Azimuthal angular correlations in high energy processes in QCD

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Deeply Inelastic Scattering (DIS) probing hadron structure

Kinematic Invariants

\[ Q^2 = -q^2 = -(k_{\mu} - k'_{\mu})^2 \]

\[ Q^2 = 4E_eE_e'\sin^2\left(\frac{\theta_e'}{2}\right) \]

\[ y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right) \]

\[ x = \frac{Q^2}{2pq} = \frac{Q^2}{sy} \]

\[ s \equiv (p + k)^2 \]
Deep Inelastic Scattering

QCD: scaling violations

\[ F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 x q(x, Q^2) \]

early experiments (SLAC,...):
scale invariance of hadron structure

large number of gluons at small \( x \)
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space.

hadron/nucleus becomes a dense system of gluons: **gluon saturation**

Physics of strong color fields in QCD, multi-particle production—possibly discover novel universal properties of theory in this limit.
Perturbative QCD breaks down at small $x$

“attractive” bremsstrahlung vs. “repulsive” recombination

included in pQCD

not included in pQCD (collinear factorization)

energy $\sim 1/x$

$$\frac{\alpha_s}{Q^2} \frac{xG(x, Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$
QCD at high energy: saturation

- Geometric Scaling
- Non-perturbative region
- Perturbative region
- $Q^2_s(Y)$
- $\Lambda^2_{QCD}$
- $\alpha_s \sim 1$
- $\alpha_s \ll 1$
Probing saturation in high energy collisions

“nucleus-nucleus” (dense-dense)
initial multiplicity/energy density

“proton-nucleus” (dilute-dense)
production spectra, correlations

DIS
structure functions (diffraction)
NLO di-hadron/jet correlations
3-hadron/jet angular correlations

need quite a bit of modeling

much less modeling
Probing saturation via correlations

polar angle (long-range rapidity correlations)

azimuthal angle (back to back)

signatures in production spectra

multiple scattering via Wilson lines:
  \( p_t \) broadening

x-evolution via JIMWLK:
  suppression of spectra/away side peaks
di-hadron correlations in dA at RHIC

Recent STAR measurement (arXiv:1008.3989v1):

Tuchin, NPA846 (2010)
A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)
T. Lappi + H. Mantysaari, NPA908 (2013)

signature of gluon saturation?

shadowing+energy loss?

Azimuthal angular correlations in DIS
Di-hadron azimuthal correlations in DIS

Electron Ion Collider..., A. Accardi et al., arXiv:1212.1701

Zheng-Aschenauer-Lee-Xiao, PRD89 (2014) 7, 074037
3-parton production in DIS

\[ \gamma^* T \rightarrow q \bar{q} g X \]

+ radiation from anti-quark

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans
PLB761 (2016) 229
NPB920 (2017) 232
target (proton, nucleus) as a classical color field
quark propagator in the background color field: Wilson line $V$

\[ \sim gA + (gA)^2 + (gA)^3 + \cdots \]

\[ S_F(q,p) \equiv (2\pi)^4 \delta^4(p-q) \left[ S_F^0(p) + S_F^0(q) \tau_f(q,p) S_F^0(p) \right] \quad \text{with} \quad S_F^0(p) = \frac{i}{p^+ + i\epsilon} \]

\[ \tau_f(q,p) \equiv (2\pi)\delta(p^+ - q^+) \gamma^+ \int d^2x_t e^{i(q_t-p_t)\cdot x_t} \left\{ \theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1] \right\} \]

\[ V(x_t) = \hat{p} e^{ig \int dz^+ A^- (z^+, x_t)} \quad \text{similar for gluon propagator} \]
spinor helicity methods

massless quarks: helicity eigenstates

\[ u_\pm(k) = \frac{1}{2} (1 \pm \gamma_5) u(k) \]
\[ v_\mp(k) = \frac{1}{2} (1 \pm \gamma_5) v(k) \]

helicity operator

\[ h \equiv \vec{\sigma} \cdot \hat{p} = \begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix} \]

\[ \vec{\Sigma} \cdot \hat{p} u_\pm(p) = \pm u_\pm(p) \]
\[ -\vec{\Sigma} \cdot \hat{p} v_\pm(p) = \pm v_\pm(p) \]

