Supradegeneracy: Population inversion without pumping

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Abstract

In traditional thermodynamics, population inversion and steady-state currents are considered hallmarks of nonequilibrium. It is shown these phenomena can arise spontaneously, without nonequilibrium forcing, when the statistical degeneracy of the system increases rapidly enough to dominate the standard Boltzmann exponential. Numerical simulations support these predictions. Physical systems might demonstrate this statistically *supradegenerate* effect.

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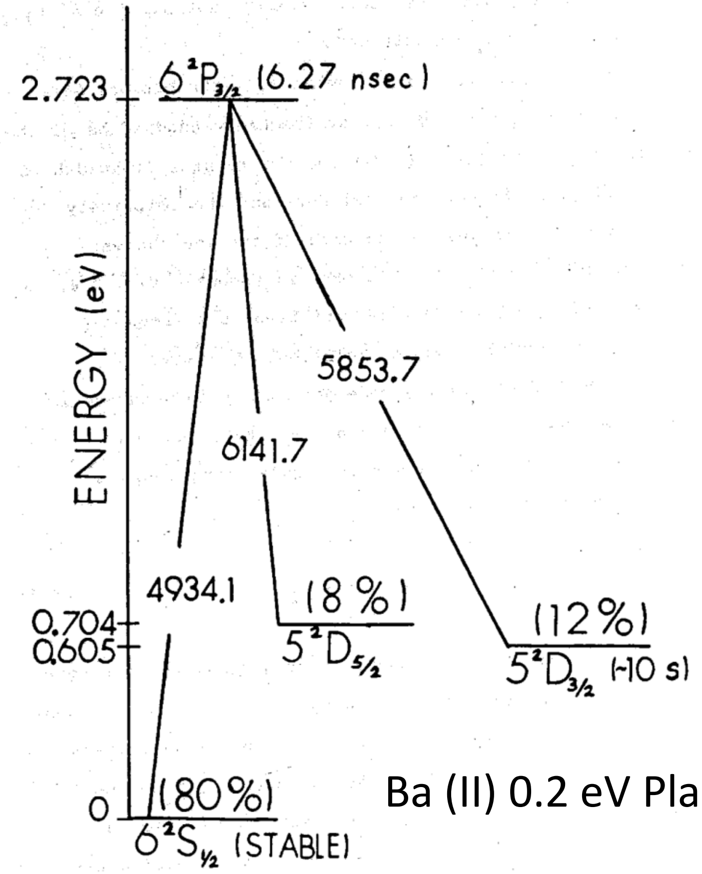
Canonical Statistical Mechanics

$$\frac{P_{\rm n+1}}{P_{\rm n}} = \frac{g_{n+1}}{g_n} e^{-\epsilon\beta}$$

$$g_i \sim 1$$

$$\epsilon \gg kT = \beta^{-1}$$

$$\frac{P_{n+1}}{P_n} < 1$$



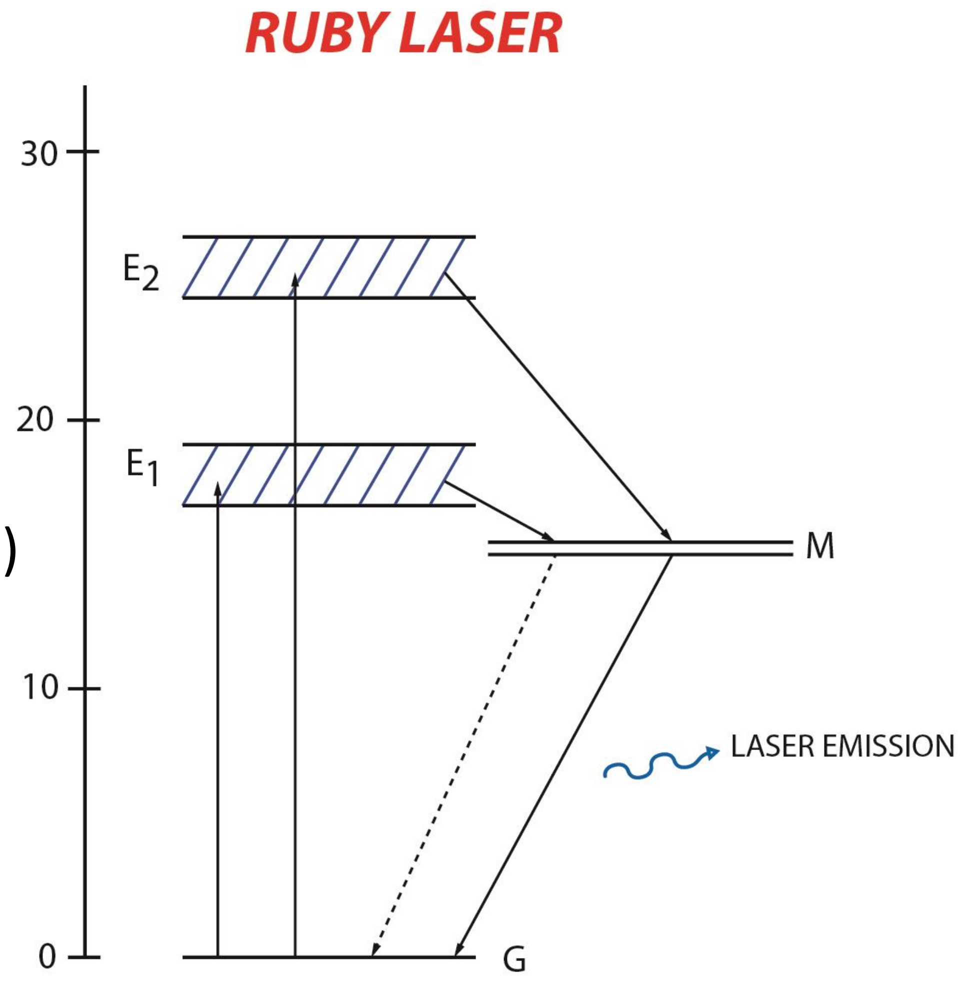
Ba (II) 0.2 eV Plasma

Population Inversion

$$\textbf{Equilibrium: } \frac{P_{n+1}}{P_n} < 1$$

$$\textbf{Nonequilibrium: } \frac{P_{n+1}}{P_n} > 1$$

(Optical pumping, electrical discharge, chemical reaction)



Supradegeneracy

$$\frac{P_{n+1}}{P_n} = \frac{p^{n+1}}{p^n} e^{-\epsilon\beta} = pe^{-\epsilon\beta} = e^{\ln(p)-\epsilon\beta}$$

If $(\ln(p) - \epsilon\beta) > 0,$

Then $\frac{P_{n+1}}{P_n} > 1$

Criterion for Supradegeneracy:

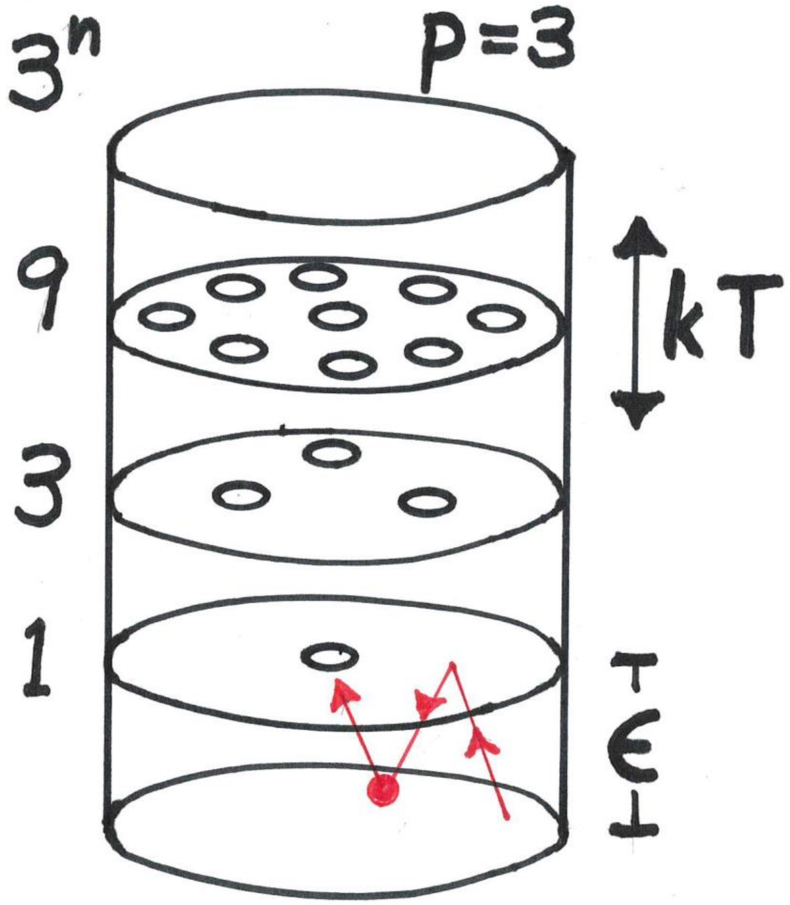
$$\boxed{(\ln(p) - \epsilon\beta) > 0}$$

