Gluons, Heavy and Light Quarks in the QCD Vacuum

Mirzayusuf Musakhanov

National University of Uzbekistan

6th International Conference on New Frontiers in Physics (ICNFP 2017) 17-29 August 2017
Talk based on:

QCD vacuum. Action and topological charge. 
(Negele et al 1999).

- Instanton = tunneling path between the C-S states ⇒
- Instanton Liquid Model (ILM) ~ collective coordinates $\zeta$, $DA \to n(\rho)D\zeta$ (Shuryak1981, Diakonov-Petrov1983).
Instanton vs hadron sizes. ILM & DLM.

Instanton size $n(\rho)$ – lattice vs ILM (Millo, Faccioli 2011).
ILM $\bar{\rho} \sim 0.3$ fm (Shuryak 1981, Diakonov-Petrov 1983).
DLM $\bar{\rho} \sim 0.5$ fm (Diakonov 2009, Shuryak et al 2015).
Quarkonium states and its sizes in non-relativistic potential model (see Satz2012).

<table>
<thead>
<tr>
<th>State</th>
<th>( J/\psi )</th>
<th>( \chi_c )</th>
<th>( \psi' )</th>
<th>( \Upsilon )</th>
<th>( \chi_b )</th>
<th>( \Upsilon' )</th>
<th>( \chi'_b )</th>
<th>( \Upsilon'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>size ( r ) [fm]</td>
<td>0.25</td>
<td>0.36</td>
<td>0.45</td>
<td>0.14</td>
<td>0.22</td>
<td>0.28</td>
<td>0.34</td>
<td>0.39</td>
</tr>
</tbody>
</table>

- \( r_{J/\psi} = 0.25 \text{ fm} \), \( r_{\Upsilon} = 0.14 \text{ fm} \).
- A model estimates of nucleon quark core size \( r_N \sim 0.3 - 0.5 \text{ fm} \) (see Weise1985).
- Small quark core size hadrons are insensitive to the confinement, we may safely apply ILM.
ILM and its main parameters.

- **Sum ansatz** \( A = \sum_{\pm} A_{\pm} \), \( N_+ = N_- = N/2 \).
- **Average instanton size** \( \rho \) and inter-instanton distance \( R = (V/N)^{1/4} \);
- **Estimates:**
  - \( R \approx 0.89 \text{ fm}, \rho \approx 0.36 \text{ fm} \) – lattice;
  - \( R \approx 1.00 \text{ fm}, \rho \approx 0.33 \text{ fm} \) – phenomenological;
  - \( R \approx 0.76 \text{ fm}, \rho \approx 0.32 \text{ fm} \) – our estimate with account of \( 1/N_c \) corrections correspond ChPT (Goeke et al 2007).

Thus within 10 – 15% uncertainty different approaches give similar estimates.

- **Packing parameter** \( \pi^2 (\rho/R)^4 \sim 0.1 - 0.3 \) ⇒ Independent averaging over instanton positions and orientations.
Light quarks in ILM.

Zero modes \((\hat{\rho} + g \hat{A}_\pm) \Phi_{\pm,0}(x, \zeta_\pm) = 0\) dominance ⇒
light quarks partition function \(Z[\xi^+, \xi] = \)

\[
= \int D\zeta \text{Det}_{\text{low}}(\hat{\rho} + g \hat{A} + im) \exp \left( -\xi^+ (\hat{\rho} + g \hat{A} + im)^{-1} \xi \right) =
\]

\[
= \int D\zeta \prod_f D\psi_f D\psi^\dagger_f \exp \int \left( \psi^\dagger_f (\hat{\rho} + im_f) \psi_f + \psi^\dagger_f \xi_f + \xi^+_f \psi_f \right)
\]

\[
\times \prod_f \left\{ \prod_{+}^{N^+} V_{+,f}[\psi^\dagger, \psi] \prod_{-}^{N^-} V_{-,f}[\psi^\dagger, \psi] \right\}, \quad V_{\pm,f}[\psi^\dagger, \psi] =
\]

\[
= i \int dx \left( \psi^\dagger_f(x) \hat{\rho} \Phi_{\pm,0}(x; \zeta_\pm) \right) \int dy \left( \Phi^\dagger_{\pm,0}(y; \zeta_\pm)(\hat{\rho} \psi_f(y) \right).
\]

\(\psi^\dagger, \psi\) are constituent quarks.