\[ u_+(k) = v_-(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^-} e^{-i\phi_k} \end{bmatrix} \]
\[ u_-(k) = v_+(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{i\phi_k} \end{bmatrix} \]

\[ e^{\pm i\phi_k} = \frac{k_x \pm ik_y}{\sqrt{2k^+ k^-}} = \sqrt{2} \frac{k_t \cdot \epsilon^\pm}{k_t} \]

\[ k^\pm = \frac{E \pm k_z}{\sqrt{2}} \]

with \( e^{\pm i\phi_k} \) and \( \epsilon^\pm \) given by:

\[ n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0) \]
\[ \bar{n}^\mu = (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0) \]

and

\[ \epsilon_\pm = \frac{1}{\sqrt{2}} (1, \pm i) \]
spinor helicity methods

notation:

\[ |i^\pm> \equiv |k_i^\pm> \equiv u_\pm(k_i) = v_\mp(k_i) \quad <i^\pm| \equiv <k_i^\pm| \equiv \bar{u}_\pm(k_i) = \bar{v}_\mp(k_i) \]

basic spinor products:

\[ <i \, j> \equiv <i^- |j^+> = \bar{u}_-(k_i) \, u_+(k_j) = \sqrt{|s_{ij}|} e^{i \phi_{ij}} \quad \cos \phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|} k_i^+ k_j^+} \]
\[ [i \, j] \equiv <i^+ |j^-> = \bar{u}_+(k_i) \, u_-(k_j) = -\sqrt{|s_{ij}|} e^{-i \phi_{ij}} \quad \sin \phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|} k_i^+ k_j^+} \]

with

\[ s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j \]
\[ = - <i \, j> \, [i \, j] \]

and

\[ <ii> = [ii] = 0 \quad <ij> = [ij] \equiv 0 \]

charge conjugation

\[ <i^+ |\gamma^\mu|j^+> = <j^- |\gamma^\mu|i^-> \]

Fierz identity

\[ <i^+ |\gamma^\mu|j^+> <k^+ |\gamma^\mu|l^+> = 2[i^k] <l^j> \]

any off-shell momentum

\[ k^{\mu} \equiv \bar{k}^{\mu} + \frac{k^2}{2k^+} \, n^\mu \quad \text{where} \quad \bar{k}^{\mu} \text{ is on-shell} \quad \bar{k}^2 = 0 \]

any on-shell momentum

\[ \not{p} = |p^+| <p^+| + |p^-| <p^-| \]
Diagram A1

Numerator: Dirac Algebra

\[ a_1 \equiv \bar{u}(p)(k)(p + k)k_1(l)(k_1 - l)v(q) \]

longitudinal photons

\[ a_1^{L;+-+} = -\frac{\sqrt{2} Q}{[n \bar{k}] l^+ [n p] < k p > [n p] < n \bar{k}_1 > [n \bar{k}_1] < n q > \]

\[ (\langle n \bar{k}_1 > [n \bar{k}_1] - l^+ < n \bar{n} > [n \bar{n}] \rangle) \]

with

\[ \langle np \rangle = -[np] = \sqrt{2p^+} \]

quark anti-quark gluon helicity: + - +

transverse photons: +

\[ a_1^{\perp;+;+-+} = -\frac{\sqrt{2}}{[n k]} [pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n}k_1 > [k_1 n] < nq > \]
Diagram A3

Numerator: Dirac Algebra

longitudinal photons quark anti-quark gluon helicity: + - +

$$a_3^{L;+--} = \frac{\sqrt{2}Q}{l^+ [nk^2]} [pn] \left( < n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n] \right) < \bar{k}_2 \bar{k}_1 > [\bar{k}_1 n]$$

$$= \begin{cases} \frac{2^4 Q (l^+)^2 (z_1 z_2)^{3/2}}{z_3} \left[ z_3 k_1 t \cdot \epsilon - (z_1 + z_3) k_2 t \cdot \epsilon \right] 
\end{cases}$$

the rest is some standard integrals, we know how to compute the numerators efficiently