or

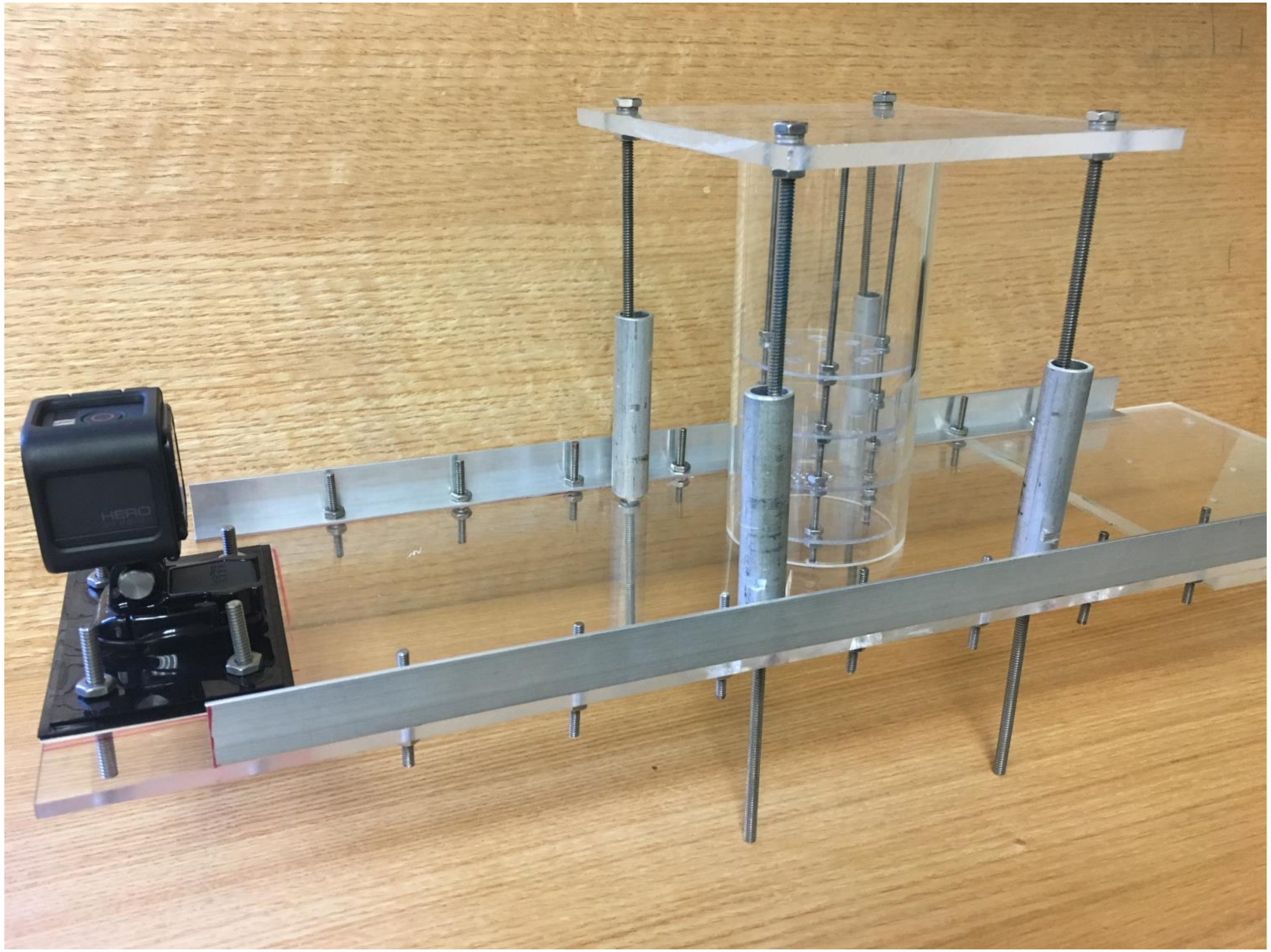
$$\boxed{\ln(p) > \frac{\epsilon}{kT}}$$

Population inversion without pumping

Supradegeneracy Model



Supradegeneracy Model



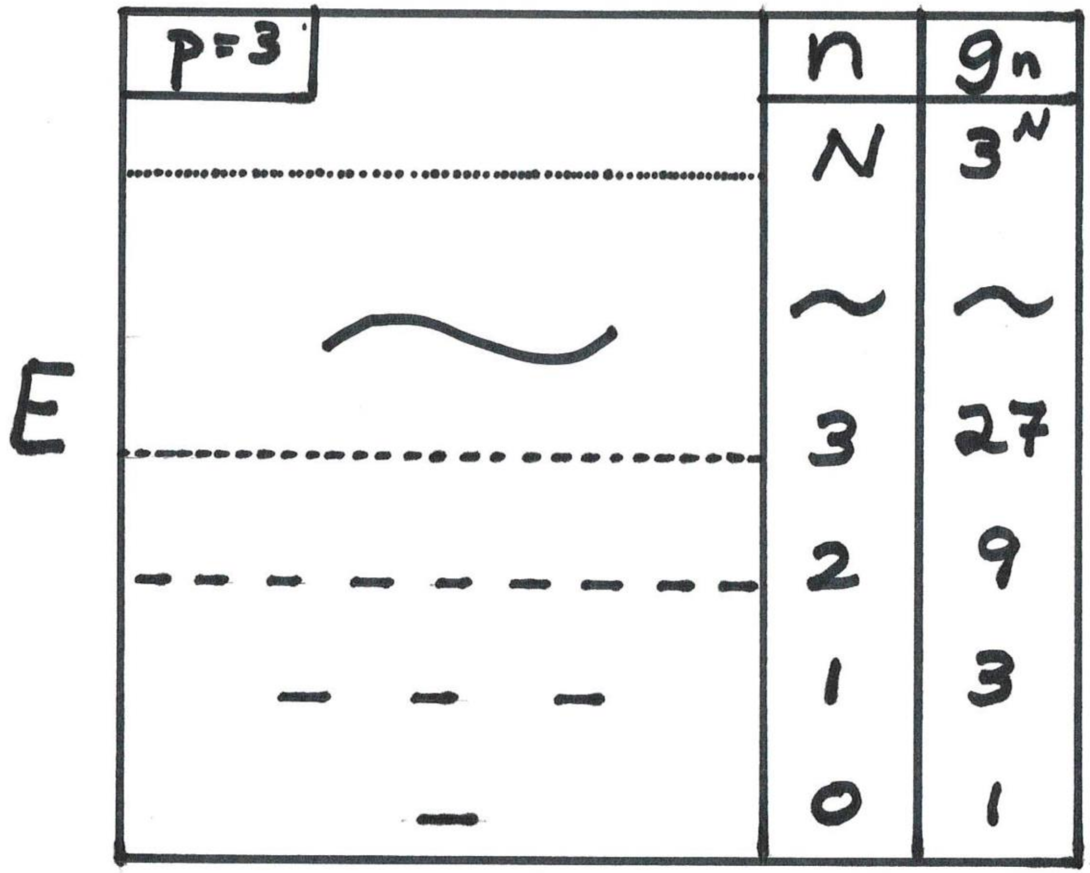
Basic Supradegeneracy Model (BSM)

(1) $\epsilon = \text{const.} \quad (\sim kT)$

$$\ln(p) - \epsilon\beta > 0$$

$$\epsilon < kT\ln(p)$$

(2) $g_n = p^n$

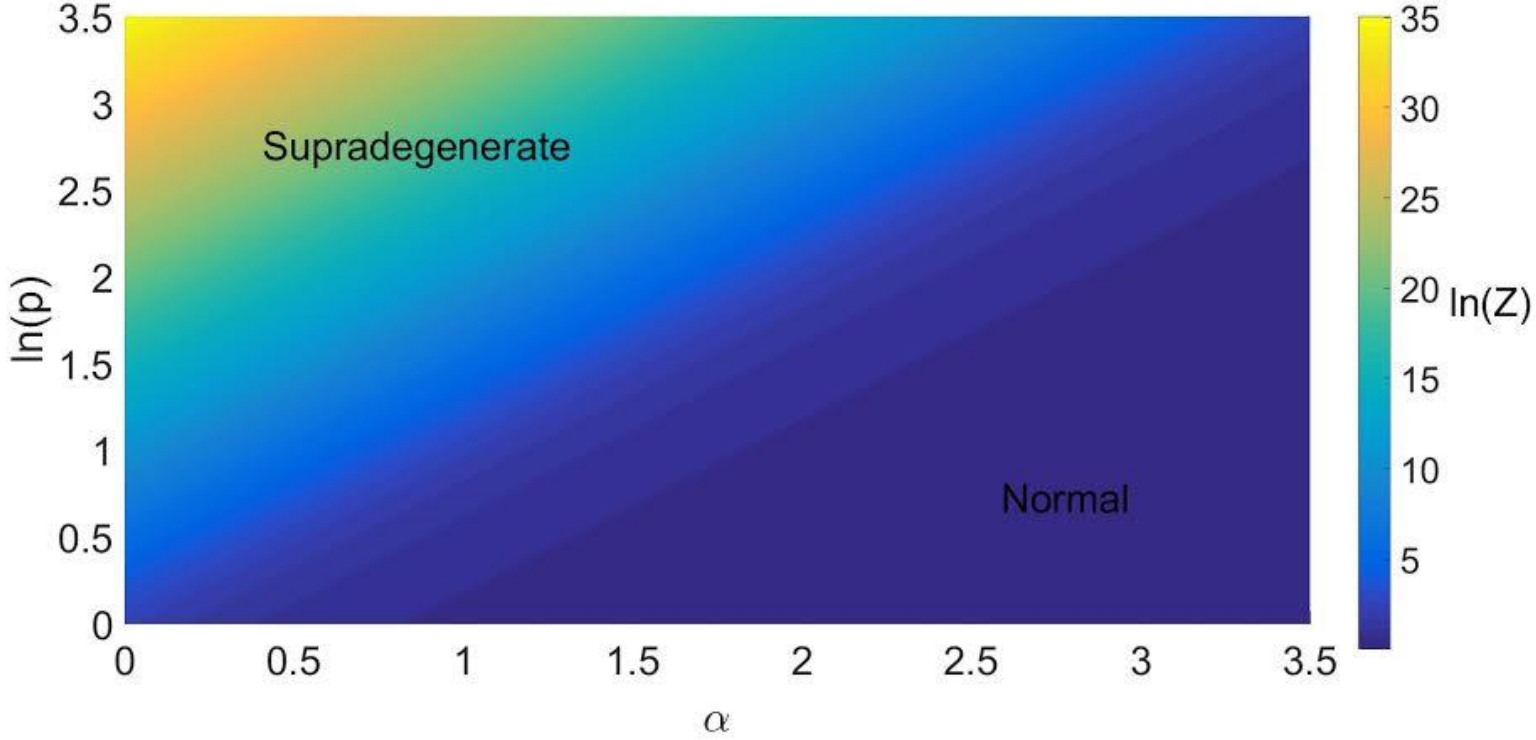


Partition Function (BSM)

