Gluons, Heavy and Light Quarks in the QCD Vacuum

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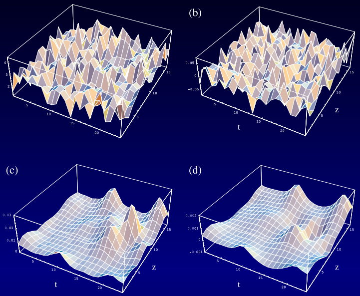
Outline

Talk based on:

- Dynamical gluon mass in the instanton vacuum model, M.M.,O.Egamberdiev,e-Print:arXiv:1706.06270[hep-ph];
- Heavy-heavy and heavy-light quarks interactions generated by QCD vacuum, M.M., EPJ Web Conf. 137 (2017) 03013, e-Print: arXiv:1703.07825 [hep-ph];
- Instanton effects on the heavy-quark static potential, U. Yakhshiev et al, Chinese Physics C41(2017)083102, e-Print: arXiv:1602.06074 [hep-ph];
- Heavy-light quarks interactions in QCD vacuum, M.M., PoS BaldinISHEPPXXII(2015)012, e-Print: arXiv:1412.4472 [hep-ph];
- Low energy constants of chi PT from the instanton vacuum model, K. Goeke et al, Phys.Rev. D76 (2007) 076007, e-Print: arXiv:0707.1997 [hep-ph].

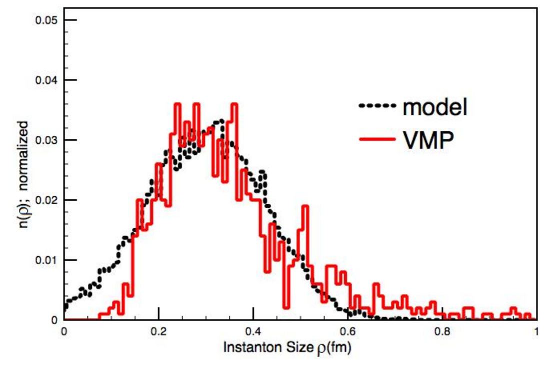
QCD vacuum. Action and topological charge.

(Negele et al 1999).



- Instanton = tunneling path between the C-S states $\Rightarrow$
- Instanton Liquid Model (ILM) $\sim$ collective coordinates ζ, $DA \to n(\rho)D\zeta$ (Shuryak1981, Diakonov-Petrov1983).
- KvBLL instantons $\sim$ dyons can describe large instantons $\Rightarrow$ Liquid Dyon Model (LDM)(Diakonov2009, Shuryak et al 2015) $\Rightarrow$ confinement–deconfinement. Small size instantons still in terms of collective coordinates.

Instanton vs hadron sizes.ILM & DLM.



Instanton size $n(\rho)$ – lattice vs ILM (Millo,Faccioli2011).
ILM $\bar{\rho} \sim 0.3\,\mathrm{fm}$(Shuryak1981, Diakonov-Petrov1983).
DLM $\bar{\rho} \sim 0.5\,\mathrm{fm}$ (Diakonov2009, Shuryak et al 2015).

Instanton vs hadron sizes.

Quarkonium states and its sizes in non-relativistic potential model (see Satz2012).

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
mass [Gev]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
size r [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

- $r_{J/\psi} = 0.25\, fm, \;\; r_\Upsilon = 0.14\, fm.$
- A model estimates of nucleon quark core size $r_N \sim 0.3 - 0.5$ fm (see Weise1985).
- Small quark core size hadrons are insensitive to the confinement, we may safely apply ILM.

ILM and its main parameters.

- Sum ansatz $A = \sum_{\pm} A_{\pm}$, $N_{+} = N_{-} = N/2$.
- Average instanton size ρ and inter-instanton distance $R = (V/N)^{1/4}$;
- Estimates:
  - $R \approx 0.89\ fm$, $\rho \approx 0.36\ fm$ – lattice;
  - $R \approx 1.00\ fm$, $\rho \approx 0.33\ fm$ – phenomenological;
  - $R \approx 0.76\ fm$, $\rho \approx 0.32\ fm$ – our estimate with account of $1/N_c$ corrections correspond ChPT (Goeke et al 2007).

