Supradegeneracy: Population inversion without pumping
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Abstract

In traditional thermodynamics, population inversion and steady-state currents are considered hallmarks of nonequilibrium. It is shown these phenomena can arise spontaneously, without nonequilibrium forcing, when the statistical degeneracy of the system increases rapidly enough to dominate the standard Boltzmann exponential. Numerical simulations support these predictions. Physical systems might demonstrate this statistically supradegenerate effect.

Daniel P. Sheehan, University of San Diego
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P319 S/M Class (Fall 2016)
\[ \frac{P_{n+1}}{P_n} = \frac{g_{n+1}}{g_n} e^{-\epsilon/\beta} \]

\[ g_i \sim 1 \]

\[ \epsilon \gg kT = \beta^{-1} \]

\[ \frac{P_{n+1}}{P_n} < 1 \]
Population Inversion

Equilibrium: \( \frac{P_{n+1}}{P_n} < 1 \)

Nonequilibrium: \( \frac{P_{n+1}}{P_n} > 1 \)

(Optical pumping, electrical discharge, chemical reaction)
Supradegeneracy

\[
\frac{P_{n+1}}{P_n} = \frac{p^{n+1}}{p^n} e^{-\epsilon\beta} = p e^{-\epsilon\beta} = e^{\ln(p) - \epsilon\beta}
\]

If \((\ln(p) - \epsilon\beta) > 0\),

Then \(\frac{P_{n+1}}{P_n} > 1\)

Criterion for Supradegeneracy:

\[
(\ln(p) - \epsilon\beta) > 0
\]

or

\[
\ln(p) > \frac{\epsilon}{kT}
\]

Population inversion without pumping
Supradegeneracy Model
Supradegeneracy Model
Basic Supradegeneracy Model (BSM)

(1) \( \epsilon = \text{const.} \quad (\sim kT) \)

\[ \ln(p) - \epsilon \beta > 0 \]

\[ \epsilon < kT \ln(p) \]

(2) \( g_n = p^n \)
Partition Function (BSM)

\[ Z_{BSM} = \frac{e^{\gamma(N+1)} - 1}{e^\gamma - 1} = e^{\gamma N/2} \frac{\sinh \left( \frac{\gamma(N+1)}{2} \right)}{\sinh \left( \frac{\gamma}{2} \right)} \]

\[ \gamma \equiv \ln(p) - \epsilon \beta = \ln(p) - \alpha \]

Number of Levels
N=10
Limit: \( N \gamma \gg 1 \)

\[
Z_{BSM} \simeq \exp(\gamma N)
\]

Helmholtz free energy:

\[
F \equiv -\frac{1}{\beta} \ln(Z) = N \epsilon - kTN \ln(p) = U - TS
\]

Energy: \( U = N \epsilon \)

Multiplicity: \( \Omega = p^N \)

Entropy:

\[
S = k \ln(\Omega) = k \ln(p^N) = k \ln(p^{U/\epsilon})
\]

Specific heat: \( C_v = \left( \frac{\partial U}{\partial T} \right) = Nk \ln(p) \)

Chemical potential: \( \mu = \beta^{-1} \ln(\frac{N}{Z_{BSM}}) \)

(\( N \) = number of indistinguishable, non-interacting particles)
Occupation Probability

\[ Pr(n) = \exp\left(\frac{\ln(p) - \alpha n}{2}\right) \]

\( N=10 \)

\( n=1 \)

\( n=3 \)

\( n=5 \)

\( n=10 \)
Occupation Probability

N=10

\[ \alpha = \ln(3) \]

- Normal Degenerate
- Superdegenerate

\( \Pr(\alpha, p, n) \)

\( n \):

0 2 4 6 8 10

\( p = 1 \)

\( p = 2 \)

\( p = 3 \)

\( p = 4 \)

\( p = 5 \)
Level Occupation (BSM)

\[ < n > = \frac{\partial}{\partial \gamma} (\ln(Z)) = \]

\[ \frac{(N+1) \exp[\gamma(N+1)]}{(\exp[\gamma(N+1)]+1)} - \frac{1}{\exp[\gamma]-1} \]

**Limit:** \( \gamma \gg 1 \)

\[ < n > \rightarrow N + 1 \]
Density of States

\[ g_{\text{BSM}}(E) = \exp\left[\frac{\ln(p)}{\epsilon} E\right], \quad (0 \geq E \geq \epsilon N) \]  
(Superdegeneracy)

\[ g_{\text{3D}}(E) = CE^{1/2} \]  
(Maxwell-Boltzmann ideal gas, electrons in solid)
Particle Distribution

\[ g_{BSM}(E)n_B(E) = e^{\mu\beta}e^{\left(\frac{\ln(p)}{\epsilon} - \beta\right)E} \]

\[(0 \leq E \leq E_{max})\]