$$Z_{\text{BSM}} = \frac{e^{\gamma(N+1)}-1}{e^{\gamma}-1} = e^{\gamma N/2}\frac{\sinh\left(\frac{\gamma(N+1)}{2}\right)}{\sinh\left(\gamma/2\right)}$$

$$\gamma \equiv \ln(p) - \epsilon\beta = \ln(p) - \alpha$$

Number of Levels
N=10



BSM Thermodynamics

Limit: $N\gamma \gg 1$

$$Z_{\text{BSM}} \simeq \exp(\gamma N)$$

Helmholtz free energy:

$$\text{F} \equiv -\frac{1}{\beta}\ln(Z) = N\epsilon - kTN\ln(p) = U - TS$$

Energy: $U = N\epsilon$

Multiplicity: $\Omega = p^N$

Entropy:

$$S = k\ln(\Omega) = k\ln(p^N) = k\ln(p^{U/\epsilon})$$

Specific heat: $C_v = (\frac{\partial U}{\partial T}) = Nk\ln(p)$

Chemical potential: $\mu = \beta^{-1}\ln(\frac{\mathcal{N}}{Z_{\text{BSM}}})$

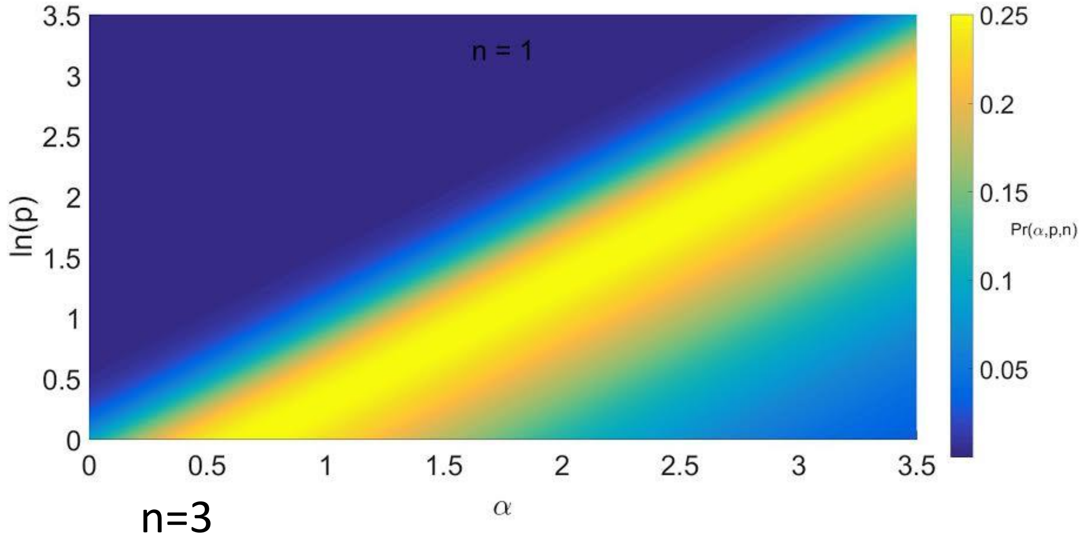
($\mathcal{N}$ = number of indistinguishable, non-interacting particles)