add up the amplitudes, add, square... : get (trace of) products of Wilson lines
structure of Wilson lines: amplitude

\[
\begin{align*}
\left( \begin{array}{c}
\end{array} \right)_{ij} &= \left[ V^{\dagger}(y_t) V(x_t) t^a \right]_{ij} \\
\left( \begin{array}{c}
\end{array} \right)_{ij} &= \left[ V^{\dagger}(y_t) t^b V(x_t) \right]_{ij} U^{ba}(z_t)
\end{align*}
\]
Dipoles at large $N_c$ : Balitsky-Kovchegov eq \[ T \equiv 1 - S \]

\[
\frac{d}{dy} T(x_t - y_t) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2} \times \\
[T(x_t - z_t) + T(z_t - y_t) - T(x_t - y_t) - T(x_t - z_t)T(z_t - y_t)]
\]

\[ \tilde{T}(p_t) \to \log \left[ \frac{Q_s^2}{p_t^2} \right] \quad \text{saturation region} \]

\[ \tilde{T}(p_t) \to \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right] \gamma \quad \text{extended scaling region} \]

\[ \tilde{T}(p_t) \to \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right] \quad \text{pQCD region} \]

*Rummukainen-Weigert, NPA739 (2004) 183*

*NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)*
Quadrupole: \( Q(r, \bar{r}, \bar{s}, s) \equiv \frac{1}{N_c} \langle Tr\ V(r)\ V^\dagger(\bar{r})\ V(\bar{s})\ V^\dagger(s) \rangle \)

3-parton azimuthal angular correlations

**multiple scattering:**
**broadening** of the peak

**x-evolution:**
**reduction** of magnitude
some thoughts/ideas/......

cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

Munier, Peigne, Petreska, arXiv:1603.01028

\[ \frac{dI}{dz} \rightarrow \frac{d\sigma^{a+A\rightarrow a+g+X}}{dydy'd^2p_t} \]

the difference between a nuclear target and a proton target is the medium induced energy loss

used to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

can also do this for di-jets in DIS

(3-parton production/2-parton production)
SUMMARY

**CGC is a systematic approach to high energy collisions**
- high gluon density: re-sum multiple soft scatterings
- high energy: re-sum large logs of energy (rapidity or log 1/x)

**Leading Log CGC works (too) well**
- it has been used to fit a wealth of data; ep, eA, pp, pA, AA

**Precision (NLO) studies are needed**
- available for DIS, single inclusive forward production in pp, pA

**Azimuthal angular correlations offer a unique probe of CGC**
- 3-hadron/jet correlations should be even more discriminatory
Azimuthal correlations in DIS

Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037
QCD phase space

a general formalism for all $x, Q^2$

toward unifying JIMWLK with DGLAP?
long-range rapidity correlations: the ridge

Initial state vs final state?
if final state, early or late times?

AA at RHIC

PP at LHC
The quadrupole

\[ Q(r, \bar{r}, \bar{s}, s) \equiv \frac{1}{N_c} < Tr V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) > \]

line config.: \[ r = \bar{s}, \quad \bar{r} = s, \quad z \equiv r - \bar{r} \]
square config.: \[ r - \bar{s} = \bar{r} - s = r - \bar{r} = \cdots \equiv z \]

“naive” Gaussian: \[ Q = S^2 \quad S(r, \bar{r}) \equiv \frac{1}{N_c} < Tr V(r) V^\dagger(\bar{r}) > \]

Gaussian
\[ Q_\parallel(z) \approx \frac{N_c + 1}{2} \left[ S(z) \right]^{\frac{N_c+2}{N_c+1}} - \frac{N_c - 1}{2} \left[ S(z) \right]^{\frac{N_c-2}{N_c-1}} \]

\[ Q_{sq}(z) = \left[ S(z) \right]^2 \left[ \frac{N_c + 1}{2} \left( \frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left( \frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right] \]

Gaussian + large \( N_c \)
\[ Q_\parallel(z) = S^2(z)[1 + 2 \log[S(z)]] \]
\[ Q_{sq}(z) = S^2(z) \left[ 1 + 2 \ln \left( \frac{S(z)}{S(\sqrt{2}z)} \right) \right] \]