Small packing parameter ⇒ independent averaging:

\[
V_{\pm}[\psi^\dagger, \psi] = \int d\zeta \prod_f V_{\pm,f}[\psi^\dagger, \psi]
\]

⇒ non-local \((\sim \rho)\) t’Hooft-like vertex with \(2N_f\)-legs.
Spontaneous Breaking of the Chiral Symmetry.

$Z$ in saddle-point approximation (leading order on $1/N_c$) $\to$ SBCS $\to$ Dynamical quark mass $M(q)$: ILM at $\rho = 0.33 \text{ fm}, R = 1 \text{ fm}$ vs lattice (Bowman2005).

$M(0) \approx 360 \text{ MeV} \sim$ strength of light quark-instanton interaction!! $\Rightarrow$ Even at $1/N_c$ leading order successful reproducing of quark condensate, pion and nucleon properties etc!! (see eg Diakonov2002).

Next to leading order $1/N_c$ corrections. Successful reproducing of Low Energy Constants of ChPT (Goeke et al 2007).
Heavy quarks in ILM.

ILM heavy quark propagator: \( w = \int D\zeta (\theta^{-1} - iA_4)^{-1}, \)

\(< t_2|\theta|t_1 > = \theta(t_2 - t_1).\)

Pobyli\v{c}tca Eq. (DPP1989) \( w^{-1} = \theta^{-1} + \sum_i \int d\zeta_i (w - a_i^{-1})^{-1}. \)

Solution in lowest order on density, effective parameter of expansion: \( \rho^4/R^4 \sim 0.012 \)

\[
\begin{align*}
  w^{-1} &= \theta^{-1} - \frac{N}{2} \text{tr}_c \sum_{\pm} \theta^{-1}(w_{\pm} - \theta)\theta^{-1} + O(N^2/V^2),
\end{align*}
\]

where single (anti)instanton \( w_{\pm} = (\theta^{-1} - iA_{\pm,4})^{-1}. \)

Instanton media contribution to the heavy quark mass

\[
\Delta m_Q = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c, \quad i_0(0) = 0.55.
\]

At \( \rho = 0.33 \text{ fm}, \quad R = 1 \text{ fm} \)

\( \Delta m_Q \approx 70 \text{ MeV} \sim \text{ strength of a heavy quark-instanton interaction!!} \)
Heavy quark-antiquark potential in ILM.

Static central and spin-dependent parts of the potential from ILM averaged Wilson loop (DPP1989) by means of Pobylitsa Eq.

\[ V(\vec{r}) = V_C(r) + V_{SS}(r)(\vec{S}_Q \cdot \vec{S}_Q) + V_{LS}(r)(\vec{L} \cdot \vec{S}) \]

\[ + V_T(r) \left[ 3(\vec{S}_Q \cdot \vec{n})(\vec{S}_{\bar{Q}} \cdot \vec{n}) - \vec{S}_Q \cdot \vec{S}_{\bar{Q}} \right], \]

where

\[ V_{SS}(r) = \frac{1}{3m_Q^2} \nabla^2 V_C(r), \quad V_{LS}(r) = \frac{1}{2m_Q^2} \frac{1}{r} \frac{dV_C(r)}{dr}, \]

\[ V_T(r) = \frac{1}{3m_Q^2} \left( \frac{1}{r} \frac{dV_C(r)}{dr} - \frac{d^2V_C(r)}{dr^2} \right). \]
Solid curve $\sim$ Set I $\rho = 0.33$ fm and $R = 1$ fm
$\sim$ phenomenology (Shuryak1981, Diakonov-Petrov1983),
Dashed one $\sim$ Set II $\rho = 0.36$ fm, $R = 0.89$ fm $\sim$ lattice (Chu et al1994, Negele1998, DeGrand2001, Faccioli-DeGrand2003),
with $1/N_c$ corrections (MM et al2007) $m_c = 1275$ MeV.
ILM contribution to the charmonium states.

\[ \Delta M_{c\bar{c}} = M_{c\bar{c}} - 2m_c \text{ in [MeV].} \]