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates.
- Packing parameter $\pi^2(\rho/R)^4 \sim 0.1 - 0.3$ $\Rightarrow$ Independent averaging over instanton positions and orientations.

Light quarks in ILM.

Zero modes $(\hat{p}+g\hat{A}_\pm)\Phi_{\pm,0}(x,\zeta_\pm)=0$ dominance $\Rightarrow$ light quarks partition function $Z[\xi^+,\xi]=$

$$= \int D\zeta\, \mathrm{Det}_{low}(\hat{p}+g\hat{A}+im)\exp\left(-\xi^+(\hat{p}+g\hat{A}+im)^{-1}\xi\right) =$$

$$= \int D\zeta \prod_f D\psi_f D\psi_f^\dagger \exp\int\left(\psi_f^\dagger(\hat{p}\,+\,im_f)\psi_f+\psi_f^\dagger\xi_f+\xi_f^+\psi_f\right)$$

$$\times\prod_f\left\{\prod_+^{N_+}V_{+,f}[\psi^\dagger,\psi]\prod_-^{N_-}V_{-,f}[\psi^\dagger,\psi]\right\},\quad V_{\pm,f}[\psi^\dagger,\psi]=$$

$$= i\int dx\left(\psi_f^\dagger(x)\,\hat{p}\Phi_{\pm,0}(x;\zeta_\pm)\right)\int dy\left(\Phi_{\pm,0}^\dagger(y;\zeta_\pm)(\hat{p}\,\psi_f(y)\right).$$

$\psi^\dagger,\psi$ are constituent quarks.

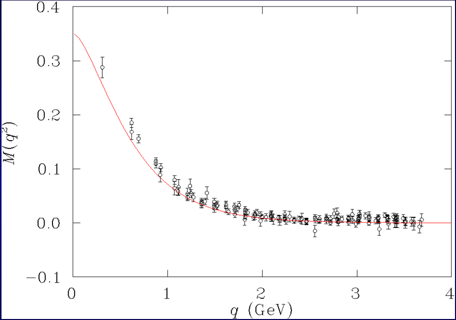
Small packing parameter $\Rightarrow$ independent averaging:

$V_\pm[\psi^\dagger,\psi]=\int d\zeta_\pm\prod_f V_{\pm,f}[\psi^\dagger,\psi]$

$\Rightarrow$ non-local ($\sim\rho$) t'Hooft-like vertex with $2N_f$-legs.

Spontaneous Breaking of the Chiral Symmetry.

Z in saddle-point approximation (leading order on $1/N_c$) $\rightarrow$ SBCS $\rightarrow$ Dynamical quark mass $M(q)$: ILM at $\rho = 0.33\, fm$, $R = 1\, fm$ vs lattice (Bowman2005).



$M(0) \approx 360$ MeV $\sim$ strength of light quark-instanton interaction!! $\Rightarrow$ Even at $1/N_c$ leading order successful reproducing of quark condensate, pion and nucleon properties etc!! (see eg Diakonov2002).
Next to leading order $1/N_c$ corrections. Successful reproducing of Low Energy Constants of ChPT (Goeke et al 2007).

Heavy quarks in ILM.

ILM heavy quark propagator: $w = \int D\zeta(\theta^{-1} - iA_4)^{-1}$, $< t_2|\theta|t_1 >= \theta(t_2 - t_1)$.
Pobylitca Eq. (DPP1989) $w^{-1} = \theta^{-1} + \sum_i \int d\zeta_i(w - a_i^{-1})^{-1}$.
Solution in lowest order on density, effective parameter of expansion: $\rho^4/R^4 \sim 0.012$

$$w^{-1} = \theta^{-1} - \frac{N}{2}\mathrm{tr}_c \sum_{\pm} \theta^{-1}(w_{\pm} - \theta)\theta^{-1} + O(N^2/V^2),$$

where single (anti)instanton $w_{\pm} = (\theta^{-1} - iA_{\pm,4})^{-1}$.
Instanton media contribution to the heavy quark mass

$$\Delta m_Q = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c, \;\; i_0(0) = 0.55.$$

At $\rho = 0.33\,\mathrm{fm}, R = 1\,\mathrm{fm}$
$\Delta m_Q \approx 70\,\mathrm{MeV} \sim$ strength of a heavy quark-instanton interaction!!