Occupation Probability

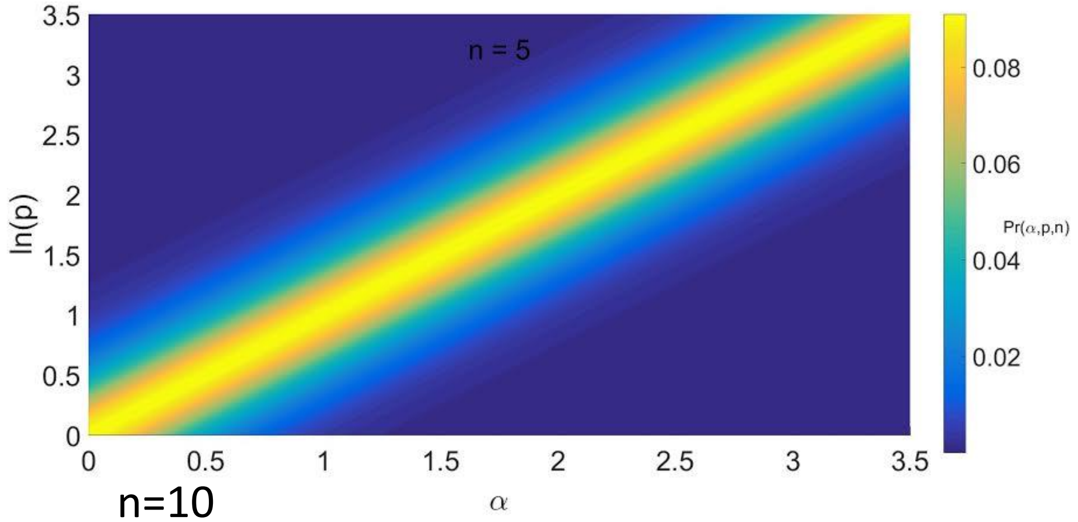
N=10

$$Pr(n) = \frac{\exp[(\ln(p)-\alpha)n]}{Z}$$

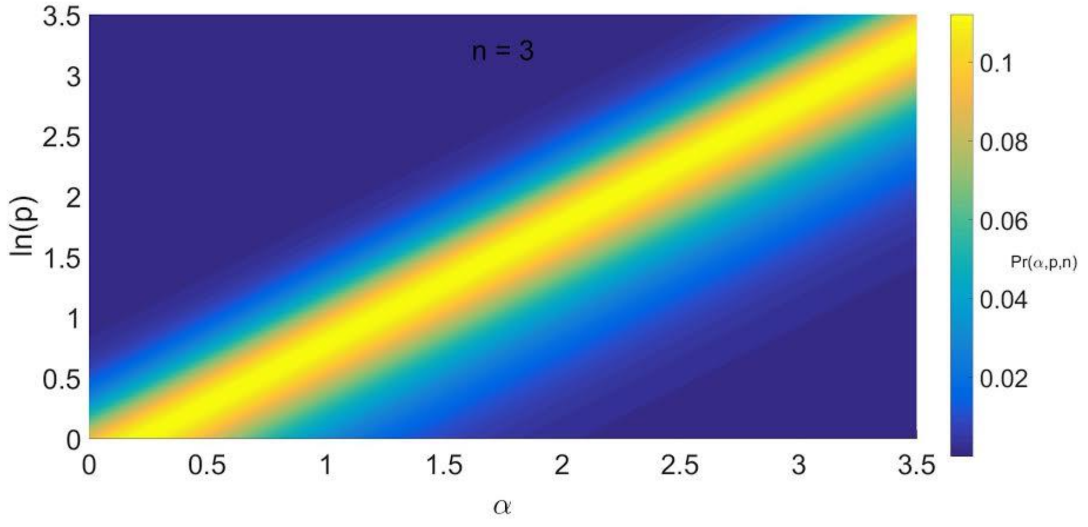
n=1



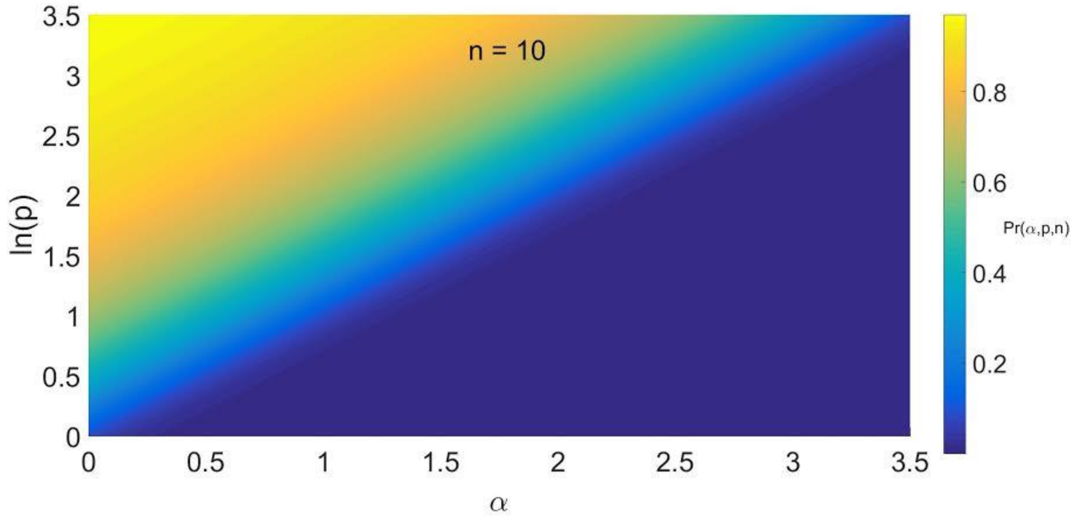
n=5



n=3

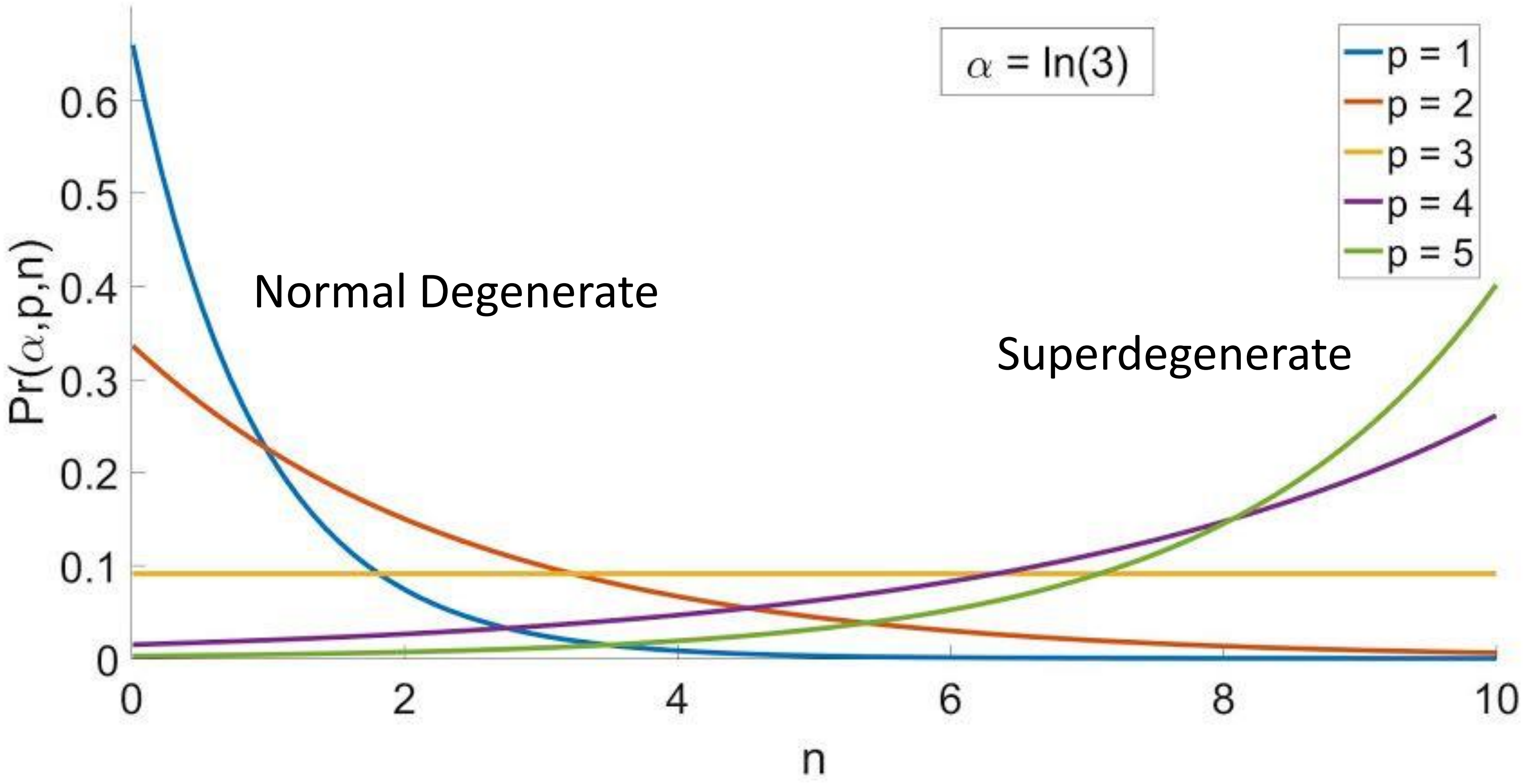


n=10



Occupation Probability

N=10



Level Occupation (BSM)

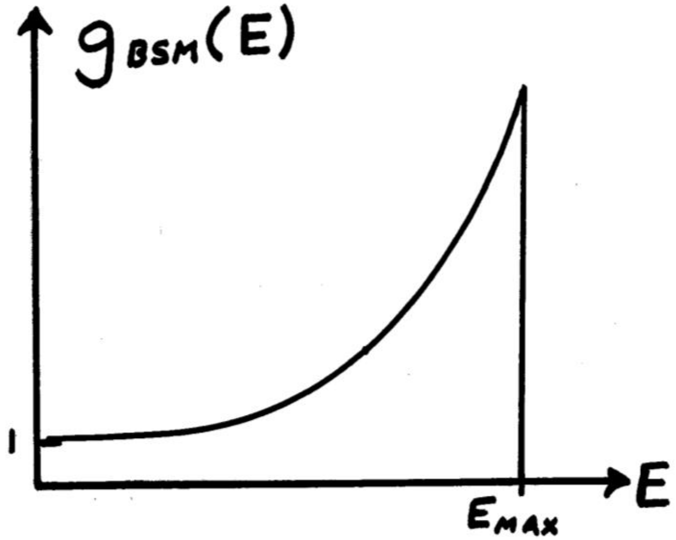
$$< n >= \frac{\partial}{\partial\gamma}(\ln(Z)) =$$

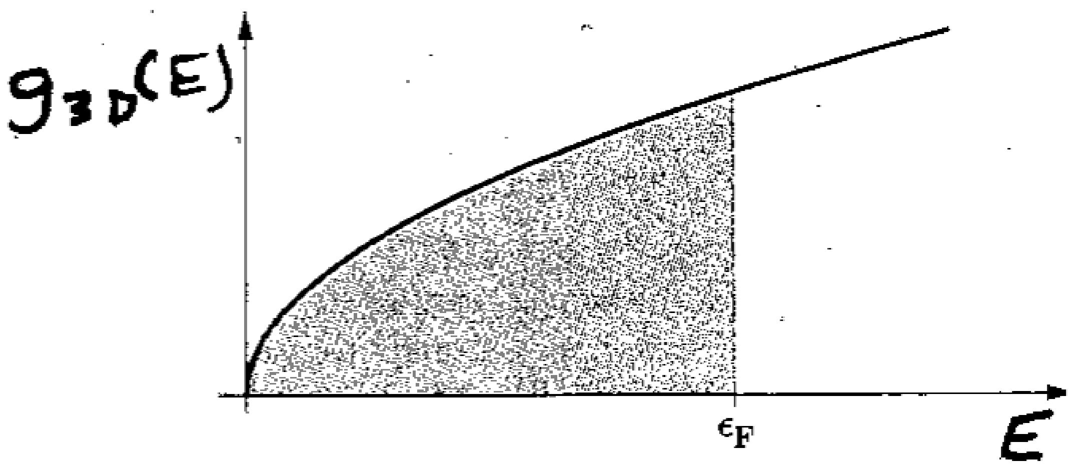
$$\frac{(N+1)\exp[\gamma(N+1)]}{(\exp[\gamma(N+1)]+1} - \frac{1}{\exp[\gamma]-1}$$

Limit: $\gamma \gg 1$

$$< n > \rightarrow N + 1$$

Density of States





$$g_{\rm BSM}(E) = \exp[\frac{\ln(p)}{\epsilon}E], \quad (0 \geq E \geq \epsilon N)$$

(Superdegeneracy)

$$g_{\rm 3D}(E) = CE^{1/2}$$

(Maxwell-Boltzmann ideal gas, electrons in solid)