<table>
<thead>
<tr>
<th>( \Delta M_{c\bar{c}}(J^P) )</th>
<th>Set I</th>
<th>Set IIb</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta M_{\eta_c}(0^-) )</td>
<td>118.81</td>
<td>203.64</td>
<td>433.6 ± 0.6</td>
</tr>
<tr>
<td>( \Delta M_{J/\psi}(1^-) )</td>
<td>119.57</td>
<td>205.36</td>
<td>546.916 ± 0.11</td>
</tr>
<tr>
<td>( \Delta M_{\chi_{c0}}(0^+) )</td>
<td>142.43</td>
<td>250.86</td>
<td>864.75 ± 0.31</td>
</tr>
</tbody>
</table>

One can see that the instanton effects are not small \( \sim 30 - 40\% \) in comparison with the experimental data and strongly depend on instanton liquid parameters.

The planned complete calculations of the potential include the account of gluon modification in ILM/DLM and confinement, since DLM pretend to describe QCD vacuum on the large distances too.
Scalar "gluons" in ILM.

No zero modes in $\Delta_i^{-1} = (p + A_i)^2$ and $\Delta^{-1} = (p + \sum_i A_i)^2$

$\Rightarrow$ existence of the propagators

$\Delta = (p^2 + \sum_i (\{p, A_i\} + A_i^2) + \sum_{i\neq j} A_i A_j)^{-1}$,

$\Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1}$,  $\Delta_0 = p^{-2}$.

Define:

$\tilde{\Delta} = (p^2 + \sum_i (\{p, A_i\} + A_i^2))^{-1}$.

The propagator in ILM is $\bar{\Delta} \equiv \langle \tilde{\Delta} > = \int D\zeta \Delta$.

Start first from $\tilde{\Delta}$.

The extension of Pobylitsa Eq. is

$\tilde{\Delta}^{-1} - \Delta_0^{-1} = \sum_i \langle \tilde{\Delta} + (\Delta_i^{-1} - \Delta_0^{-1})^{-1} \}^{-1} >$

Effective parameter of instanton density expansion in fact is

$\rho^4/R^4 \sim (1/3)^4 = 0.012$.

At the first order on density we have

$\tilde{\Delta}^{-1} - \Delta_0^{-1} = N \Delta_0^{-1}(\bar{\Delta}_i - \Delta_0)\Delta_0^{-1}$,

With same accuracy $\bar{\Delta} = \tilde{\Delta}$. 
Scalar "gluon" dynamical mass in ILM.

Well-known result for the $\Delta_I$ (Brown et al 1978) ⇒

\[
M_s(q) = \left[ \frac{3\rho^2}{(N_c^2 - 1)R^4} 4\pi^2 \right]^{1/2} q\rho K_1(q\rho)
\]

where the form-factor $q\rho K_1(q\rho)$

At $\rho = 0.33$ fm, $R = 1$ fm

\[
M_s(0) = 256 \text{ MeV}
\]
Gluons in ILM. Zero-modes problem.

Over single instanton fluctuations quadratic form of the effective action is \((a_\mu M_{\mu\nu}^I a_\nu)\). There is a \(4N_c\) zero-modes \(M_{\mu\nu}^I \phi_{\nu}^i = 0\) which are the fluctuations along of the collective coordinates \(\zeta_I\). \(P_{\mu\nu}^I\) – zero-modes projection operator.

The single instanton gluon propagator \(S_{\mu\nu}^I\) is

\[
M_{\mu\nu}^I S_{\nu\rho}^I = \delta_{\mu\nu} - P_{\mu\nu}^I
\]

The explicit solution was given by (Brown et al 1978).

To extend Pobylitsa Eq. introduce artificial gluon mass \(m\).

Now define \(G_{m,\rho\nu}^I\) and \(g_{m,\mu\nu}^I\) with request \(\lim_{m \to 0} g_{m,\mu\nu}^I = S_{\mu\nu}^I\), where

\[
(M_{\mu\rho}^I + m^2 \delta_{\mu\rho})g_{m,\rho\nu}^I = \delta_{\mu\nu} - P_{\mu\nu}^I.
\]

Accordingly (Brown1978) \(G_{m,\rho\nu}^I = g_{m,\rho\nu}^I + \frac{1}{m^2} P_{\rho\nu}^I\).

It is clear that \(G_{m,\mu\rho}^{-1} = (M_{\mu\rho}^I + m^2 \delta_{\mu\rho})\).
Dynamical gluon mass in ILM.

