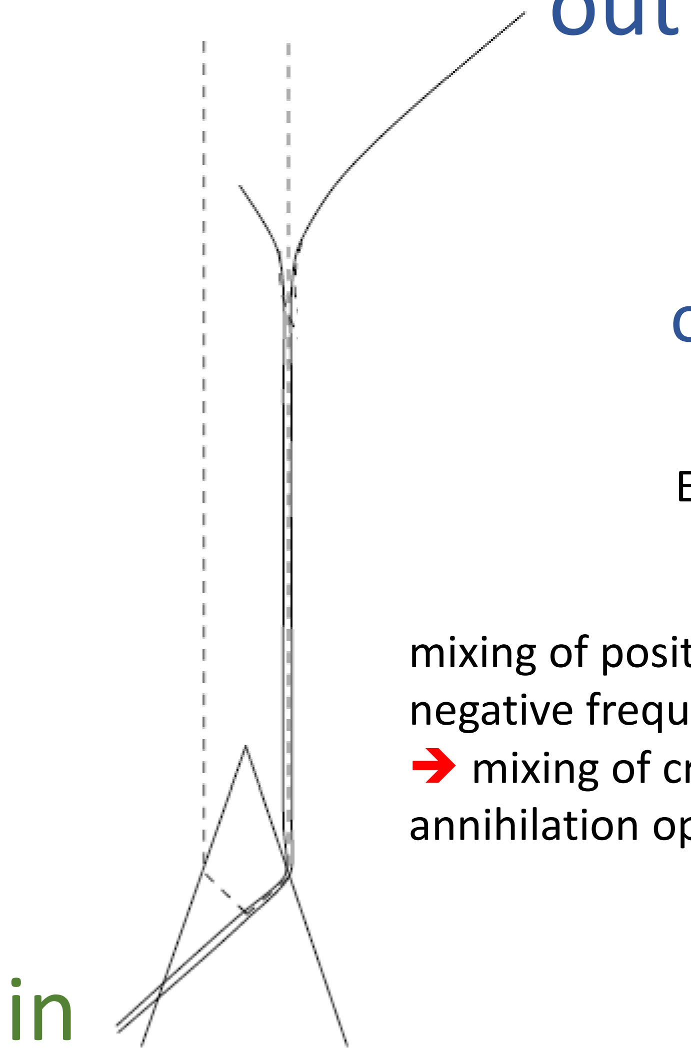
➔ Spontaneous emission from the vacuum!

Black hole ➔ Hawking radiation

$$\omega_{in} \sim e^{\kappa t} \omega_{out}$$

surface gravity of the black hole

Negative frequency waves



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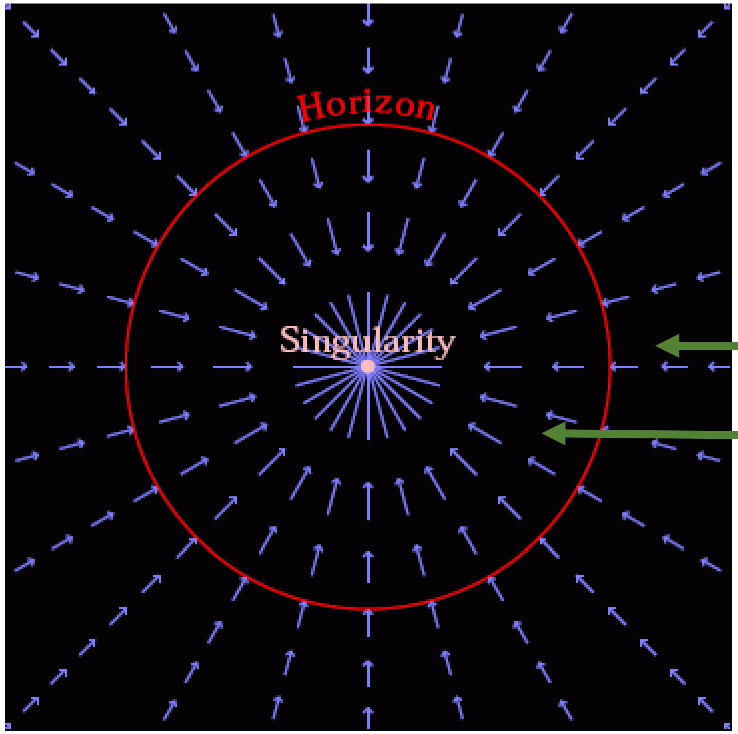
➔ mixing of creation and annihilation operator

➔ Spontaneous emission from the vacuum!

Black hole ➔ Hawking radiation

Does this effect occur only in a gravitational setup?

Spacetime flow at the horizon



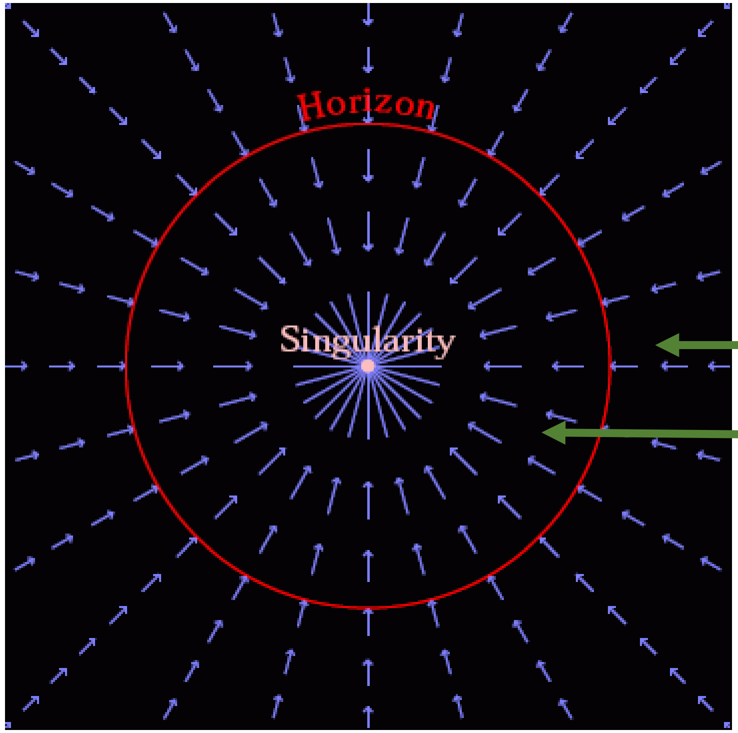
Subluminal flow of spacetime

Superluminal flow of spacetime

$$ds^2 = -dt^2 + (dr + \beta dt)^2 + r^2\left((d\theta)^2 + sin^2\theta(d\phi)^2\right)$$

Space flows radially inwards at velocity $\beta = \sqrt{2M/r}$.
At the horizon, $\beta = c$: nothing can escape from the inside of a black hole.
t ever increases towards the horizon -> light from infalling objects redshifts

Spacetime flow at the horizon



Subluminal flow of spacetime

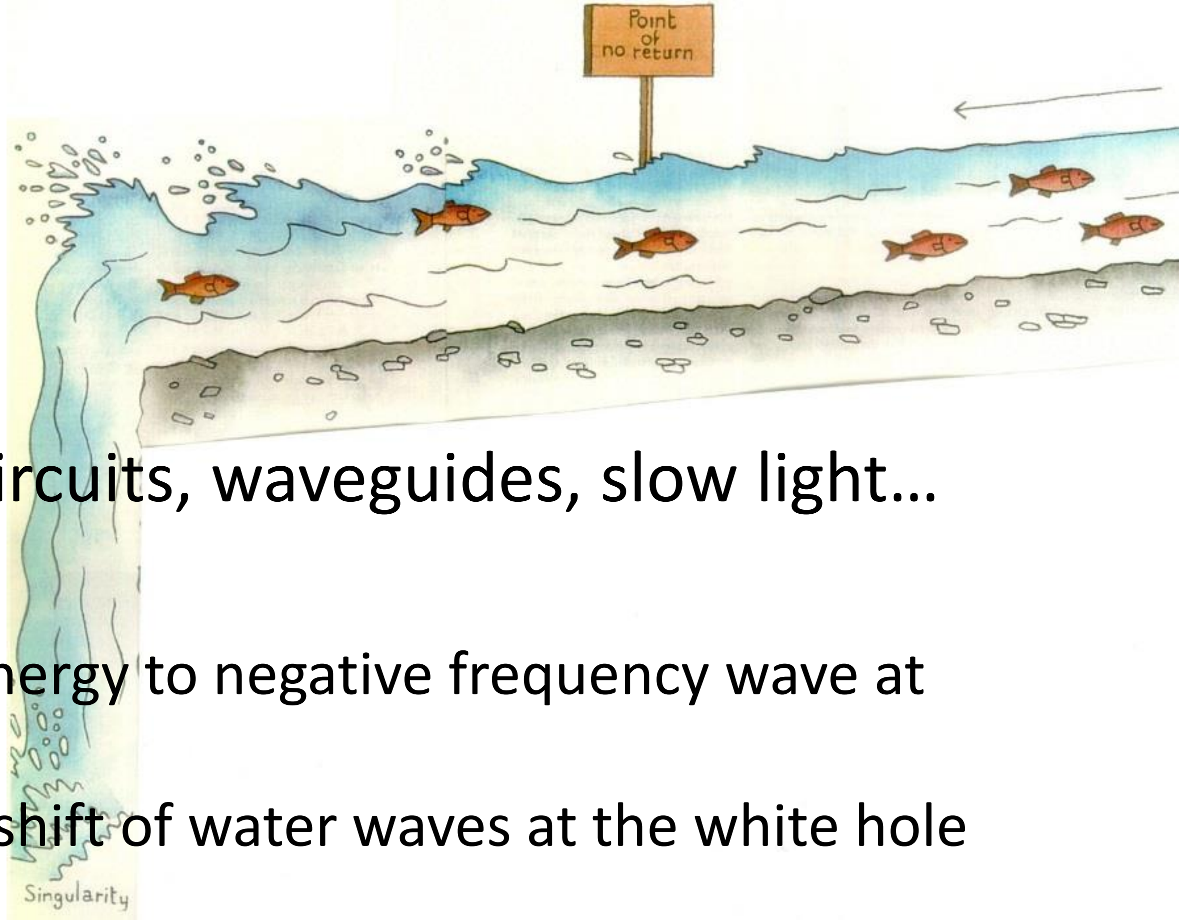
Superluminal flow of spacetime

Event horizon separates region of sub from superluminal flow

Unruh (1974): spacetime as a moving fluid – the waterfall model

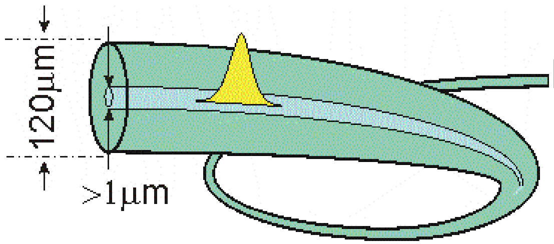
Spacetime flow at the horizon – in the lab

Unruh, 1981: Flow gradient can be realized in laboratories + analogue systems can produce HR

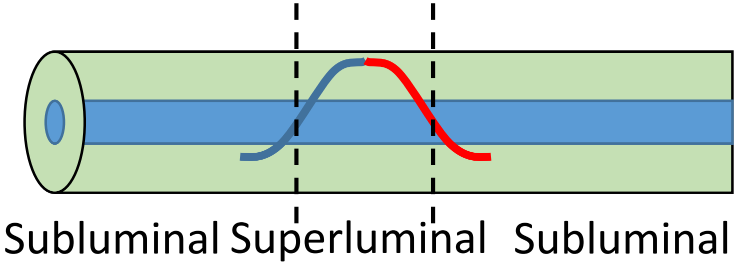


- Superfluid helium, superconducting circuits, waveguides, slow light…
- Water waves
  - Rousseaux *et al*. NJP 2008. Transfer of energy to negative frequency wave at the white hole horizon.
  - Weinfurtner *et al*. PRL 2011. Frequency shift of water waves at the white hole horizon.
  - Rousseaux *et al*. PRL 2016. Noise correlations across the water-wave horizon.
- Phonons in a BEC
  - Steinhauer Nat Phys 2016. Hawking radiation at the black hole horizon?