Particle Distribution

$$g_{\mathrm{BSM}}(E)n_B(E) = e^{\mu\beta}e^{(\frac{\ln(p)}{\epsilon}-\beta)E}$$

$$(0 \leq E \leq E_{\mathrm{max}})$$

Fermi-Dirac

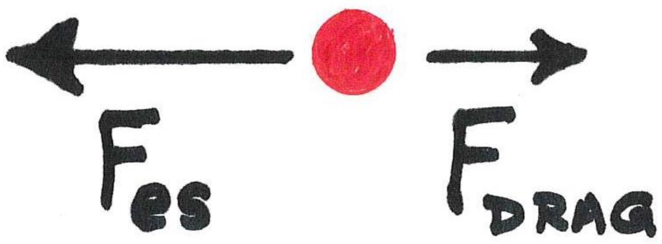
Bose-Einstein

Maxwell-Boltzmann

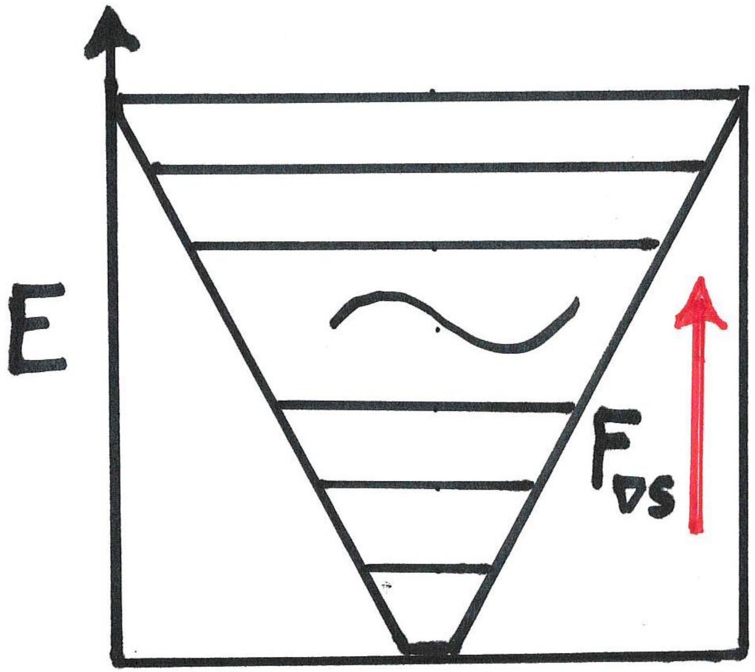
Superdegenerate Runaways

Plasmas (Dreicer Limit)

$$F_{\text{e.s.}} > F_{\text{drag}}$$



Superdegenerate Energy Ladder



Entropic Force

$$\boxed{F_{\nabla S} \equiv T\nabla S}$$

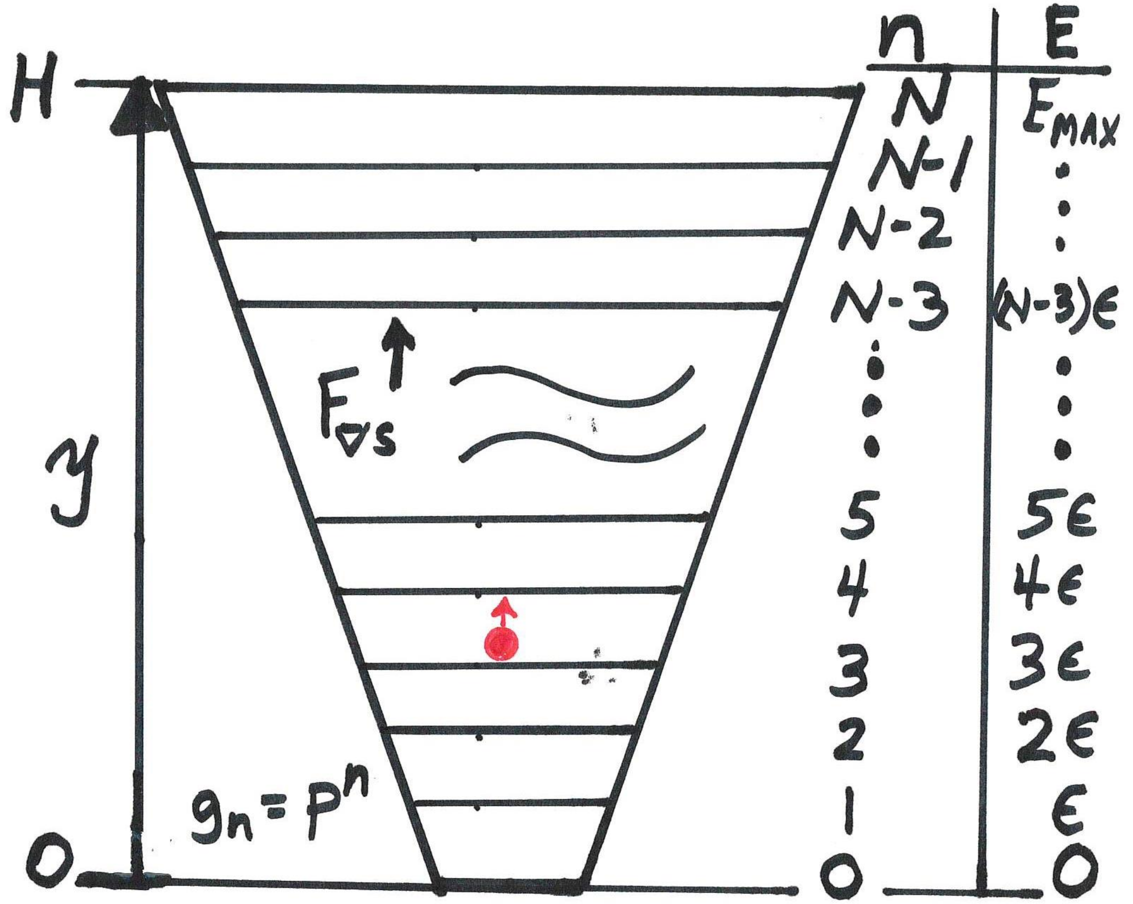
For BSM:

$$\frac{n}{N} = \frac{E_n}{E_{max}} = \frac{y}{H}$$

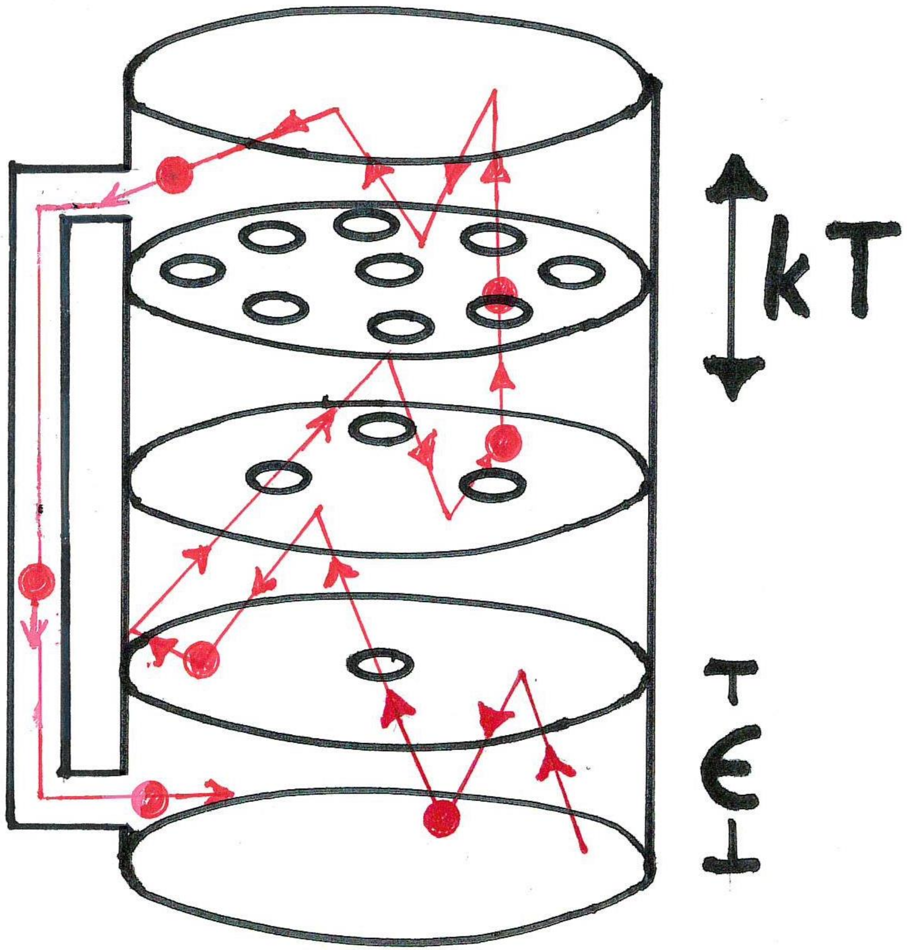
$$\Omega(y) = p^n = \exp[\frac{N}{H}\ln(p)y]$$

$$F_{\nabla S} = T\frac{\partial S}{\partial y} = \frac{N}{H}\ln(p)kT$$

$$\boxed{F_{\nabla S} = (\frac{\ln(p)}{\epsilon\beta})(\frac{E_{max}}{H})}$$



Particle and Energy Currents



BSM: Numerical Studies

$$Pr_{\uparrow} = \frac{\chi_{\uparrow}}{\chi_{\uparrow}+\chi_{\downarrow}+\chi_{\leftrightarrow}}$$

$$Pr_{\downarrow} = \frac{\chi_{\downarrow}}{\chi_{\uparrow}+\chi_{\downarrow}+\chi_{\leftrightarrow}}$$