Fermi-Dirac \hspace{1cm} Bose-Einstein

Maxwell-Boltzmann
Superdegenerate Runaways

Plasmas (Dreicer Limit)

\[ F_{\text{e.s.}} > F_{\text{drag}} \]

Superdegenerate Energy Ladder
Entropic Force

\[ F_\nabla S \equiv T \nabla S \]

For BSM:

\[ \frac{n}{N} = \frac{E_n}{E_{\text{max}}} = \frac{y}{H} \]

\[ \Omega(y) = p^n = \exp\left[\frac{N}{H} \ln(p)y\right] \]

\[ F_{\nabla S} = T \frac{\partial S}{\partial y} = \frac{N}{H} \ln(p)kT \]

\[ F_{\nabla S} = \left( \frac{\ln(p)}{\epsilon \beta} \right) \left( \frac{E_{\text{max}}}{H} \right) \]
Particle and Energy Currents
**BSM: Numerical Studies**

\[ Pr_\uparrow = \frac{\chi_\uparrow}{\chi_\uparrow + \chi_\downarrow + \chi_{\leftrightarrow}} \]

\[ Pr_\downarrow = \frac{\chi_\downarrow}{\chi_\uparrow + \chi_\downarrow + \chi_{\leftrightarrow}} \]

\[ Pr_{\leftrightarrow} = \frac{\chi_{\leftrightarrow}}{\chi_\uparrow + \chi_\downarrow + \chi_{\leftrightarrow}} \]

\[ \chi_\uparrow = (pe^{-\epsilon/\beta})\eta_\uparrow \]

\[ \chi_\downarrow = (1)\eta_\downarrow \]

\[ \chi_{\leftrightarrow} = q_{nn}\eta_{\leftrightarrow} \]

**BSM Energy Ladder**

Diagram showing the energy ladder with states \((n), (n-1), (n+1)\) and transition probabilities between them.
Particle and Energy Currents

\[ f_{\uparrow} = P r_{\uparrow} f_{\text{att}} \]
\[ f_{\downarrow} = P r_{\downarrow} f_{\text{att}} \]
\[ f_{\text{att}} \simeq f_{\text{vib}} \]
\[ f_{\text{net}} = f_{\uparrow} - f_{\downarrow} \]

Particle Current:
\[ I_p = N f_{\text{net}} \]

Energy Current:
\[ I_E = N \epsilon (f_{\uparrow} - f_{\downarrow}) = N \epsilon f_{\text{vib}} \frac{x_{\uparrow} - x_{\downarrow}}{x_{\uparrow} + x_{\downarrow} + x_{\leftrightarrow}} \]

Energy Current (Reduced Model):
\[ I_{E,\text{red}} = (N f_{\text{vib}}) \frac{\alpha (pe^{-\alpha} - 1)}{pe^{\alpha} + 1} \]
Energy Current

\[ I_{E,\text{red}} = \left( \frac{N f_{\text{vib}}}{\beta} \right) \frac{\alpha(p e^{-\alpha} - 1)}{(p e^{\alpha} + 1)} \]
Stochastic Dynamics Simulation

- Temperature (energy units)
- Level probability
- Total current
- Leak strength

Graphs showing the relationship between temperature and level probability, and leak strength and total current.
Near Precedents for Superdegeneracy

Hydrogen

Particle in 3D Box

\[ g_{\text{BSM}}(E) = \exp \left[ \frac{\ln(p)}{\epsilon} E \right] \]

\[ g_{3\text{D}}(E) = CE^{1/2} \]

\[ \frac{\partial \ln[g_{3\text{D}}(E)]}{\partial E} > \frac{\partial \ln[g_{\text{BSM}}(E)]}{\partial E}; \quad (E \sim 0) \]
Thermosynthetic Life

Fig. 1 Schematic of proposed biochemical machinery for thermosynthetic life. Charge cycles clockwise: diffusively up through the pyramid and ballistically down the conduction ladder through the reaction center, where high-energy chemical products are formed.
Experimental Supradegenerate Systems
Summary and Prospects

Unexplored thermodynamic regime

\((E, \beta, \mu, \ln(p))\)

Criterion: \(\ln(p) > \epsilon/kT\)

Novel phenomena: population inversion without pumping; nonequilibrium currents; runaway particles

Equilibrium vs. nonequilibrium

Foundations of thermodynamics

Quantum: connections and challenges