Creating analogue horizons with moving Refractive Index Front (RIF)



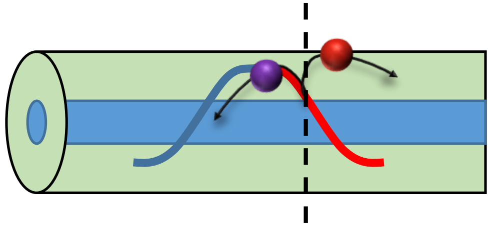
Philbin *et al.*, Science (2008)



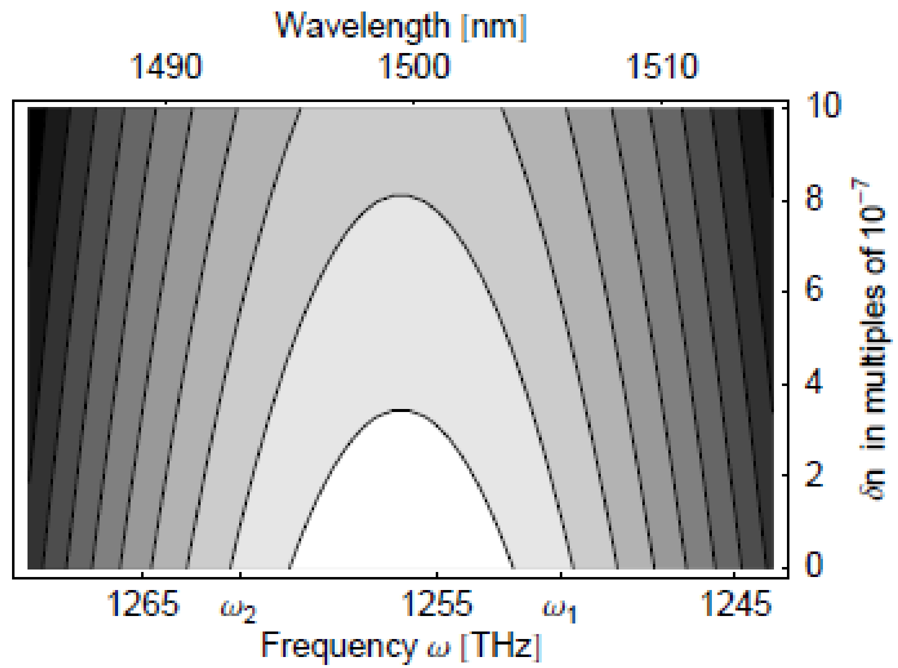
Pulse creates RIF → modifies speed of light by Kerr effect

Moving analogue horizon in medium at rest

Resulting HR temperature can be very high (up to 1000K)



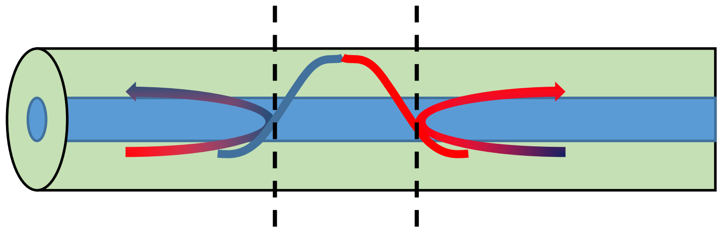
Optical analogue and RIF



Philbin *et al.*, Science (2008)

Dispersion: $\omega = f(\lambda) \rightarrow v_p \neq v_g \rightarrow {}^{\omega}/_{k} \neq {}^{\partial\omega}/_{\partial k}$

$$\omega = f(refractive\ index)$$



Back of pulse blueshifts incoming light

Front of pulse redshifts incoming light

Moving frame: incoming wave slows down, turns around frequency shifted ≡ event horizon

Theoretical advances

Recent efforts in optical event horizons

Rubino *et al.* PRL (2012), Robertson PRE 90 (2014), Belgiorno *et* al. PRD 91 (2015), Linder *et al.* PRD 93 (2016)

Finazzi and Carusotto, PRA 89 (2013)

➔model for spontaneous emission

at a moving RIF in a nonlinear dielectric

➔Light emitted from the vacuum

in the presence and absence of a horizon

Jacquet and König, PRA 92 (2015)

➔RIF can act as black hole, white hole and horizonless emitter

➔First complete spectrum of light from vacuum

mixing of positive and negative frequencies

PHYSICAL REVIEW A **92**, 023851 (2015)

Quantum vacuum emission from a refractive-index front

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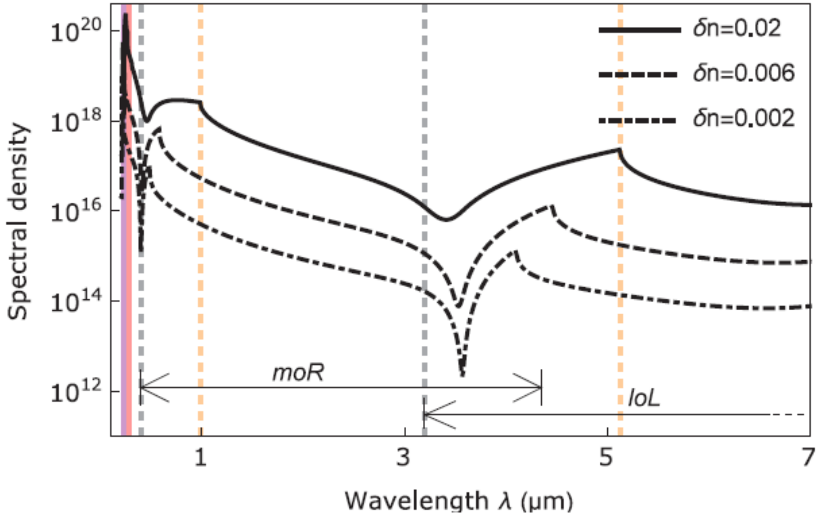
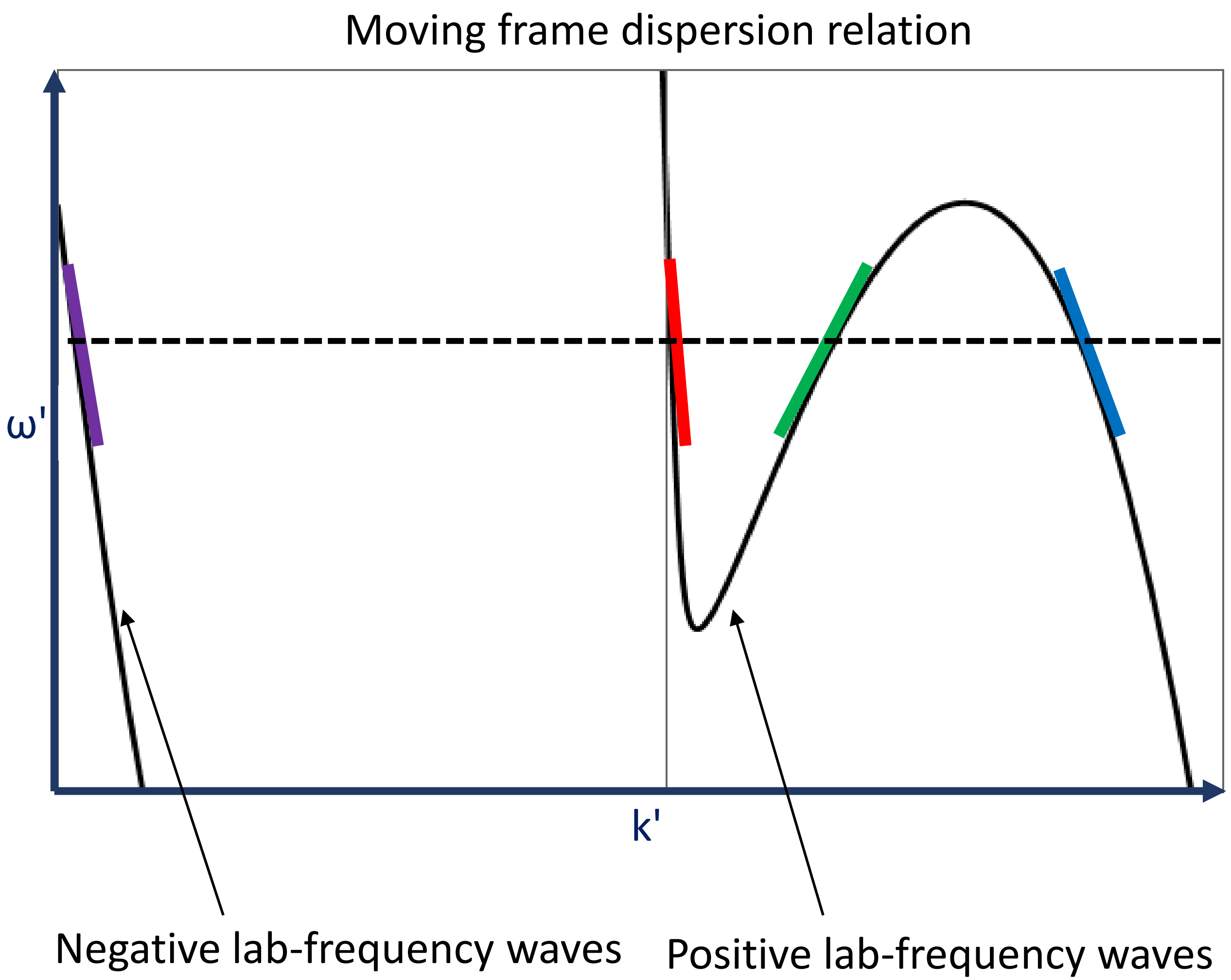
School of Physics and Astronomy, SUPA, University of St. Andrews, North Haugh, St. Andrews KY16 9SS, United Kingdom
(Received 28 April 2015; published 28 August 2015)

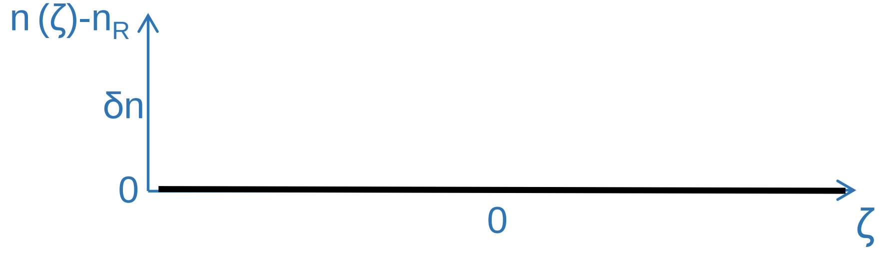
A moving boundary separating two otherwise homogeneous regions of a dielectric is known to emit radiation from the quantum vacuum. An analytical framework based on the Hopfield model, describing a moving refractive-index step in 1 + 1 dimensions for realistic dispersive media has been developed by S. Finazzi and I. Carusotto [Phys. Rev. A **87**, 023803 (2013)]. We expand the use of this model to calculate explicitly spectra of all modes of positive and negative norms. Furthermore, for lower step heights we obtain a unique set of mode configurations encompassing black-hole and white-hole setups. This leads to a realistic emission spectrum featuring black-hole and white-hole emission for different frequencies. We also present spectra as measured in the laboratory frame that include all modes, in particular a dominant negative-norm mode, which is the partner mode in any Hawking-type emission. We find that the emission spectrum is highly structured into intervals of emission with black-hole, white-hole, and no horizons. Finally, we estimate the number of photons emitted as a function of the step height and find a power law of 2.5 for low step heights.

DOI: 10.1103/PhysRevA.92.023851 PACS number(s): 42.50.Nn, 42.65.Hw, 04.62.+v, 42.50.Xa