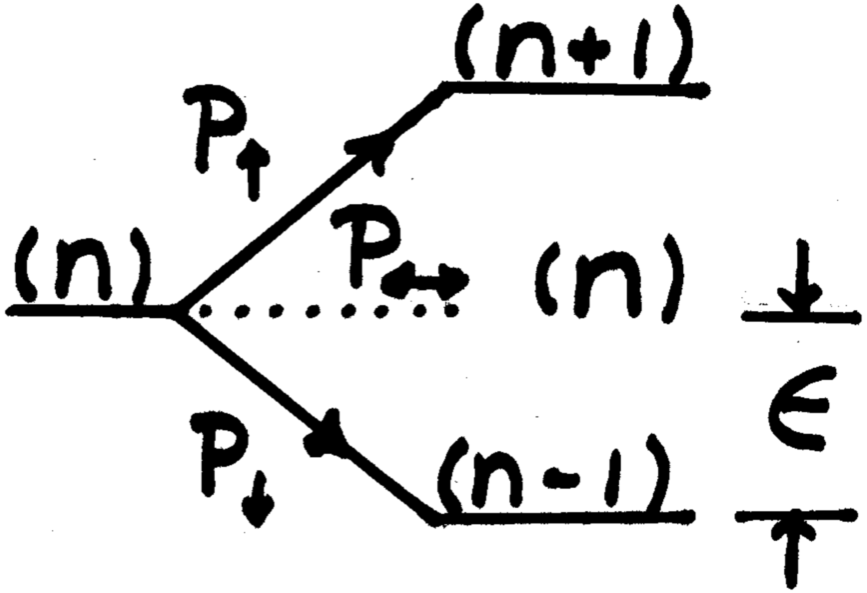
$$Pr_{\leftrightarrow} = \frac{\chi_{\leftrightarrow}}{\chi_{\uparrow}+\chi_{\downarrow}+\chi_{\leftrightarrow}}$$

$$\chi_{\uparrow} = (pe^{-\epsilon\beta})\eta_{\uparrow}$$

$$\chi_{\downarrow} = (1)\eta_{\downarrow}$$

$$\chi_{\leftrightarrow} = q_{\mathrm{nn}}\eta_{\leftrightarrow}$$

BSM Energy Ladder



Particle and Energy Currents

$$f_{\uparrow} = Pr_{\uparrow} f_{\text{att}}$$

$$f_{\downarrow} = Pr_{\downarrow} f_{\text{att}}$$

$$f_{\text{att}} \simeq f_{\text{vib}}$$

$$f_{\text{net}} = f_{\uparrow} - f_{\downarrow}$$

Particle Current:

$$I_{\text{p}} = \mathcal{N} f_{\text{net}}$$

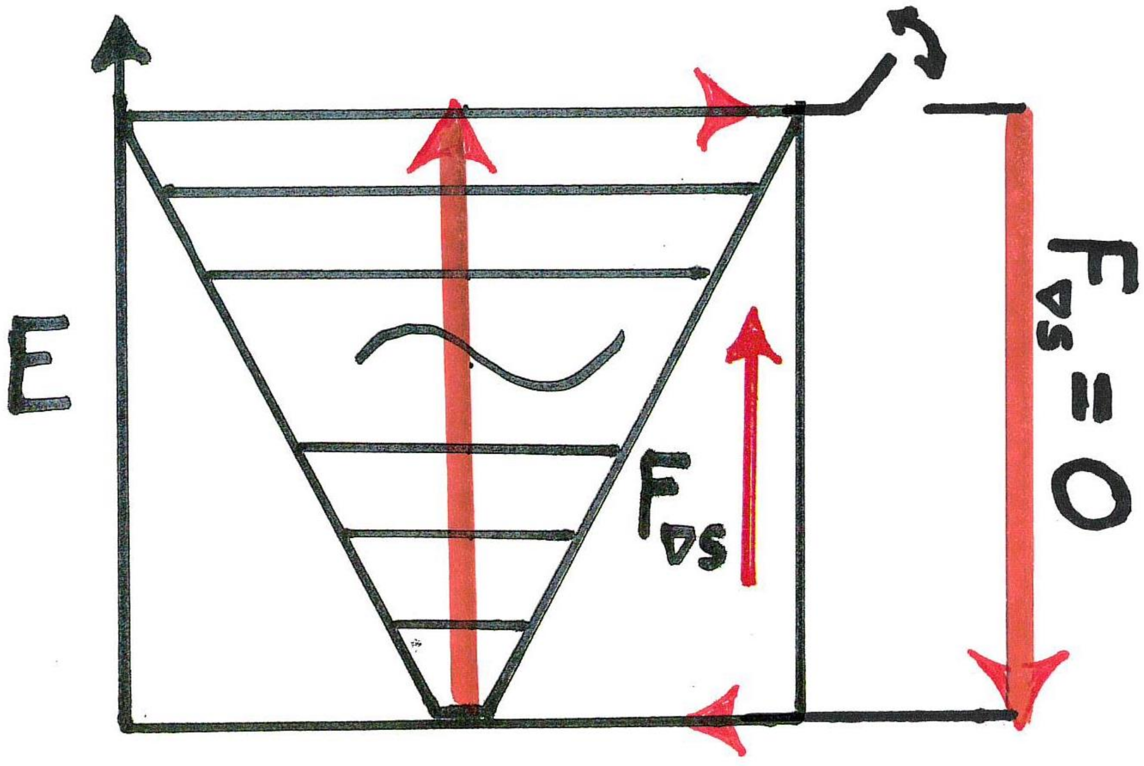
Energy Current:

$$I_{\text{E}} = \mathcal{N}\epsilon(f_{\uparrow} - f_{\downarrow}) = \mathcal{N}\epsilon f_{\text{vib}} \frac{\chi_{\uparrow} - \chi_{\downarrow}}{\chi_{\uparrow} + \chi_{\downarrow} + \chi_{\leftrightarrow}}$$

Energy Current (Reduced Model):

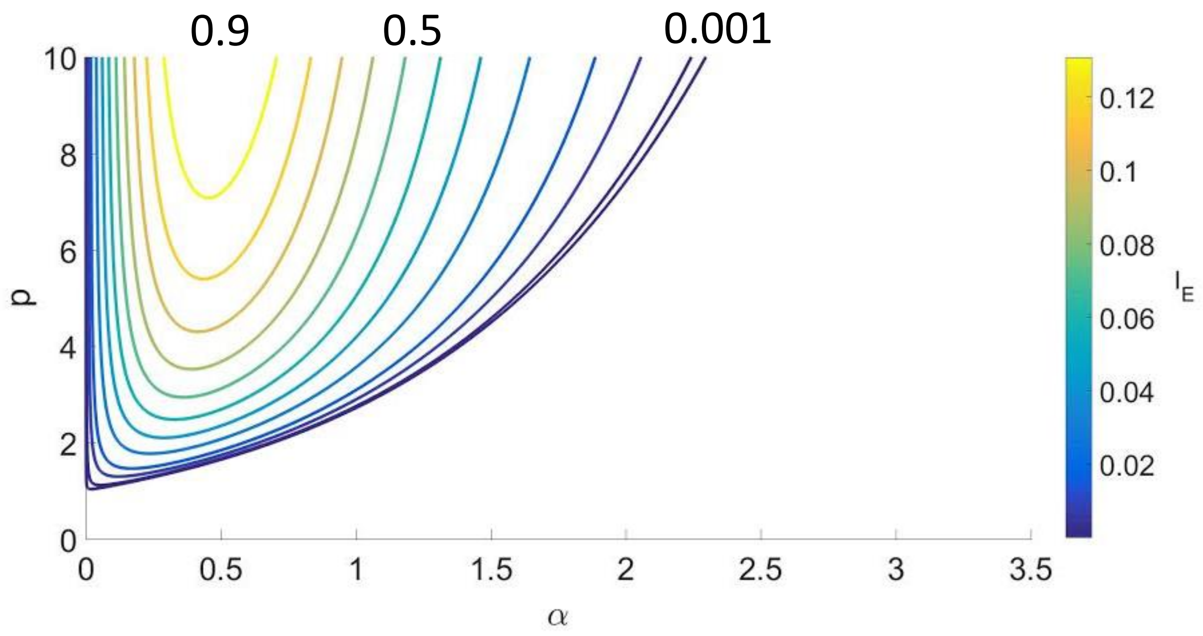
$$I_{\text{E,red}} = \left(\frac{\mathcal{N} f_{\text{vib}}}{\beta}\right) \frac{\alpha(pe^{-\alpha} - 1)}{(pe^{\alpha} + 1)}$$