Repeat the way to Pobylitsa Eq. for ILM "scalar" gluon propagator $\tilde{\Delta}$ and neglect by $O(\rho^8/R^8)$ terms $\Rightarrow$

$$\tilde{G}_{m,\rho\nu} - G^0_{m,\rho\nu} = N(\tilde{G}^I_{m,\alpha\nu} - G^0_{m,\rho\nu})$$

at $m \to 0$ limit

$$M^2_g \delta_{\rho\nu} = NS^{0-1}_{\rho\sigma}(S^I_{\sigma\mu} - S^0_{\sigma\mu})S^{0-1}_{\mu\nu}$$

From well-known result for the $S^I_{\sigma\mu}$ (Brown et al 1978) we conclude

$$M^2_g(q) = 2M^2_s(q).$$

At $\rho = 0.33$ fm, $R = 1$ fm

$$M_g(0) = 362 \text{ MeV}.$$  

It is essentially modify $Q\bar{Q}$ one-gluon exchange potential at the distances $r \sim M_g^{-1} \sim 0.5$ fm. It might be important for the charmonium properties.
Heavy-light quarks interactions in ILM.

Account of light quarks: \( D\zeta \Rightarrow D\zeta \text{Det}_{\text{low}}(\hat{p} + g\hat{A} + im). \)

Heavy quark propagator is

\[
\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left( \psi_f^\dagger (\hat{p} + im_f) \psi_f \right) \prod_\pm \left( \frac{V_\pm[\psi^\dagger, \psi]}{N_\pm} \right)^{N_\pm} < T|w[\psi, \psi^\dagger]|0 >, 
\]

where \( w[\psi, \psi^\dagger] = \)

\[
\prod_\pm \left( \frac{V_\pm[\psi^\dagger, \psi]}{N_\pm} \right)^{-N_\pm} \int D\zeta (\theta^{-1} - iA_4)^{-1} \prod_f \prod_\pm^{N_\pm} V_{\pm, f}[\psi^\dagger, \psi]
\]

Solution of extended Pobilitca Eq. is \( w^{-1}[\psi, \psi^\dagger] = \)

\[
= \theta^{-1} - \frac{N}{2} \sum_\pm \frac{1}{V_\pm[\psi^\dagger, \psi]} \Delta_{H, \pm}[\psi^\dagger, \psi] + O(N^2/V^2),
\]

\[
\Delta_{H, \pm}[\psi^\dagger, \psi] = \int d\zeta_\pm \prod_f V_{\pm, f}[\psi^\dagger, \psi] \theta^{-1}(w_\pm - \theta)\theta^{-1}.
\]

\( \Rightarrow \) heavy \((Q)\)-light quarks(\(\psi\)) interaction term

\[
S_{Q\psi} = -\lambda \sum_\pm Q^\dagger \Delta_{H, \pm}[\psi^\dagger, \psi] Q
\]
Heavy–light quarks interactions \((N_f = 1)\).

\[
S_{Q\psi} = i \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^3 q}{(2\pi)^3} (2\pi)^4 \delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q}) \delta(k_{2,4} - k_{1,4})
\]

\[
(M(k_1)M(k_2))^{1/2} \Delta m_Q R^4 \frac{i_0(q\rho)}{i_0(0)} \left[ \frac{2N_c^2 - N_c}{2N_c^2 - 2} \psi^+(k_1)\psi(k_2)Q^+Q
\]

\[
+ \frac{N_c^2 - 2N_c}{2N_c^2 - 2} (\psi^+(k_1)QQ^+\psi(k_2) + \psi^+(k_1)\gamma_5 QQ^+\gamma_5 \psi(k_2)) \right]
\]

First term is heavy quark–light meson interaction term, while second and third terms – \(Qq\) mesons degenerated on parity. It is similar to (Chernyshev et al. 94, 95).
Light quarks contribution to the $Q\bar{Q}$ potential. 