Heavy quark-antiquark potential in ILM.

Static central and spin-dependent parts of the potential from ILM averaged Wilson loop (DPP1989) by means of Pobylitsa Eq.

$$V(\vec{r}) = V_C(r) + V_{SS}(r)(\vec{S}_Q \cdot \vec{S}_{\bar{Q}}) + V_{LS}(r)(\vec{L} \cdot \vec{S})$$

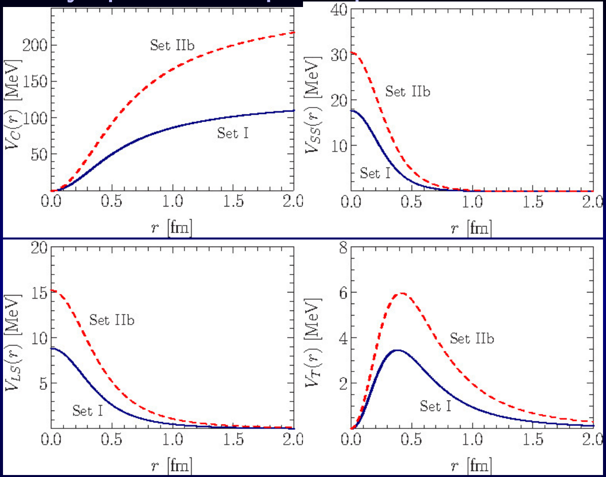
$$+V_T(r)\left[3(\vec{S}_Q \cdot \vec{n})(\vec{S}_{\bar{Q}} \cdot \vec{n}) - \vec{S}_Q \cdot \vec{S}_{\bar{Q}}\right],$$

where

$$V_{SS}(r) = \frac{1}{3m_Q^2}\nabla^2 V_C(r), V_{LS}(r) = \frac{1}{2m_Q^2}\frac{1}{r}\frac{dV_C(r)}{dr},$$

$$V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_C(r)}{dr} - \frac{d^2 V_C(r)}{dr^2}\right).$$

Heavy quark-antiquark potential in ILM.



Solid curve $\sim$ Set I $\rho = 0.33$ fm and $R = 1\,\mathrm{fm}$ $\sim$ phenomenology (Shuryak1981, Diakonov-Petrov1983), Dashed one $\sim$ Set II $\rho = 0.36$ fm, $R = 0.89\,\mathrm{fm}$ $\sim$ lattice (Chu et al1994, Negele1998, DeGrand2001, Faccioli-DeGrand2003), with $1/N_c$ corrections (MM et al2007) $m_c = 1275$ MeV.

ILM contribution to the charmonium states.

$\Delta M_{c\bar{c}} = M_{c\bar{c}} - 2m_c$ in [MeV].

$\Delta M_{c\bar{c}}(J^P)$	Set I	Set IIb	Exp.
$\Delta M_{\eta_c}(0^-)$	118,81	203,64	$433,6 \pm 0.6$
$\Delta M_{J/\psi}(1^-)$	119,57	205,36	$546,916 \pm 0.11$
$\Delta M_{\chi_{c0}}(0^+)$	142,43	250,86	$864,75 \pm 0.31$

One can see that the instanton effects are not small $\sim 30-40\,\%$ in comparison with the experimental data and strongly depend on instanton liquid parameters.

The planned complete calculations of the potential include the account of gluon modification in ILM/DLM and confinement, since DLM pretend to describe QCD vacuum on the large distances too.

Scalar "gluons" in ILM.