Dispersion relation of the dielectric in the frame of the pulse

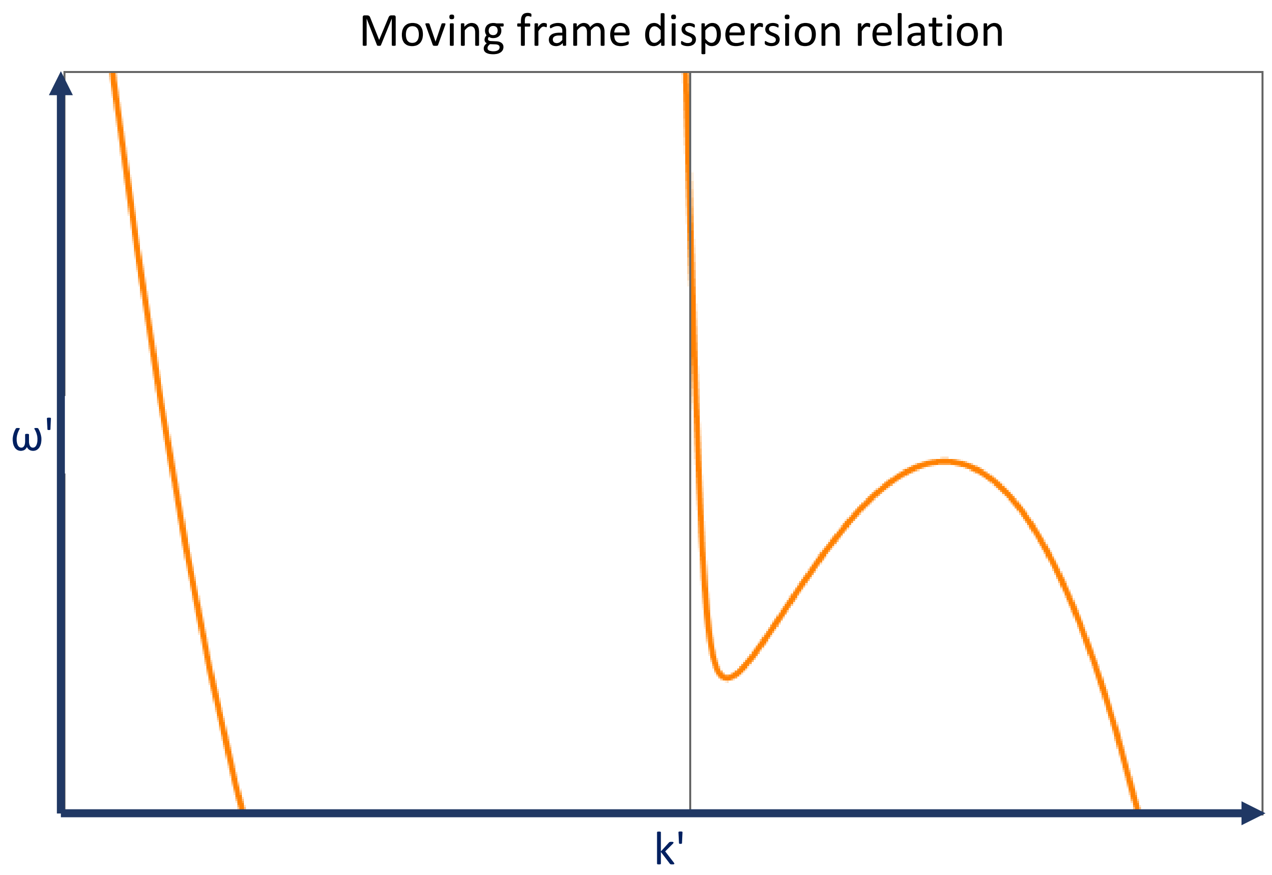


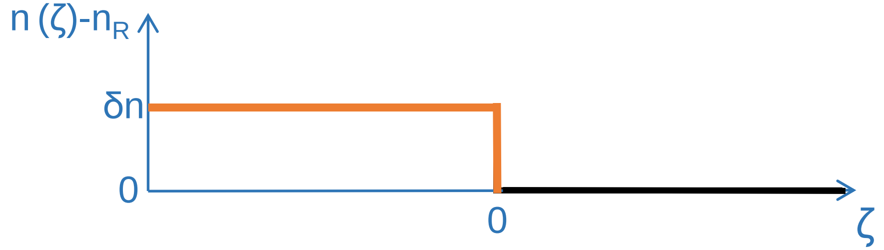


Conserved co-moving frequency ω'

Group velocity $\frac{\partial \omega'}{\partial k'}$

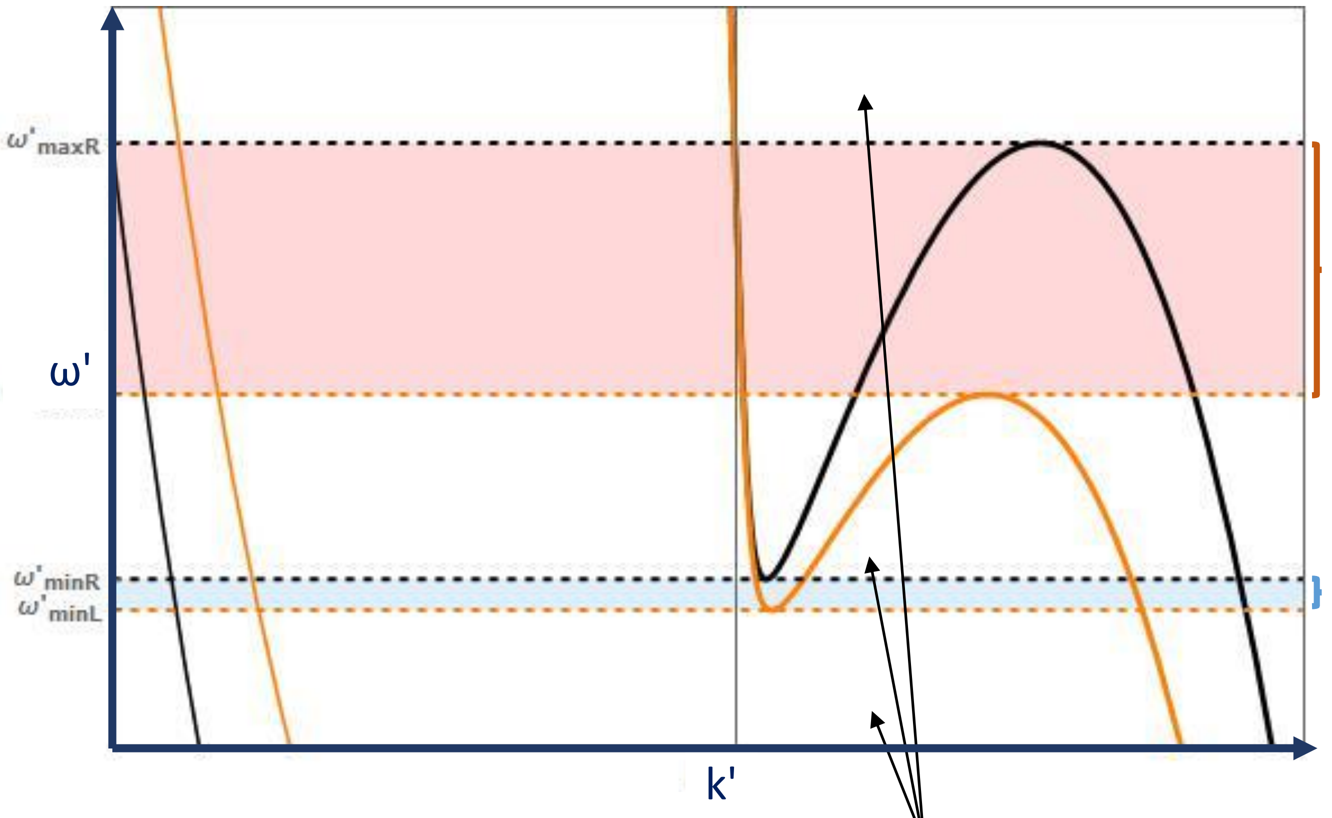
Dispersion relation of the dielectric in the frame of the pulse



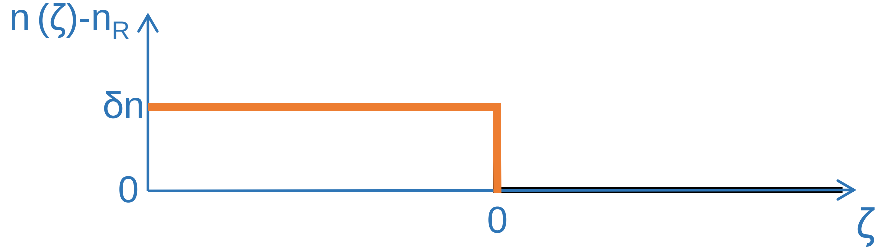


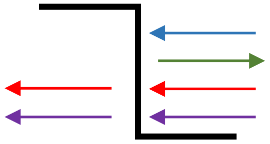
Simultaneous black- and white-hole horizons!

Moving frame dispersion relation

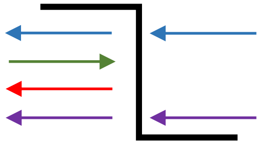


Horizonless frequency intervals





Analogue black hole horizon



Analogue white hole horizon

Frequency ω' is conserved

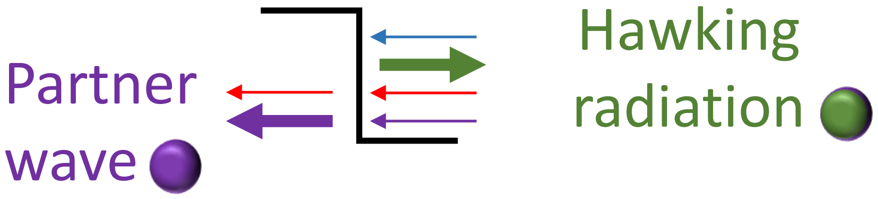
➔waves can scatter into other waves that have same ω'

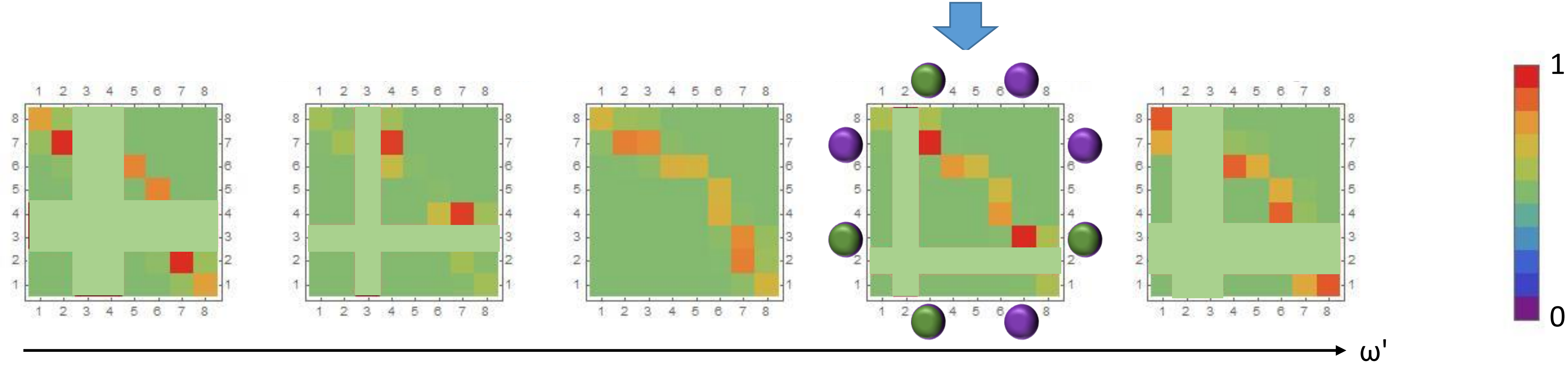
Spontaneous emission of light

Incoming modes scatter into outgoing modes
➔ mixing positive and negative norm waves

Quasi pair-wise emission

Analogue black-hole horizon

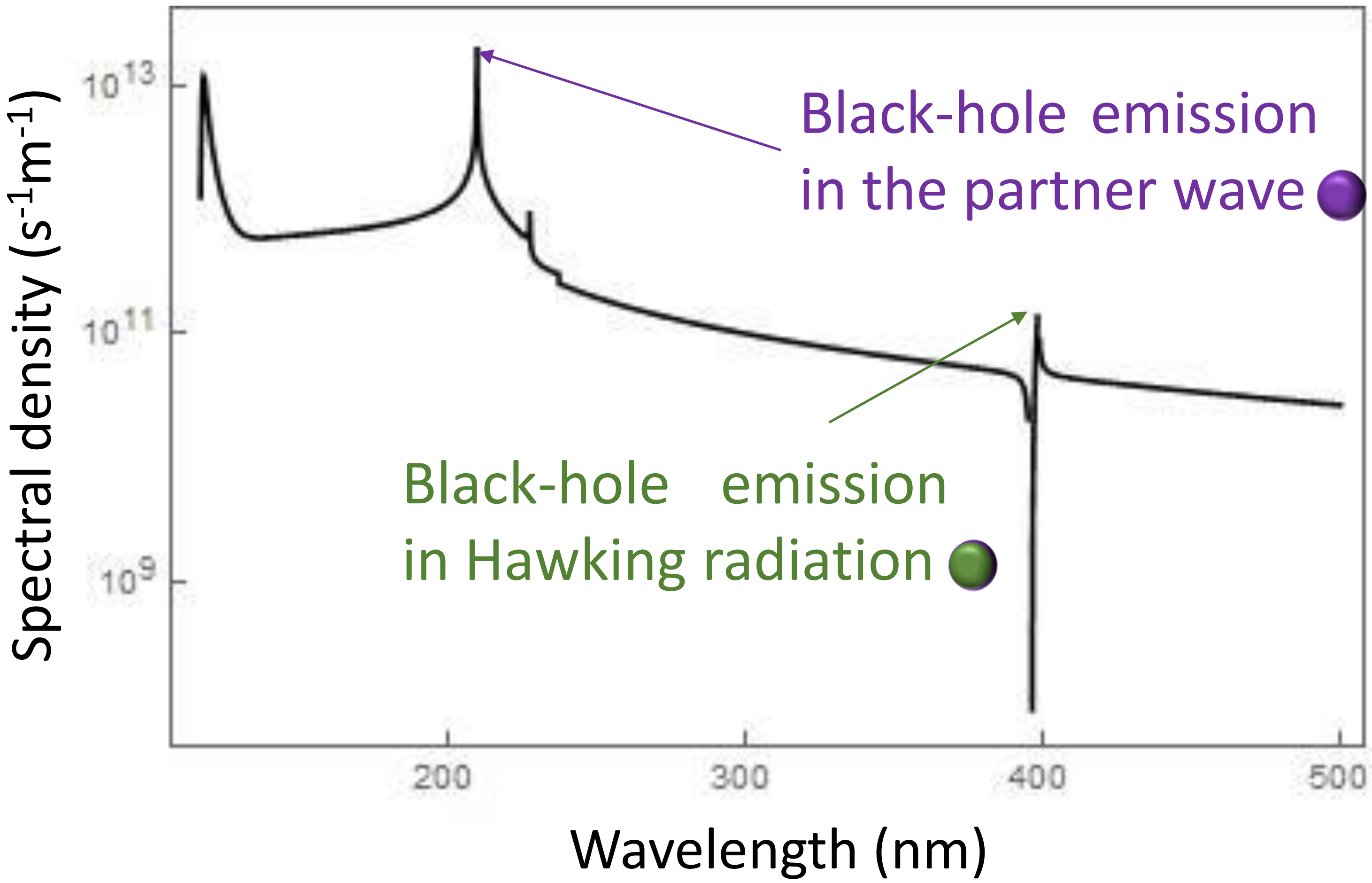




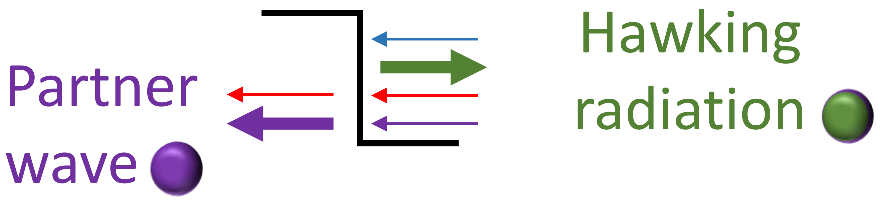
Mode-mode correlation map in the moving frame