Now averaged Wilson loop is given by

$$\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + im_f)\psi_f\right) \prod_\pm \left(V_\pm[\psi^\dagger, \psi]\right)^{N_\pm}$$

$$\text{tr} T|W[\psi, \psi^\dagger]|0 >$$, where $< T|W[\psi, \psi^\dagger]|0 >$

$$= \prod_\pm \left(V_\pm[\psi^\dagger, \psi]\right)^{-N_\pm} \int D\zeta \prod_{f}^{N_\pm} \prod_\pm V_{\pm, f}[\psi^\dagger, \psi]$$

$$P \exp(i \int_{L_1} dx_4 A_4) P \exp(i \int_{L_2} dx_4 A_4)$$

Solution of extended Pobilititca Eq.

$$W^{-1}[\psi, \psi^\dagger] = w_1^{-1}[\psi, \psi^\dagger](\times) w_2^{-1, T}[\psi, \psi^\dagger]$$

$$-\frac{N}{2} \sum_\pm \left(V_\pm[\psi^\dagger, \psi]\right)^{-1} \int d\zeta \prod_f V_{\pm, f}[\psi_f^\dagger, \psi_f]$$

$$\left(\theta^{-1} \left(w_\pm^{(1)} - \theta\right) \theta^{-1}\right) (\times) \left(\theta^{-1} \left(w_\pm^{(2)} - \theta\right) \theta^{-1}\right)^T + O\left(\frac{N^2}{V^2}\right)$$
Heavy quark–antiquark potential $V_{lq}$, generated by light quarks ($N_f = 1$).

Figure: Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, at Set II $\rho = 0.36$ fm, $R = 0.89$ fm.
Quarkonium light hadron transitions.

Charmonium sizes \( r_c \sim 0.4 \text{ fm} \), bottomonium sizes \( r_b \sim 0.2 \text{ fm} \).

Hadronic transitions at the assumption \( \lambda_g \gg r_c, r_b \Rightarrow \) multipole expansion (see eg Voloshin12):

But \( \lambda_g \approx \rho = 0.33 \text{ fm} \sim r_c, r_b \).

What are an instanton corrections?
Quarkonium pion transitions \((Q^+ Q)' \to (Q^+ Q) \pi \pi (N_f = 2)\).

Essential part of heavy–light quarks interactions at \(N_f = 2\) – the co-product of colorless heavy \(Q^+ Q\) and light \(\psi^+ \psi\) quarks factors \(\Rightarrow\) heavy quark–pion interaction action:

\[
S_{Q\pi} = -\frac{i}{2} \Delta m_Q R^4 f_{\pi Q}^2 \int d^4x \text{tr} \partial_\mu U^\dagger(x) \partial_\mu U(x) \\
\times \int e^{-ipx} \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} i_0(p\rho) / i_0(0) Q^\dagger(p_2) Q(p_1)
\]

where \(f_{\pi Q}^2 \simeq 0.3 f_\pi^2\) and pions \(\vec{\phi}\) are given by \(U = \exp(i\vec{\tau} \vec{\phi})\). Similar heavy quark-antiquark – pion interaction term \(S_{QQ\pi}\). Both of these terms give a contribution to the quarkonium pion transitions and needs detailed investigations.
Discussion.

- Instantons with sizes $\rho \sim$ hadron quark core sizes $r$ give most essential contribution to their properties. In this case ILM is applicable.
- The strength of a heavy quark-instanton interaction is defined by $\Delta m_Q \sim \text{packing parameter } \rho^{-1} \sim 70 \text{ MeV}$, at $\rho = 0.33 \text{ fm}, R = 1 \text{ fm}$.
- The strength of a gluon-instanton interaction is much more large and given by the dynamical gluon mass $M_g \sim (\text{packing parameter})^{1/2} \rho^{-1} \sim 362 \text{ MeV}$.
- The analogous quantity for light quarks is $M \sim (\text{packing parameter})^{1/2} \rho^{-1} \sim 360 \text{ MeV}$.
- Light quarks much more strongly interact with instantons then heavy one due to zero-modes.
- Instantons naturally generate also heavy-light quarks interaction, which might be important for the heavy quarkonium and heavy-light quarks systems properties. It can be responsible for the SBCS effects in heavy quarks physics.
Future work.

- Extend the calculations of heavy-heavy quarks potential with ILM modified gluons.
- Take into account light quarks in the observables of heavy quark physics:
  - $Q\bar{Q}$ pions transitions;
  - heavy-light mesons etc.
- Consider ILM generated gluon-light quarks interactions and related problems of exotic hadrons.

Thank you for the attention.