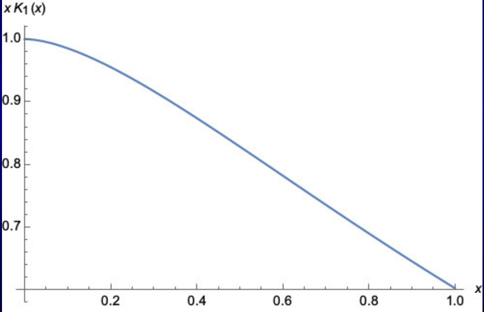
No zero modes in $\Delta_I^{-1} = (p + A_I)^2$ and $\Delta^{-1} = (p + \sum_i A_i)^2$
$\Rightarrow$ existence of the propagators
$\Delta = (p^2 + \sum_i(\{p, A_i\} + A_i^2) + \sum_{i\neq j} A_i A_j)^{-1}$,
$\Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1}$, $\Delta_0 = p^{-2}$.
Define:
$\tilde{\Delta} = (p^2 + \sum_i(\{p, A_i\} + A_i^2))^{-1}$.
The propagator in ILM is $\bar{\Delta} \equiv < \bar{\Delta} > = \int D\zeta\, \Delta$.
Start first from $\bar{\tilde{\Delta}}$.
The extension of Pobylitsa Eq. is
$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} = \sum_i < \{\bar{\tilde{\Delta}} + (\Delta_i^{-1} - \Delta_0^{-1})^{-1}\}^{-1} >$
Effective parameter of instanton density expansion in fact is
$\rho^4/R^4 \sim (1/3)^4 = 0.012$.
At the first order on density we have
$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} = N\Delta_0^{-1}(\bar{\Delta}_I - \Delta_0)\Delta_0^{-1}$,
With same accuracy $\bar{\Delta} = \bar{\tilde{\Delta}}$.

Scalar "gluon" dynamical mass in ILM.

Well-known result for the Δ_I (Brown et al 1978) $\Rightarrow$

$$M_s(q) = [\frac{3\rho^2}{(N_c^2-1)R^4}4\pi^2]^{1/2} q\rho K_1(q\rho)$$

where the form-factor $q\rho K_1(q\rho)$



At $\rho = 0.33\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$

$$M_s(0) = 256\ MeV$$

.

Gluons in ILM. Zero-modes problem.

Over single instanton fluctuations quadratic form of the effective action is $(a_\mu M^I_{\mu\nu} a_\nu)$, There is a $4N_c$ zero-modes $M^I_{\mu\nu}\phi^i_\nu = 0$ which are the fluctuations along of the collective coordinates ζ_I. $P^I_{\mu\nu}$ – zero-modes projection operator.
The single instanton gluon propagator $S^I_{\mu\nu}$ is
$M^I_{\mu\nu} S^I_{\nu\rho} = \delta_{\mu\nu} - P^I_{\mu\nu}$
The explicit solution was given by (Brown et al 1978).
To extend Pobylitsa Eq. introduce artificial gluon mass m.
Now define $G^I_{m,\rho\nu}$ and $g^I_{m,\mu\nu}$ with request $\lim_{m\to 0} g^I_{m,\mu\nu} = S^I_{\mu\nu}$,
where $(M^I_{\mu\rho} + m^2\delta_{\mu\rho}) g^I_{m,\rho\nu} = \delta_{\mu\nu} - P^I_{\mu\nu}$.
$(M^I_{\mu\rho} + m^2\delta_{\mu\rho}) G^I_{m,\rho\nu} = \delta_{\mu\nu}$
Accordingly (Brown1978) $G^I_{m,\rho\nu} = g^I_{m,\rho\nu} + \frac{1}{m^2} P^I_{\rho\nu}$.
It is clear that ${G^I}^{-1}_{m,\mu\rho} = (M^I_{\mu\rho} + m^2\delta_{\mu\rho})$.

Dynamical gluon mass in ILM.

Repeat the way to Pobylitsa Eq. for ILM "scalar" gluon propagator $\bar\Delta$ and neglect by $O(\rho^8/R^8)$ terms $\Rightarrow$

$$\bar G_{m,\rho\nu} - G^0_{m,\rho\nu} = N(\bar G^I_{m,\alpha\nu} - {G^0}_{m,\rho\nu})$$

at $m \to 0$ limit

$$M_g^2 \delta_{\rho\nu} = N {S^0}^{-1}_{\rho\sigma}(\bar S^I_{\sigma\mu} - {S^0}_{\sigma\mu}){S^0}^{-1}_{\mu\nu}$$

From well-known result for the $S^I_{\sigma\mu}$ (Brown et al 1978) we conclude

$$M_g^2(q) = 2M_s^2(q).$$

At $\rho = 0.33\,\mathrm{fm}, R = 1\,\mathrm{fm}$

$$M_g(0) = 362\, MeV.$$

It is essentially modify $Q\bar Q$ one-gluon exchange potential at the distances $r \sim M_g^{-1} \sim 0.5\, fm$. It might be important for the charmonium properties.