Spontaneous emission of light

Lab Frame spectrum of spontaneous emission in bulk fused silica



Incoming modes scatter into outgoing modes ➔ mixing positive and negative norm waves



Spectral correlations?
➔ on the arxiv at the end of the month

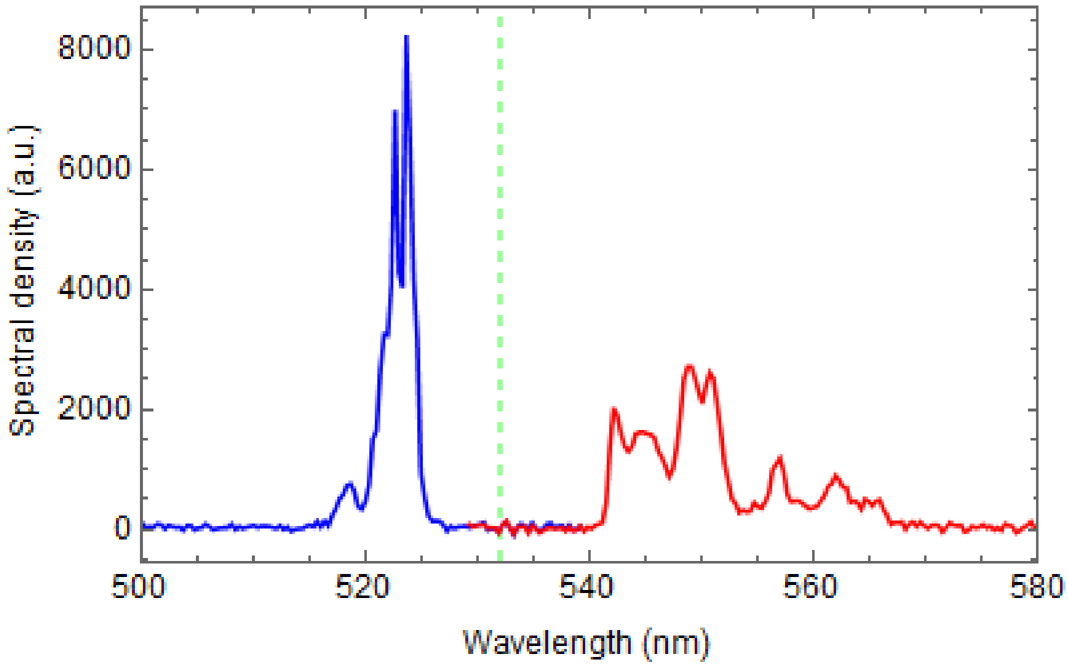
Scattering of a positive norm CW at the horizon

- Scattering of coherent state:

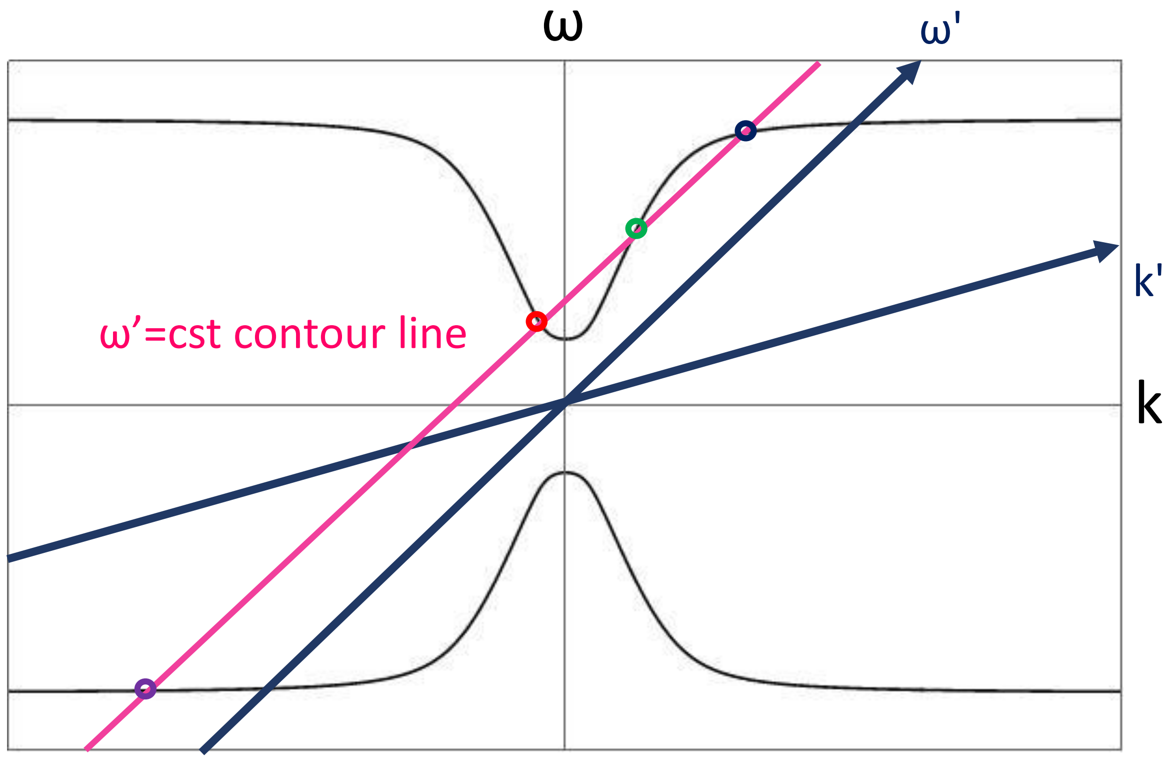
$$\langle \alpha_{laser} | \hat{n} | \alpha_{laser} \rangle = \overbrace{\mathrm{a}}^{\text{Spontaneous emission}} + \underbrace{\left|\beta^{laser,}\right|^2 |\alpha_{laser}|^2}_{\text{Parametric amplification}}$$

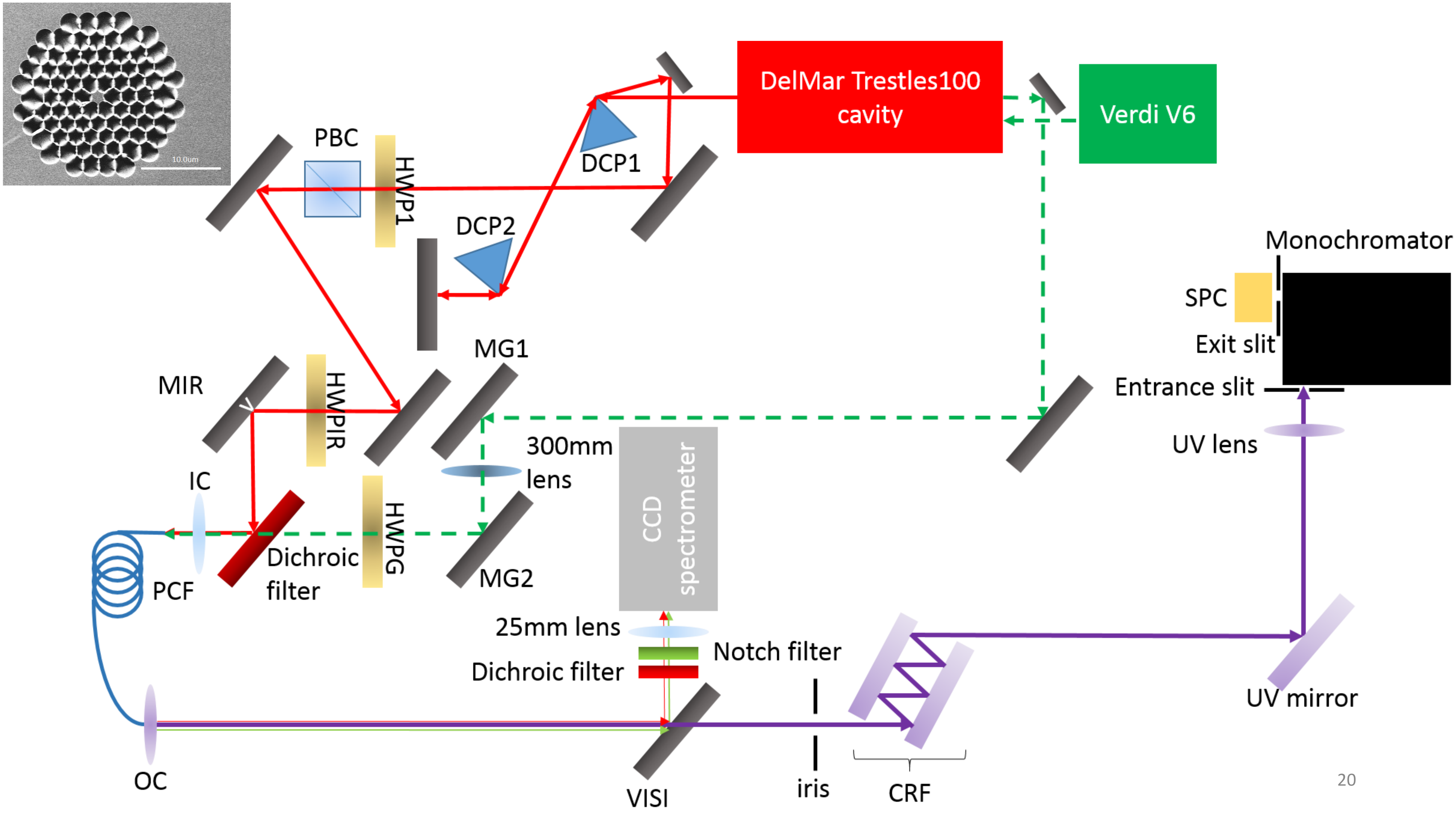
Scattering of a positive norm CW at the horizon

- Scattering of coherent state:



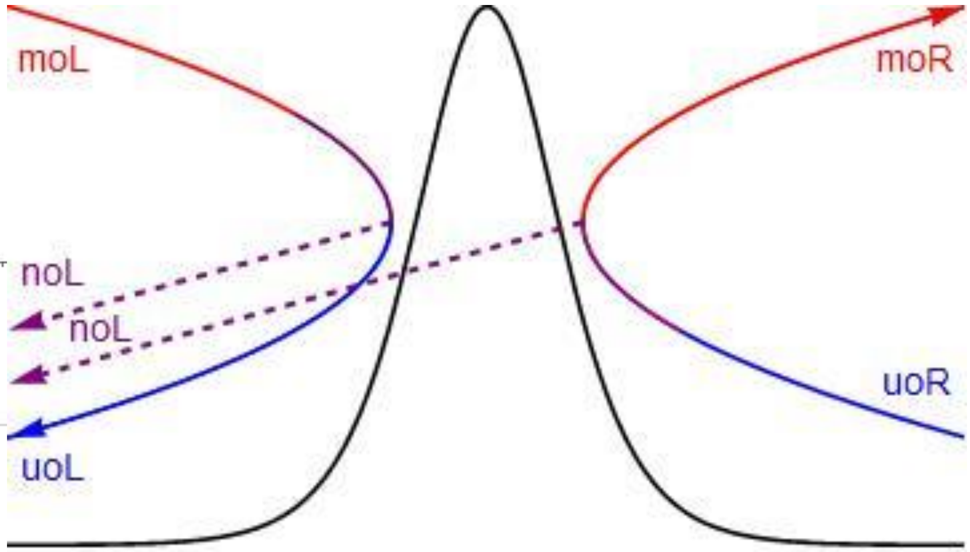
Laboratory frame dispersion relation



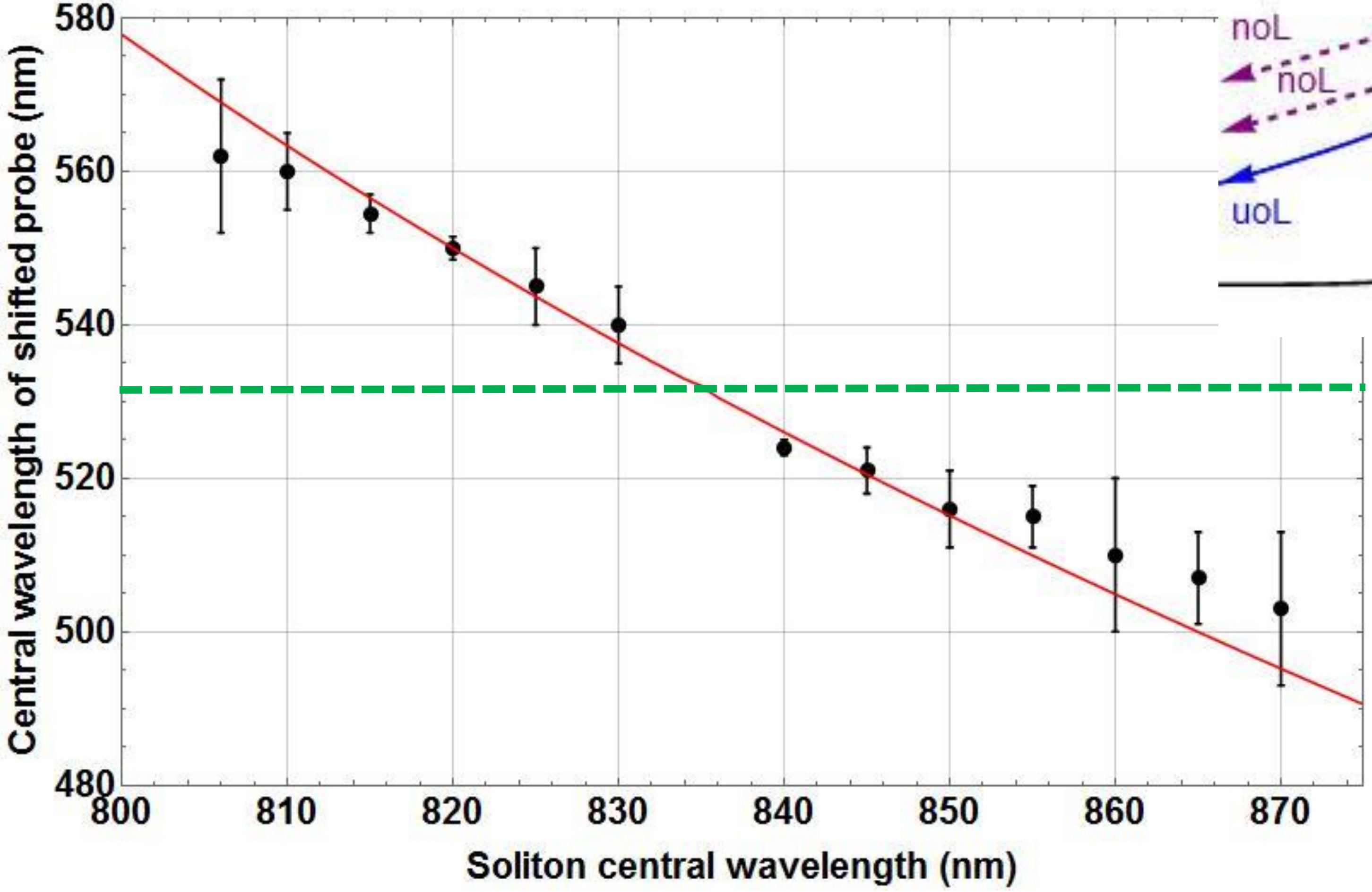


10.0um
PBC
HWP1
DCP1
DelMar Trestles100 cavity
Verdi V6
DCP2
MG1
MIR
HWPIR
Monochromator
SPC
Exit slit
Entrance slit
UV lens
300mm lens
CCD spectrometer
IC
Dichroic filter
HWPG
MG2
PCF
25mm lens
Notch filter
Dichroic filter
UV mirror
OC
VISI
iris
CRF

Scattering of positive norm CW at horizon



Frequency shifting of a 532nm CW
at a horizon in PCF NL 1.5-670



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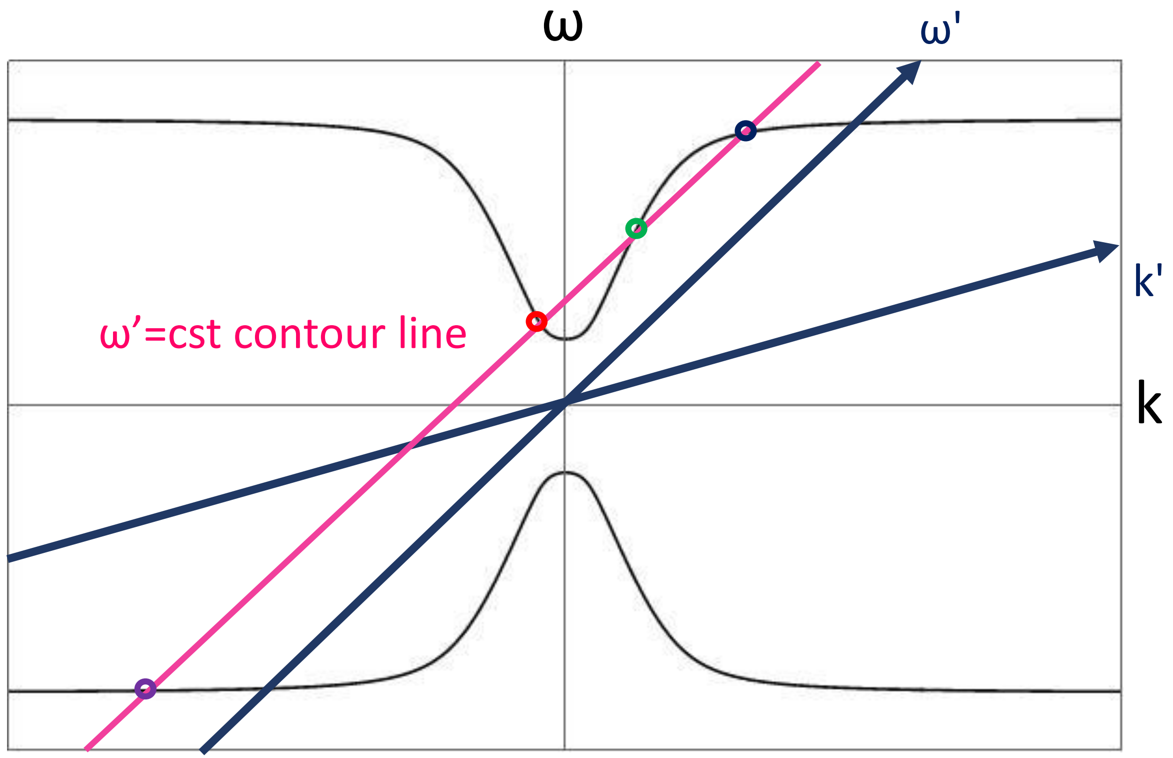
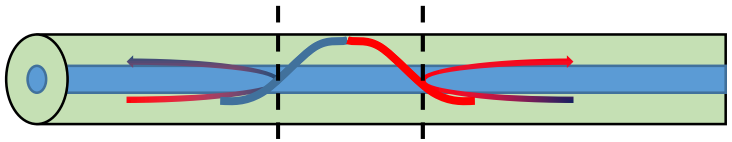
Laboratory frame dispersion relation

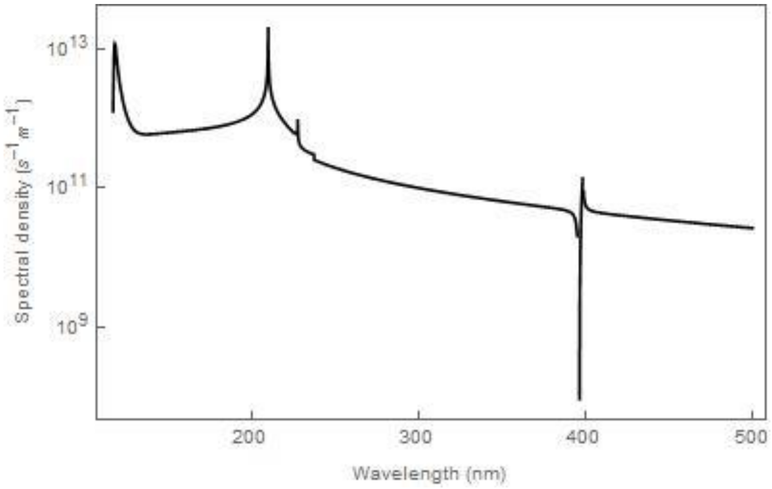


Expect about 0.1 photon/second at 222nm

Negative frequency at the horizon

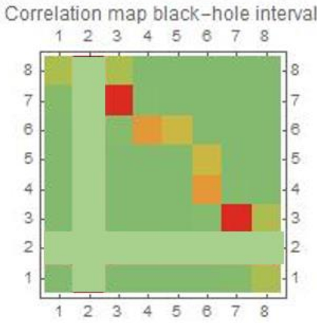
Scattering of light at a refractive index front

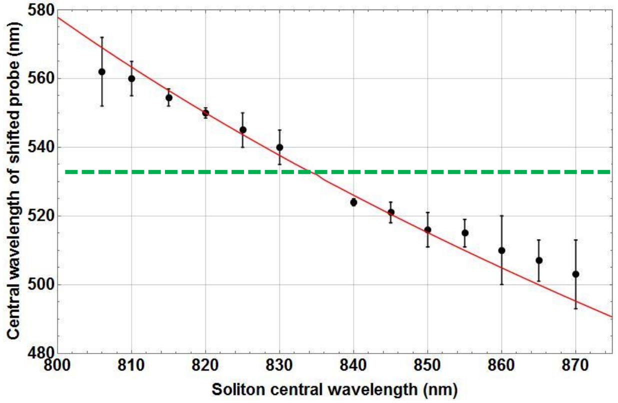




- Pulse in nonlinear dielectric creates an analogue system to black and white holes
- The front of the pulse can act as a black hole, a white hole, or a horizonless emitter
- Pairs emitted via the Hawking effect can be observed in the optical frequency range, partner in the UV and Hawking radiation in the visible.

➔ Towards the experimental measurement of pairs





Light-matter interaction in a dispersive medium

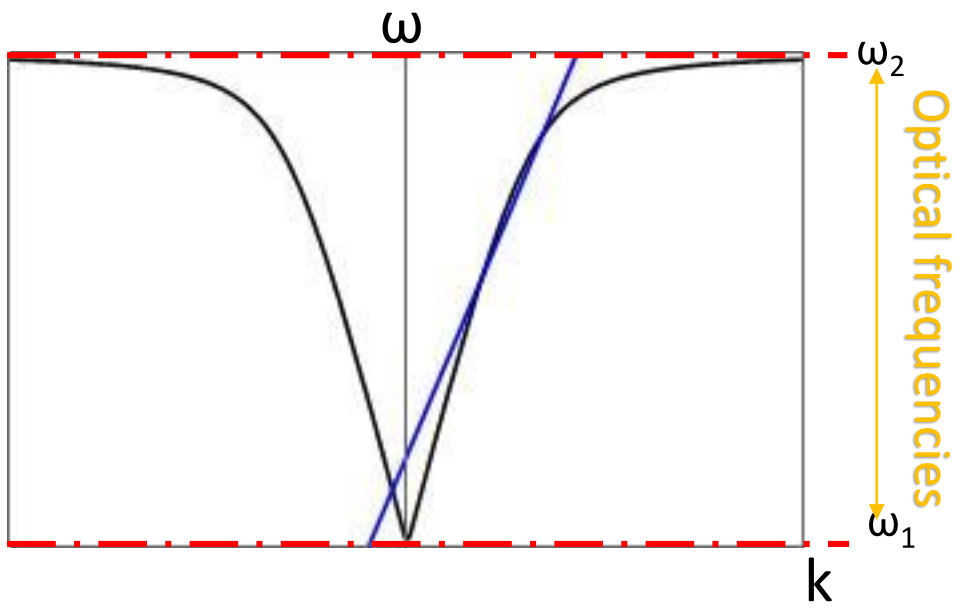
Lagrangian for light matter interaction:

$$\mathcal{L} = \frac{(\partial_t A)^2}{8\pi c^2} - \frac{(\partial_x A)^2}{8\pi} + \sum_{i=1}^{3}\left(\frac{(\partial_t P_i)^2}{2\kappa_i {\omega_i}^2} - \frac{P_i^2}{2\kappa_i} + \frac{A}{c}\partial_t P_i\right)$$

Sellmeier dispersion relation:

$c^2k^2=\omega[1+\sum_{i=1}^{3}(4\pi\kappa_i)/(1-\omega^2/\omega_i^2)]$

Laboratory frame dispersion relation



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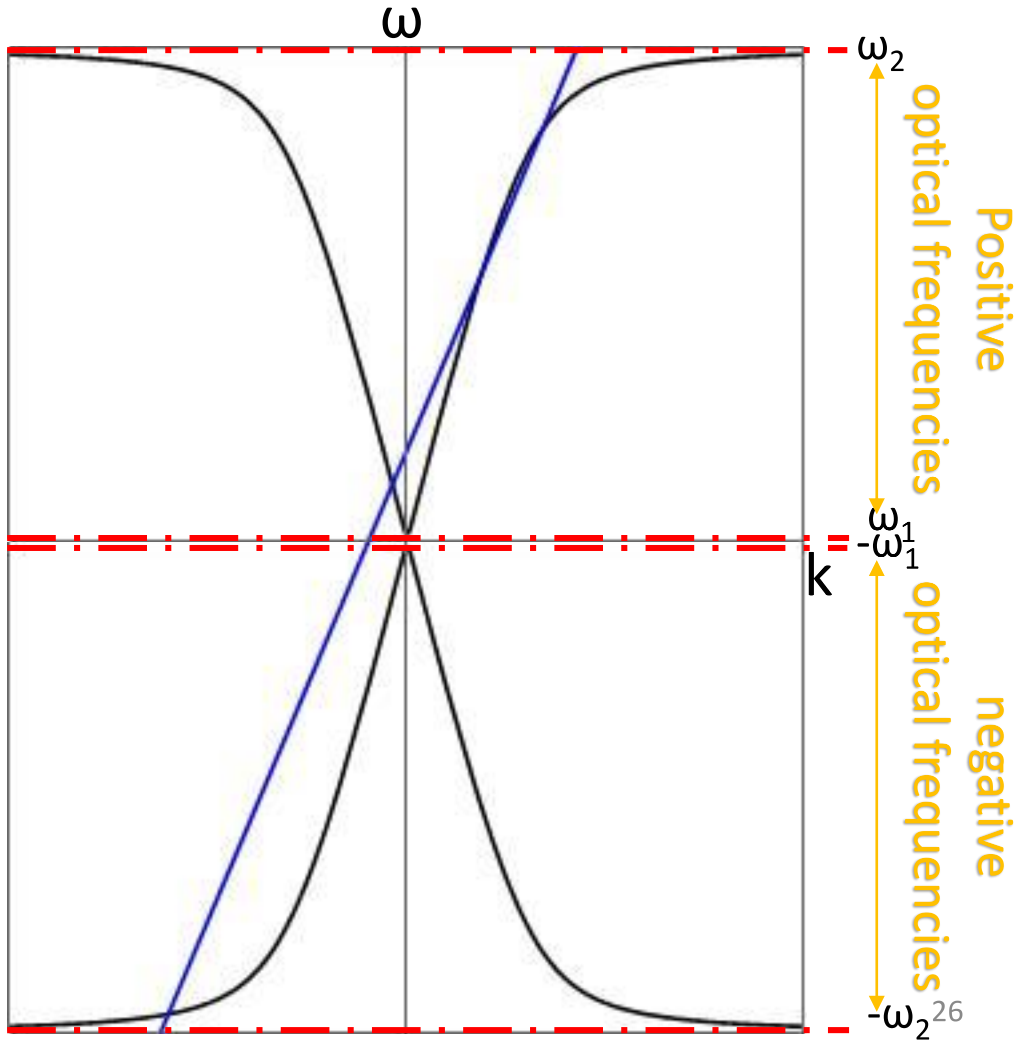

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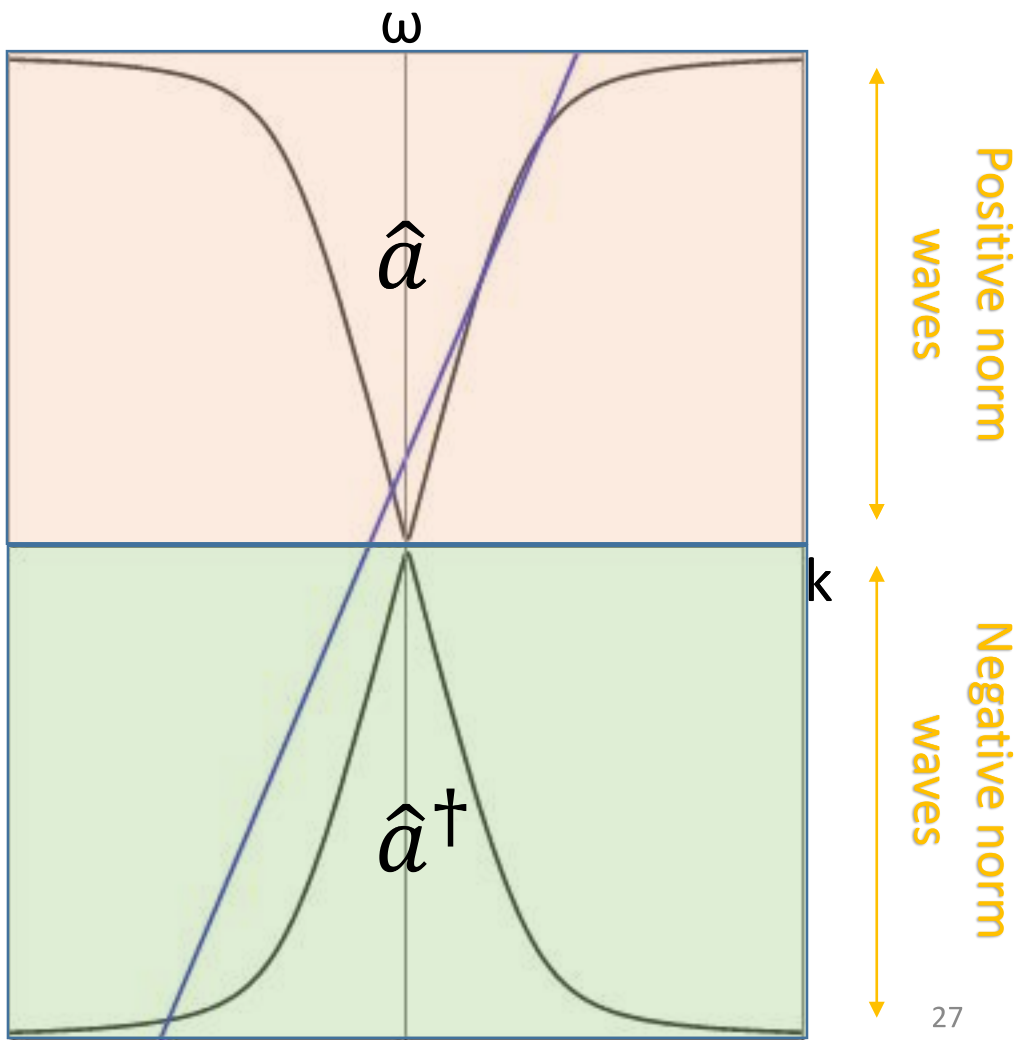
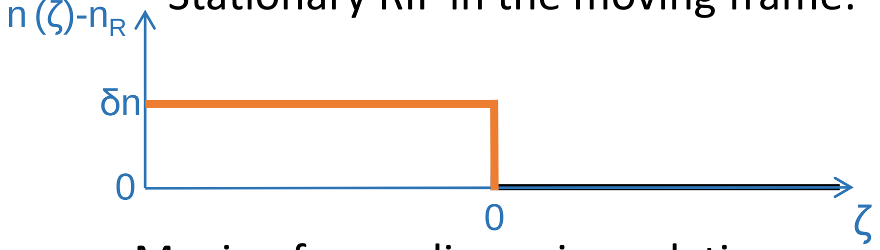


Light-matter interaction in a dispersive medium

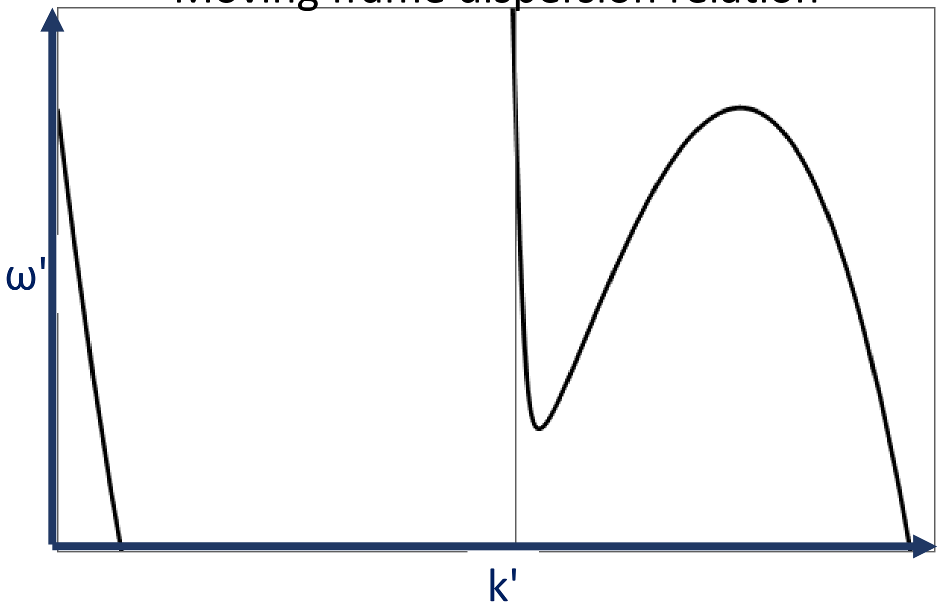
Lorentz boost to frame moving at speed u:

$$\omega' = \gamma(\omega - uk)$$
$$k' = \gamma(k - \omega\, u/c^2)$$

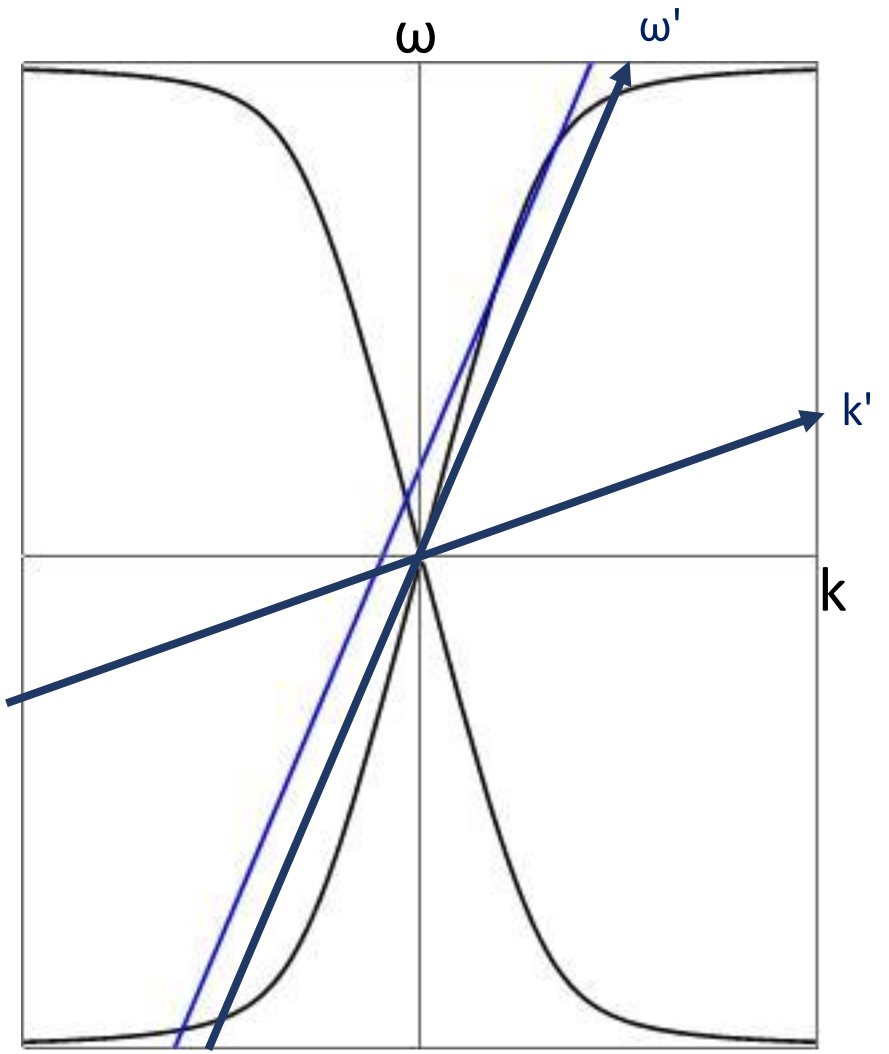
Stationary RIF in the moving frame:

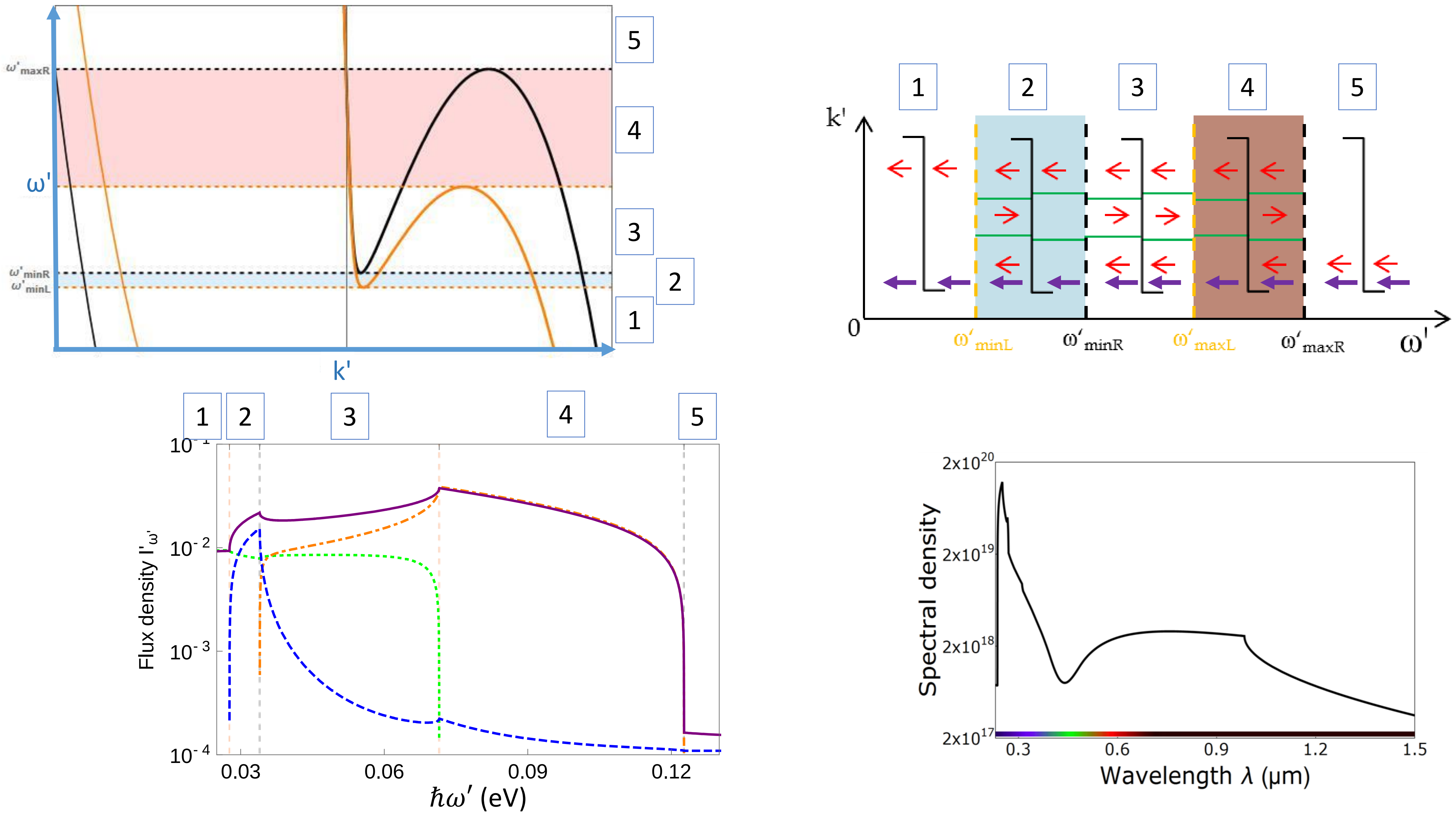


Moving frame dispersion relation

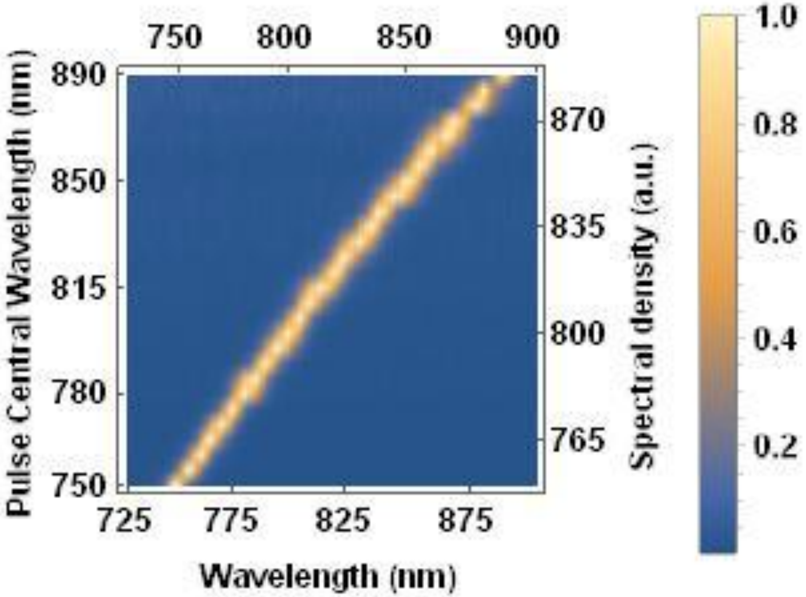


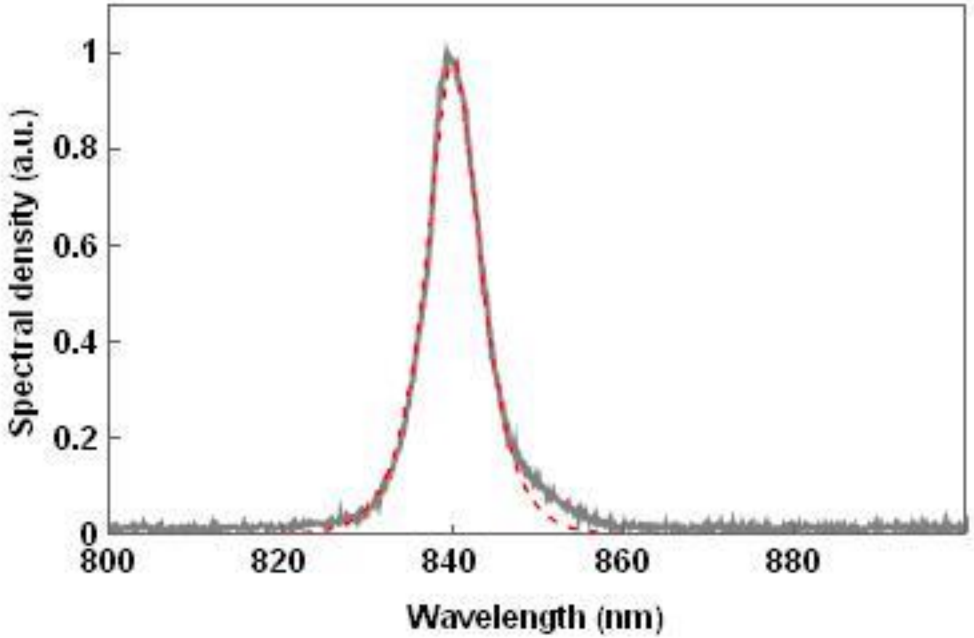
Laboratory frame dispersion relation

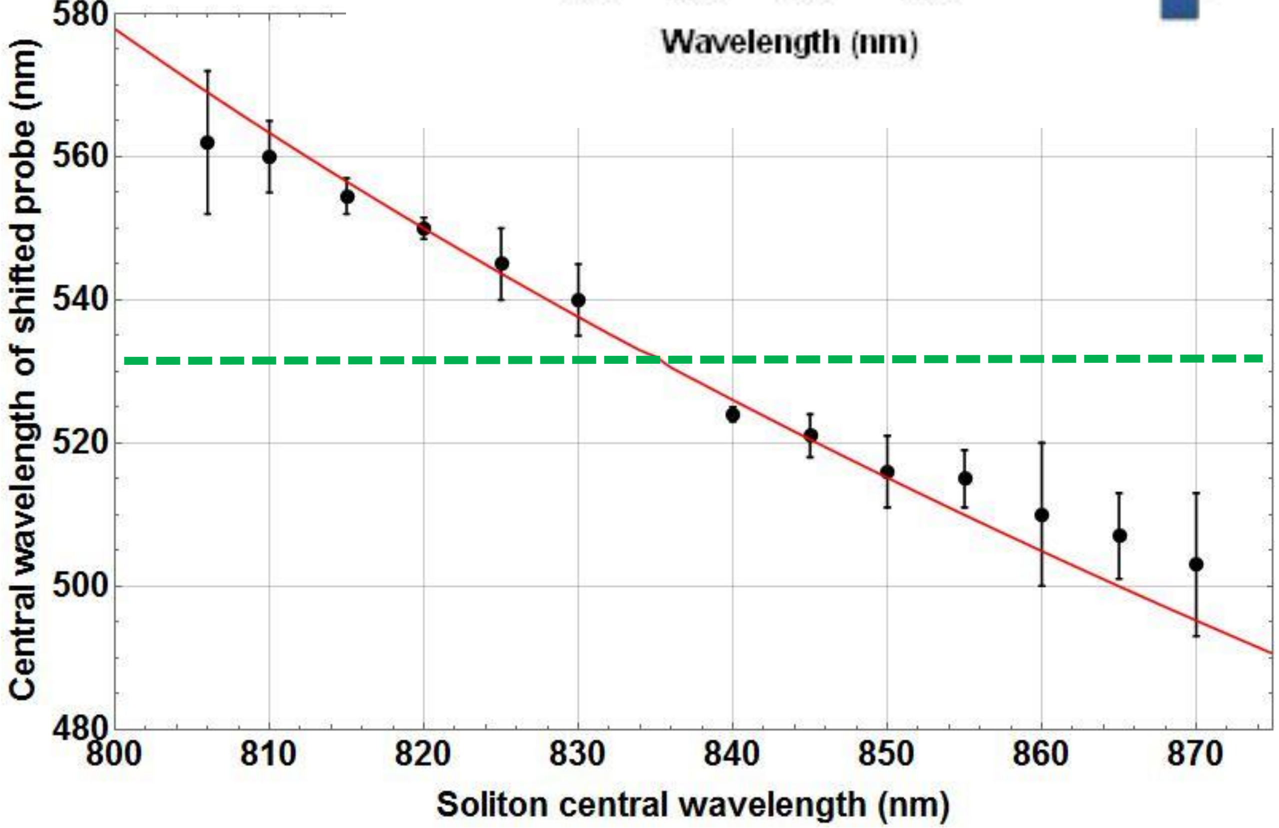




ω'maxR
ω'
ω'minR
ω'minL
k'
5
4
3
2
1
k'
0
ω'minL
ω'minR
ω'maxL
ω'maxR
ω'
Flux density Γ'ω'
10-1
10-2
10-3
10-4
0.03
0.06
0.09
0.12
ℏω' (eV)
Spectral density
2x10^20
2x10^19
2x10^18
2x10^17
0.3
0.6
0.9
1.2
1.5
Wavelength λ (μm)

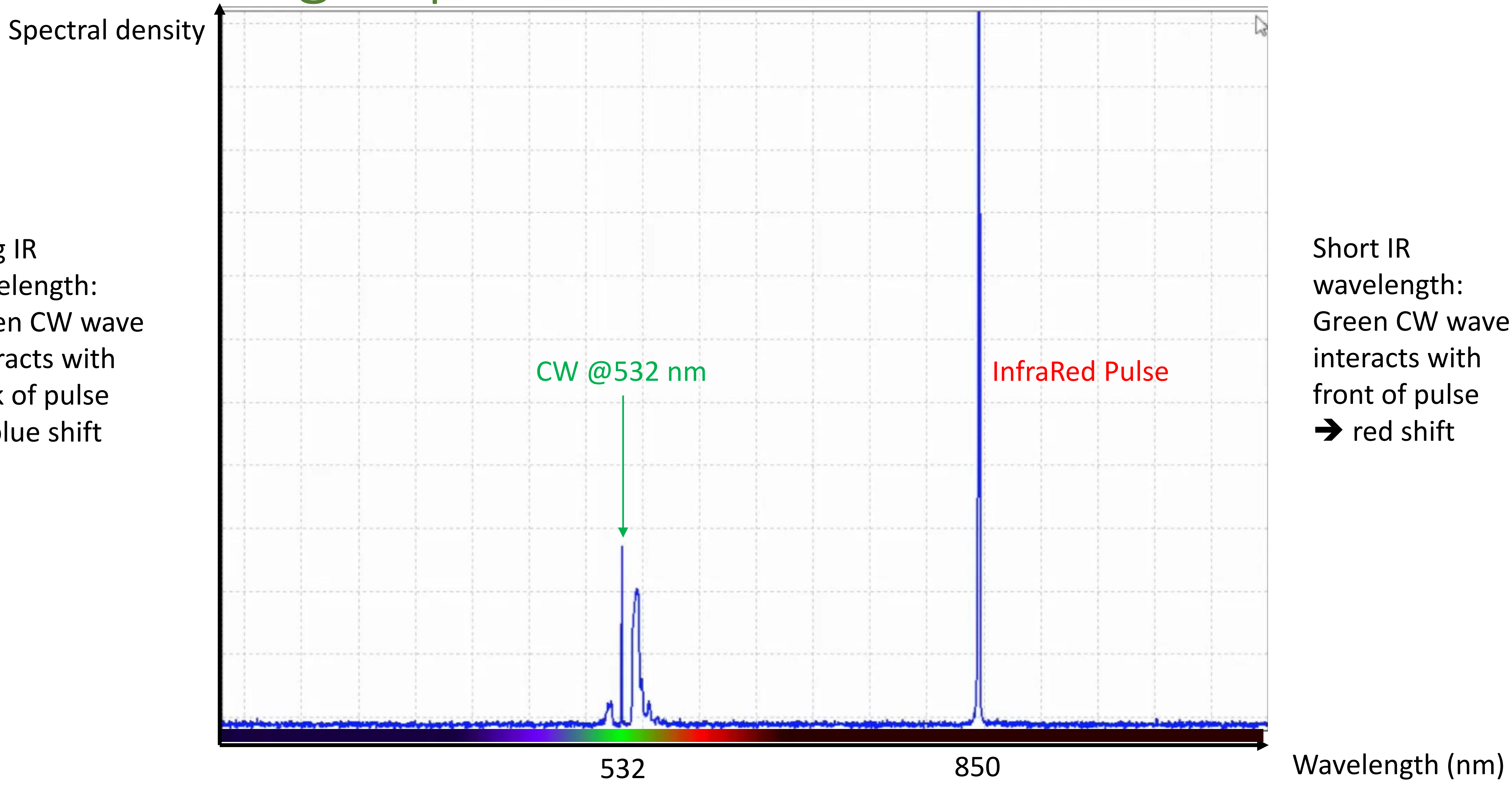

750 800 850 900
890 850 815 780 750
870 835 800 765
725 775 825 875
Pulse Central Wavelength (nm)
Spectral density (a.u.)
Wavelength (nm)
1.0 0.8 0.6 0.4 0.2