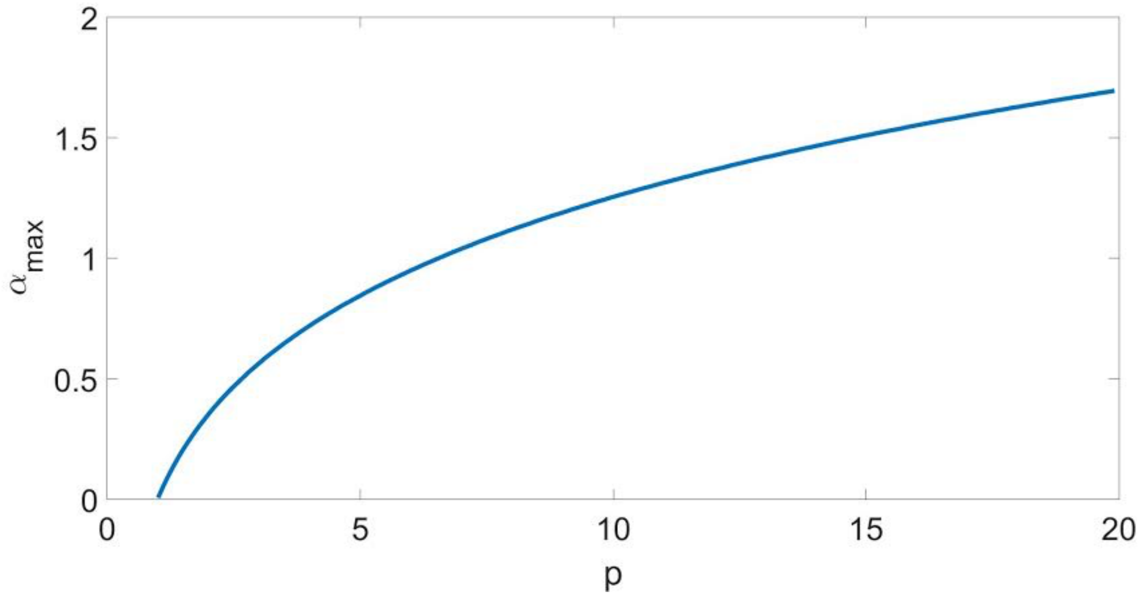
BSM Energy Ladder



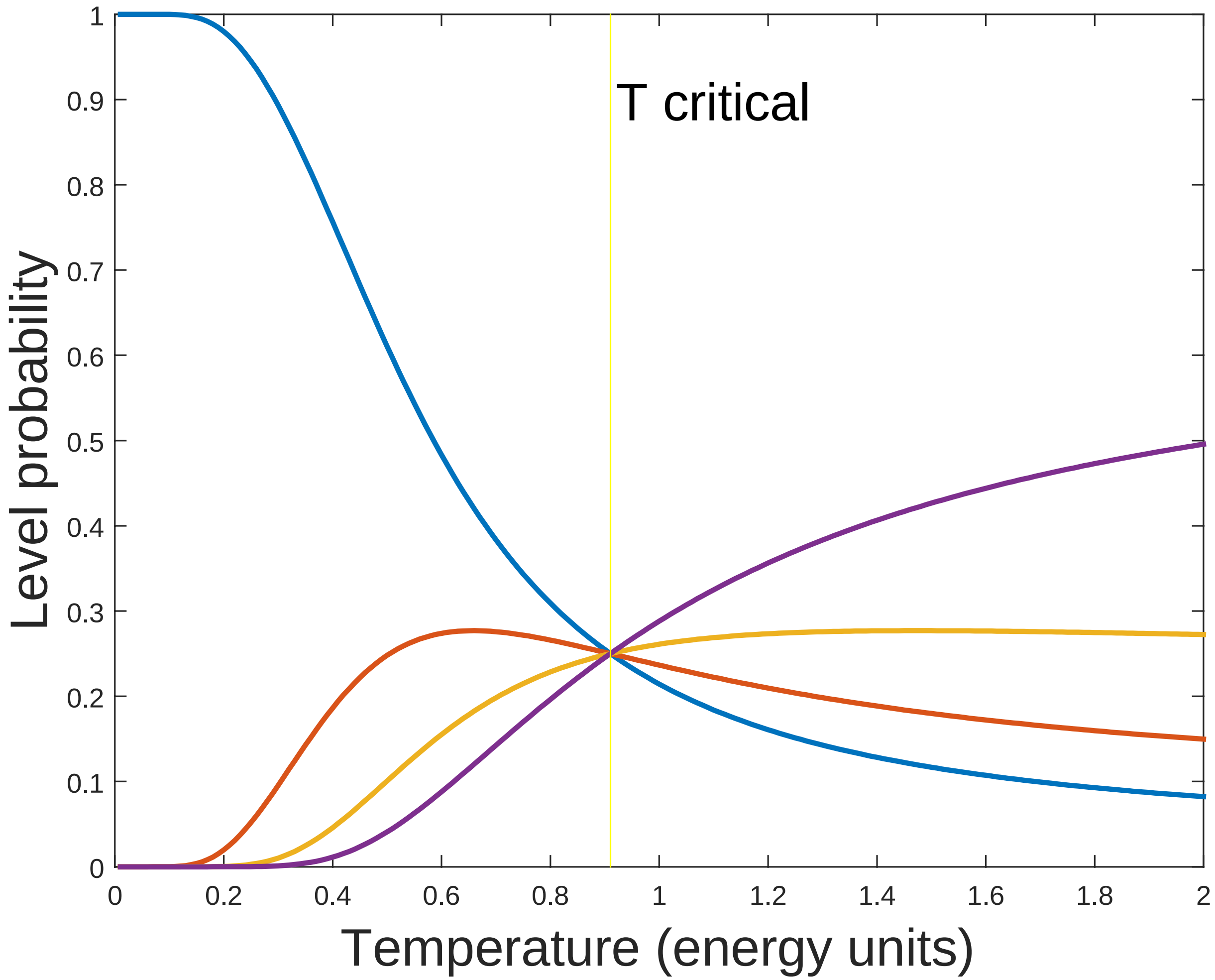
Energy Current

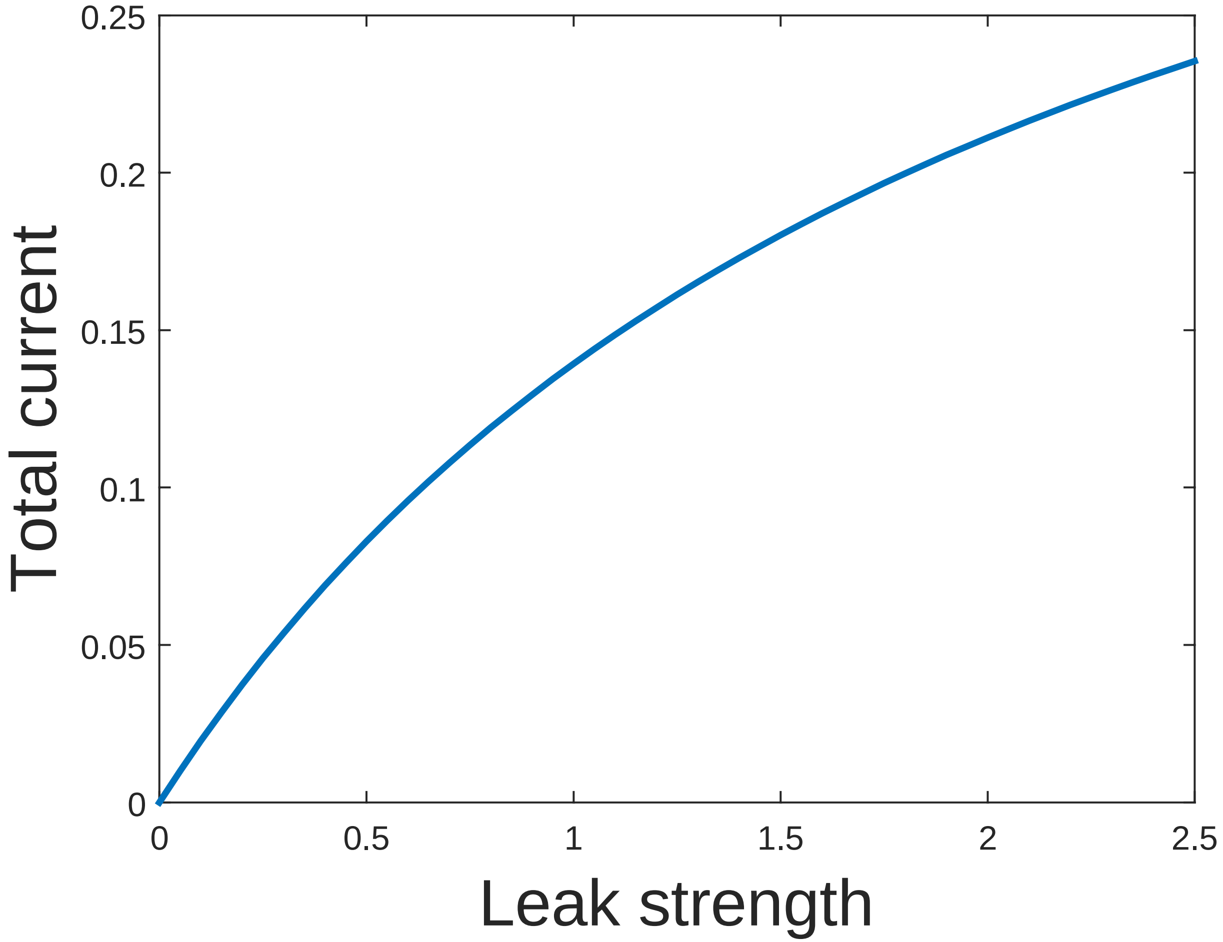
$$I_{\mathrm{E,red}} = \left(\frac{\mathcal{N} f_{\mathrm{vib}}}{\beta}\right)\frac{\alpha(pe^{-\alpha}-1)}{(pe^{\alpha}+1)}$$