Heavy-light quarks interactions in ILM.

Account of light quarks: $D\zeta \Rightarrow D\zeta\, \mathrm{Det}_{low}(\hat p + g\hat A + im)$.

Heavy quark propagator is

$\int \prod_f D\psi_f D\psi_f^\dagger \exp\int \left(\psi_f^\dagger(\hat p + im_f)\psi_f\right) \prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{N_\pm}$

$< T|w[\psi,\psi^\dagger]|0 >$,

where $w[\psi,\psi^\dagger] =$

$\prod_\pm \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{-N_\pm} \int D\zeta\,(\theta^{-1} - iA_4)^{-1} \prod_f \prod_\pm^{N_\pm} V_{\pm,f}[\psi^\dagger,\psi]$

Solution of extended Pobilitca Eq. is $w^{-1}[\psi,\psi^\dagger] =$

$$= \theta^{-1} - \frac{N}{2}\sum_\pm \frac{1}{\overline{V_\pm[\psi^\dagger,\psi]}}\Delta_{H,\pm}[\psi^\dagger,\psi] + O(N^2/V^2),$$

$$\Delta_{H,\pm}[\psi^\dagger,\psi] = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger,\psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}.$$

$\Rightarrow$ heavy (Q)-light quarks(ψ) interaction term

$$S_{Q\psi} = -\lambda \sum_\pm Q^\dagger \Delta_{H,\pm}[\psi^\dagger,\psi] Q$$

Heavy–light quarks interactions ($N_f = 1$).

is

$$S_{Q\psi} = i \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^3q}{(2\pi)^3} (2\pi)^4 \delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q}) \delta(k_{2,4} - k_{1,4})$$

$$(M(k_1)M(k_2))^{1/2} \Delta m_Q R^4 \frac{i_0(q\rho)}{i_0(0)} \left[\frac{2N_c^2 - N_c}{2N_c^2 - 2} \psi^+(k_1)\psi(k_2)Q^+Q \right.$$

$$\left. + \frac{N_c^2 - 2N_c}{2N_c^2 - 2} (\psi^+(k_1)QQ^+\psi(k_2) + \psi^+(k_1)\gamma_5 QQ^+\gamma_5\psi(k_2)) \right]$$

First term is heavy quark–light meson interaction term, while second and third terms – Qq mesons degenerated on parity. It is similar to (Chernyshev etal94,95).

Light quarks contribution to the $Q\bar{Q}$ potential.

Now averaged Wilson loop is given by

$$\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f\right) \prod_\pm \left(\overline{V_\pm[\psi^\dagger, \psi]}\right)^{N_\pm}$$

$\mathrm{tr} < T|W[\psi, \psi^\dagger]|0 >$, where $< T|W[\psi, \psi^\dagger]|0 >=$

$$= \prod_\pm \left(\overline{V_\pm[\psi^\dagger, \psi]}\right)^{-N_\pm} \int D\zeta \prod_f \prod_\pm^{N_\pm} V_{\pm,f}[\psi^\dagger, \psi]$$

$$P \exp(i \int_{L_1} dx_4 A_4) P \exp(i \int_{L_2} dx_4 A_4)$$

Solution of extended Pobilitca Eq.

$$W^{-1}[\psi, \psi^\dagger] = w_1^{-1}[\psi, \psi^\dagger](\times) w_2^{-1,T}[\psi, \psi^\dagger]$$

$$-\frac{N}{2} \sum_\pm \left(\overline{V_\pm[\psi^\dagger, \psi]}\right)^{-1} \int d\zeta_\pm \prod_f V_{\pm,f}[\psi_f^\dagger, \psi_f]$$

$$\left(\theta^{-1}\left(w_\pm^{(1)} - \theta\right)\theta^{-1}\right)(\times)\left(\theta^{-1}\left(w_\pm^{(2)} - \theta\right)\theta^{-1}\right)^T + O\left(\frac{N^2}{V^2}\right)$$

Heavy quark–antiquark potential V_{lq}, generated by light quarks ($N_f = 1$).