1 0.8 0.6 0.4 0.2 0
Spectral density (a.u.)
800 820 840 860 880
Wavelength (nm)

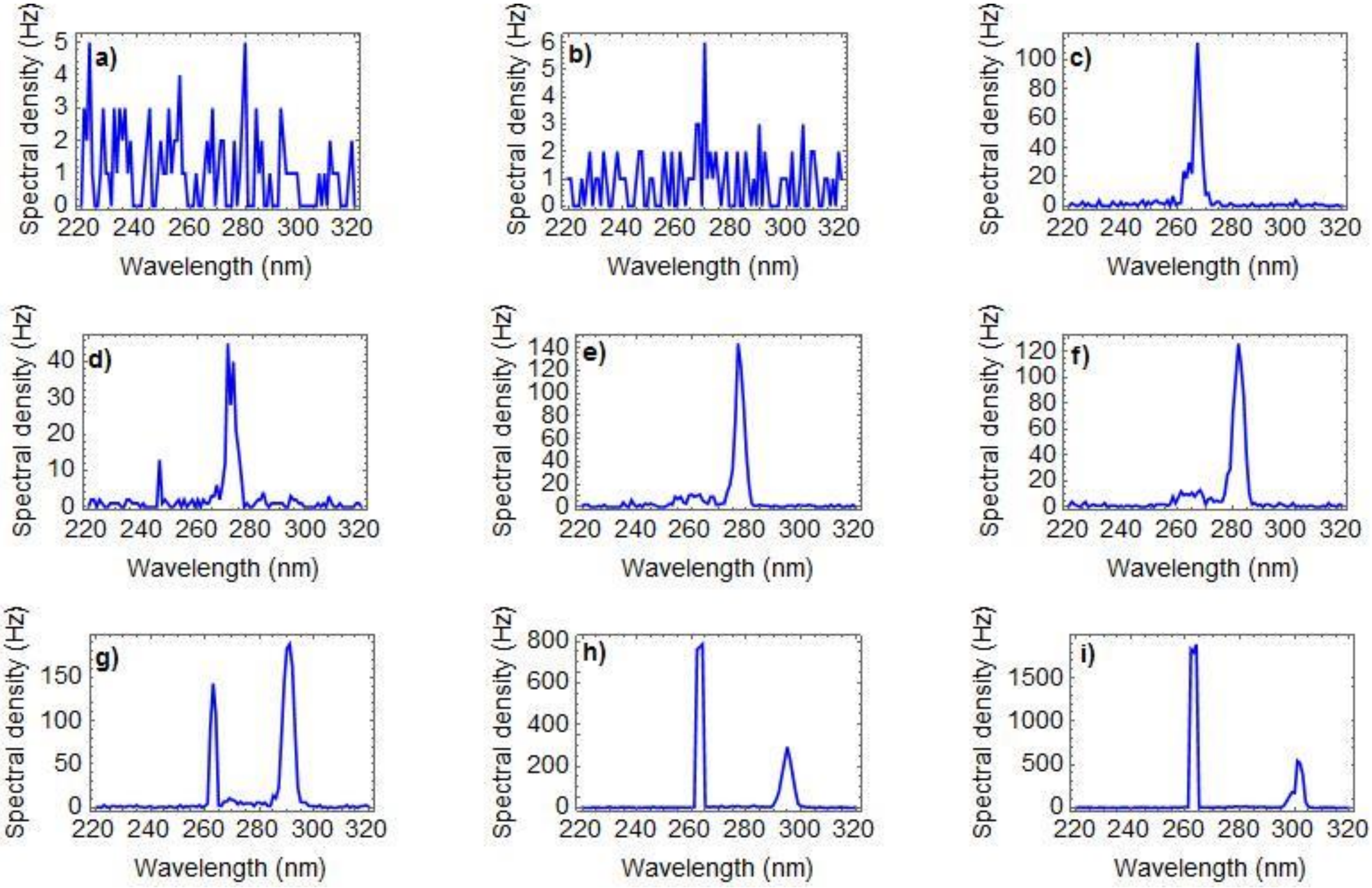

580 560 540 520 500 480
Central wavelength of shifted probe (nm)
800 810 820 830 840 850 860 870
Soliton central wavelength (nm)

Scattering of positive norm CW at horizon

Long IR wavelength: Green CW wave interacts with back of pulse ➔ blue shift



Short IR wavelength: Green CW wave interacts with front of pulse ➔ red shift


a)
b)
c)
d)
e)
f)
g)
h)
i)
Spectral density (Hz)
Wavelength (nm)
220 240 260 280 300 320

Negative frequency waves – nonlinear optics

Coupling to waves that have negative laboratory frequency is possible

➔ creation of photon pairs

➔ linear scattering process

Fourier analysis: all light waves oscillate with both positive and negative frequencies

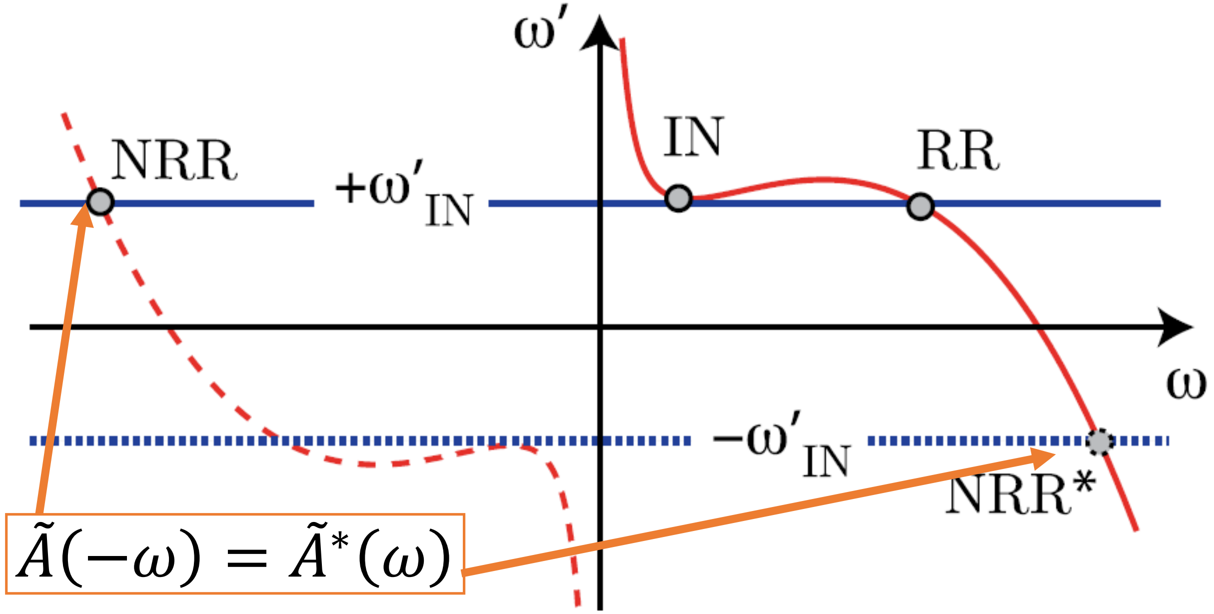
$$A = \int_{-\infty}^{+\infty} \tilde{A}(\omega) e^{-i\omega t} d\omega$$

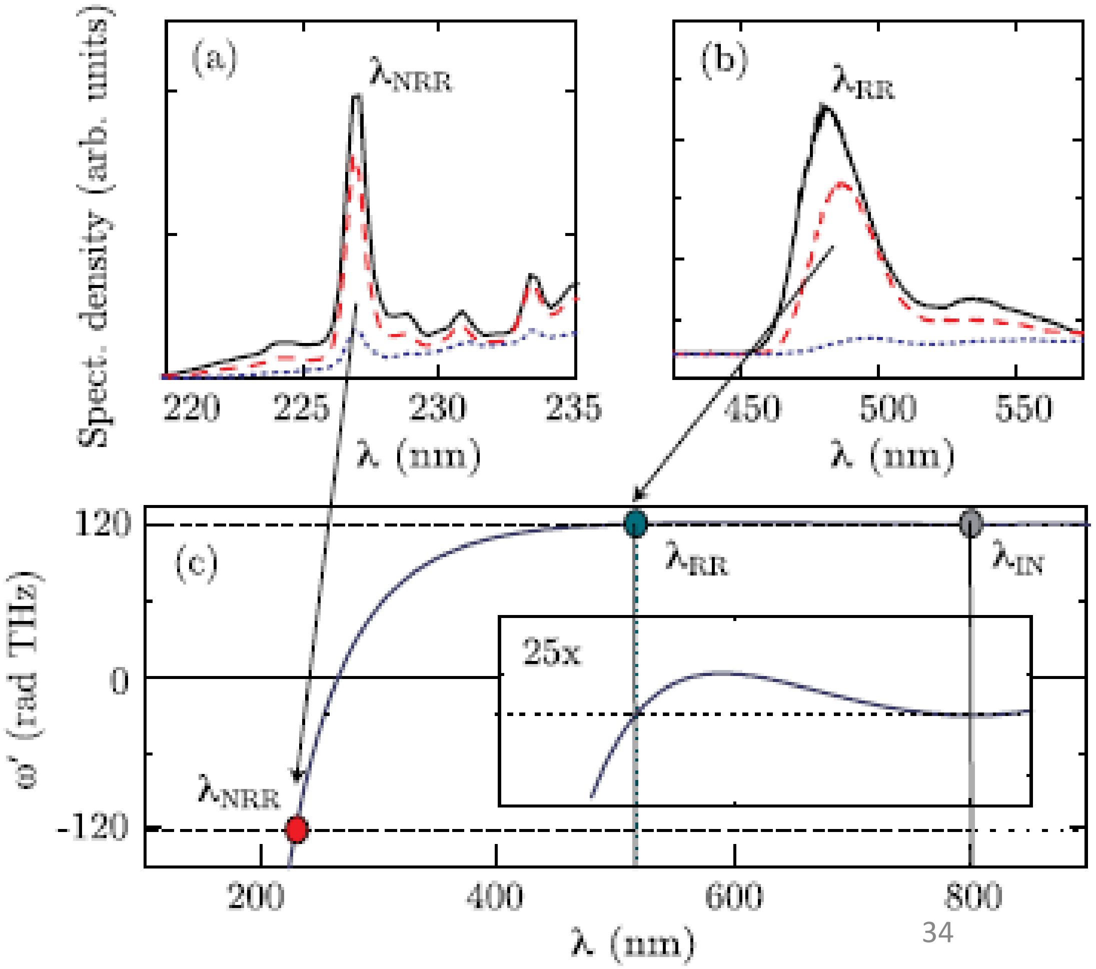
Electric field is real ➔ $\tilde{A}(-\omega) = \tilde{A}^*(\omega)$

Negative frequency waves – nonlinear optics

Momentum conservation in the lab frame ≡ energy conservation in the moving frame

Couple 7-fs pulse into few mm-long PCF
Rubino, PRL (2012), McLenaghan, NJP (2014)





Negative frequency waves recently observed in water experiments Weinfurtner, PRL (2011), Rousseaux, NJP (2008),