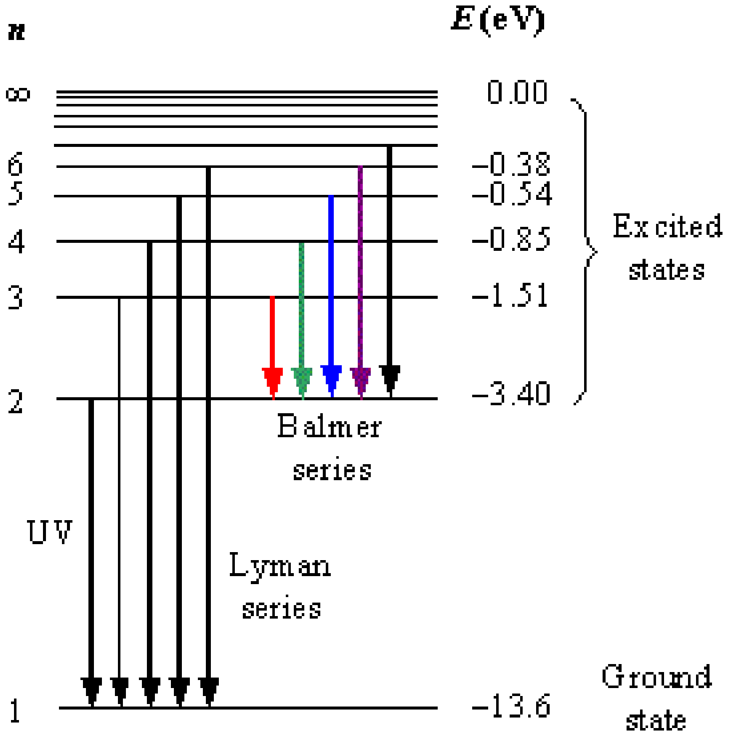
Stochastic Dynamics Simulation

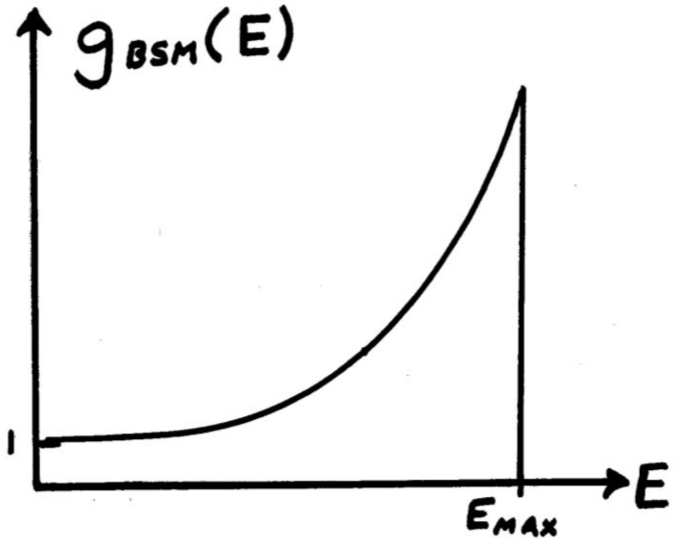




Near Precedents for Superdegeneracy

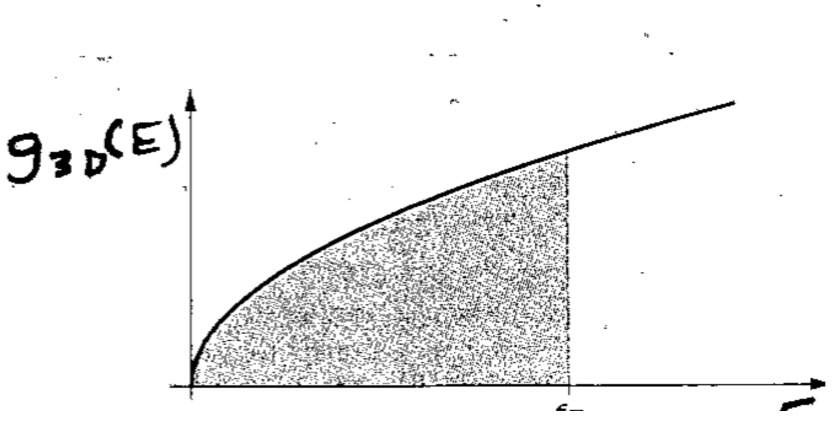
Hydrogen





$$g_{\mathrm{BSM}}(E) = \exp[\frac{\ln(p)}{\epsilon}E]$$

Particle in 3D Box



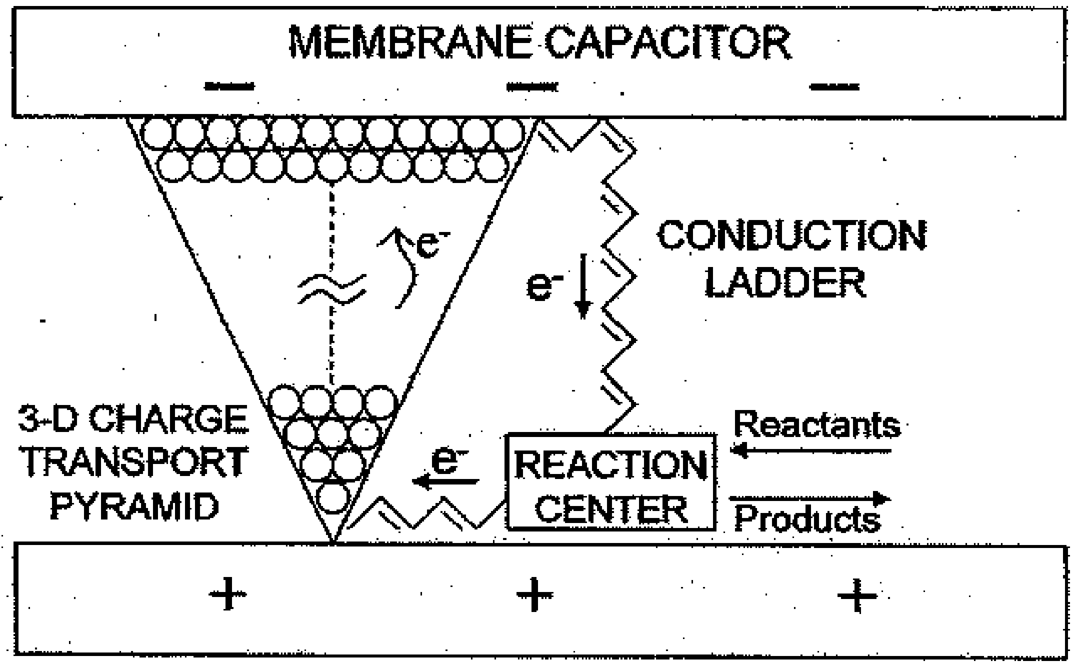
$$g_{\mathrm{3D}}(E) = CE^{1/2}$$

$$\frac{\partial \ln[g_{3D}(E)]}{\partial E} > \frac{\partial \ln[g_{\mathrm{BSM}}(E)]}{\partial E}; \qquad (E \sim 0)$$

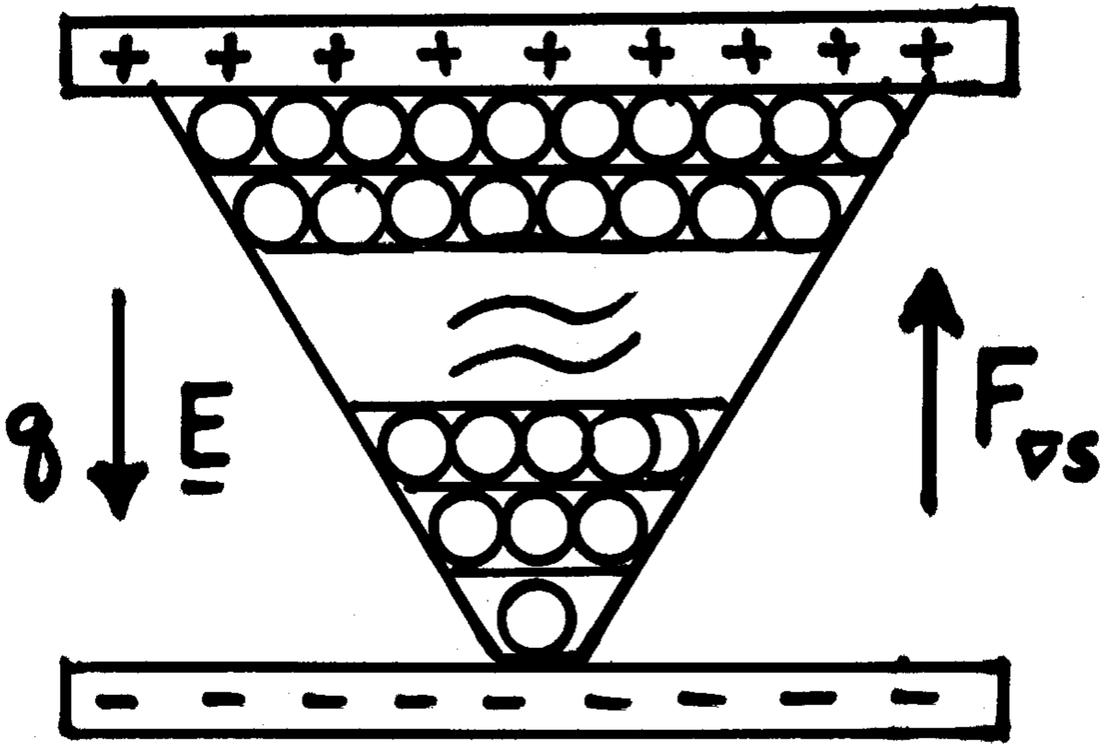
Thermosynthetic Life

 Found Phys (2007) 37: 1774–1797

Fig. 1 Schematic of proposed biochemical machinery for thermosynthetic life. Charge cycles clockwise: diffusively up through the pyramid and ballistically down the conduction ladder through the reaction center, where high-energy chemical products are formed



Experimental Supradegenerate Systems



Summary and Prospects

Unexplored thermodynamic regime

$$(E, \quad \beta, \quad \mu, \quad \ln(p))$$

Criterion: $\ln(p) > \epsilon/kT$

Novel phenomena: population inversion without pumping; nonequilibrium currents; runaway particles

Equilibrium vs. nonequilibrium

Foundations of thermodynamics

Quantum: connections and challenges