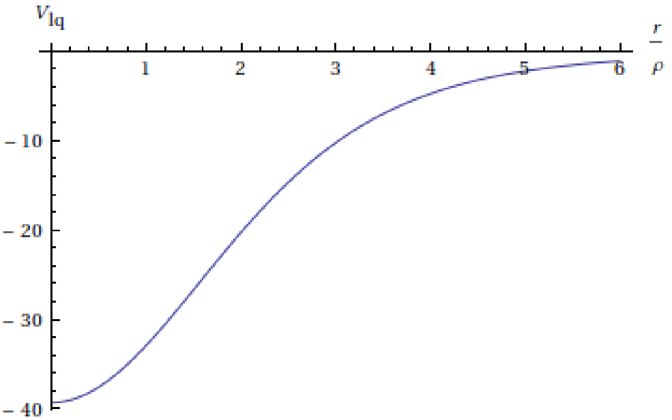
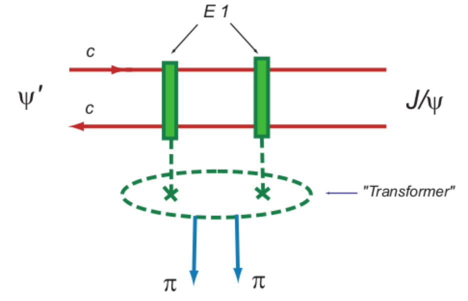


Figure: Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, at Set II $\rho = 0.36$ fm, $R = 0.89\,\mathrm{fm}$.

Quarkonium light hadron transitions.

Charmonium sizes $r_c \sim 0.4\, fm$, bottomonium sizes $r_b \sim 0.2\, fm$. Hadronic transitions at the assumption $\lambda_g >> r_c, r_b \Rightarrow$ multipole expansion (see eg Voloshin12):



But $\lambda_g \approx \rho = 0.33\, fm \sim r_c, r_b$.
What are an instanton corrections?

Quarkonium pion transitions $(Q^+Q)' \to (Q^+Q)\,\pi\pi$ $(N_f = 2)$.

Essential part of heavy–light quarks interactions at $N_f = 2$ –the co-product of colorless heavy Q^+Q and light $\psi^+\psi$ quarks factors $\Rightarrow$ heavy quark–pion interaction action:

$$
\begin{aligned}
S_{Q\pi} &= -\frac{i}{2}\Delta m_Q R^4 f_{\pi Q}^2 \int d^4x\, \mathrm{tr}\, \partial_\mu U^\dagger(x)\partial_\mu U(x) \\
&\times \int e^{-ipx}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4} i_0(p\rho)/i_0(0) Q^\dagger(p_2)Q(p_1)
\end{aligned}
$$

where $f_{\pi Q}^2 \simeq 0.3 f_\pi^2$ and pions $\vec\phi$ are given by $U = \exp(i\vec\tau\vec\phi)$.
Similar heavy quark-antiquark – pion interaction term $S_{QQ\pi}$.
Both of these terms give a contribution to the quarkonium pion transitions and needs detailed investigations.

Discussion.

- Instantons with sizes $\rho \sim$ hadron quark core sizes r give most essential contribution to their properties. In this case ILM is applicable.
- The strength of a heavy quark-instanton interaction is defined by $\Delta m_Q \sim packing\ parameter\ \rho^{-1} \sim 70\, MeV$, at $\rho = 0.33\, fm$, $R = 1\, fm$.
- The strength of a gluon-instanton interaction is much more large and given by the dynamical gluon mass $M_g \sim (packing\ parameter)^{1/2}\ \rho^{-1} \sim 362\, MeV$.
- The analogous quantity for light quarks is $M \sim (packing\ parameter)^{1/2}\ \rho^{-1} \sim 360\, MeV$.
  Light quarks much more strongly interact with instantons then heavy one due to zero-modes.
- Instantons naturally generate also heavy-light quarks interaction, which might be important for the heavy quarkonium and heavy-light quarks systems properties. It can be responsible for the SBCS effects in heavy quarks physics.

Future work.

- Extend the calculations of heavy-heavy quarks potential with ILM modified gluons.
- Take into account light quarks in the observables of heavy quark physics:
    - $Q\bar{Q}$ pions transitions;
    - heavy-light mesons etc.
- Consider ILM generated gluon-light quarks interactions and related problems of exotic hadrons.

Thank you for the attention.